one sees that the second order term effectively "doubles" the slip-flow velocity when the local Knudsen number approaches unity. It should be noted, however, that if there is an external velocity imposed in the same direction, the ratio given by (A-2) is no longer as simple as (A-4), and actually the ratio is much smaller because the denominator of the ratio contains the surface velocity. The exact effect due to this second order term is difficult to assess unless the problem is solved completely. However, the effect of the second order term on the velocity components can be examined.

The approximate form of the Navier-Stokes equations to be solved are the same as the previous problems and are as follows:

$$\frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial z^2}$$
$$\frac{\partial p}{\partial y} = \mu \frac{\partial^2 v}{\partial z^2} \qquad \text{(A-5)}$$
$$\frac{\partial p}{\partial z} = 0$$

The corresponding boundary conditions for the foregoing three equations are as follows:

$$u(z=0) = U + \lambda \frac{\partial u}{\partial x}\Big|_{z=0} - \frac{\lambda^2}{2}\frac{\partial^2 u}{\partial x^2}\Big|_{z=0} + \ldots$$

$$u(z=h) = -\lambda \frac{\partial u}{\partial x}\Big|_{z=h} - \frac{\lambda^2}{2}\frac{\partial^2 u}{\partial x^2}\Big|_{z=h} - \ldots$$

$$v(z=0) = \lambda \frac{\partial v}{\partial y}\Big|_{z=0} - \frac{\lambda^2}{2}\frac{\partial^2 v}{\partial y^2}\Big|_{z=0} + \ldots \qquad \text{(A-6)}$$

$$v(z=h) = -\lambda \frac{\partial v}{\partial y}\Big|_{z=h} - \frac{\lambda^2}{2}\frac{\partial^2 v}{\partial y^2}\Big|_{z=h} - \ldots$$

Solving equations (A-5) for u and v with the boundary conditions (A-6), the following equations are obtained:

$$u = \frac{\partial p}{\partial x}[z^2 - hz - h\lambda - \lambda^2] + U\left[1 - \frac{\lambda + z}{h + 2\lambda}\right] \qquad \text{(A-7)}$$

$$v = \frac{1}{2\mu}\frac{\partial p}{\partial y}[z^2 - hz - h\lambda - \lambda^2] \qquad \text{(A-8)}$$

Upon evaluating equation (A-8) at $z=0$ and $z=h$, one sees that the effect of the second order slip term is to "double" the slip velocity at the boundaries as the local Knudsen number approaches unity. This effect is most evident in the transverse direction since there is no Couette flow in that direction. Presently, the authors have begun to modify the first-order approximation theory to include the second order effects. Reynolds equation based on the u and v velocities given by equations (A-7) and (A-8) have been derived and solved numerically. The results will be published once the study is completed. For the present investigation, the additional slip due to the second order effects at high local Knudsen numbers will be approximated by introducing the surface accommodation coefficient. Since the present existing theory is already an approximation of the real phenomenon, another approximation to account for the second order slip effects may be as good as considering both slip terms.

DISCUSSION

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This work confirms the validity of the slip flow theory for hydrodynamic lubrication at clearances down to 0.075 micron. By employing helium as the lubricant, Knudsen numbers of order one may be demonstrated without masking the slip effects with high bearing number influences. The paper should be a welcomed addition to the body of carefully taken experimental data for very low flying sliders.

As the authors point out, the occurrence of Knudsen numbers of order unity requires the consideration of second order effects in the slip boundary conditions. Most of the experimental data presented in the paper falls below the theoretical load/spacing curve derived from only first order slip effects for spacings below 0.25 micron. We can estimate the influence of a second order slip correction by deriving a model lubrication equation.

By solving equations (A-5) together with the second order slip conditions (A-1) given in the paper, the resulting dimensionless lubrication equation appears as

$$\frac{\partial}{\partial x}\left(h^3 P \frac{\partial P}{\partial x}\right) + 6K_\infty \frac{(2-\sigma)}{\sigma}\frac{\partial}{\partial x}\left(h^2 \frac{\partial P}{\partial x}\right) + 6K_\infty^2\left(\frac{2-\sigma}{\sigma}\right)\frac{\partial}{\partial x}\left(h\frac{\partial \ln P}{\partial x}\right)$$
$$+ \left(\frac{L}{w}\right)^2 \left\{\frac{\partial}{\partial y}\left(h^3 P\frac{\partial P}{\partial y}\right) + 6K_\infty\left(\frac{2-\sigma}{\sigma}\right)\frac{\partial}{\partial y}\left(h^2\frac{\partial P}{\partial y}\right) + 6K_\infty^2\left(\frac{2-\sigma}{\sigma}\right)\frac{\partial}{\partial y}\left(h\frac{\partial \ln P}{\partial y}\right)\right\} = \Lambda\frac{\partial Ph}{\partial x} \qquad \text{(B-1)}$$

where $K_\infty = \frac{\lambda_a}{h_{\min}}$

For the case of high Λ, higher derivatives in x may be neglected in equation (B-1), resulting in a negligible error in predicted load of order $0\cdot(1/\Lambda)$. This simplified equation is given by

$$\frac{\partial}{\partial y}\left(h^3 P\frac{\partial P}{\partial y}\right) + 6K_\infty\left(\frac{2-\sigma}{\sigma}\right)\frac{\partial}{\partial y}\left(h^2\frac{\partial P}{\partial y}\right) + 6K_\infty^2\left(\frac{2-\sigma}{\sigma}\right)\frac{\partial}{\partial y}\left(h\frac{\partial \ln P}{\partial y}\right) = \Lambda^*\frac{\partial Ph}{\partial x} \qquad \text{(B-2)}$$

When the clearance is only a function of x, equation (B-2) simplifies to

$$\frac{h^3}{2}\frac{\partial^2 p^2}{\partial y^2} + 6K_\infty\left(\frac{2-\sigma}{\sigma}\right)h^2\frac{\partial^2 P}{\partial y^2} + 6K_\infty^2\left(\frac{2-\sigma}{\sigma}\right)h\frac{\partial^2 \ln P}{\partial y^2} = \Lambda^*\frac{\partial Ph}{\partial x} \qquad \text{(B-3)}$$

TERM 1 TERM 2

The term identified as TERM 1 represents the influence of

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first order slip effects on the transverse flow. TERM 2 accounts for the second order influence of slip on the transverse flow and appears as a gradient diffusion of logarithmic pressure. TERM 2 tends to augment the side leakage and as pointed out by the authors, the second order slip correction produces a decrease in bearing load at fixed clearance. Incorporating the second order effects should move the theoretical load/spacing curve toward the experimental data. It will be interesting to observe if this change alone accounts for the deviation between first order slip theory and experiment.

It would be appreciated if the authors would discuss briefly the approximations involved in deriving the second order slip boundary conditions. Also, could the authors comment on the applicabilty of the second order slip conditions for Knudsen numbers of the order of one?