is such that the barlike character is lost. Other applications may exist, however.

In the state of plane stress for uniform thickness, σ_z remains the intermediate principal stress (except at the boundaries $r = a, b$ of the bar, where $\sigma_r = \sigma_z = 0$; $\sigma_\theta = \pm\sigma$ which present no difficulties). Indeed, the point is not even mentioned by Swida nor in another recent paper by Eason.[5]

It is easily checked through equations (9) and (10) of the paper that, for the particular form of h selected, again no difficulties of this type are encountered; however, for general h this need not be the case.

It should be added that, in view of Eason's recent paper,[5] comparisons between plane stress (for uniform h) and plane strain (when $\nu = 1/2$) are now possible, employing the von Mises yield criterion. The case of variable thickness, however, remains a point for further investigation.

Author's Closure

The author appreciates the interesting and pertinent remarks by Mr. Murch, and thanks him for the additional references.

[5] G. Eason, "The Elastic-Plastic Bending of a Curved Bar by End Couples in Plane Stress," *The Quarterly Journal of Mechanics and Applied Mathematics*, vol. 13, 1960, pp. 334–358.

Buckling of Circular Cones Under Axial Compression[1]

PAUL SEIDE.[2] In this paper an attempt is made to prove the formula

$$P_{cr} = P_{cyl\,\infty} \cos^2\alpha = \frac{2\pi\, Et^2 \cos^2\alpha}{[3(1 - \nu^2)]^{1/2}} \qquad (1)$$

is the lowest value of P_{cr} that satisfies the stability determinant of Seide[3] for values of Poisson's ratio ν other than zero. The reasoning used is that, if "the value of P_{cr} is smaller than $P_{cyl\,\infty}$ $\cos^2\alpha$, the stability determinant will contain Bessel functions of complex numbers" and "such a determinant is *not likely* to yield a real value for the critical load." When equation (1) is substituted into the stability determinant, it is found that pairs of columns are identical, which means that the determinant is identically equal to zero, without regard to the locations of the radius-thickness ratios of the ends of the cone.

Unfortunately, such a procedure is invalid since the stability determinant is not correct for P_{cr} equal to $P_{cyl\,\infty}$ $\cos^2\alpha$. For this case the solution of the differential equation of Seide[3] is different from that given for P_{cr} greater than $P_{cyl\,\infty}$ $\cos^2\alpha$ and the correct stability determinant may vanish only for certain combinations of values of the radius-thickness ratio at both ends of the cone. The stability determinant for P_{cr} less than $P_{cyl\,\infty}$ $\cos^2\alpha$ can be put into real form, and at this point it does not appear to be possible to state *a priori* that this determinant will not vanish for some combination of values of load coefficient, semi-vertex angle, and radius-thickness ratio of both ends. Only for ν equal to zero was it possible to obtain the result of equation (1) without resorting to large-scale numerical investigation of the equations. It is the writer's belief, however, that the correct minimum for Poisson's ratios other than zero differs by only a very few per cent from this result, as indicated by some calculations being made at the present time for the case of combined axial compression and external pressure.

The test results are also open to question, as is usually the case when more than one investigator has worked on the buckling of shells under axial compression. Tests have been made at Space Technology Laboratories by Victor Weingarten and recorded recently.[4] These tests indicate that the correction coefficient applicable to conical shells under axial compression may be considerably greater than that given by the Kanemitsu-Nojima equation using the average radius of convective-thickness ratio, and thus even greater than the results using the greatest radius of curvature. In a private discussion with the author it appeared that this discrepancy might be due to the somewhat unusual end conditions of his tests which consisted of the cone resting on a spherical surface. It is debatable whether this restraint would be equivalent to the bulkheads rigid in their own plane that is assumed by the theory of Seide[3] and approximated in Morgan, et al.,[4] since inward motion of the shell walls would not necessarily be prevented.

Authors' Closure

The authors wish to thank Dr. Seide for his discussion.

He is entirely correct that the stability determinant as given by Seide[3] is not correct for P_{cr} equal to $P_{cyl\infty} \cos^2\alpha$, and therefore the author's reasoning to show that the formula

$$P_{cr} = P_{cyl\infty} \cos^2\alpha = \frac{2\pi E t^2 \cos^2\alpha}{[3(1 - \nu^2)]^{1/2}}$$

is the lowest value of P_{cr} that satisfies the stability determinant of Seide[3] is invalid for values of Poisson's ratio ν other than zero. The reason the afore-mentioned stability determinant is invalid in this case is due to the fact the required four independent solutions (Eq. 17, Seide[3]) of the second-order equations (Eqs. 15, Seide[3]) degenerate into only two independent solutions and thus no longer represent a complete solution.

It should be pointed out, however, that the afore-mentioned erroneous reasoning by the authors does not invalidate their test results and conclusions as presented in the paper.[1]

The authors believe that the boundary conditions for their cones approximated pinned supports. During testing of these cones, no apparent displacements normal to the generator at the boundaries were observed. Only after very large displacements in postbuckling range was the friction insufficient to prevent these normal displacements.

Tests of the buckling of cylinders and cones reported recently by Space Technology Laboratories, Inc. (STL)[4] resulted in higher buckling correction coefficients for cylindrical shells under axial compression than those reported previously in numerous papers.[5,6,7] As noted by Mr. Seide, the test results for cones obtained at STL show higher buckling coefficients than those reported by the authors. These observations would seem to indicate a general discrepancy between the STL results and those of others, not just a discrepancy between the STL results and those results of the authors.

[1] By Leslie Lackman and Joseph Penzien, published in the September, 1960, issue of the JOURNAL OF APPLIED MECHANICS, vol. 27, TRANS. ASME, vol. 82, Series E, pp. 458–460.

[2] Member of the Technical Staff, Space Technology Laboratories, Inc., Los Angeles, Calif. Assoc. Mem. ASME.

[3] Paul Seide, "Axisymmetric Buckling of Circular Cones Under Axial Compression," JOURNAL OF APPLIED MECHANICS, vol. 23, TRANS. ASME, vol. 78, 1956, pp. 625–628.

[4] E. J. Morgan, Paul Seide, and V. I. Weingarten, "Semiannual Report on Development of Design Criteria for Elastic Stability of Thin Shell Structures," Space Technology Laboratories Report STL/TR-59-0000-09959, July 1-December 31, 1959.

[5] S. B. Batdorf, M. Schildcrout, and M. Stein, "Critical Stress of Thin-Walled Cylinders in Axial Compression," NACA TN 1343, 1947.

[6] S. Kanemitsu and N. Nojima, "Axial Compression Tests of Thin Circular Cylinders," a Master of Science thesis, California Institute of Technology, 1939.

[7] L. A. Harris, H. S. Suer, W. T. Skene, and R. J. Benjamin, "The Stability of Thin-Walled Unstiffened Circular Cylinders Under Axial Compression Including the Effects of Internal Pressure," Preprint No. 660, Institute of the Aeronautical Sciences.