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Time-Integral Variational Principles for Nonlinear Nonholonomic Systems[1]

B. Ravindra[2].

1 Introduction

The author presents a concise account of the time-integral variational principles for nonholonomic systems and also gives a geometrical interpretation of the discrepancies found between the equations of motion obtained using the variational approach and those derived from the d'Alembert-Lagrange principle. He rightly points out the lack of appreciation of these nuances in the engineering community, as has also been observed recently by Hagedorn (1997). The author follows Hertz (1894) in distinguishing the straightest (mechanical) and the shortest (variational equations for the geodesics or "mathematical" as termed by the author) and arrives at the well-known conclusion that the lack of commutativity of the variational and integral operators is the source of the discrepancy. While the author rightly emphasizes the significance of the geometric view point, some of the exciting, recent developments on the new geometry arising out of the problems of "nonholonomic" mechanics and related connections to control theory are not mentioned. Since these aspects are of considerable theoretical and practical value to the practitioners of mechanics, a brief description with relevant references is provided here.

Riemann's Habilitationsvortrag in 1854 was a watershed in the history of geometry. Since then, deep relations between geometry and mechanics have been discovered by Levi-Civita, Ricci, Weyl, among others. The problems of nonholonomic mechanics also served as a source of inspiration for the creation of a new language and new geometric objects. A simple example of nonholonomic structure is the contact structure. It is interesting to note that this arose from the work of Gibbs on his graphical methods in thermodynamics of fluids (1873). However, it is the pioneering work of Caratheodory in 1909 on foundations of thermodynamics that provided the necessary impetus for further work. In his analysis of second law of thermodynamics and entropy, Caratheodory needed a theorem that any two points on a contact manifold may be connected by an admissible curve. An extension of Caratheodory's work is due to Chow (1939) and Rashevsky (1938). As pointed by Hermann (1963), this theorem, often referred to as Chow theorem, provides the differential geometric picture of the accessibility problem in control theory. It is a remarkable fact that, though Caratheodory's theorem has a kinematic nature, it can be used to define a nonholonomic metric known as Carnot-Caratheodory metric (Gromov, 1995). The geodesic problems arising out of such a metric have many interesting features (Montgomery, 1994). The geodesics, which do not satisfy the usual geodesic equations are termed as the "abnormal minimizers" and there exists a natural interpretation of these via Pontrjagin's maximum principle. It may be noted that one already encounters singular extremals in Caratheodory's calculus of variations. The history of nonholonomic mechanics and associated controversies reminds one the need for a suitable language and notation. It appears that E. Cartan has already advocated this in 1935. As Laplace said, "Such is the advantage of a well-constructed language that its simplified notation often becomes the source of profound theories." The controversy between Leibniz and Newton over the invention of calculus is well known. Historians point out that despite the claims of priority, the notation adopted by Leibniz was superior and this fact played a major role in the advancement of mathematics in the Continent as opposed to England (Babbage, 1830). Nonholonomic geometry might provide a suitable notation and language to resolve the controversies in nonholonomic mechanics.

It is indeed remarkable that Hertz already cited the theorem due to Caratheodory without proof (Hertz, 1894). Hertz's *Principles of Mechanics* was written with the idea of eliminating force as a fundamental concept in Mechanics and with a program of giving an interpretation of Ether in terms of "hidden motions" and "hidden masses." While this book was superseded by the development of relativity and quantum mechanics, Hertz's own introduction to his mechanics is philosophically profound and influenced Boltzmann and Wittgenstein. In this book, Hertz clearly mentioned the possibility of using higher dimensional geometry in mechanics and introduced a clear distinction between holonomic and nonholonomic constraints. He was also aware of the fact that application of Hamilton's principle to nonintegrable equations is problematic, in particular he mentioned that when three-dimensional bodies roll on one another without slipping, nonintegrable equations necessarily occur. He wrote "For rolling without slipping does not contradict either the principle of energy or any other generally accepted law of physics. The process is one which is so nearly realised in the visible world that even integration machines are constructed on the assumption that it strictly takes place." While this reference to integrating machines is quite curious, one finds an echo in Roger Brockett's foreword to a book on dynamics and control of multibody systems (Marsden et al., 1989):

"In my case, I had been fascinated for some time with the idea of nonholonomic constraints, partly because of the role they play in the planar integration scheme found in Vannevar Bush's differential analyzer and partly because they seemed mysterious. As I attempted to find a comfortable way of thinking about them, and the related matter of the Caratheodory statement of the second law of thermodynamics, I eventually realized that there was a wide variety of ways in which geometrical ideas could be helpful in forming a unified view of control."

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[2] Institute fur Mechanik II, Darmstadt University of Technology, Darmstadt, D-64289, Germany.

It is hoped that the control theoretic perspective sheds light on the pathological aspects surrounding the variational problems of the nonholonomic systems.

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Author's Closure[3]

I would like to thank B. Ravindra for his scholarly and constructive comments. The reasons that I did not include the modern geometrical developments he is referring to are that, first, for all that one would need a monograph or an extensive review article (à la ASME *Applied Mechanics Reviews*), not a modest size paper like the one discussed; and, second, I am not that well familiar with those developments so as to be able to present them in an accessible fashion. My impression is that, outside of some mathematicians and control theorists, most other engineers are quite uncomfortable with the mathematical jargon currently used to express them; i.e., the kind utilized by such authors as (alphabetically): Arnold, Cartan, Gallissot, Griffiths, Hermann, Lewis, Marsden, Ratiu, Simo, Smale, et al. All these formalisms, in addition to constituting an intimidating "moat" that prevents the rest of us from getting enlightened and participating on those modern developments, they also seem to constitute an expensive and low-yield investment of effort and time. My sincere hope is that someone more knowledgeable and competent than I would present the subject in simple and readable form (i.e., without epsilonics and/or set theoretic straightjackets)—*that* would be a real contribution! What us nonmathematicians need today is (inverting the title of F. Klein's famous book) *advanced mathematics, and mechanics, from an elementary viewpoint,* even at the expense of rigor; otherwise we will keep drifting towards a dynamical tower of Babel. Perhaps B. Ravindra would choose to carry out such a useful task!

Enhanced Elastic Buckling Loads of Composite Plates With Tailored Thermal Residual Stresses[1]

Ch. Sarath Babu and T. Kant[2]. The authors present an innovative idea of tailoring the manufacturing-introduced thermal residual stresses in order to enhance the buckling strength of laminated composite plates and have given some interesting results. To demonstrate the effect of the tailored thermal residual stresses on the buckling load of symmetric composites, the authors have considered a square graphite/epoxy plates with two types (types I and II) of reinforcement that introduce nonzero thermal residual stress resultants when the plates cool to room temperature after the cure process. The basic laminate considered is a $[90/0]_s$ cross-ply having a/h ratio equal to 600 (a = 360 mm; thickness of each ply, t = 0.15 mm) where a and h denote width and thickness of the laminate, respectively.

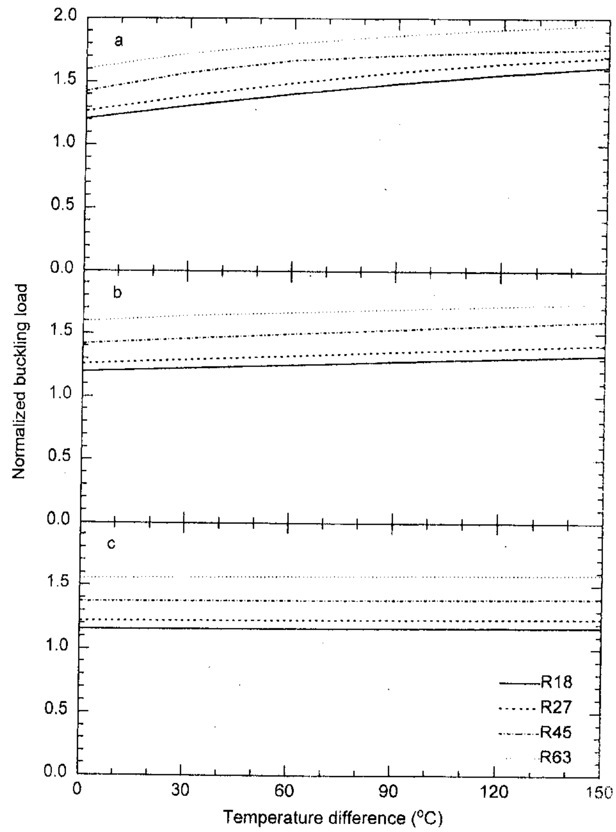

Fig. 1 Normalized buckling loads of plates (type I) R18, R27, R45, and R63 as a function of temperature difference; (*a*) a/h = 600, (*b*) a/h = 300, and (*c*) a/h = 100

Buckling loads of these plates are determined using finite element formulation within the context of linear stability analysis and the eigenvalue problem for λ is obtained as

$$\{[K] + [K_G^R] - \lambda[K_G^Q]\}\{\delta\} = 0 \qquad (1)$$

where $[K]$, $[K_G^R]$, and $[K_G^Q]$ are, respectively, the global stiffness matrix, global geometric stiffness matrix due to thermal residual stress resultants, and global geometric stiffness matrix due to stress resultants caused by displacement loading.

The buckling loads of reinforced plates are normalized with respect to a uniform $[90/0]_s$ plate having the same mass as that of the reinforced plate. The authors have shown that the normalized buckling loads of plates with type I reinforcement increase by about 30 percent and that of plates with type II reinforcement increase by about 300 percent, as the temperature difference is increased from 0°C to 150°C. Though the results presented are interesting, the effect of the thickness of the laminate on enhancement of the buckling load is not presented. Since the authors have considered a very thin laminate (a/h = 600), the increase in normalized buckling load due to increase in temperature difference is very high. However, in case of laminates having higher thickness than the one considered, the

[3] J. G. Papastavridis, School of Mechanical Engineering, Georgia Institute of Technology, Atlanta, GA 30332-0405.

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[2] Department of Civil Engineering, IIT Bombay, Powai, Mumbai-400 076, India. e-mail: tkant@civil.iitb.ernet.in