DISCUSSION

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A mathematical model for the pitting fatigue life of pinion and gear or entire meshing train is needed for more accurately representing the reliability of mechanical transmissions. This load-life model for gears can then be combined with rolling element bearing life models for overall reliability studies or trade-off studies of high reliability systems such as helicopter transmissions. It is doubtful that such a model will replace the accepted design tools of the AGMA [31, 32][3] for commercial quality gears.

Using the Lundberg-Palmgren model [17, 18] for rolling element bearings is a good choice because of the similarity of concentrated contacts in gears and bearings. One should proceed with caution since this model is basically formulated for subsurface oriented fatigue caused by reversing sub-surface shear stresses. Liu [33] established that the maximum shear stress occurs on the surface whenever the coefficient of traction equals or exceeds 0.1. Thus the gear mesh must be designed to avoid scoring [34] or surface damage before the present model is valid. The same problem exists with rolling element bearings and has been compromised by using life adjustment factors [30] and the authors reasonably suggest this procedure for gears. The existence of acceptable life values by the present model does not rule out the possibility of surface oriented pitting or scoring.

It is difficult to understand the authors choice of c and h values identical with those of Lundberg-Palmgren [17, 18] and a Weibull slope of $e = 1.5$ resulting in a load-life exponent of $p = 1.5$. The load-life exponent reported by the authors in [16] is $p = 3.38$ which is consistent with the industry standard [35] for line contact in roller bearings. The Weibull slope has considerable scatter in testing. The National Bureau of Standards study in 1956 [36] for establishing the agreement within the ball bearing industry showed that e varied between 0.5 and 3.25 with a median of 1.43 for 213 separate tests. Air melt vacuum degassed steels are cleaner with finer and more dispersed nonmetallic inclusions and one may expect the Weibull slope to be higher indicating less scatter within a test group. There is no reason however to expect less scatter between tests than indicated in [36]. More statistical data is needed.

The further development of the gear load-life model should follow the procedures as well as the mathematics of Lundberg-Palmgren [17–18]. The present paper and [21] agree upon the load-life exponent and dependence on the face width when expressed in algebraic terms [22].

$$L = \left(\frac{W_{tm}}{W_t}\right)^p \tag{25}$$

$$p = \frac{c - h + 1}{2e} \tag{26}$$

$$W_{tM} \sim f^{\frac{c-h-1}{c-h+1}} \tag{27}$$

If one accepts an average value of the Weibull slope, e, from gear testing results, then two equations exist to determine the material constants from test data. Load-life tests can be obtained from the present NASA test rig. Tests which vary only the face width, f, are needed and perhaps could be made on the present NASA test rig by not offsetting the test gears. Thus c and h may differ from rolling bearing theory if indeed the average Weibull slopes are greater than 10/9 and 9/8 used in [17, 18]. The additional test data will give a basis for the material constant K_2. All additional testing should use the present test lubricant as it has been demonstrated by NASA to remain stable over long periods of time.

The Lundberg-Palmgren models [17, 18] were formulated on the basis of extensive test data obtained over many years by a number of bearing companies. A similar sharing or collection of gear test data will be needed to gain wide acceptance of a gear load-life model. The need for such a model is real and the authors should be congratulated for their contribution to this ongoing task.

Additional References

31 Anon., "Surface Durability (Pitting) of Spur Gear Teeth," AGMA Standard 210.02, Jan. 1965.

32 Anon., " Surface Durability (Pitting) of Helical and Herringbone Gear Teeth, AGMA Standard 211.02, Feb. 1969.

33 Liu, C. K., "Stresses and Deformations Due to Tangential and Normal Loads on an Elastic Solid With Applications to Contact Stresses," Doctoral Thesis, University of Illinois, 1950.

34 Anon., "Gear Scoring Design Guide for Aerospace Spur and Helical Power Gears," AGMA Information Sheet 217.01, Oct. 1965.

35 Anon., "Method of Evaluating Load Ratings for Roller Bearings," AFBMA Sect. No. 11, July 1960.

36 Lieblein, J., and Zelen, M., "Statistical Investigation of the Fatigue Life of Deep-Groove Ball Bearings," *Journal of Research*, National Bureau of Standards, Vol. 57, NR 5, Nov. 1956.

E. J. Bodensieck[4]

The authors have made a commendable effort toward establishing a new rating system for compressive fatigue of gear teeth patterned after the AFBMA system for bearings. Their results, however, differ so widely from established gear design practices that some comparisons are in order.

1 The power loss indicated by the oil flow rate and temperatures is 0.919 percent. Using Shipley's mechanical advantage method [37][5] for the tooth proportions of this paper, the average friction coefficient is 0.05. An average friction coefficient calculated by integrating the instantaneous power loss, using either Kelley and Benedict [38] or Bodensieck [39] for instantaneous friction coefficients, yields average friction coefficients of 0.022 and 0.024 respectively. This would lead one to believe that the mesh power, and hence tooth loading might be higher than reported. Iteration of either method (2) or (3) suggests that the tooth load could have been about 3586.0 N (805 lb) instead of 1670.0 N (363 lb).

Differences of this magnitude are possible with rotating vaned actuators due to centrifugal effects in the actuator cavities if the oil is vented, either intentionally or inadvertently from the outer surface of the unloaded side of the vanes. Calibration of the torque applier at operating speed is necessary to relate the torque to supply pressure.

2 The stress of 1.71×10^9 N/m^2 (248000 psi) at 1.14×10^7 cycles is low compared to the values from either AGMA Standard 411.02 [40] adjusted to 90% reliability, or calculated by the method of AGMA Technical Paper 229.19 [41]. Both of these methods show that properly manufactured gears of the quality described by the authors should have a reliability of 90 percent at compressive stresses of 2.55×10^9 N/m^2 (370000 psi). The stress ratio squared (2.226) correlates fairly well with the load ratio (2.218) from my first comment.

3 If one were to use the stress at 1.14×10^7 cycles and the S-N curve exponent of 3.0 for a design basis the results are somewhat startling. It is not uncommon to be required to design for 99.0% reliability at 5×10^{10} cycles or more in either today's aircraft engines, or in marine steam turbine propulsion gearing. Extending the S-N curve to this point by this paper's values gives an allowable stress of 1.04×10^8 N/m^2 (15150. psi). From the tabulated

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[5] Numbers 37–42 in brackets designate Additional References at end of discussion.

values in AGMA 411.02 the allowable stress is 9.75×10^8 N/m^2 (141000. psi) Although not so stated in the AGMA 411.02 standard, the values tabulated were derived from several hundred tests for 99.97 percent reliability.

Further extension of the paper's method to marine gears requires a correction for hardness. Using the relationship given by Bamberger, et al. [42] the allowable stress for 1.31×10^{11} cycles, or 25 years service in a steam turbine ship propulsion high speed pinion, would be 3.4×10^7 N/m^2 (4938. psi). The limits set by marine underwriting groups is in the order of ten times this value.

Similarly if we look at values for actuators, which may see 100,000. cycles of tooth loading in their required life, the method of this paper would allow 8.3×10^9 N/m^2 (1.204×10^6 psi) which is unrealistically high.

In conclusion it seems appropriate to ask the authors if they have a means available to calibrate the torque applier used in the tests at the operating speed. I would further suggest that an examination of the Lundberg-Palmgren theory with the word "significant" included in the "Significantly Stressed Volume" statement might prove revealing and valuable. The determination of what base stress level is "significant" in compressive fatigue would be a valuable contribution toward closing the gap between theory and empirical rating methods for both gears and rolling element bearings.

Additional References

37 Shipley, "The Efficiency of Involute Spur Gears," ASLE, Jan. 1959 (also McGraw-Hill *Gear Handbook,* pp. 14-3 to 14-6)

38 Kelley and Benedict, "Instantaneous Coefficients of Gear Tooth Friction," ASME/ASLE Conference 1960.

39 Bodensieck, "Specific Film Thickness as an Index of Gear Tooth Surface Deterioration," AGMA Aerospace Committee, Sept. 1965 (Revised)

40 AGMA Standard 411.02 Design Procedure for Aircraft Engine and Power Take-off Spur and Helical Gears, 1966

41 Bodensieck, "A Stress-Life-Reliability Rating System for Gear and Rolling Element Bearing Compressive Stress, and Gear Root Bending Stress," AGMA Semiannual Meeting, Nov. 1974.

42 Bamberger, et al., "Life Adjustment Factors for Ball and Roller Bearings," ASME Rolling Element Committee, 1971.

Authors' Closure

The thoughtful comments of Messrs. Rumbarger and Bodensieck are much appreciated. The authors agree with the general comments and observations by Mr. Rumbarger. Present use of the contents of the paper is directed to high quality aviation and especially helicopter transmissions. One of the problems with correlating the theory to experimental results in the open literature is one of woefully inadequate reporting of test conditions. If it is known in advance that a correlation study with a Lundberg-Palmgren type gear life model is desired, then perhaps gear researchers would report the needed parameters. Most of the gear life data reported is simply a correlation of life with contact stress. Often the points on an S-N diagram are for gears of various sizes. Therefore, the important effects of stressed volume and depth to the maximum orthogonol reversing shear stress cannot be accounted for due to lack of information. It is hoped that the methods of this paper will be examined by those researchers who are conducting gear test programs.

We welcome Mr. Rumbarger's emphasis that acceptable life according to the surface fatigue life model presented does not rule out failure by other mechanisms. Experience has shown that the mode of failure will change from pitting fatigue to surface distress, wear, or scoring as the surface tangential force increases within the area of contact. According to Liu [33], the reversing orthogonal shear stress is not affected (in total peak-to-peak amplitude or in depth of occurrence) as surface traction coefficient increases to 0.1 or higher. However, the maximum shear stress and the maximum octahedral stress do increase (although not significantly) and approach the surface. According to equation (2), when the critical shearing stress is at or near the surface, no significant life of the gear system can be expected. Since surface traction and pitting fatigue life are lubrication related, it may be possible to relate them together in a lubrication life adjustment factor similar to that of [30]. As a result, the authors decided to adhere to the original approach of Lundberg and Palmgren.

The abnormally low value for p was obtained due to a purely mathematical application of the relation between c, h, e, and p as given in [17]. If $c = 10\frac{1}{3}$, $h = 2\frac{1}{3}$, and $c = 3$ are used in equation (26), then a load-life exponent p which equals 1.5 results. This value we now know to be contrary to test results since the authors have recently finished a series of gear life tests using different loads. The results show that the load-life exponent should be about 4. If additional tests were run by changing the face width of the gears, then the exponent in equation (27) could be determined, as suggested by Mr. Rumbarger. However, this would not help in determining c and h, since the same linear combination of c and h is contained in that exponent and also in the exponent p. What is needed is a test that would provide linearly independent relations. Such a test would be to vary the size of the gears and, hence, the curvature sum.

$$L \propto \Sigma\rho^{-\frac{(c-h-1)}{2e}} \tag{28}$$

Based upon oil flow rate and temperature, Mr. Bodensieck implies that the actual gear loads may be higher than those reported. If all the losses as determined by a heat balance on the oil are attributed to sliding friction, then Mr. Bodensieck would be correct. However, a majority of the losses with the test gear system are caused by windage, support bearings, oil churning and viscous drag of the rotating components. Hence, in order to achieve an accurate gear loading it was necessary to install strain gages on the drive shaft near the test gear. The strain gages were used to establish the strain output for various levels of static torque. The pressure necessary to produce the same level of torque under rotating conditions was established by correlating hydraulic pressure with the strain level desired during rotation at 10,000 rpm. This method resulted in establishing the 1670 N (363 lb) gear load.

We would also emphasize as Mr. Bodensieck did in his third comment that extrapolation of the theory beyond the range of experimental validation is dangerous. As pointed out in the paper, there is evidence showing that the Weibull slope may be dependent on contact stress level. For instance in reference [15] at 100,000 psi, $e = 1$ and at 300,000 psi, $e = 3$. If this is true, then the theory presented here will penalize the allowed stress for extended life requirements. This statement is based on using equation (26) to get the load-life exponent. If the more reasonable value of four (4) for the load-life exponent is used, we think Mr. Bodensieck's fears about using the life prediction theory for very short or very long lived gears will be allayed.

The authors wish the data upon which AGMA 411.02 is based were made available in the open literature. Such data with the necessary gear dimensions and loads would provide a valuable check on the use of Lundberg-Palmgren methods for gear surface fatigue life. We encourage Mr. Bodensieck and others who have such data to publish it.