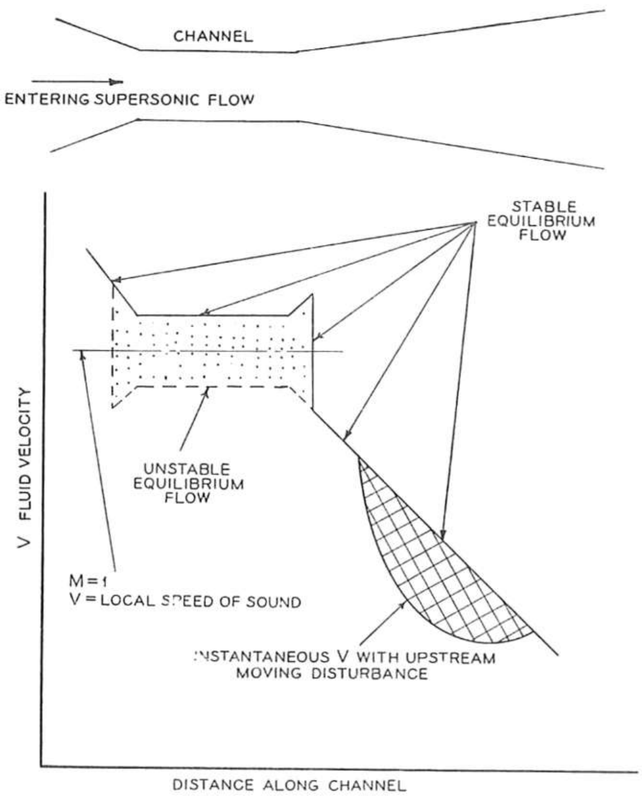

Fig. 1 Diagrammatic Representation of Long-Throat Supersonic Diffuser

from the rear of the channel. If the effect of this disturbance is to move the shock to a position upstream from the unstable-equilibrium shock position, then the shock will move out ahead of the diffuser and the starting procedure will have to be repeated. If, on the other hand, the disturbance is not sufficiently strong to move the shock as far as the upstream (unstable) equilibrium position, then it is shown in the reference that the shock will return to its original stable-equilibrium position. It is also shown in this report that the minimum disturbance necessary to move the shock just to the unstable-equilibrium position is obtained roughly when the "area of the disturbance" on a velocity-length plot (the cross-hatched area in Fig. 1), is equal to the dotted area on the same figure.

It will be seen that the use of the long throat increases the dotted area, and thus increases the maximum disturbance level, which may be tolerated in the diffuser without forcing the shock to jump out upstream. Alternatively, it is possible to increase the back pressure or the efficiency of the diffuser, thus reducing the strength of the equilibrium shock, without reducing the dotted area, when a long throat is used, and the disturbance level is maintained constant. This latter situation is apparently the case in the experiments reported in the subject paper and explains the increase in efficiency observed with the use of long throats.

It is clear that this one-dimensional frictionless theory cannot give the whole story of the shock motions in the diffuser, and this apparently is illustrated in the paper by the authors' observations that the shock position is stable in constant-area channels, whereas this theory predicts only neutral stability for this case. Thus it may be that, for a considerable part of the time, the shock would be found in the long throat of these diffusers and that the mixing mechanism postulated by the authors might be responsible for a part of the observed improvements in efficiency.

However, the authors observed the occurrence of "unstable points" of higher efficiency only with the long-throat diffuser. We take this to mean that they observed more efficient shock positions, which could be obtained only for limited periods of time, after which a sufficiently large disturbance would come along and force the shock past the throat and the unstable-equilibruim position, and so necessitate another starting procedure. Such unstable points have been observed previously in other investigations of supersonic diffusers but usually the difference between maximum unstable and maximum stable efficiencies was quite small. However, in the long-throat diffusers, this effect is emphasized because of the ability of these diffusers to resist relatively large disturbances and so to be disturbed only by comparatively rare occurrences. It is thus seen that this phenomenon finds explanation only from the disturbance point of view and not from the mixing point of view presented by the authors.

Authors' Closure

The idealized analysis presented by Professor Kantrowitz probably helps to explain some of the phenomena encountered in diffuser instability. However, there is some question whether or not any explanation of the phenomena Professor Kantrowitz refers to can be defended completely on the basis of either boundary-layer effects or an analysis basis on one-dimensional unsteady frictionless-flow theory.

The "mixing point of view" that Professor Kantrowitz attributes to the authors appears to be of his own making. It probably comes from the statement by the authors that "once separation occurred, the stream did not again fill the passage for a distance equal to 8 to 12 diameters of the tube." This statement was intended to describe the observations that normal shocks were usually not present in the straight section but that a separation of the stream from the tube walls usually occurred in their stead. This observation was made through a series of high-speed schlieren photographs similar to those contained in the subject paper. It was occasionally observed that the stream would sometimes detach itself from only one of the walls of the rectangular passageway under observation.

Stability of Linear Oscillating Systems With Constant Time Lag[1]

H. Poritsky.[2] Fashions come and go even in technical sciences. Nowadays it is customary to treat every problem by means of Laplace's transforms, and no author feels that he has done his subject justice without them. It is worth pointing out, therefore, that the stability problem in question can be handled directly without Laplace transforms.

Assuming that the linear difference-differential equation of the free system has exponential solutions $e^{\lambda t}$, it is found on substituion that λ must satisfy a certain transcendental equation. The similarity between ordinary linear systems, resulting in ordinary differential equations with constant coefficients, and the linear systems with a constant time lag, considered by the author, lies in the fact that in each case the solution of the homogeneous system, corresponding to free oscillations, can be obtained by means of exponentials. The difference lies in the fact that in ordinary systems one is led to an algebraic equation for the solution of these exponentials, while in a system with a constant time lag, one is led to a proper transcendental equation. The stability of the free system thus leads to the location of roots λ in the negative-real half of the complex plane.

[1] By H. I. Ansoff, published in the June, 1949, issue of the Journal of Applied Mechanics, Trans. ASME, vol. 71, pp. 158–164.

[2] Engineering General Division, General Electric Company, Schenectady, N. Y. Mem. ASME.

Where more intimate analysis, possibly requiring the introduction of the Laplace or Fourier integral, is needed, is in showing that the general vibration of the system is expressible in terms of the exponential solutions, and in treating systems with applied forces.

M. Satche.[3] Mathematically, the question is to find out the conditions to be fulfilled so that the roots of the characteristic equation

$$p^2I + pR + Spe^{-p\tau} + K = 0 \qquad [1]$$

have negative real parts.

On account of the $e^{-p\tau}$ term, this equation is not an algebraic one, and does not allow using the Routh-Hurwith criterion.

However, it is possible to extend the use of the Nyquist criterion in a simple manner.

Generally speaking any equation may be written in the form

$$f(p) - g(p) = 0 \qquad [2]$$

The Cauchy-Rouche theorem states that, for a Bromwich contour followed by p

$$\text{Variation of argument } [f(p) - g(p)] = 2\pi(N - P) \qquad [3]$$

N being the number of zeroes, and P the number of poles lying inside the contour.

Assuming $P = 0$, if we want $N = 0$, the variation of the argument must be nought.

If we draw the diagrams of $f(p)$ and $g(p)$ when p follows the Bromwich contour in the p-plane, $f(p) - g(p)$ argument is the angle α (see Fig. 1, herewith), between the vector $M_1 M_2$, joining corresponding points of the diagrams and the X-axis.

It is easy to compute the total variation of this angle when the

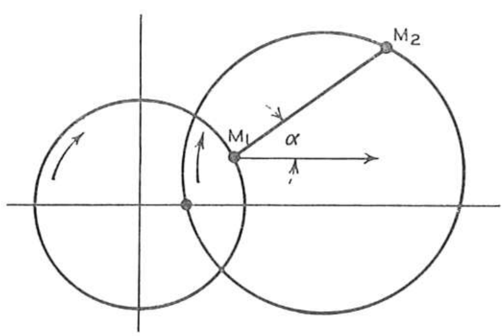

Fig. 1

two diagrams are simple curves as straight lines, circles, parabolas, etc., and then to state it equal zero.

The Nyquist criterion corresponds to the case when one of these diagrams is reduced to a simple point.

Application. Let us write Equation [1] of this discussion in the following form

$$\left(\frac{R}{S} + p\,\frac{I}{S} + \frac{K}{pS}\right) - (-e^{-p\tau}) = 0$$

Then

$$f(p) = \frac{R}{S} + p\,\frac{I}{S} + \frac{K}{pS}$$

$$g(p) = -e^{-p\tau}$$

[3] Ingenieur à la Société Alsthom, 38 avenue Kléber, Paris, France.

As we have added the pole $p = 0$, the Bromwich contour must not enclose the point 0, Fig. 2 of this discussion.

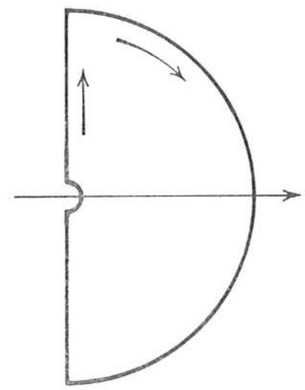

Fig. 2

When p follows this contour, $f(p)$ is a straight line turning into a semicircle on the right-hand side, $g(p)$ is a circle centered on the origin whose radius is unity, Fig. 3, herewith.

We must work out two different cases as follows:

$R > S$ the two diagrams are quite apart; the system is stable.

$R < S$ the system is stable if M_1 is placed on the left-hand side of the straight line AB, when M_2 lies between A and B.

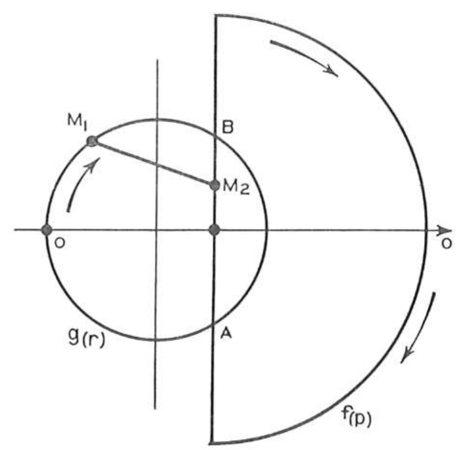

Fig. 3

The parameters ω_A and ω_B of the points A and B on the $f(p)$ diagram are

$$\omega_A = \omega_0\left(\sqrt{m^2 + 1} - m\right)$$

$$\omega_B = \omega_0\left(\sqrt{m^2 + 1} + m\right)$$

with

$$\omega_0 = \sqrt{\frac{K}{I}} \qquad 2m = \sqrt{\frac{S^2 - R^2}{K^2}}$$

We must then have

$$\cos \omega\tau > -\frac{R}{S}$$

when ω answers the following inequality

$$\omega_0\left(\sqrt{m^2 + 1} - m\right) < \omega < \omega_0\left(\sqrt{m^2 + 1} + m\right)$$

Thence, practically, when m is not too small the only condition to be fulfilled is

$$\cos \left[\tau\omega_0 \left(\sqrt{m^2 + 1} + m\right)\right] > -\frac{R}{S}$$

It will be observed that this method may be easily extended to the other cases handled by the author.

Author's Closure

The author hastens to agree with Dr. Poritsky's claim that the substitution method is equally adequate in arriving at the characteristic equation. However, having been brought up in a generation of engineers to whom the use of transforms is a direct and simple method of solving linear differential equations, he fails to see why the Laplace transform should be reserved for more "intimate" analysis. The question is, perhaps, of esthetic nature and the answer depends on whether it is better taste to use older less streamlined tools on simpler problems in order to avoid giving an impression of pseudosophistication.

Dr. Satche's solution represents, in the author's opinion, a very elegant approach to the problem.

Stress Concentration Around a Triaxial Ellipsoidal Cavity[1]

A. Nadai.[2] The analysis presented in this paper must be considered as one of the outstanding advances of recent years in the theory of elasticity, hence the authors should be commended highly for the precise mathematical attack on a classical problem which had not yet been solved in the mathematical theory of elasticity. There is a group of nice problems in this theory which might be solved by making use of certain functions with which only the professional mathematicians are acquainted. Since the authors have made use of the elliptic functions of Jacobi, perhaps it may be worthy of mention that there is another group of questions in the theory of elasticity which might be attacked by making use of the theta functions. The writer has in mind the problem of an extended plate, bent by uniform pressure, resting on columns whose centers coincide with the intersection points of two families of equidistant parallel lines perpendicular to each other. It is obvious that this case introduces double periodic functions. In the writer's experiences some excellent use could be made of the theta functions for expressing precisely the deflected middle surface of the plate.

Engineers not being at all familiar with the functions introduced by the authors may agree with the writer that it would be of advantage for the authors to include a section in their most valuable analysis, devoted to a brief review of a few important properties of the three elliptic functions of which they made such excellent use in their paper. This would enable a larger group of engineers to make a more profitable study of the authors' exact analysis. The suggestion is merely submitted for possible consideration by the authors.

W. P. Roop.[3] Although these calculations are valid only within elastic limits, yet by suggestion and analogy they indicate something about the conditions of fracture, at least in the brittle mode. It was with this in mind as a partial objective that the work was first planned. It was thought at that time that notched geometries favored brittleness through increase in triaxiality by reducing the intensity of shearing action in its proportion to primary tension. The importance of the lowest of the three principal stresses was recognized. When compressive components of stress are absent, a simplified measure of triaxiality has sometimes been taken to be the ratio of the smallest to the greatest of the three principal stresses, although a better measure of the force causing plastic flow is the octahedral shearing stress.

The data which have been presented in this paper indicate that the ratio of the lowest to the highest among the principal stresses in the cases investigated did not rise above, say, $1/5$ or $1/4$. In such a ratio there seems to be little encouragement to brittleness.

It has been supposed that if a fracture stress limit existed at a level only a little above that of the flow stress, even a small increase in triaxiality might tip the scales in favor of brittleness. However, this only leads us into arguments, which seem to have brought us nowhere, about the influence of triaxiality on fracture stress. A strong opinion exists to the effect that the concept of fracture stress is better abandoned altogether.[4]

It was indicated that the authors were surprised to find the greatest concentration in tangential stress at a point on the surface of the cavity where the curvature in a meridional section of that surface was a little smaller (radius larger) than at other points. This does indeed seem anomalous. Since this excess in concentration is not great, however, and since in actual cavities all the concentrations at points of high notch acuity are modified by plastic flow at moderate load levels, it seems that the judgment of the authors was sound when they contented themselves with verifying their calculated results. It is possible, is it not, that the differences are small enough to lie within the limits of precision of the numerical work?

The importance of the paper lies, not in the precision of the numerical values of the concentrations, but in the approximate evaluation of the third principal stress. For this the precision of the work was ample to establish the main result, as follows:

In the cases studied, triaxiality remained moderate in value, nowhere even distantly approaching full equality of the three principal stresses. This applies to a block of metal of infinite extent in all three dimensions, and therefore, a fortiori, to an infinite plate of finite thickness.

If brittleness in a plate is to be explained by triaxiality, therefore, it can only be through some auxiliary hypothesis to account for the great effect of low levels of triaxiality.

This is a notable achievement. Fortunately for them, it leaves the speculators quite free in devising suitable auxiliary hypotheses.

Authors' Closure

The authors greatly appreciate Dr. Nadai's comments and suggestions. Unfortunately, the inclusion in the paper of an expository section, dealing with elliptic functions, did not appear compatible with the current space limitations of the Journal of Applied Mechanics.

In connection with Captain Roop's observations regarding the significance of the results from the viewpoint of failure considerations, reference is made to Prof. L. H. Donnell's discussion[5] which the present discusser evidently has in mind. In this context, conjectures as to the role of triaxiality in embrittlement, which rest solely upon the "small" triaxiality of the prevailing stress distribution without taking into account the characteristic properties of the material, would seem unwarranted.

The authors cannot share Captain Roop's expectation that the

[1] By M. A. Sadowsky and E. Sternberg, published in the June, 1949, issue of the Journal of Applied Mechanics, Trans. ASME, vol. 71, pp. 149–157.

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[3] Structural Research Laboratory, Swarthmore College, Swarthmore, Pa.

[4] "Fracturing of Metals," American Society for Metals, 1948, p. 29.

[5] Journal of Applied Mechanics, Trans. ASME, vol. 70, 1948, pp. 87–88.