Journal of Applied Mechanics

Discussion

Unconventional Internal Cracks Part 1: Symmetric Variations of a Straight Crack, and Part 2: Methods of Generating Simple Cracks[1]

Z.-b-Kuang.[2] Professor Wu published two papers that are also of interest to me. In a scientific report of Xi'an Jiaotong University, 1976, in a Chinese journal *Acta Mechanica Sinica*, 1979, No. 2, and in the "Selected Papers of Scientific Research," 1978-1980 of Xi'an Jiaotong University, we published papers entitled "Stress Analysis For Plane Curved Polygonal Defects Containing Cusps Only," and so on. In those papers we modified the contour of Muskhelishvili's contour integral formulas as shown in Fig. 1.

Using the behavior of stresses near corner points, we prove that for any polygonal defects the formulas of complex stress functions $\varphi(\zeta), \phi(\zeta)$ can be expressed as follows:

$$\varphi_0(\zeta)-\frac{1}{2\pi i}\int_{c_1}\frac{\omega(\sigma)}{\overline{\omega'(\delta)}}\,\frac{\overline{\varphi_0'(\sigma)}\,d\sigma}{\sigma-\zeta}=-\frac{1}{2\pi i}\int_{C_1}\frac{f_0(\sigma)\,d\sigma}{\sigma-\zeta} \tag{1}$$

$$\psi_0(\zeta)-\frac{1}{2\pi i}\int_{C_2}\frac{\overline{\omega(\sigma)}}{\omega'(\sigma)}\,\frac{\overline{\varphi_0'(\sigma)}\,d\sigma}{\sigma-\zeta}=-\frac{1}{2\pi i}\int_{C_2}\frac{\overline{f_0(\sigma)}\,d\sigma}{\sigma-\zeta} \tag{2}$$

$$\varphi(\zeta)=\varphi_0(\zeta)+\Gamma R\zeta-\frac{X+iY}{2\pi(1+\kappa)}\,ln\zeta \tag{3}$$

$$\psi(\zeta)=\psi_0(\zeta)+\Gamma' R\zeta+\frac{\kappa(X-iY)}{2\pi(1+\kappa)}\,ln\zeta \tag{4}$$

$$f_0(\sigma)=i\int_{t_0}^{t}(X_n+iY_n)\,ds-\Gamma R\sigma-\bar{\Gamma}' R\frac{1}{\sigma}+\frac{X+iY}{2\pi}\,ln\sigma$$
$$+\frac{X-iY}{2\pi(1+K)}\,\frac{\sigma\omega(\sigma)}{\overline{\omega'(\sigma)}}-\Gamma\bar{R}\,\frac{\omega(\sigma)}{\overline{\omega'(\sigma)}} \tag{5}$$

where the meaning of the symbols $\varphi(\zeta), \omega(\zeta)$ . . . . . can be understood from the literature [1].

Because near a general corner point $\varphi(\zeta)$, $\psi(\zeta)$, and $\omega(\zeta)$ are nonrational functions, equations (1) and (2) can only be solved by numerical methods. But, in the neighborhood of a cusp $\varphi(\zeta)$, $\psi(\zeta)$, and $\omega(\zeta)$ are all rational functions and therefore the exact solution of equations (1) and (2) can be obtained.

[1] By C. H. Wu, and published in the March and June issues of the ASME Journal of Applied Mechanics, Vol. 49, pp. 67–68, and 383–388.

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In those papers we discussed some particular problems. The body is applied a uniform load at infinity and the faces of flaw are free. The tangent of cusp which is of interest to us is parallel to the axis OX. Therefore, the complex stress intensity factor can be expressed by

$$K=K_1-iK_2=2\sqrt{\pi}\,\frac{\varphi'(\sigma_j)}{\sqrt{\omega''(\sigma_j)}} \tag{6}$$

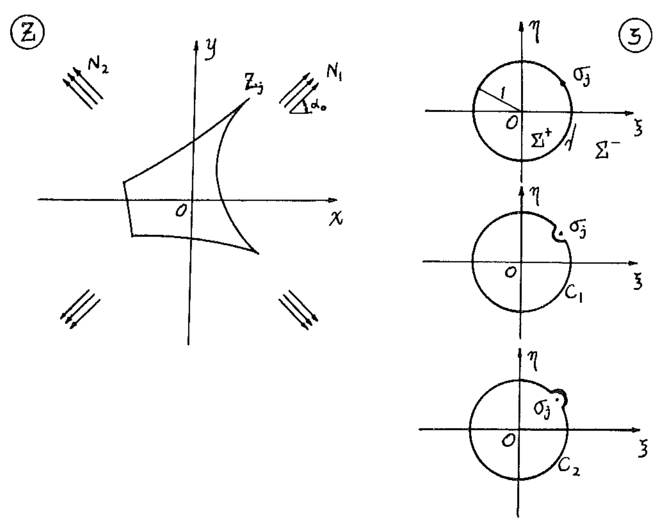

Fig. 1

1 Airfoil Flaw

The mapping function which is well known in hydrodynamics

$$Z=\omega(\zeta)=\frac{1}{2}e^{i\beta}[(\rho\zeta+\alpha)+(\rho\zeta+\alpha)^{-1}] \tag{7}$$

transforms the airfoil flaw in the Z-plane onto the unit circle γ in the ζ-plane, and transforms the exterior region of the flaw onto Σ^- (Fig. 2). In equation (7) ρ, α, and β are parameters, as shown in Fig. 2. By varying these parameters we can obtain various shapes of airfoil flaw. Substituting (7) into (1) and (2) we obtain

$$K=\sqrt{\pi}e^{\frac{i\beta}{2}}\left\{\Gamma+\bar{\Gamma}'e^{-i\beta}+\frac{(\rho^2-\alpha\bar{\alpha})^2}{[(\rho^2-\alpha\bar{\alpha})^2-\alpha^2]}\left[\Gamma+\frac{2}{\rho}e^{i\beta}\bar{\varphi}_0'\left(-\frac{\rho}{\alpha}\right)\right]\right\} \tag{8}$$

$$\bar{\varphi}_0'\left(-\frac{\rho}{\alpha}\right)=\left\{1-\frac{\alpha^2\bar{\alpha}^2}{[(\rho^2-\alpha\bar{\alpha})^2-\alpha^2][(\rho^2-\alpha\bar{\alpha})^2-\bar{\alpha}^2]}\right\}^{-1}$$

$$\left\{\Gamma' R\frac{\alpha^2}{\rho^2}+\frac{\Gamma R e^{-2i\beta}\alpha^2}{(\rho^2-\alpha\bar{\alpha})^2-\bar{\alpha}^2}-\frac{\bar{\Gamma}'\bar{R}\alpha^2\bar{\alpha}^2 e^{-2i\beta}}{\rho^2[(\rho^2-\alpha\bar{\alpha})^2-\bar{\alpha}^2]}\right.$$

$$\left.+\frac{\Gamma\bar{R}\alpha^2\bar{\alpha}^2}{[(\rho^2-\alpha\bar{\alpha})^2-\alpha^2][(\rho^2-\alpha\bar{\alpha})^2-\bar{\alpha}^2]}\right\} \tag{9}$$

For the symmetrical airfoil flaw we have

$$\alpha=\bar{\alpha}=-a, \quad \beta=0, \quad \rho=1+a \tag{10}$$

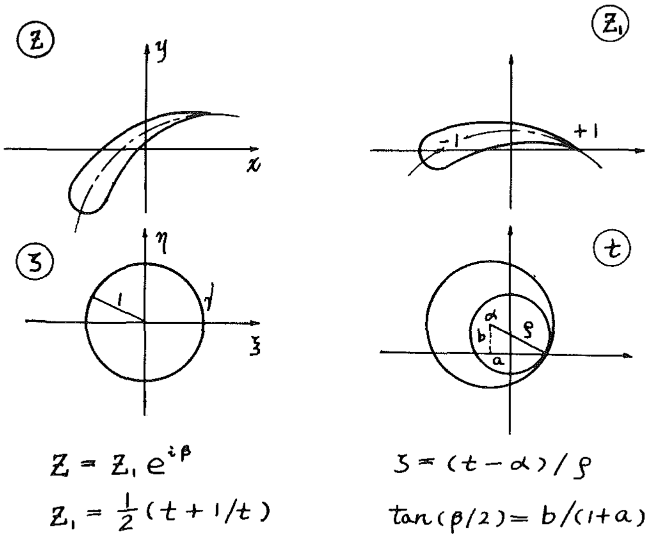

Fig. 2

2 Lip Flaw

The mapping function that transforms the lip flaw whose length is 2 in the Z-plane onto the unit circle in the ζ-plane is

$$Z=\omega(\zeta)=\frac{1}{2}\rho i\left[\zeta+\frac{m}{\zeta}-\frac{\zeta}{\rho^2(\zeta^2+m)}\right] \tag{11}$$

$$\rho(1+m)=a, \quad \rho(1-m)=1 \tag{12}$$

where ρ and m are parameters, as shown in Fig. 3. The stress intensity factor now takes the form

$$K=\frac{\sqrt{\pi}}{1+m}\left[\Gamma+\bar{\Gamma}'-m\Gamma+\frac{(1+m)^3}{(1+m^2)^2}(\Gamma+m\Gamma'-m^2\Gamma)\right.$$

$$\left.-\frac{m^2(1+m)}{(1+m^2)^2}(\Gamma'-\bar{\Gamma}')\right] \tag{13}$$

For any lip flaw with length 2 we can select the following approximate mapping function

$$Z=\frac{1}{2}\left(t+\frac{1}{t}\right), \quad t=i\rho\left(\zeta+\sum_{1,3}^{N}A_n\zeta^{-n}\right) \tag{14}$$

where

$$\rho\left(1+\sum_{1,3}^{N}A_n\right)=a, \quad \rho\left[1+\sum_{1,3}^{N}(-1)^{\frac{n+1}{2}}A_n\right]=1 \tag{15}$$

here a is a selected constant and ρ and A_n are two real coefficients that are determined in such a way so that the approximate configuration is as close as possible to the true configuration.

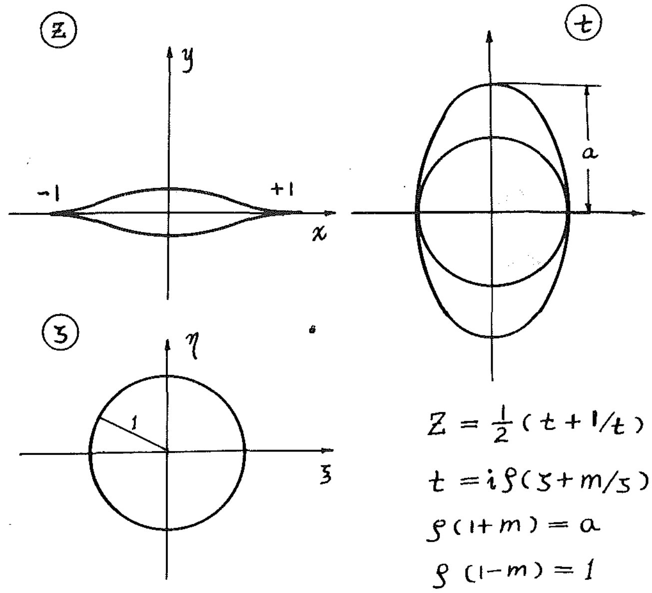

Fig. 3

3 Hypocycloidal Polygon Flaw With $n+1$ Cusps

The mapping function for this case is

$$Z=\omega(\zeta)=R\left(\zeta+\frac{1}{n\zeta^n}\right) \tag{16}$$

and the stress intensity factor is given by

$$K=\frac{2\sqrt{\pi R}}{\sqrt{n+1}}\left(2\Gamma+\frac{n^2\bar{\Gamma}'+n(n-2)\Gamma'}{n^2-n+2}\right) \tag{17}$$

The foregoing discussed flaw can be easily applied to solve some interesting elastic inclusion problems.

From this discussion we can conclude that the formulas (1) and (2) are quite general and useful, and can be applied to corner points that are not cusps.

Reference

[1] Muskhelishvili, N. I., *Some Basic Problems of the Mathematical Theory of Elasticity*, translated by J. R. M. Radokk, Groningen, P. Noordhoff Ltd., 1963.

A Further Examination on the Application of the Strain Energy Density Theory to the Angled Crack Problem[1]

G. C. Sih[2] and E. Madenci[3]. The paper is another one of the author's numerous attempts[4] on the angle crack and elliptical notch problem. Presumably, the objective is to help identify a realistic failure criterion. This, however, cannot be accomplished merely by comparing experimental data with theoretical predictions because the scatter on critical load measurements often overshadow the small differences between different failure criteria. The main purpose of this discussion is to clarify that the dilemmas of the S-theory as

[1] Chang, K. J., and published in the June 1982 issue of the ASME JOURNAL OF APPLIED MECHANICS, Vol. 49, pp. 377–385.

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[4] Refer to references [33–36] in the author's paper.