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Discrete Structure in Velocity Space

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The introduction of discreteness into the space-time continuum is perhaps one of the most interesting ideas in recent high energy particle physics. In fact, some authors[1] have discussed this possibility, though the relation between their ideas and the experimental data has not been well treated. In a previous paper,[2] we have attempted progress in this direction and proposed a theory in which we establish a relation between the observed enhancements in the distribution of the longitudinal Lorentz angles for the meson clusters emitted from cosmic ray jet interactions[3] and the simplest three-dimensional integral Lorentz transformation.[4] It is remarkable in this theory that there exists a specific (minimum) velocity $v=(\sqrt{8}/3)c$ corresponding to the Lorentz angle $\theta=\cosh^{-1}3=1.7628$ $(=\theta_h)$ or the Lorentz factor $\gamma=3$. It is the purpose of the present paper to call attention to the possible existence of another specific velocity $(\sqrt{3}/2)c$.

The velocity space is very important for the kinematical analysis of the multi-particle final states resulting from high energy particle collisions, since it is the very space where hodograph methods are available.[5] Previously[2] we used the longitudinal Lorentz angle $\theta=\tanh^{-1}(v_{\parallel}/c)$, and found that the distribution of particles in this one-dimensional space is uniform with mesh size $\theta\cong1.7$, as pointed out by Hasegawa,[3] suggesting the presence of discrete velocity levels. This mesh size was almost equal to the minimum Lorentz angle θ_h, i.e., the Lorentz angle induced by the simplest three-dimensional integral Lorentz transformation. However, from the theoretical viewpoint, there is no reason to exclude the four-dimensional case where the minimum velocity should be smaller than the three-dimensional case, i.e., $v=(\sqrt{3}/2)c$, $\theta=\cosh^{-1}2=1.31696$ $(=\theta_m)$, $\gamma=2$. This indicates that there exists another kind of particle distribution with mesh size θ_m.

As θ_m is the minimum possible Lorentz angle, it is inferred that the reactions with mesh size θ_m occur most frequently. But the observation of the signals which come from discrete velocity levels are concentrated strongly at mesh size smaller than θ_m for many reasons, for example, a) the stronger secondary effects which mediate between observed mesons and primary objects, b) the stronger overlapping of clusters in velocity space. Though Hasegawa[3] measured the mesh size directly for the reactions with mesh size θ_h, the same method may not be applicable to the events with mesh size θ_m.

Now if we knew where to take the point of origin in θ-space, we would have a double advantage. The first would be of a fundamental nature: we should have some information on the further physical meaning of the θ-levels. The second would be one of a practical nature: events with different values of the normalization parameter—it might be the incident energy, or it might be the survivor energy—would be superposed and the levels could be analysed with high signal-to-noise ratio.

At present, however, we do not know which parameter we should choose for normalization. Further, sometimes—especially in the case of cosmic ray events—the information on some parameters are lacking. Therefore we will have to proceed in two steps. When the relevant information is available, we shall make several tentative guesses about the origin point and shall analyse the levels from the one-particle θ_i-distribution. When they are not available, we shall make the analysis using the two-particle longitudinal correlation function, i.e., the one in terms of $|\theta_i-\theta_j|$ for each event, where θ_i and θ_j represent the Lorentz angles of mesons i and j, respectively.[6] By this method we shall have to be content with lower signal-to-noise ratio, but, on the other hand, we shall have a more model-independent result. Summing a sufficiently large number of $|\theta_i-\theta_j|$ data, separated into groups according to incident energy and multiplicity, an empirical characteristic distribution will be obtained for each group. We expect that each distribution will have peaks at $|\theta_i-\theta_j|=n\theta_m$ and $n\theta_h$ $(n=1, 2, \cdots)$.

The analysis of the data available at present has been made by one of the authors (J. I.), and the result will be published elsewhere. The authors are very eager to have, for analysis, sufficiently good statistics of jet data containing information as comprehensive as possible.

1) D. Finkelstein, *Coral Gables Conference on Fundamental Interactions at High Energy*, ed. T. Gudehus et al. (Gordon and Breach, Science Pub., Jan. 1969).
S. D. Drell, *Proceedings of the Conference on Expectations for Particle Reactions at the New Accelerators, University of Wisconsin, April 1970.*
2) K. Mori and H. Nakamura, The paper HE-30, *Proceedings of the 11th International Conference on Cosmic Rays, Budapest, 1969*, Acta Physica Academiae Scientiarum Hungaricae **29**, Suppl. **3** (1970), 131; Prog. Theor. Phys. **41** (1969), 1378; **43** (1970), 965, 1213.
3) S. Hasegawa, Prog. Theor. Phys. **26** (1961), 150; **29** (1963), 128.
See also Prog. Theor. Phys. Suppl. No. 33 (1965); No. 47 (1971).
4) A. Schild, Can. J. Math. **1** (1949), 29.
H. S. M. Coxter and G. J. Whitrow, Proc. Roy. Soc. **A201** (1950), 417.
5) A. Sommerfeld, Verh. Dtsch. Phys. Ges. **11** (1909), 577.
6) J. Iwai, Soryushiron Kenkyu (mimeographed circular in Japanese) **44** (1970), 197.