under heteroscedasticity because it implicitly assumes that the true residuals all have the same distribution. We can assume instead that the distributions are different, say H_i for the ith residual and estimate each H_i in some way. Professor Wu's suggestion amounts to estimating the joint distribution of the residuals by a distribution having marginal mean 0 and variance $r_i^2/(1 - w_i)$. There are two problems with his suggestion: a) The resampled residuals are uncorrelated but not necessarily independent, as they should be, and more importantly, b) only the first two moments of this distribution are specified, so there is no hope of capturing higher-order effects. A method that estimated each H_i with the empirical distribution function of the residuals in some neighborhood of the ith point might hold more promise.

Acknowledgment. I would like to thank Brad Efron for suggesting mean squared error as a relevant measure.

REFERENCES

EFRON, B. (1979). Bootstrap methods: Another look at the jackknife. *Ann. Statist.* **7** 1–26.

EFRON, B. (1985). Bootstrap confidence intervals for a class of parametric problems. *Biometrika* **72** 45–58.

EFRON, B. (1986). Better bootstrap confidence intervals. To appear in *J. Amer. Statist. Assoc.*

DEPARTMENT OF PREVENTIVE MEDICINE
AND BIOSTATISTICS AND DEPARTMENT OF STATISTICS
UNIVERSITY OF TORONTO
TORONTO, ONTARIO M5S 1A8
CANADA

NEVILLE WEBER

University of Sydney

Wu's paper should be praised for clarifying the relationship between the various weighted and unweighted versions of the jackknife and the bootstrap and for giving simple examples to demonstrate the shortcomings of various methods. The paper also stresses the role of the jackknife in regression analysis as a means of obtaining an estimator for the covariance matrix of the least squares estimator, $\tilde{\beta}$, which is robust against heteroscedastic errors. This feature of the jackknife has not received enough emphasis in regression literature.

As noted by Weber (1986), Hinkley's weighted jackknife variance estimator, $V_{H(1)}$, is effectively the robust estimator proposed by White (1980), commonly used in econometrics. The new estimator $V_{J(1)}$ has the consistency property of $V_{H(1)}$, but also has the advantage of being unbiased when the model has homoscedastic errors.

The bootstrap procedure in regression does not lead to a robust, consistent estimator of the covariance matrix of $\hat{\beta}$. The bootstrap method based on resampling the residuals has the obvious shortcoming of imposing a linear model structure on the resampled values, forcing the error terms to be independent and identically distributed. Such a procedure loses any variation in the distributions of the original error terms, e_i. The procedure suggested in Section 7 is an attempt to ensure that the error terms associated with x_i in the bootstrap samples capture some of the possible dependence on x_i. This approach leads to the "bootstrap" estimator for the covariance of $\tilde{\beta}$ being $V_{J(1)}$, and so this resampling method is as effective as the weighted jackknife in this regard. The potential advantage the bootstrap procedure has over the jackknife in general is in approximating the distribution of $(\hat{\beta} - \beta)$. In the case where the e_i's are independent and identically distributed the procedure suggested in Section 7 forces some arbitrary distribution on y_i^* through t_i^*, and the actual distribution of the e_i's is lost. Thus, the usefulness of this approach in generating confidence intervals by approximating the distribution of $(\hat{\beta} - \beta)$ is in question. Perhaps the jackknife–bootstrap hybrid is the answer to this problem and this model certainly deserves more investigation.

The bootstrap percentile method for calculating confidence intervals in regression has been investigated by Robinson (1985). He compared the bootstrap approach to the exact confidence intervals obtained by inverting permutation tests and suggested an adjustment to the bootstrap percentile method to improve its coverage probability.

The use of t-confidence intervals in the simulation study for the parameter $\theta = -\beta_1/2\beta_2$ should give moderate results with normal errors and β_2 away from 0. Weber and Welsh (1983) found that the standardised distribution of the jackknife statistic for θ can be very skewed and so one would not expect the symmetric t-confidence interval to give reasonable coverage in general. The adjusted percentile methods appear to be the appropriate way of obtaining nonparametric confidence intervals for θ.

REFERENCES

ROBINSON, J. (1985). Nonparametric confidence intervals in regression—the bootstrap and randomization methods. In *New Perspectives in Theoretical and Applied Statistics* (M. Puri, J. P. Vilaplana and W. Wertz, eds.). Wiley, New York.

WEBER, N. C. (1986). The jackknife and heteroskedasticity—consistent variance estimation for regression models. *Econom. Lett.* **20** 161–163.

WEBER, N. C. and WELSH, A. H. (1983). Jackknifing the general linear model. *Austral. J. Statist.* **25** 425–436.

WHITE, H. (1980). A heteroskedasticity—consistent covariance matrix estimator and a direct test for heteroskedasticity. *Econometrica* **48** 817–838.

DEPARTMENT OF MATHEMATICAL STATISTICS
UNIVERSITY OF SYDNEY
SYDNEY, N.S.W. 2006
AUSTRALIA

H. P. WYNN AND S. M. OGBONMWAN

The City University and University of Benin

Along with commenting on this authoritative paper, we wish to make a plea for an approach to the computational problems of resampling and simulation