

Discrete temporal solitons along a chain of nonlinear coupled microcavities embedded in photonic crystals

Demetrios N. Christodoulides and Nikos K. Efremidis

Department of Electrical and Computer Engineering, Lehigh University, Bethlehem, Pennsylvania 18015

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We demonstrate that spatiotemporal discrete solitons are possible in nonlinear photonic crystal structures. Analysis indicates that these states can propagate undistorted along a series of coupled resonators or defects by balancing of the effects of discrete lattice dispersion with material nonlinearity. In principle, these self-localized entities are capable of exhibiting very low velocities, depending on the coupling coefficient among successive microcavities. This class of solitons can follow any preassigned path in a three-dimensional environment. © 2002 Optical Society of America

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Photonic crystals are artificial microstructures in which the refractive index is periodically modulated at a length scale comparable to the wavelength of operation.^{1,2} For specific crystal configurations, this index periodicity can lead to a complete photonic bandgap (in a certain range of frequencies), thus inhibiting wave propagation in all three directions.¹ As noted in several studies, the presence of gaps in the macroscopic dispersion relation of such periodic structures introduces novel features that one can exploit to control the propagation of light. In this respect, photonic crystals are highly promising in terms of integrating useful optical components such as waveguides, couplers, cavities, and filters on the same substrate.²

Recently, a new type of optical waveguide that involves a periodic sequence of coupled high- Q resonators was proposed.^{3,4} In such a system, waveguiding is accomplished via light hopping or tunneling among successive microcavities that effectively act as defects within the crystal. Interestingly, this process has much in common with electronic transport in crystalline solids when it is described within the so-called tight-binding approximation.³⁻⁷ Optical wave propagation in both linear and $\chi^{(2)}$ nonlinear coupled-resonator optical waveguides has also been investigated.^{8,9} Of course, like any other waveguide, including those in photonic crystals,¹⁰ coupled-resonator optical waveguides are dispersive elements and can thus cause significant pulse broadening during propagation. These group-velocity dispersion effects are of course expected to be more pronounced for relatively short pulses (i.e., when the pulse occupies only a few cavities). Therefore, it will be of interest to find ways to counteract dispersive effects in such systems.

In this Letter we show that spatiotemporal discrete solitons can propagate undistorted along a chain of coupled nonlinear high- Q cavities or defects that are embedded in a photonic crystal structure. Such states are possible as a result of the balance between the effect of discrete lattice dispersion with that of material nonlinearity. These self-localized entities are capable of exhibiting very low group velocities, depending on the coupling strength among successive microcavities, and in principle they can remain immobile like frozen

bubbles of light. In addition, this class of solitons can be effectively navigated along any preassigned path in a three-dimensional environment. Methods of optimizing their transport efficiency when they encounter sharp bends are also discussed.

We begin our analysis by considering a periodic sequence of identical coupled high- Q microcavities or defects, similar to that shown schematically in Fig. 1. In principle, these defects can confine light in either two- or three-dimensional geometries, provided that they are surrounded by an appropriate photonic bandgap structure.² The distance between successive resonators (or primitive cells) is D , and the material is taken here to be Kerr nonlinear. Furthermore, we assume that each cavity-defect is single mode,¹¹ oscillating at an eigenfrequency ω_0 . The electromagnetic mode of each resonator in isolation is given by $\mathbf{E} = \mathbf{E}_0(\mathbf{r})\exp(-i\omega_0 t)$ and $\mathbf{H} = \mathbf{H}_0(\mathbf{r})\exp(-i\omega_0 t)$, where $\mathbf{E}_0(\mathbf{r})$ and $\mathbf{H}_0(\mathbf{r})$ represent the cavity eigenmodes. Evidently, because of proximity, finite coupling exists between successive defects. To describe this coupling process as well as the effects arising from nonlinearity, we derive the nonlinear equations of motion, using the Lorentz reciprocity theorem. To do so, let us assume that the presence of the other cavities near a particular site perturbs the total permittivity from ϵ to ϵ' . In general, the perturbed fields $\mathbf{E}' = \mathbf{E}_0'(\mathbf{r}, t)\exp(-i\omega_0 t)$ and $\mathbf{H}' = \mathbf{H}_0'(\mathbf{r}, t)\exp(-i\omega_0 t)$ obey $\nabla \times \mathbf{E}_0' = \mu_0(i\omega_0 \mathbf{H}_0' - \partial \mathbf{H}_0'/\partial t)$ and $\nabla \times \mathbf{H}_0' = \epsilon'(-i\omega_0 \mathbf{E}_0' + \partial \mathbf{E}_0'/\partial t)$. By applying the divergence theorem to the quantity $\nabla \cdot (\mathbf{E}_0^* \times \mathbf{H}_0' + \mathbf{E}_0' \times \mathbf{H}_0^*)$ and by using Maxwell's equations, we obtain the Lorentz reciprocity relation:

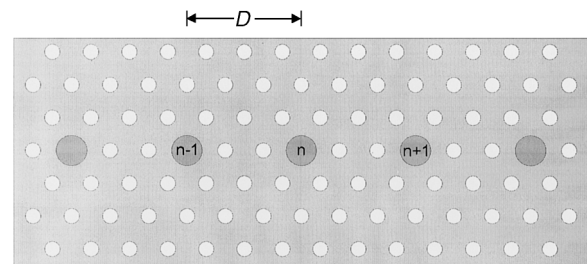


Fig. 1. Array of microcavities or defects embedded in a photonic crystal structure. The distance between elements is D .

$$\begin{aligned} & \oint \mathbf{ds} \cdot (\mathbf{E}_0^* \times \mathbf{H}_0' + \mathbf{E}_0' \times \mathbf{H}_0^*) \\ &= \iiint d\mathbf{v} \left[i\omega_0(\epsilon' - \epsilon)\mathbf{E}_0' \cdot \mathbf{E}_0^* - \epsilon'\mathbf{E}_0^* \cdot \frac{\partial \mathbf{E}_0'}{\partial t} \right. \\ & \quad \left. - \mu_0\mathbf{H}_0^* \cdot \frac{\partial \mathbf{H}_0'}{\partial t} \right]. \quad (1) \end{aligned}$$

Since \mathbf{E}_0 and \mathbf{H}_0 represent bound modes that vanish at infinity, the surface integral of Eq. (1) is equal to zero. Next, we express the perturbed fields as a time-varying superposition of the cavities' bound states, e.g., $\mathbf{E}_0'(\mathbf{r}, t) = \sum a_m(t)\mathbf{E}_{0m}$ and $\mathbf{H}_0'(\mathbf{r}, t) = \sum a_m(t)\mathbf{H}_{0m}$, where the eigenfunctions $\mathbf{E}_{0m} = \mathbf{E}_0(\mathbf{r} - \mathbf{r}_m)$ and $\mathbf{H}_{0m} = \mathbf{H}_0(\mathbf{r} - \mathbf{r}_m)$ are localized at the lattice points. If, in Eq. (1), we let $\mathbf{E}_0 = \mathbf{E}_{0n}$ and $\mathbf{H}_0 = \mathbf{H}_{0n}$, and keeping in mind that the material is Kerr nonlinear ($n^2 = n_0^2 + 2n_0n_2|E|^2$), we obtain the discrete nonlinear evolution equations:

$$i \frac{da_n}{dt} + \sum c_{mn}a_m + \gamma|a_n|^2a_n = 0, \quad (2)$$

where the linear coupling coefficients c_{mn} are

$$c_{mn} = \frac{\omega_0 \iiint d\mathbf{v}(\epsilon' - \epsilon)\mathbf{E}_{0n}^* \cdot \mathbf{E}_{0m}}{\iiint d\mathbf{v}(\mu_0|\mathbf{H}_{0n}|^2 + \epsilon|\mathbf{E}_{0n}|^2)} \quad (3)$$

and the self-phase modulation strength, γ , is given by

$$\gamma = \frac{2n_0n_2\epsilon_0\omega_0 \iiint d\mathbf{v}|\mathbf{E}_{0n}|^4}{\iiint d\mathbf{v}(\mu_0|\mathbf{H}_{0n}|^2 + \epsilon|\mathbf{E}_{0n}|^2)}. \quad (4)$$

If we now consider only nearest-neighbor interactions (as, for example, in a straight chain of resonators), Eq. (2) takes the form

$$i \frac{da_n}{dt} + \Delta\omega a_n + c(a_{n+1} + a_{n-1}) + \gamma|a_n|^2a_n = 0. \quad (5)$$

$\Delta\omega = c_{mm}$ represents a small shift in the eigenfrequency ω_0 that arises from the presence of neighboring cavities. As a result, the effective eigenfrequency of each resonator in this chain is $\omega_0' = \omega_0 - \Delta\omega$. In addition, $c = \pi/2\tau_c$ (in inverse time units) is the coupling strength between successive sites, where τ_c is the time required for one cavity to completely couple its energy to its neighbors (in the linear regime). Note that the linear part of Eq. (5) is in agreement with the results of a recent study concerning coupled defects in photonic crystals.¹²

Equation (5) describes the evolution dynamics of the optical field in a nonlinear chain of resonators or microcavities. The equation has the form of a discrete nonlinear Schrödinger equation that is known to exhibit discrete soliton (DS) solutions. It is noteworthy that, so far, in nonlinear optics the only other system that happens to support (spatial) DS states is that of nonlinear waveguide arrays.¹³ However, unlike their spatial cousins,^{13,14} the DSs reported here are by nature spatiotemporal entities. The dispersive properties of this lattice become apparent if one considers the linear dispersion curve of Eq. (5). This curve can be obtained by use of the discrete plane-wave solution, $\exp[i(\Omega t - Kx_n)]$, at low amplitudes, where $x_n = nD$

and Ω and K are its angular frequency and wave number, respectively. In this case, one readily finds that $\Omega = 2c \cos(KD) + \Delta\omega$, which, in turn, describes the photonic band structure of this lattice within the Brillouin zone.

In general, Eq. (5) does not exhibit closed-form solutions. Yet, in two limiting cases (for broad and highly localized pulses), this equation can be accurately treated analytically. For example, for broad enough solitons, the so-called long-wavelength approximation can be employed. In this case, to understand better how these discrete solitons can propagate in a nonlinear resonator array let us assume that the discrete amplitudes a_n can be written as $a_n = \Phi_n \exp\{iqn + it[\Delta\omega + 2c \cos(q)]\}$, where q represents a phase difference among successive microcavities. If the optical field varies slowly from site to site (long-wavelength approximation), then one can use a Taylor series expansion, i.e., $\Phi_{n\pm 1} = \Phi \pm D\Phi_x + (D^2/2)\Phi_{xx}$, where x denotes a continuous coordinate along the resonator chain and Φ is now a continuous envelope encompassing the discrete field distribution. Substituting these expressions into Eq. (5), we obtain a nonlinear Schrödinger equation:

$$i\left(\frac{\partial \Phi}{\partial t} + v_e \frac{\partial \Phi}{\partial x}\right) + cD^2 \cos(q) \frac{\partial^2 \Phi}{\partial x^2} + \gamma|\Phi|^2\Phi = 0, \quad (6)$$

where $v_e = 2cD \sin(q)$ is the wave's group velocity in this system. The traveling soliton solutions of Eq. (6) are readily found to be

$$\Phi = \Phi_0 \operatorname{sech}\left(\frac{x - v_e t}{x_0}\right) \exp(i\mu t). \quad (7)$$

In Eq. (7), Φ_0 represents the peak amplitude of this traveling soliton and x_0 is its spatial width, also related to its pulse width τ_0 via $x_0 = v_e \tau_0$. Furthermore, $\Phi_0^2 = 2cD^2 \cos(q)/\gamma x_0^2$ and $\mu = cD^2 \cos(q)/x_0^2$. Equation (7) clearly demonstrates that spatiotemporal DSs can indeed propagate at a group speed v_e in lattices of nonlinear coupled defects or resonators. These entities are possible as a result of the balance between the effects of nonlinearity and that of temporal dispersion that arises from the linear coupling among discrete sites. The optical field profile of such a moderately confined discrete soliton is shown

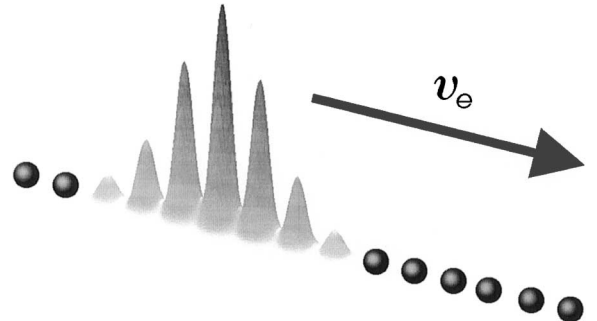


Fig. 2. Temporal discrete soliton propagating along a nonlinear chain of coupled resonators at a group speed v_e .

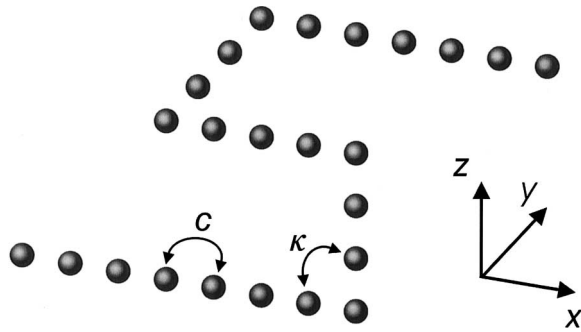


Fig. 3. Three-dimensional pathway of coupled resonators.

schematically in Fig. 2. The soliton eigenvalue μ in Eq. (7) represents an important quantity, since it relates the carrier frequency ω_c of the exciting laser to the rest of the soliton parameters, i.e., $\omega_c = \omega_0 - \Delta\omega - cD^2 \cos(q)/x_0^2$. Thus, given ω_c and x_0 , the discrete phase shift q as well as the other soliton parameters can be uniquely determined. In principle, the soliton group velocity v_e can be very low for very small coupling coefficients or for $q \approx 0, \pi$. If the phase shift is $q = 0$, the group velocity is $v_e = 0$, and thus the DS becomes in essence immobile (frozen light). However, for $0 \leq q \leq \pi/2$, when the lattice dispersion is anomalous [$cD^2 \cos(q) > 0$] this DS propagates at $v_e = 2cD \sin(q)$. For $q = \pi$, the dispersion is normal (for $c > 0$), and thus immobile staggered dark solitons are expected to exist.¹⁵ Similarly, when the nonlinearity of the chain is of the defocusing type, dark solitons are allowed at $q = 0$ and staggered bright solitons are expected to exist at the end of the Brillouin zone¹⁶ ($q = \pi$) when again $c > 0$. The regime close to $q \approx \pi/2$ is also of interest, since to first order the dispersion is close to zero, thus allowing dispersion management as well as dispersion-free propagation in the linear regime.¹⁷ In addition to the weakly or moderately localized DS states as approximately described by Eq. (7), other considerably more confined solutions of Eq. (5) are also known to exist. This latter type of DS is associated with nonlinear defect states that have altogether different transport properties because of Peierls–Nabarro effects. In this case, the discrete field distribution (in self-focusing systems with $q = 0$) has approximately the form $a_n \approx a_0 \exp(-|n|D/x_0)$.

Finally, we would like to mention that this class of solitons can be navigated along any preassigned path in a three-dimensional environment. This could occur, for example, in a three-dimensional chain of nonlinear coupled resonators (completely surrounded by a photonic crystal), as depicted in Fig. 3. As shown in Refs. 18 and 19, reflection losses in such a discrete system arise when the soliton traverses a sharp bend. This is because the finite coupling strength κ between the two sites around the corner plays an important role in this process. Yet these bending losses can be essentially eliminated by appropriately engineering the corner site of the bend. Following the analysis in Ref. 18, these losses are eliminated by slightly detuning the

corner site by an amount $\Delta\Omega$ in the effective eigenfrequency of the resonator according to

$$\frac{\Delta\Omega}{c} = - \frac{\left(\frac{2\kappa}{c}\right) \left[1 + \frac{\kappa}{c} \cos(q)\right]}{[1 - (\kappa/c)^2]}. \quad (8)$$

The required detuning $\Delta\Omega$ at the corner can be obtained by changing either the dimensions or the index composition of the microcavity.

In conclusion, we have shown that temporal discrete solitons can propagate along a chain of nonlinear coupled resonators or defects that are embedded in a photonic crystal structure. This class of solitons can effectively follow any preassigned path in a three-dimensional environment.

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