Discussion of Particle Markov chain Monte Carlo methods by Christophe Andrieu, Arnaud Doucet, Roman Holenstein

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We would like to thank the authors for a very interesting paper. Consider a d-dimensional diffusion processes X_t governed by the stochastic differential equation (SDE)

$$dX_t = \alpha(X_t, \theta)\, dt + \sqrt{\beta(X_t, \theta)}\, dW_t$$

where W_t is standard Brownian motion. It is common to work with the Euler-Maruyama approximation with transition density $f_\theta(\cdot|x)$ such that

$$(X_{t+\Delta t}|X_t = x) \sim \mathrm{N}\left(x + \alpha(x, \theta)\Delta t\,,\ \beta(x, \theta)\Delta t\right).$$

For low frequency data, the observed data can be augmented by adding $m - 1$ latent values between every pair of observations. For observations on a regular grid, $y_{1:T} = (y_1, \ldots, y_T)'$ that are conditionally independent given $\{X_t\}$ and have marginal probability density $g_\theta(y|x)$, inferences are made via the posterior distribution $\theta, x_{1:T}|y_{1:T}$ using Bayesian MCMC techniques. Due to high dependence between $x_{1:T}$ and θ, care must be taken in the design of an MCMC scheme. A joint update of θ and $x_{1:T}$ or a carefully chosen reparameterisation (Golightly & Wilkinson 2008) can overcome the problem. The PMMH algorithm described in the paper allows a joint update of parameters and latent data. Given a proposed θ^*, the algorithm can be implemented by running an SMC algorithm targeting $p(x_{1:T}|y_{1:T}, \theta^*)$ using only the ability to forward simulate from the Euler-Maruyama approximation.

To compare the performance of the PMMH scheme with the method of Golightly & Wilkinson (2008) (henceforth referred to as the GW scheme), consider inference for an SDE governing $X_t = (X_{1,t}, X_{2,t})'$ with

$$\alpha(X_t, \theta) = \begin{pmatrix} \theta_1 X_{1,t} - \theta_2 X_{1,t} X_{2,t} \\ \theta_2 X_{1,t} X_{2,t} - \theta_3 X_{2,t} \end{pmatrix},\ \beta(X_t, \Theta) = \begin{pmatrix} \theta_1 X_{1,t} + \theta_2 X_{1,t} X_{2,t} & -\theta_2 X_{1,t} X_{2,t} \\ -\theta_2 X_{1,t} X_{2,t} & \theta_2 X_{1,t} X_{2,t} + \theta_3 X_{2,t} \end{pmatrix}.$$

This is the diffusion approximation of the stochastic Lotka-Volterra model (Boys, Wilkinson & Kirkwood 2008). We analyse a simulated dataset of size 50 with $\theta = (0.5, 0.0025, 0.3)$, corrupted by adding a zero mean Gaussian noise. Independent Uniform $U(-7, 2)$ priors were taken for each $\log(\theta_i)$. The GW scheme and the PMMH sampler were implemented for 500,000 iterations, using a random walk update with Normal innovations to propose $\log(\theta^*)$, with the variance of the proposal being the estimated variance of the target distribution, obtained from a preliminary run. The PMMH scheme was run for $N = 200$, $N = 500$ and $N = 1000$ particles and in all cases, discretisation was set by taking $m = 5$.

Computational cost scales roughly as $1 : 8 : 20 : 40$ for GW : PMMH ($N = 200 : 500 : 1000$). For $N = 1000$ particles, the mixing of the chain under the PMMH scheme is comparable to the GW scheme; see Figure 1. Despite the extra computational cost of

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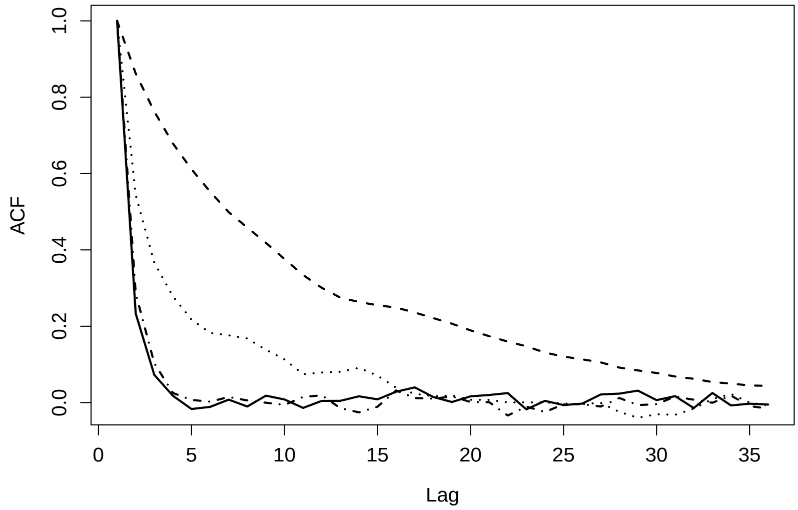

Figure 1: ACF of θ_1 from the output of the GW scheme (solid line) and PMMH schmes with $N = 200$ (dashed line), $N = 500$ (dotted line) and $N = 1000$ (dot-dashed line).

the PMMH scheme, unlike the GW scheme the PMMH algorithm is easy to implement and requires only the ability to forwards simulate from the model. This extends the utility of particle Markov chain Monte Carlo to a very wide class of models where evaluation of the likelihood is difficult (or even intractable), but forward simulation is possible.

References

Boys, R. J., Wilkinson, D. J. & Kirkwood, T. B. L. (2008), 'Bayesian inference for a discretely observed stochastic-kinetic model', *Statistics and Computing* **18**, 125–135.

Golightly, A. & Wilkinson, D. J. (2008), 'Bayesian inference for nonlinear multivariate diffusion models observed with error', *Computational Statistics & Data Analysis* **52**, 1674–1693.