

A conservative and good correlation with experiments for up to 10⁵ cycles of failure for nine of the twelve materials has been found. For the other three cases, agreement was satisfactory, though slightly nonconservative. This is attributed to insufficient experimental results or inaccurate property information.

References

1 L. F. Coffin, Jr., "A Study of the Effects of Cyclic Thermal Stresses in Ductile Metals," TRANS. ASME, vol. 76, 1954, pp. 931-950.

2 J. F. Tavernelli and L. F. Coffin, Jr., "A Compilation and Interpretation of Cyclic-Strain Fatigue Tests," Trans. ASM, vol. 51, 1959, pp. 438-453.

3 L. F. Coffin, Jr., and J. Tavernelli, "The Cyclic Straining and Fatigue of Metals," *Trans. AIME*, vol. 215, 1959, pp. 794–807.

4 A. Johansson, "Fatigue of Steels at Constant Strain Amplitude and Elevated Temperatures," Proceedings, Colloquium on Fatigue, Stockholm, Sweden, 1955.

5 J. H. Gross and R. D. Stout, "Plastic Fatigue Behavior of High-Strength Pressure Vessel Steels," The Welding Journal, vol. 34, 1955, Research Supplement, pp. 161-S to 166-S.

B. F. Langer, Correspondence to Members of ASME Boiler and Pressure Vessel Committee, Special Committee to Review Code Stress Basis, Task Group on Fatigue, July 17, 1959.

Metals Handbook, American Society of Metals, 1948, (a) p. 811, (b) p. 1047, (c) p. 565, (d) p. 817, (e) p. 823, (f) p. 119. 8 C. G. Goetzel, "Some Properties of Oxygen-Free, High-Con-

ductivity Copper," Trans. ASM, vol. 27, no. 2, 1939.

Mechanical Properties of Metals and Alloys, Circular C447, U. S. Department of Commerce, National Bureau of Standards, p. 177.

10 Titanium Metal Pamphlet 214.

T. J. Baker, private communication. 11

DISCUSSION

S. S. Manson³

Introduction

The report by Tavernelli and Coffin represents a very valuable contribution to the state of the art of estimating fatigue properties of materials from a minimum quantity of experimental information. The writer has recently been engaged in a similar pursuit of methods of estimating fatigue properties, and it is the purpose of this discussion to outline some of the results that have been obtained by a different method, as well as to indicate how the authors' equation (as well as the alternate ones proposed by

³ Chief, Materials and Structures Division, NASA Lewis Research Center, Cleveland, Ohio.

the writer) can be effectively used in optimizing material properties for fatigue application.

Alternate Relations

Before discussing the alternative approach it is instructive to consider the discrepancies that exist between the experimental data points shown in the report, and the lines representing the various materials according to the equation

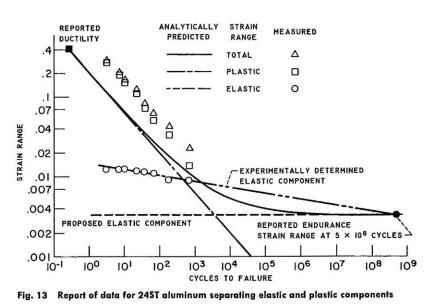
$$\Delta \sigma = \frac{ED}{2} N^{-1/2} + 2\sigma_{\rm end}$$

These discrepancies are clearly evident in the low cyclic life range for many of the materials analyzed; in the range of high cyclic life, the discrepancies are hidden by the very condensed stress scale used. It is therefore desirable to examine somewhat more closely the correlation for one of the materials, as an aid in discussing potential improvements.

Fig. 13 shows the analysis for 24ST aluminum. The scale here is double-logarithmic which is more conventional for this type of data representation, and avoids excessive condensation of the data in the high cyclic life range. The plot is made on the basis of strain rather than "nominal" stress, but conversion to stress could readily be made by multiplying by elastic modulus. Using the equation proposed in the report, predictions for plastic and elastic components of strain are shown by the dot-dash and dotted lines, respectively. The experimentally determined components of plastic and elastic strain range are shown by the squares and circles, respectively. For both components there are discrepancies between the predictions and the experimental values. In the case of the plastic component, the representation of which by the term $\epsilon_p = D/2N_f^{-1/2}$ has been suggested before by Coffin, does not coincide with the data primarily because the intercept at $N_f = 1/4$ is not D, as hypothesized by him. For this material, and for others investigated by the writer, some additional discrepancy occurs because the slope of the line representing the plastic component is not universally -1/2, as implied by

the expression
$$\epsilon_p = \frac{D}{2} N_f^{-1/2}$$
.

The elastic component is represented in the relation proposed in the report by a horizontal line passing through the strain range associated with the endurance limit. As seen in Fig. 13 (and typical of other materials) a better representation is by an inclined double-dot-dash straight line. Both lines have the common point at the value chosen for the endurance limit, but in the



Journal of Basic Engineering

DECEMBER 1962 / 537

The assumptions used in the report for estimating the plastic and elastic components are generally of conservative nature. That is, at a given life the strain range allowed by the proposed equation is usually lower than the experimental value. Still, it is preferable to improve the correlation by the use of a more accurate equation, and to introduce conservativeness of any desired magnitude by application of conventional safety factors rather than acceptance of inaccuracy in the relation itself.

The earliest published proposal for a power law relationship between cyclic life and plastic strain was made by the writer in 1952 (ref. [12]⁴) in the form $\epsilon_p = MN_f^z$, where both M and zwere to be regarded as material constants. Later (ref. [13]) Coffin proposed the same relation, identifying M with the ductility in the tensile test, M = D/2, and z as a universal constant, approximately equal to -1/2 for all materials. These are reasonable approximations, but in view of the discrepancies that have appeared in the correlations of data for a large number of materials that have since been tested, it is perhaps more reasonable to return to the more general relation $\epsilon_p = MN_f^z$, treating M and zas material constants. Since more experimental data are now available, the relation between M and z with other readily measurable properties can now better be sought.

Another factor that has become apparent in recent years is that an improvement in the life relationship can be obtained by relating life to total strain range instead of plastic strain only. The improvement lies largely in the higher life range where the elastic strain becomes significant compared to, and at high life even much greater than, the plastic strain. In the paper under discussion this modification to the life equation is made by adding an elastic term which is *constant* over the entire life range, as evidenced by the horizontal dotted line in Fig. 13. As also indicated, however, in Fig. 13 the elastic strain can better be represented as a variable with life, and as a reasonable first approximation, as a linear relation with life.⁵ Observation of this general characteristic for a large number of materials has recently led the writer to propose (ref. [14]) that a suitable form for the elastic term is

$$\Delta \epsilon_{el} = \frac{G}{E} N_f^{\gamma}$$

Thus the total strain range, $\Delta \epsilon$, which is the sum of the elastic and plastic components, becomes

$$\Delta \epsilon = M N_f^{\epsilon} + \frac{G}{E} N_f^{\gamma} \tag{4}$$

where M, z, G, and γ are to be regarded as material constants.

From a practical viewpoint there would appear to be relatively little objection to regarding the coefficients and exponents as material constants determinable from a few fatigue tests. In the limit, only two fatigue tests are required, since each fatigue test establishes a point on both the elastic and plastic line, and only two points are required to establish a straight line. Practically, of course, it is desirable to establish the lines by many more tests than two, especially since the most important region for establishment of the plastic line is the low cyclic life range, whereas the most important region for the elastic line is the high cyclic life range. The economic and technical importance of many designs more than justifies the experimental program necessary for accurate determination of the material constants, and it is felt by the writer that the experimental approach is the proper one for important applications. In some cases, however, it is desirable to estimate the constants from readily available material properties normally measured in the conventional tensile test. In an attempt to determine relationships from which engineering approximations may be made, the writer and his co-workers have conducted a series of fairly extensive fatigue tests on a large number of materials representing a number of classes of importance in current technology. The complete program is to be described in a forthcoming NASA report; in brief, the findings were as follows:

(1) For the determination of the elastic line two points can be drawn:

(a) At 1/4 cycle life the elastic strain range can be estimated as approximately $2.5\sigma_f/E$, where σ_f is the fracture stress in the tensile test—that is, the load at fracture divided by the area as measured after fracture; and E is the elastic modulus.

(b) At a life of 10^5 cycles, the elastic strain range is approximately $0.90\sigma_u/E$, where σ_u is the conventional ultimate tensile strength.

(2) Having determined the elastic line, the plastic line is determined as follows:

(a) At a life of 10 cycles the plastic strain is approximately ${}^{1}_{4}D^{3/4}$, where D is the conventional logarithmic ductility as defined in the report under discussion.

(b) At 10^4 cycles the plastic strain is determined from the elastic strain range at this life according to the relationship

$$(\epsilon_p)_{10}^4 = 0.0069 - 0.525(\Delta \epsilon_{el})_{10}^6$$

where $(\Delta \epsilon_{tl})_{10}^4$ is determined from the elastic line at 10⁴ cycles according to the method of (1) above. This relation follows from the approximate constancy at 10⁴ cycles of total strain range elastic plus plastic—for most materials studied. That is, a total strain range of approximately 1 percent results in failure in 10⁴ cycles for most of the materials investigated.

The above relationships can be used to determine graphically the elastic and plastic lines, as well as their sum, thus providing the graphical relation between strain range and cyclic life. Alternatively, the procedure can be applied analytically, providing relations for M, z, G, and γ in terms of the more commonly measured quantities. Thus, as a first approximation

$$\Delta \epsilon = \epsilon_{el} + \epsilon_p = \frac{G}{E} N_f^{\gamma} + M N_f^z \qquad (4)$$

or

$$\Delta \sigma = E \Delta \epsilon = G N_f^{\gamma} + M E N_f^z \tag{5}$$

where $\Delta \sigma$ is the nominal stress range (strain range multiplied by elastic modulus in cases where plastic strain occurs) and

$$G = \frac{9}{4} \sigma_u \left(\frac{\sigma_f}{\sigma_u}\right)^{0.9} \tag{6}$$

$$\gamma = -0.08 - 0.18 \log \left(\frac{\sigma_f}{\sigma_u}\right) \tag{7}$$

$$M = 0.83D \left[1 - 82 \left(\frac{\sigma_u}{E} \right) \left(\frac{\sigma_f}{\sigma_u} \right)^{0.18} \right]^{-1/3}$$
(8)

$$z = -0.52 - \frac{1}{4} \log D + \frac{1}{3} \log \left[1 - 82 \left(\frac{\sigma_u}{E} \right) \left(\frac{\sigma_f}{\sigma_u} \right)^{0.18} \right]$$
(9)

⁴Numbers in brackets from 12 to 14 designate References at end of this discussion.

⁵ For a more accurate approach see writer's discussion to "Design of Pressure Vessels for Low Cycle Fatigue," by B. F. Langer, JOURNAL OF BASIC ENGINEERING, TRANS. ASME, Series D, vol. 84, 1962, p. 389.

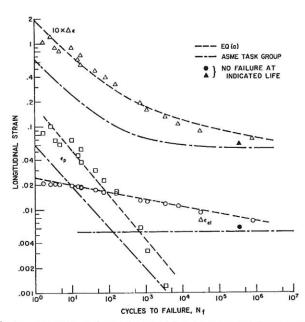


Fig. 14 Comparison of experimentally determined total strain range and components of elastic and plastic strain with predictions by approximate methods for 52100 steel

It is interesting to note that the plastic exponent z contains the term -0.52 (corresponding to -1/2 assumed as a universal value for all materials by the authors), but it also contains correction terms related to ductility, fracture stress, and ultimate tensile strength.

A comparison of the approximate relation described previously with that proposed in the report under discussion is presented in a forthcoming NASA report. Only brief mention of the results will be given here. Fig. 14 shows the comparison for SAE 52100 steel. In this case the correlation shows considerable favor for the method indicated here, in spite of the fact that no fatigue measurements are involved in the new method outlined; whereas the method of the subject report requires knowledge of an endurance limit. For other materials the discrepancy between the two methods was not so great. Figs. 15 and 16 show an overall comparison between the two methods. These curves show the ratios of the predicted strains to the measured strains for a number of materials. The predicted strains were obtained by the two methods using measured tensile properties. For purposes of computing the predicted life relationship according to the method of the authors' report under discussion, the endurance limit was taken as the extrapolated value at 10⁷ cycles. Thus the method of the subject report was given the benefit of knowledge of a good estimate of the endurance limit at 107 cycles, and should therefore correlate well at the high cyclic lives. The "measured" value of strain at each life was taken as the value on the best curve drawn through the experimental data. It can be seen that in general the method of the present discussion yields better correlation.

An alternate approach can be taken which retains some of the improved accuracy of the above analysis, while also retaining the extreme simplicity of the relation discussed in the authors' report. Since the main source of error in the low and intermediate cyclic life range arises from the assumption of the intercept of the plastic line (i.e., a value of D at N = 1/4), an improved relation can be obtained by choosing the intercept as $1/4D^{3/4}$ at 10 cycles, and retaining the assumption that the slope z is -1/2. Under this condition

$$\epsilon_p = 0.8D^{3/4} N^{-1/2} f \tag{10}$$

To obtain the final relation between life and total strain, the

Journal of Basic Engineering

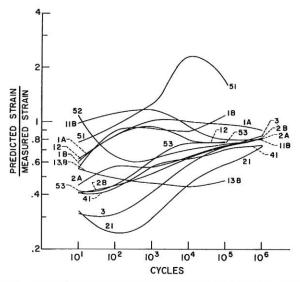


Fig. 15 Ratios of predicted total strain to measured total strain. Predictions based on method using tensile ductility and endurance limits as discussed by Coffin and Tavernelli. See Fig. 16 for material code.

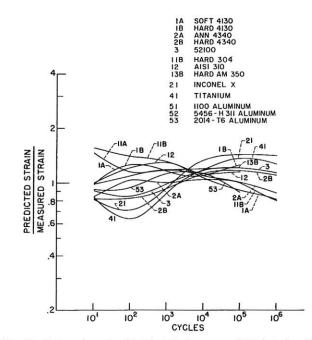


Fig. 16 Ratios of predicted total strain to measured total strain. Predictions based on method using Eq. (5).

elastic strain is assumed, as in the report to be a constant over the life range, that is $\Delta \epsilon_{\epsilon l} = \frac{2\sigma_{end}}{E}$. Thus the stress range becomes

$$\Delta \sigma = E \Delta \epsilon = 0.8 E D^{3/4} N_f^{-1/2} + 2\sigma_{end} \qquad (11)$$

Often, when preliminary estimates are being made, the value of σ_{end} might not be available. In those cases σ_{end} may be approximated by $\sigma_{end} = \frac{1}{3}\sigma_u$ and Equation (11) becomes

$$\Delta \sigma = E \Delta \epsilon = 0.8 E D^{3/4} N_f^{-1/2} + \frac{2\sigma_u}{3}$$
(12)

Improved accuracy in the elastic component of strain range can be obtained by assuming it to be variable with life rather than constant over the entire life range. According to Eq. (7) the most conservative value of slope is $\gamma = -0.08$. Hence, if the

DECEMBER 1962 / 539

endurance limit σ_{end} is specified at a life N_{erd} , a line of slope 0.08 is passed through this point in the plane of $\Delta \epsilon_{el}$ versus N on logarithmic coordinates. Thus

$$\epsilon_{el} = \frac{2\sigma_{end}}{E} \left(\frac{N_f}{N_{end}}\right)^{-.08} \tag{13}$$

Combining equations (10) and (13)

$$\Delta \sigma = E \Delta \epsilon = 0.8 E D^{3/4} N_f^{-1/2} + 2\sigma_{\text{end}} \left(\frac{N_f}{N_{\text{end}}}\right)^{-0.08}$$
(14)

If the endurance limit is not known then the ultimate tensile strength can be used to obtain a point on the elastic line. Under this condition the elastic strain is $\frac{0.90\sigma_u}{E}$ at 10⁵ cycles, but since the point is taken at the relatively short life of 10⁵ cycles, greater conservatism will be obtained at the higher lives by choosing a steeper slope. For an arbitrarily chosen value of $\gamma = -0.1$ (although another value may be optimum) the elastic component becomes

$$\Delta \epsilon_{el} = \frac{0.90\sigma_u}{E} \left(\frac{N_f}{10^5}\right)^{-0.1} = \frac{2.85\sigma_u}{E} N_f^{-0.1}$$
(15)

and the life equation becomes

$$\Delta \sigma = E \Delta \epsilon = 0.8 E D^{3/4} N_f^{-1/2} + 2.85 \sigma_u N_f^{-0.1}$$
(16)

Fig. 17 shows a comparison between predicted and measured strains for a number of materials in which the predictions are based on Eq. (16). Comparing Fig. 17 to 15 and 16, it can be seen that Eq. (16) represents a satisfactory approximation for estimating life. Use is made of only very commonly measured properties—ductility and ultimate tensile strength—whereas the equation discussed in the report requires knowledge of an endurance limit. If the latter is known, Eq. (14) will, in general, serve better to approximate the life relationship.

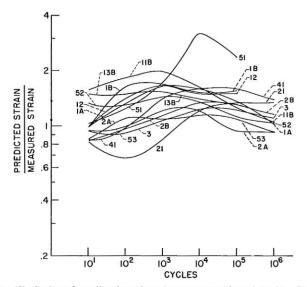


Fig. 17 Ratios of predicted total strain to measured total strain. Predictions based on Eq. (16). See Fig. 16 for material code.

Application to a Optimization of Material Properties

Closed-form equations such as discussed by the authors, or the alternates (11), (12), and (16) can be extremely useful in determining heat-treatments and/or cold work to optimize the condition of a material toward resisting a specified strain range. Usually such mechanical or thermal treatment can be used to increase strength (endurance or static ultimate) at the expense of reduc-

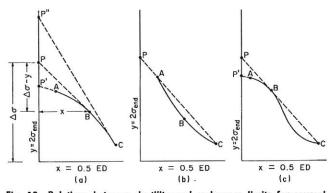


Fig. 18 Relations between ductility and endurance limits for several hypothetical materials as influenced by heat-treatment or cold work. Choice of optimum treatment for maximum fatigue life can be determined from such curves by simple geometric constructions.

tion in ductility. The optimum compromise between strength and ductility can be obtained by differentiation. For example, using the equation discussed in the report

$$\Delta \sigma = E \Delta \epsilon = \frac{ED}{2} N_f^{-1/2} + 2\sigma_{\text{end}} \qquad (17)$$

If a value of $\Delta \epsilon$ (or $\Delta \sigma$) is specified, the problem becomes that of selecting the optimum combination of D and σ_{end} to make N_f a maximum for the specified value of $\Delta \sigma$. Thus, calling ED/2= x, and $2\sigma_{end} = y$ for convenience, and differentiating with respect to x

$$\frac{d\Delta\sigma}{dx} = N_f^{-1/2} - \left(\frac{x}{2}N_f^{-1/2}\right)\frac{dN_f}{dx} + \frac{dy}{dx}$$
(18)

But $\frac{d\Delta\sigma}{dx} = 0$ since $\Delta\sigma$ is a fixed number, and $\frac{dN_f}{dx} = 0$ since we are seeking a maximum value of N_f . Thus $dy/dx = -N_f^{-1/2}$, which can be substituted into Eq. (17) to get

$$-\frac{dy}{dx} = \frac{\Delta\sigma y}{x} \tag{19}$$

The graphical significance of Eq. (19) is shown in Fig. 18(a). If ABC is the plot of corresponding values of endurance limit and ductility achievable by varying the heat-treatment, and the applied nominal stress $\Delta \sigma$ is located at point P on the vertical axis, the optimum heat-treatment will be at point B obtained by drawing a tangent from P to ABC. For any stress less than that at P'(the intersection with the vertical axis of the tangent drawn at the end point A), the optimum condition is at A; for any stress greater than that at P'' (the intersection with the vertical axis of the tangent at the end point C), the optimum condition is at C. If the curve ABC is convex, as in Fig. 18(b), Eq. (19) defines a minimum in life rather than a maximum, and therefore does not serve to identify optimum treatments. It can readily be shown that for nominal stresses greater than that at P (the intersection of the secant AB with the vertical axis) the optimum condition for the material is at C, while for nominal stresses less than that at P the optimum condition is at A. If the curve has both convex and concave regions, as in Fig. 18(c), the tangent BC is first drawn from the end point C, intersecting the vertical axis at P. The tangent at A intersects the vertical axis at P'. For stresses below P', the condition at A is optimum. Between P' and P the optimum condition is obtained by constructing a tangent, as in Fig. 17(a). For nominal stresses greater than that at P, the optimum condition is at C.

While the above analysis is based on the life Equation (17), similar analyses are possible based on Equations (11), (12), (14), or (16). In all cases the abscissa x = 0.5ED is replaced by x = 0.8ED^{1/4}. The ordinate $y = 2\sigma_{end}$ applies to Equation (11) but is replaced by $y = 2\sigma_u/3$ for Eq. (12), $2\sigma_{end} (N_{end})^{0.08}$ for Eq. (14), and $y = 2.85\sigma_u$ for Eq. (16). For Eqs. (11) and (12) the analysis is identical to that described, except for the change in the coordinates. For Eqs. (14) and (16), however, the analysis becomes more complicated. It can be shown, however, that if the curve is concave, as in Fig. 17(a), the optimum treatment for a given applied stress can still be obtained, but in a slightly more indirect way. An arbitrary point B is chosen, and a tangent constructed, which intersects the vertical axis at P. The distance OP then becomes $\Delta \sigma [-dy/dx]^{\alpha/(0.5+\alpha)}$ where dy/dx is the slope at point B, $\Delta \sigma$ the nominal stress range for which the condition at point B is optimum, and α is the exponent associated with the elastic part of the stress range equations, being equal to -0.08 for Eq. (14) and -0.1 for Eq. (16). It is thus possible to make a plot of the optimum combinations of strength and ductility versus applied "nominal" stress.

Additional References

12 S. S. Manson, "Behavior of Materials Under Conditions of Thermal Stress" Heat Transfer Symposia, Univ. of Michigan Engrg. Res. Inst., 1953, pp. 9-75. Also published as NACA TN 2933, 1954.
I. F. Coffin, Jr., "A Study of Cyclic-Thermal Stresses in a Ductile Metal," TRANS. ASME, vol. 76, 1954, pp. 931-950.
S. S. Manson, "Thermal Stresses in Design-Part 19: Cyclic

Life of Ductile Materials," Machine Design, July 7, 1960, p. 139.

Authors' Closure

The authors appreciate Mr. Manson's extended discussion to their paper and his willingness to report some of his own original work here. We, as well as others interested in this subject matter, are aided invaluably by his searching examination of the paper. Although our position has not changed as a consequence of the points raised, nevertheless, Manson's approach to the problem unquestionably will find many supporters.

Summarizing briefly Mr. Manson's position as we see it, he first takes issue with three assumptions contained in the derivation of Equation (3) of our paper. These are:

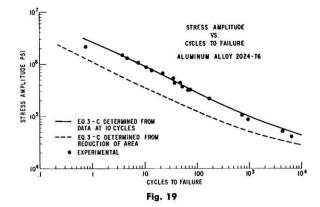
(a) The use of the true strain at fracture for the plastic strain range $\Delta \epsilon_n$ at $N = \frac{1}{4}$ cycles;

(b) the use of -1/2 as the exponent of N in Equation (3);

(c) the use of the strain range associated with the endurance limit as the elastic strain range application for failure in a relatively few cycles.

In place of these assumptions he proposes his own method [represented analytically by Equation (5)] which requires the determination of four constants based on observation of the behavior of the cyclic-strain fatigue and tensile data of several materials tested in his laboratory.

With respect to Manson's criticisms of assumptions employed in Equation (3), the use of the fracture ductility in that equation is indeed somewhat arbitrary. However, as indicated in the paper, the assumption has been demonstrated to be conservative [2], a fact considered to be advantageous to those interested in the application of the equation to design. The disparity between experiment and theory in the Figs. 1-12 is largely traceable to this approximation. As indicated in Table II, reference [2], aluminum alloys such as 24ST (2024-T6) generally give the poorest agreement when comparing extrapolated plastic strain range-cycles to failure data to fracture ductility at one-quarter cycle. Other metals give much better agreement when such extrapolations are made, as reference [2] also indicates, and these will give a much better fit to Equation (3). Fig. 1 for commercially pure aluminum is one such case. Mr. Manson has not particularly improved the situation by his ductility modification of Equation (10) since the basic difficulty regarding the general use of fracture ductility for cyclic strain application is not re-



moved. Rather he has eliminated the conservative features of the procedure given by Equation (3). Equation (3) can be shown to produce a very close fit to experimental data for cases where the ductility is not in good agreement with extrapolated cyclicstrain data at 1/4 cycle by identifying the constant C with cyclic-strain data. For example, Fig. 19 shows the kind of agreement possible for Equation (3) when C is determined from the plastic strain-range required to produce failure in ten cycles. Although agreement is good, one is now required to make a low cycle fatigue test.

With respect to Manson's second objection, that of using -1/2as the exponent of N, it can be said that experimental data strongly supports the use of $-\frac{1}{2}$ providing no time-temperature dependent effects operate to influence the results. When creep effects appear, however, the exponent is dependent on the frequency of cycling.6

The authors concur that somewhat better agreement with experiment would result if elastic effects were accounted for by a cyclic dependent term rather than a cyclic independent one as used in Equation (3). As pointed out in the discussion, the error involved in the simplification is generally small and the additional complications of reducing this error along the lines suggested by Mr. Manson do not appear warranted, except perhaps for materials of low ductility. Now with respect to Mr. Manson's method for predicting low cycle fatigue behavior the results as indicated in Fig. 16 appear to be quite impressive. However, one is somewhat disturbed by the high degree of empiricism involved in the selection of the constants, and the obscure physical significance in the determination. On the other hand, use is made of the interesting, but not new, observation that a 1 percent total strain range produces failure in 10,000 cycles for a wide variety of metals. This may be as useful as the assumption used in our paper, that the fracture ductility fits the low cycle fatigue curves at 1/4 cycle. It could, in fact, serve as an alternative means for determining C in Equation (3).

There is also concern that the empirically determined constants have been derived specifically for highly alloyed or heavily cold worked metals and alloys such as those indicated in Fig. 16. In such cases the constant 2C does not agree too well with fracture ductility at N = 1/4. Thus use of the quantity $1/4D^3/4$ (where D is the logarithmic ductility) at 10 cycles may be considerably in error for the more ductile metals where the agreement between 2C and the fracture ductility is close. It would be interesting to see the results of the method for such metals as annealed 304 or 347 stainless steel, 1100 aluminum (not included in Fig. 16), mild steel, copper, etc.

The authors await with interest publication of the experimental data and the detailed analysis of these data which lead to the results described in Mr. Manson's discussion.

⁶ L. F. Coffin, Jr., "Low Cycle Fatigue-A Review," to be published in Materials Research.