

Disk-Drive-Like Operations in the Hippocampus

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Abstract

The rapid computation of re-playable memories within the hippocampus in the form of spike sequences is a near computer-like operation. Information can be encoded once during the initial experience, and replayed numerous times after in a compressed-time representation [1–8]. Theta oscillations, sharp-wave ripples, and attractor dynamics have been posited to collectively play a role in the formation and replay of memories. However, the precise interplay between these dynamical states remains elusive. Here, we show that the memory formation dynamics and operations of the hippocampus are not just computer-like, but map directly onto the dynamics and operations of a disk-drive. We constructed a tripartite spiking neural network model where the hippocampus is explicitly described as a disk drive with a rotating disk, an actuator arm, and a read/write head. In this Neural Disk Drive (NDD) model, hippocampal oscillations map to disk rotations in the rotating disk network while attractor dynamics in the actuator arm network point to “tracks” (spike assemblies) on the disk. The read/write head then writes information onto these tracks, which have temporally-structured spikes. Tracks can be replayed during hippocampal ripples for consolidation. We confirmed the existence of interneuron-ring-sequences, predicted by the rotating disk network, in experimental data. Our results establish the hippocampus as a brain region displaying explicit, computer-like operations. Based on the known interactions between the hippocampus and other brain areas, we anticipate that our results may lead to additional models that revisit the hypothesis that the brain performs explicit, computer-like operations.

1 Introduction

The metaphor that the brain operates as a computer has been pervasive in neuroscience since Jon von Neumann’s pioneering work in the 1950’s [9–19]. At the near simultaneous dawn of computer science and electrophysiological-based, computational neuroscience, Von Neumann postulated that the nearly discrete action potentials fired by neurons were comparable to the digital binary units or bits in the vacuum tubes and transistors of early computers [9]. Such a metaphor, if made into a concrete model, could help reach a comprehensive understanding of, and formulate predictions on, the nature of the computational operations underlying brain functions.

Unfortunately, von Neumann’s efforts were limited by the state of knowledge of the brain at the time. However, the last decades of neuroscientific work have now shed important insights into the cellular substrates and network dynamics at the nexus of brain and behaviour, laying the foundational knowledge about how some neuronal regions specialize and adapt to perform specific operations [20–24]. Notably, the hippocampus of the mammalian brain holds mnemonic information used to inform behaviour [1–8, 20, 25–46]. Likewise, modern computers hold information for further operations using dedicated components: Hard Disk Drives (HDDs) [47].

45 Accordingly, we test a more direct version of von Neumann’s “brain-as-a-computer” analogy by
46 establishing a theoretical framework where hippocampal operations and dynamics are mapped directly
47 onto those of a computer’s disk drive in the Neural Disk Model of Hippocampal Function. The NDD
48 model is a tri-partite network where each network maps onto the core components of a disk drive:
49 the rotating disk network, the read/write head network, and the actuator arm network. The NDD
50 model also successfully maps theta oscillations and sharp-wave ripples to disk-rotations while attractor
51 dynamics act as the actuator arm, pointing to individual tracks or cell assemblies. The zoo of observed
52 hippocampal replays (forward replay, reverse replay, extended replay, fragmented replay) can inter-
53 preted as specific data-accessing events in the NDD model. Finally, we detected the interneuron-ring
54 sequences in multi-unit recordings from rats, as predicted by the rotating disk network.

55 2 Results

56 2.1 Mapping Hippocampal Behaviours to Disk Drive Dynamics

57 We start by first describing hippocampal dynamics. The spike times of hippocampal pyramidal cells
58 and interneurons are organized on multiple timescales by a collection of network oscillations that are
59 observed as rhythmic fluctuations in the local field potentials (LFPs) and correlate with behavioural
60 states and memory processing stages (Figure 1A). Sequences of said spikes are observed on two time
61 scales: temporally compressed spike sequences during hippocampal sharp-wave ripples (SWRs, Figure
62 1A-B) and temporally dilated forms of these spike sequences during hippocampal theta oscillations
63 (Figure 1A,C). The SWR LFP event is a 125–250 Hz oscillatory event lasting approximately 100 ms
64 and supporting memory consolidation [1–8, 26, 48–53]. The theta oscillation, on the other hand, is a
65 5–12 Hz oscillation that dominates hippocampal LFPs, organising temporally structured firing activity
66 of pyramidal cells in support of learning during active exploration (Figure 1C) [20, 25, 27, 44, 54, 55].
67 During rest/sleep, theta-nested neural patterns are compressed and replayed in sharp wave/ripples
68 (SWRs) [1–8, 56–62]. The relation between compressed sharp-wave sequences and theta sequences may
69 occur through an oscillatory-interference mechanism where one oscillator controls spike times during
70 SWRs, and a second oscillator dilates SWR-sequences into theta sequences by creating an interference
71 pattern that dilates the sequential content of the carrier waves into the envelope phases (Figure 1C) [63].
72 This postulated interference mechanism provides a mechanistic explanation for hippocampal phase
73 precession [20, 44, 64–72], and explicitly links theta sequences during a single cycle of the theta oscillation
74 to compressed spike sequences during a SWR [63]. Indeed, spike sequences during a cycle of the theta
75 oscillation are also a compressed representation of the firing fields of individual cells, and are known
76 to have comparable compression rates to replay sequences in SWRs [48]. Hebbian plasticity allows
77 for one-shot learning of new sequences by using existing theta-sequences as a compressible temporal
78 backbone (Figure 1D). Different populations of neurons within a theta sequence can be selected by
79 biasing currents to store potential information (Figure 1E) allowing for discrete memories to be stored
80 in different populations of pyramidal neurons [73].

81 We discovered that collectively, these operations could be explicitly mapped onto those performed by
82 a computer’s Hard Disk Drive (HDD). To start constructing our brain-machine mapping of hippocampal
83 dynamics, we first catalogued the individual components and operations of a disk drive and mapped
84 these operations to the hippocampal equivalent (Figure 1F). The central operations of an HDD are
85 performed by three components: an Actuator Arm (AA), a Rotating Disk (RD), and a Read/Write head
86 (R/W) (Supplementary Material Section 1, Supplementary Figure S1, Supplementary Video 1, Figure
87 1E). The actuator arm points to a segment of the disk referred to as a track, which is further subdivided
88 into sectors (Figure 1F). The read/write head, which is located on the apex of the actuator arm, writes
89 new information in the form of bits, or reads previously stored bits on tracks and sectors. These three
90 mechanical components are described by three dynamically evolving parameters that control the disk
91 drive: the location of the actuator arm (ψ_A , A for “Arm”), the disk rotation speed (θ_S , s for “spinning”)
92 and the read/write speed of the head ($\theta_{R/W}$, R/W for “read/write”, Figure 1F). As the disk spins in
93 a single revolution (θ_S), a sequence of sectors on a track appears directly beneath the R/W head on
94 the actuator arm (Supplementary Video 1, Figure 1G). This sequence appears on the fast, intrinsic
95 time-scale of the disk rotation speed (θ_s). The sequence of sectors can be read from or written to in

96 a single disk rotation (Supplementary Video 2, Supplementary Video 3). This mimics the compressed
97 time-scale sequences which occur during hippocampal SWRs [1–8, 56, 73].

98 The sequence of sectors can also be accessed on a slower time-scale, by allowing the disk to perform
99 slightly more than a full revolution for each read/write cycle of $\theta_{R/W}$ ($\theta_{R/W} < \theta_S = \theta_S - \epsilon\theta$, Figure
100 1H, Supplementary Video 3). In this operating mode, the read/write head accesses each subsequent
101 sector on a slower time scale which mimics the dilated or behavioural time-scale sequences that occur
102 during hippocampal theta oscillations. Bit-sequences can be written to these sectors during the slower-
103 access mode and then rapidly replayed after (Figure 1I, Supplementary Video 2). This is similar to
104 the time-compressible, one-shot learning of spike-sequences observed in the hippocampus ([46, 48]).
105 Finally, different tracks are accessible by the actuator arm when the actuator arm changes its location
106 (Figure 1J). The location of the actuator arm, ψ_A behaves similarly to a line attractor. So long as no
107 force is impinged on the actuator arm, the location ψ_A is constant and thus all actuator arm positions
108 are stable. For an actual actuator arm, the range on ψ_A is restricted to some interval within $[0, \pi]$.
109 Applying a force to the actuator arm moves ψ_A to a new track. This mimics how bias currents within
110 the hippocampus, as a result of place, context, or other stimuli, can select different populations of cells
111 during replays [73]. These biasing currents may be regulated or produced by attractor dynamics [74–78].

112 2.2 Constructing the Rotating Disk, Read/Write Head, and Actuator Arm Networks

113 Having qualitatively linked the dynamics of the hippocampus to those of a disk drive, we wondered if
114 an explicit neural model could be constructed of the individual disk-drive components. Such a model,
115 if constructed, would merge two modelling paradigms into a concrete device: interfering oscillators and
116 attractor dynamics (see [79] for a complementary grid-cell model). To that end, we trained recurrent
117 spiking neural networks to display the dynamics of a disk drive as a tripartite network with a rotating
118 disk network, a read/write head network, and an actuator arm network [63, 80]. We started by mod-
119 elling the three networks individually to reproduce disk-drive dynamics constrained to the hippocampal
120 parameter ranges.

121 First, we considered the rotating disk network (Figure 2A-I). The RD network was trained with
122 techniques in machine learning (FORCE training [80, 81]) to cycle in a sequence around a ring with
123 frequency θ_S where θ_S was 9 Hz. This cyclical behaviour acts as the disk rotation of the NDD model
124 with disk rotations mapping to SWRs (Figure 2A). A single isolated cycle around the ring corresponds
125 to a single isolated SWR with a duration of approximately 100 ms (θ_S^{-1}). This rotational sequence is
126 generated by asymmetrically coupling the interneurons in the rotating disk network on the ring while
127 the interneurons are receiving a super-threshold excitatory current (Figure 2B). The interneurons that
128 are currently firing in the sequence inhibit interneurons that have fired just before, thereby maintaining
129 the sequential firing structure on the ring. This interneuron ring serves as the rotator for the entire disk.
130 Subsets of pyramidal neurons serve as individual tracks in the rotating disk network. If the collection
131 of neurons within a track receive enough excitation, they can fire unique sequences, commonly elicited
132 during hippocampal SWRs (Figure 2C-D). The excitation comes from recurrent coupling between the
133 pyramidal neurons with a strongly coupled subset of track initiators (Materials and Methods). The
134 initiator neurons bind the subset of pyramidal neurons into a track and are activated stochastically
135 (consistent with [82]) while the sequences within a track are regulated by the interneuron rotator. These
136 initiators may be related to high-firing rate, low spatial specificity CA1 pyramidal neurons [83, 84].

137 Next, we found that pyramidal cells in the rotating disk network that constitute a track can be
138 exposed sector(s)-at-a-time when a second oscillation with a slower frequency ($\theta_{R/W} = 8$ Hz) is applied
139 to the interneurons in the rotating disk network (Figure 2E). This frequency difference creates an inter-
140 ference that activates subsets of the full SWR-sequence of spikes (a track) in the rotating disk network
141 but as subsets of a theta-sequence (Figure 2G-H). This is the slower operating mode where sectors can
142 be accessed for reading/writing on a slower time-scale (Supplementary Video 3, Supplementary Video
143 4). We further observed that the interference between the θ_S and $\theta_{R/W}$ oscillations manifests as an
144 interference pattern in the voltage of individual neurons (Figure 2) [85] and the interference produces
145 internally generated theta sequences [25, 86, 87]. These internally generated theta sequences are used
146 for reading or writing to tracks. Therefore, the rotating disk network accurately mimics both the oper-
147 ations of a spinning disk and the hippocampus as it consists of a series of pyramidal neurons arranged

148 into tracks and sectors, which could be accessed on slow (behavioural) or fast (neural) time scales, and
149 an interneuron ring which “spins” the entire disk as the rotator thereby forming spike sequences.

150 With the rotating disk network constructed, we then focused on modelling the actuator arm (Figure
151 2J-L). The actuator arm of a conventional disk drive operates much like a continuous line attractor. The
152 actuator itself produces a physical force that moves the arm to a specified position, thereby selecting
153 a track (Figure 2J). Once the force on the arm stops, the arm stays in its new position over the track
154 and is stable. In that way, all possible positions of the actuator arm are stable while an applied force
155 can rapidly switch the arm’s position, thereby forming a continuous line attractor.

156 Given the correspondence between an actuator arm and an attractor network, we trained a recurrent
157 spiking network with FORCE training [80,81] to mimic the dynamics of a line attractor [88] to serve as
158 the actuator arm network (Supplementary Figure S2, Figure 2K). The actuator arm network receives
159 two inputs: the desired position (ψ_A) and velocity ($\frac{d\psi_A}{dt}$) of the actuator arm. We trained the actuator
160 arm network to integrate the velocity-like input to estimate the desired position ($\hat{\psi}_A$). During training,
161 the desired position, ψ_A was dropped during random periods to force the actuator arm network to
162 produce a position estimate with integration alone (Supplementary Figure S2). After training, we
163 found that the resulting actuator arm network produced isolated firing fields of pyramidal neurons,
164 consistent with observed hippocampal place fields (Figure 2K-L, Supplementary Figure S3). However,
165 here $\hat{\psi}_A$ refers to the estimated position of the actuator arm on the disk, rather than the estimated
166 position of an animal in physical space *per se*. Further, we observed that pyramidal neurons in the
167 actuator arm network exhibited bumps of activity, reporting the dynamics of an attractor network
168 (Figure 2L) [85]. Finally, we tested the line-attractor nature of the trained actuator arm network.
169 When no velocity inputs and position inputs are applied, the actuator network retains its last known
170 position as a stable state, similar to how a physical actuator arm stays in the last position on a real
171 HDD if the actuator no longer provides a force on the arm (Supplementary Figure S4). Thus, the
172 actuator arm network behaves as both a line-attractor network, and the actuator arm of a disk drive,
173 while simultaneously producing hippocampal features such as place-field like firing in the individual
174 cells.

175 Next, we wondered how the actuator arm network displayed its line attractor dynamics. To inves-
176 tigate this, we sorted all neurons (pyramidal and interneuron) according to their place (ψ_A) preference
177 (Supplementary Figure S3, Figure 2K). After sorting, we found that the actuator arm network main-
178 tained stable states through a self-disinhibitory motif. Pyramidal neurons with similar actuator arm
179 position preferences excited both themselves and interneurons with similar ψ_A preferences. The in-
180 terneurons then inhibited pyramidal neurons with dissimilar ψ_A preferences.

181 With the rotating disk and actuator arm networks created, we next investigated how information
182 is written onto, and read from the hippocampal “tracks” in the rotating disk network network. For a
183 physical disk drive, the read/write head on the actuator arm reads and writes bits of data (0’s or 1’s)
184 on individual tracks as different magnetic field directions (e.g. \uparrow or \downarrow) on a ferromagnetic material. We
185 thus sought to identify how the read/write head network could “write” bits of information.

186 We thus mapped the functional capability of a R/W head in a disk drive to a plausible biological
187 candidate: Hebbian plasticity (Figure 2D). We constructed a R/W spiking neural network that contains
188 plastic excitatory synaptic weights from all pyramidal neurons in the RD network and uses these plastic
189 weights to encode bits of information (Figure 2M). If enough excitatory weights from a track/sector
190 connect onto a neuron in the R/W head, then that neuron spikes. The spiking of neurons in the
191 read/write head (post-synaptic) paired with Hebbian plasticity to spikes in the rotating disk network
192 (pre-synaptic) is how the read/write head network reads or writes bits of information.

193 Bits of information are sent to the R/W network from external sources for encoding, causing spiking
194 in R/W pyramidal neurons. When the R/W neural spiking coincides with spiking from the active track
195 in the RD network, Hebbian plasticity encodes bits of information in the AA→R/W pathway. The
196 bits of information are written when the spinning disk continuously cycles (θ_S), and the read/write
197 oscillation ($\theta_{R/W}$) is active. As these oscillations sequentially expose the sectors of an individual track,
198 bits are written discretely to isolated sectors (Figure 2N, Supplementary Video 3). We found that this
199 discretization is visible as discrete assemblies when the information is subsequently replayed either when
200 $\theta_{R/W}$ is active or in full disk rotations (Figure 2N-O, Supplementary Video 4) during SWRs, and in fact
201 corresponds to the discrete sectors exposed during the theta-oscillation. This discretization of replays

202 may be linked to the observation that replays can also "jump" from location to location discretely [89].
203 This result shows that the read/write head in the Neural Disk Drive Model can use Hebbian plasticity
204 in a similar fashion to the read/write head on a physical HDD to encode bits of information onto a
205 plastic medium. Finally, we remark that it may also be possible that different phases of the $\theta_{R/W}$
206 oscillation are used separately for the reading and writing of bits [90–93].

207 2.3 The Tri-Partite Neural Disk Drive Model

208 Next, we investigated if the rotating disk, Actuator Arm, and read/write head networks that we con-
209 structed separately could be assembled together and function synergistically to store information as
210 the final Neural Disk Drive (Figure 3). We simulated these three constituent networks coupled with
211 three sets of cross-network synaptic weights (Figure 3A). The first set of (AA→RD) connections link
212 the actuator arm and rotating disk networks, selecting tracks and thus subsets of pyramidal neurons
213 to access. The second set of (RD→R/W) connections associated the RD network to the R/W head
214 network, and act as the storage media to store bits of information (Figure 3B). The third set of
215 (AA→R/W) connections, from the AA network to the R/W network, triggers synfire-chain like spiking
216 in the R/W network. This sequence of spikes represents the information to be acquired during the
217 writing state/theta oscillation, and replayed during the reading stage/SWR. These three (AA, RD and
218 R/W) networks acting together collectively operate as the hippocampal Neural Disk Drive (Figure 3C-
219 F). The R/W head network served to record/replay information as spike sequences; that is, leveraging
220 the computer analogy, sequences of bits are written/read onto a track (Figure 3C). The track is rotated
221 by the ring of interneurons that mediates the disk dynamics (Figure 3D). The actuator arm network
222 serves to perform path-integration and selects the specific track on the rotating disk network.

223 Together, these networks operate synergistically. For example, we found that the AA network can
224 create (theta) sequences longer than those contained within a single track by switching between tracks
225 (Supplementary Figure S5). The AA network can also bias which track becomes replayed during disk
226 rotation (Supplementary Figure S5). Finally, the zoo of hippocampal data-accessing events are well
227 explained by a disk-drive model of hippocampal operations. By disabling the rotating disk network
228 interneurons, replays can be converted into sequence-less reactivations [94] (Supplementary Figure S6).
229 Pre-plays, where the sequences during SWRs are observed before navigation [49–53,95], are pre-existing
230 or old data written to tracks before they are accessed during Reading/Writing phases (Supplementary
231 Figure S6). Fragmented replays [6], where replay trajectories jump in state-space correspond to mis-
232 alignment between the initial phase of the disk and the start of a bit sequence (Supplementary Figure
233 S6, Supplementary Video 5). Reverse replays based on dedicated sequence-reversion pools of interneu-
234 rons were previously considered [63]. In this scenario, the disk spins backwards to reverse the order of
235 bit/spike sequences (Supplementary Video 6). Replays of trajectories that were not previously experi-
236 enced by an animal ([96,97]) can also be constructed by reading out multiple tracks that were never
237 sequentially accessed during the writing phase. These tracks can also be individually reversed within a
238 multiple-track replay event [96,97].

239 Thus, the Actuator Arm, rotating disk, and read/write head networks work synergistically to record
240 information in a tripartite Neural Disk Drive, with core features of the hippocampal network being
241 mapped to well-defined operations and components of a HDD.

242 2.4 Detecting the Interneuron Ring Sequences Predicted by the NDD Model

243 This Neural Disk Drive model proposes that the hippocampus uses both network oscillations and
244 attractor dynamics to implement the operations performed by the rotating disk and the actuator arm of
245 a computer disk drive. Thus, we tested this theoretical prediction using empirical data, probing evidence
246 for one of the dynamical hallmarks of the NDD model: interneurons serving as the rotational backbone
247 of neural firing patterns. To proceed, we first considered in vivo hippocampal ensemble recordings
248 performed from rats trained to learn and remember three reward locations on a cheeseboard maze
249 (Materials and Methods, Figure 4A). On each day of this memory task, animals develop an effective
250 navigation path to reach the reward locations throughout the learning trials (Figure 4B). We used these
251 animal trajectories to analyze hippocampal patterns formed by sequences of spikes (Figure 4C-D) with a
252 Maximal Likelihood Estimation (MLE) based alignment algorithm (see Supplementary materials). The

MLE algorithm aligns the repeating firing fields internally to each other across learning trials (rather than to place) by applying time-shifts across trials through each firing field (Figure 4D, Supplementary Figure S7). The time shifts are then used to optimize an objective function, the maximum-likelihood estimator for the spike density parameterized by said time shifts. We found that aligning a single neuron simultaneously aligns an entire community of neurons within the learning session (Figure 4E). This included both pyramidal neurons and interneurons (Figure 4F), unveiling a repeating interneuron sequence that appeared to be stable across learning trials (Figure 4G, Supplementary Figure S7). Interneurons in both mice and rats are known to have differing phase preferences with respect to the theta oscillation in the LFP. Indeed, we found a similar result (Figure 4H-K) with interneurons spanning the $[0, 2\pi]$ range in both rats (Figure 4H-I) and mice (Figure 4J-K). However, phase preference firing of bursts in interneurons during theta oscillations, which has been previously reported ([98–100]) does not generate ring-sequences alone. Indeed, stochastic simulations of neurons with phase preferential firing of bursts in a population will not generate a ring sequence (Supplementary Figure S8). Our analysis shows that that hippocampal interneurons form the neural ring-sequences predicted by the NDD model across species and tasks.

3 Discussion

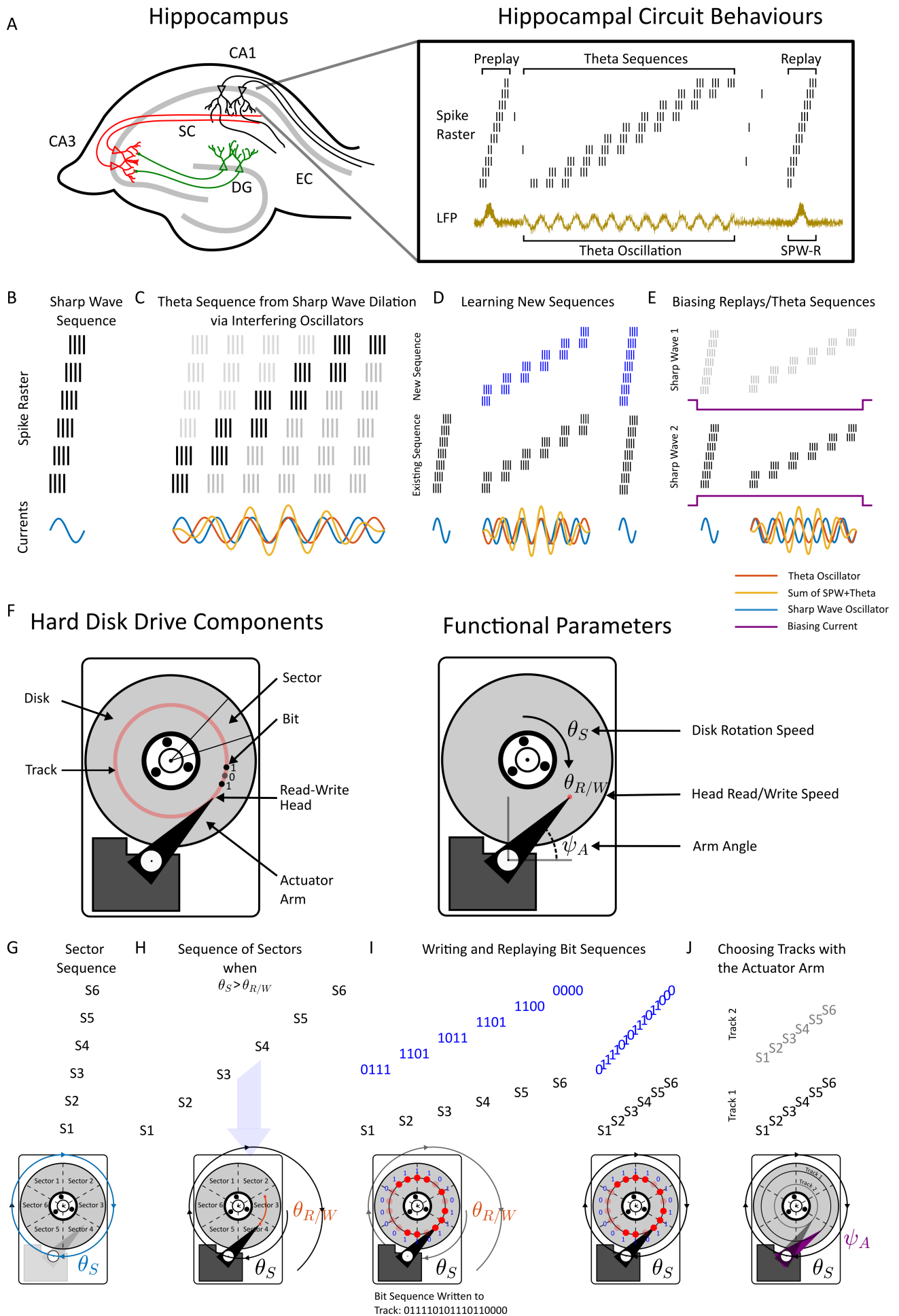
Analogies and metaphors with devices have been attempted towards obtaining useful descriptions of the brain since antiquity [14, 16–18]. Often, these analogies are a product of their times, manifesting brain function as a likeness or similarity to the dominant technological innovation of the era. Indeed, the computer analogy for brain function was proposed by Von Neumann shortly after both the first electronic computer, ENIAC (Electronic Numerical Integrator and Computer), was unveiled [101] and Hodgkin and Huxley successfully modeled the squid giant axon [102]. However, these brain analogies are often limited to observations of behaviour, and rarely directly linking the computations, components, and function of a device to the anatomy and physiology of any particular brain area [15]. This is where our contribution differs from the past by leveraging decades of in vivo experiments in the hippocampus.

We have constructed and simulated a model of hippocampal function as a Neural Disk Drive, by merging two prior modelling paradigms, attractors [74–78] and interference models [20] into one singular device. The network was constructed and simulated with 3 components: an actuator arm (Attractor), a rotating disk (Oscillator) and a read/write Head (for storing memories). This model was sufficient to reproduce some of the core behaviours of hippocampal neurons; path-integration and navigation, theta sequences and phase precession, compressed sequences and sharp-wave ripples, and the zoo of hippocampal replays. Finally, we verified one of the predictions of this model; a rotating ring-sequence of interneurons which constitute the rotator of the neural disk drive.

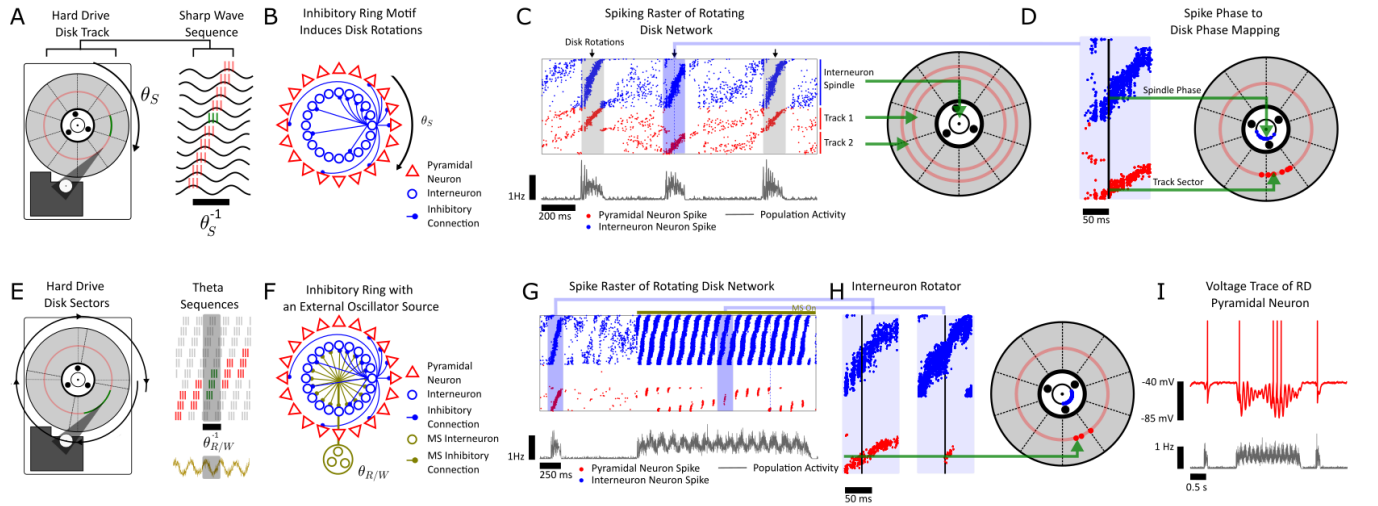
If the hippocampus does utilize oscillations and attractor dynamics similarly to how these dynamics are used in a disk drive, two natural lines of inquiry emerge. The first line of inquiry emerges from the saturation of tracks. A track on a physical disk drive can only hold so many bits. In fact, for a sufficiently large file, multiple tracks must be used to store the entire file (Supplementary Video 7). Thus, if the hippocampus stores information as a disk drive, does multi-track storage also occur? Evidence for the affirmative to this hypothesis is present in the literature in the form of so-called extended replays, where a trajectory is replayed as multiple, sequential sharp-wave-ripple complexes [28, 29]. These ripple complexes are separated by 150 ms intervals, with an inter-sharpwave-interval distribution displaying prominent peaks at multiples of 150 ms, hinting at a disk-rotation mechanism controlling SWR generation [63].

The second line of inquiry consists of the total storage space of the hippocampus, if it does indeed operate as a disk drive. In the NDD model, the storage medium used to write bits onto the RD network are the synaptic weights coupling the RD network onto the R/W head. Anatomically, these weights may be the Schaeffer Collateral connections linking CA3 to CA1, which constitutes a matrix containing at most $N_{CA3}N_{CA1}$ connections, where N_{CA3}, N_{CA1} are the number of neurons in CA3, CA1. If we think of the weights as entirely binary, and consider reasonable values of $N_{CA3} = N_{CA1} = O(10^5)$ [103], we arrive at $\approx O(10^9)$ bytes or $O(1)$ GB of storage as an upper-bound for rats. Human estimates for N_{CA3} and N_{CA1} are larger ($O(10^6)$), leaving $O(10^2)$ GB [104]. Thus, if the hippocampus does act as a disk drive, it is one of fairly limited storage, as 100 GB translates into roughly 100 hours

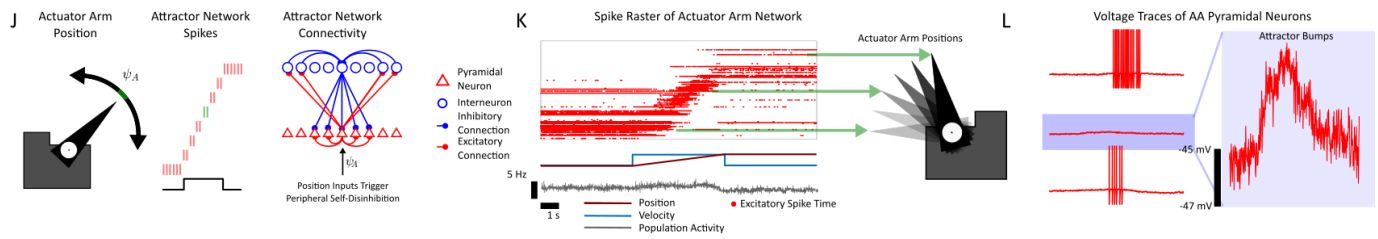
305 of video and audio with 720p resolution. Additionally, these were upper bounds that were derived
306 assuming all CA3 and CA1 neurons and that the neurons operate perfectly to transmit and store bits
307 without redundancy. Given the low values of even these optimistic bounds, the neural disk drive model
308 suggests that the hippocampus is limited in the amount of data it can store, possibly hinting that
309 the hippocampus stores some type of data-compressed representation of complex memories [105–109].
310 Alternatively, it is also possible that the low amounts of storage are only used to store recent events,
311 prior to consolidation [110–112].



The Rotating Disk Network



The Actuator Arm Network



The Read/Write Head Network

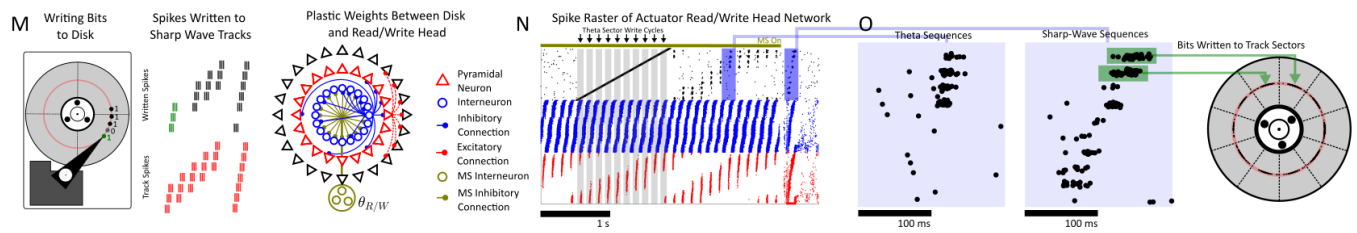
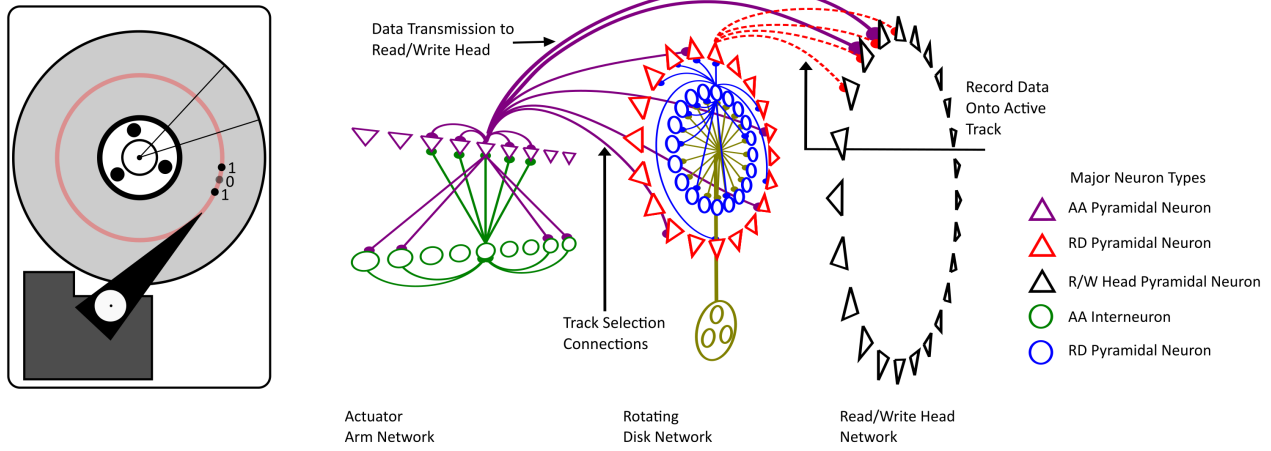
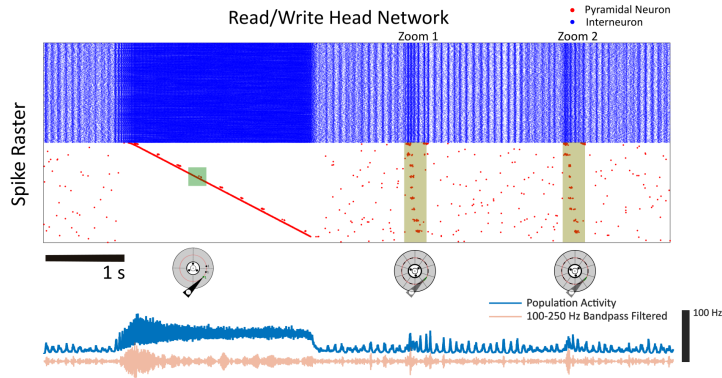
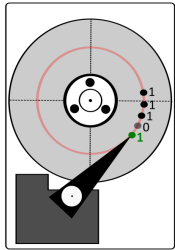


Figure 2

A Combining the Separate Networks into a Neuronal Disk Drive



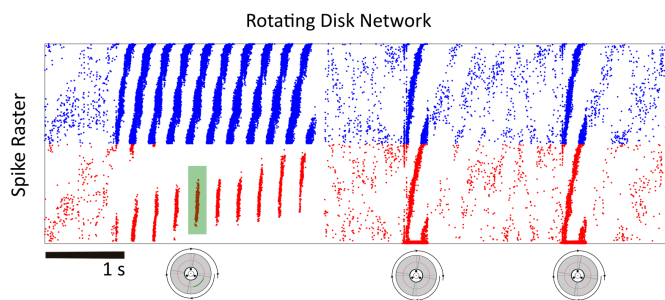
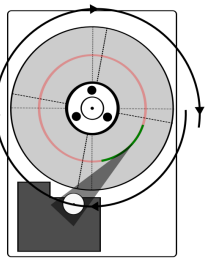
B Read/Write Head



E Zoom 1



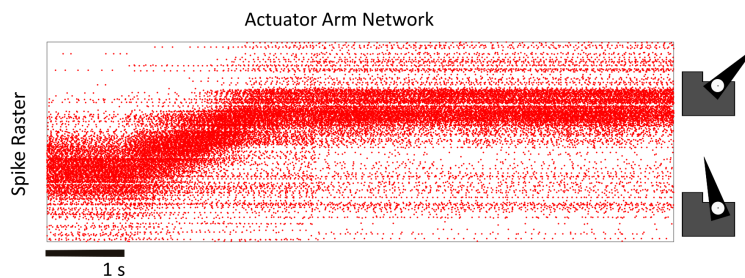
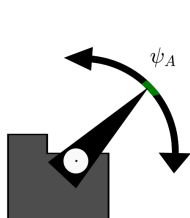
C Rotating Disk



Zoom 2



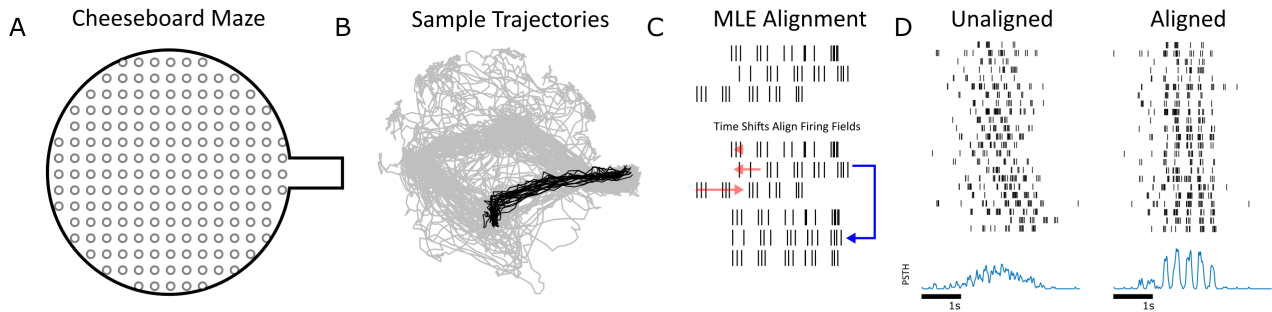
D Actuator Arm



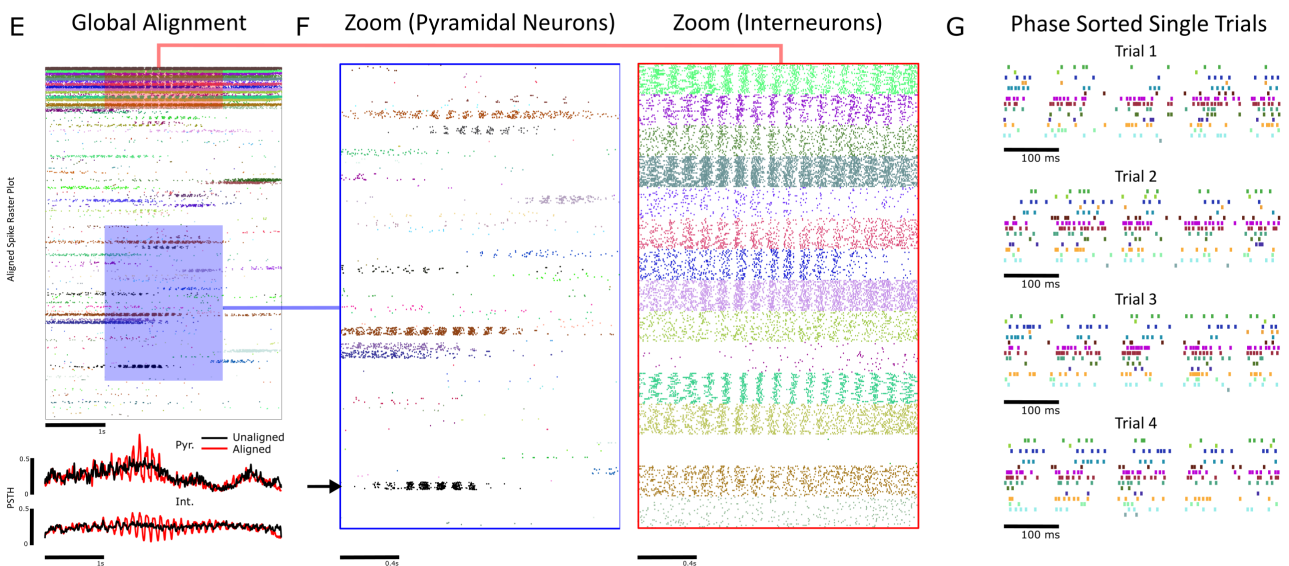
30 ms

Figure 3

Internal Alignment of Firing Fields via Stochastic Time Shifting



Alignment of a Single Neuron Leads to Global Alignment



Interneuron Theta Phase Preference Covers the Entire Unit Circle

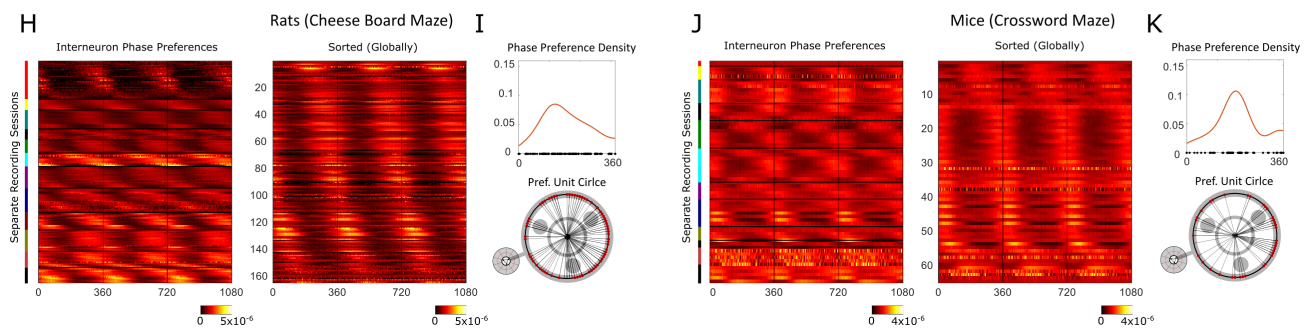


Figure 4

312 List of Supplementary Videos

313 To dynamically describe both the function of disk drives and the relation between hippocampal dy-
314 namics and disk drives, a series of animations were prepared (see below). All animations can be viewed
315 and downloaded from <https://www.nicolacomputationalneurosciencelab.com/publications>.

316 **Supplementary Video 1: The Components and Operations of a Disk Drive.** A supple-
317 mentary video that displays how the motions of the read/write head, rotating disk, and actuator arm
318 stores and accesses information.

319 **Supplementary Video 2: Writing on the Slow Time Scale.** A supplementary video that
320 shows how a disk drive can write with a frequency ($\theta_{R/W}$) that is slightly slower than the disk rotation
321 speed (θ_S). This frequency difference creates hippocampal phase precession in the Neural Disk Drive
322 model.

323 **Supplementary Video 3: Replays via Single Disk Rotations.** A supplementary video that
324 shows how a trajectory can be replayed in the Neural Disk Drive model with a single disk rotation.

325 **Supplementary Video 4: Theta Sequences via Multi-Sector Access.** A supplementary
326 video that shows the emergence of theta-sequences from a tracks worth of data. Here, the read/write
327 head accesses multiple sectors in a single disk rotation sequentially, thereby creating theta-sequences
328 with phase precessing spikes.

329 **Supplementary Video 5: Fragmented Replay via Disk-Phase Misalignment.** A supple-
330 mentary video showing how trajectories can "jump" via the misalignment of the initial phase or sector
331 of the disk with the start of a trajectory.

332 **Supplementary Video 6: Reverse replay via Counter-Clockwise Disk Rotations.** A
333 supplementary video showing how trajectories can be replayed backwards through time by spinning
334 the disk in the opposite rotation (e.g. counter-clockwise) of the initial recording (e.g. clockwise).

335 **Supplementary Video 7: Extended Replay via Multi-Track Access.** A supplementary
336 video showing how long trajectories can be decoded by using multiple tracks, with an actuator arm
337 switching between successive tracks.

338 Figure Captions

339 **Figure 1: The links between hippocampal dynamics and disk drive operations**

340 (A) The anatomical (CA3, CA1, Dentate Gyrus (DG), and Entorhinal Cortex (EC) sub-regions of the
341 hippocampus (left) along with observed hippocampal behaviours (right). Sequences of spikes occur on
342 short time scales as preplays/replays during hippocampal sharp-wave-ripples (SWRs), and on longer
343 time scales as theta sequences. The components of a theta sequence during a theta oscillation are
344 subsets of the entire preplay/replay. Preplays occur before the initial observation of a theta sequence,
345 while replays occur after the initial observation of a theta sequence. The theta oscillation is an 8-12
346 Hz oscillation displayed in the local-field potential (LFP) while SWRs are 150-250 Hz high-frequency
347 oscillations in CA1, which coincide with a large-deflection (the sharpwave) in CA3. (B) In a model
348 of hippocampal replay, sharp-wave sequences are controlled by an intra-hippocampal oscillation. The
349 oscillation is transiently activated for single cycles to trigger SWR replays. (C) By adding a secondary
350 oscillation, SWR sequences become temporally dilated into theta sequences via an interference-based
351 mechanism. (D) Dilated sharp-wave spike sequences can form a basis to learn new sequences, in
352 a one-shot, instantly compressible format. (E) Different sharp-wave sequences can be elicited by a
353 biasing current to pools of pyramidal neurons. (F) The hardware components (left) and functional
354 parameters (right) of a computer disk drive, which operates by encoding data onto a rapidly spinning
355 disk subdivided into tracks and sectors. The read/write head on the apex of the actuator arm moves to
356 encode bits onto different tracks. The disk drive has three functional parameters: the Disk Rotation
357 speed θ_S , the read/write speed of the head $\theta_{R/W}$, and the actuator arm angle ψ_A .

358 **Figure 2: Mapping hippocampus circuit components onto hard disk drive components: the rotating 359 disk, actuator arm and read/write head networks.**

360 (A) The rotating disk (RD) network contains SWR sequences that instantiate tracks in the Neural
361 Disk Drive (NDD) model of the hippocampus function. (B) An asymmetric ring of inhibition induces
362 a rotating SWR sequence with rotational speed θ_S . (C) Different subsets of RD excitatory neurons
363 instantiate tracks in the NDD model. (D) The interneuron ring acts as the primary rotator of the NDD,
364 while subsets of the RD SWR tracks act as the sectors. (E) The theta oscillations $\theta_{R/W}$ sequentially
365 expose sectors of a track to write-to. This forms theta sequences, which in the NDD model, are
366 sequentially exposed tracks of a SPR track. (F) An oscillatory inhibitory signal with a frequency of
367 $\theta_{R/W}$ triggers sector exposure during a theta oscillation. (G) During a RD SWR, all sectors of a
368 track are activated without $\theta_{R/W}$. During theta oscillations (with $\theta_{R/W}$), the sectors are sequentially
369 activated. (H) Zoom of the spike sequences in the RD network overlaid explicitly onto a disk. (I) RD
370 dynamics are present as theta oscillations in both the LFPs and in the membrane potential of individual
371 pyramidal neurons. (J) The actuator arm (AA) features dynamics of an attractor network. A torque on
372 the AA causes it to change angular position ψ_A , which activates different subsets of AA neurons. This
373 forms sequences of instantaneous rates, rather than individual spikes. When the inputs correspond
374 to velocity and place, these ongoing firing fields can be interpreted as current place fields. (K) A
375 simulation of the FORCE trained AA network with the position and velocity of the animal as inputs
376 (below). The AA pyramidal neurons (red) are sorted according to their place preference. (L) The
377 voltage traces for 3 randomly selected pyramidal neurons in the AA network. The voltage traces show
378 “ramps of activity”, which allows the neurons to modulate their ongoing firing rates to collectively
379 encode the position of the actuator arm. The position of the actuator arm can be decoded from a
380 linear combination of these rates. (M) Once a track is selected by the AA, the read/write (R/W)
381 head can write new sequences onto the track. This corresponds to updating the Schaffer collateral
382 weights in the network to store new sequential information. (N) Spike raster plot for the RD and R/W
383 head networks. The written sequences become discretized into assemblies of coactive neurons (O) The
384 discretized assemblies projected onto the sectors of a disk.

385 **Figure 3: Assembling the tripartite Neural Disk Drive model.**

386 (A) The three constituent (RD, AA, and R/W) networks are connected together to construct a tripar-
387 tite network. (B) The RD, AA and R/W networks correspond to discrete regions and pathways within
388 the hippocampus circuit. (C) The R/W network is simulated with a writing component consisting of
389 theta oscillations ($\theta_{R/W}$) and a reading phase during which SWRs occur (θ_S). (D) The RD network
390 during the writing phase and two disk rotations. The interneuron ring performs the disk rotations in
391 both the writing and reading phase. (E) The AA network, with neurons sorted according to place pref-
392 erence. The AA transitions from one location to the next. (F) Two zooms of the replayed sequences
393 that occurred during RD SWRs.

394 **Figure 4: Probing the hippocampal in-silico NDD with in-vivo ensemble recordings.**

395 (A) Schematics of the cheeseboard and the crossword mazes where rodents (rats and mice, respectively)
396 learn to navigate to reward locations while hippocampal neuron ensembles and LFPs are recorded.
397 (B) Example repeated navigational trajectories (black) used for firing pattern analyses, superimposed
398 on animal's whole path (gray) in these mazes. (C) The trajectories are aligned with a maximum
399 likelihood estimation (MLE) algorithm that maximizes the probability of recorded spiking with time-
400 shifts to each temporal firing field. (D) The firing fields, temporally unaligned (left) and aligned
401 (right). (E) Aligning a single recorded pyramidal neuron produces a global temporal alignment across
402 the ensemble of multiple recorded neurons. (F) Zoom of the aligned recorded pyramidal neurons
403 (left) and interneurons (right). The black arrow denotes the pyramidal neuron used to align the
404 entire population. (G) A sequence of interneuron ring-like spiking. (H) Temporal firing fields for a
405 population of interneurons. (I) The phase preference of firing for all interneurons is broadly distributed.
406 (J) Similarly, the firing fields for a population of interneurons recorded in mice performing another
407 memory task (the crossword maze). (K) The phase preference for all interneurons is once again,
408 broadly distributed over $[0, 2\pi]$.

409 Materials Availability

410 This study did not generate new unique reagents.

411 Data availability

412 The datasets used in this will be made available via the MRC BNDU Data Sharing Platform (<https://data.mrc.ox.ac>)
413 upon reasonable request.

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420 Materials and Methods

421 The Actuator Arm Network

422 The actuator arm network consists of 2000 excitatory (AA_E), and 2000 inhibitory (AA_I) leaky-
423 integrate-and-fire neurons:

$$\begin{aligned} \tau_m \frac{d\dot{v}_j^{AA_E}}{dt} &= -v_j^{AA_E} + RI^{AA_E} + R \sum_{j=1}^{N_E^{AA}} \omega_{ij}^{AA_E, AA_E} r_j^{AA_E}(t) + R \sum_{j=1}^{N_I^{AA}} \omega_{ij}^{AA_E, AA_I} r_j^{AA_I}(t) \\ &+ \kappa_j^{AA_E} \psi_A(t) + \gamma_j^{AA_E} \frac{d\psi_A(t)}{dt}, \quad j = 1, 2, \dots, N^{AA_E} \end{aligned} \quad (1)$$

$$\begin{aligned} \tau_m \frac{d\dot{v}_j^{AA_I}}{dt} &= -v_j^{AA_I} + RI^{AA_I} + R \sum_{j=1}^{N_E^{AA}} \omega_{ij}^{AA_I, AA_E} r_j^{AA_E}(t) + R \sum_{j=1}^{N_I^{AA}} \omega_{ij}^{AA_I, AA_I} r_j^{AA_I}(t) \\ &+ \kappa_j^{AA_I} \psi_A(t) + \gamma_j^{AA_I} \frac{d\psi_A(t)}{dt}, \quad j = 1, 2, \dots, N^{AA_I} \end{aligned} \quad (2)$$

424 where v^{AA_E} and v^{AA_I} denotes the voltage for an actuator arm excitatory and actuator arm inhibitory
425 neuron, respectively. The parameters for all neurons/synapses can be found in Table 1. Once the
426 voltage for a neuron reaches a threshold, v_{thresh} , the voltage is reset to v_{reset} .

$$v^{AA_{E/I}}(t^-) = v_{thresh}, \quad v^{AA_{E/I}}(t^+) = v_{reset} \quad (3)$$

427 Every spike is followed by an absolute refractory period, τ_{ref} during which the neuronal dynamics
428 are quenched at the reset value. The parameters $I^{AA_{E/I}}$, denote the bias currents to the neurons
429 in the E/I sub-populations, respectively. The membrane time constant, τ_m controls the integration
430 dynamics of each neuron. The parameter $R = 1 \cdot 10^9 \Omega$ serves as the resistance. The weight matrices
431 ω^{AA_E, AA_E} , ω^{AA_E, AA_I} , ω^{AA_I, AA_E} , ω^{AA_I, AA_I} denote the coupling from E to E , I to E , E to I and I to I
432 populations, respectively. These weights are trained with the FORCE algorithm, and described below.
433 The inputs $\psi_A(t)$ and $\frac{d\psi_A}{dt}$ denote the desired position of the actuator arm, and the velocity of the
434 desired position of the actuator arm. The inputs are multiplied by a set of input weights, $\kappa^{AA_{E/I}}$, and
435 $\gamma^{AA_{E/I}}$ for the actuator arm position/velocity, respectively.

436 The variables $r^{AA_{E/I}}$ are the convolved spike times for the E/I actuator arm neurons:

$$\dot{r}_j^{AA_{E/I}} = -\frac{r_j^{AA_{E/I}}}{\tau_d} + h_j^{AA_{E/I}} \quad (4)$$

$$h_j^{AA_{E/I}} = -\frac{h_j^{AA_{E/I}}}{\tau_r} + \frac{1 \cdot pA \cdot ms^2}{\tau_r \tau_d} \sum_{t_{jk}^{AA_{E/I}} < t} \delta(t - t_{jk}^{AA_{E/I}}) \quad (5)$$

437 where $t_{jk}^{AAE/I}$ denotes the k th spike fired by the j th neuron in the actuator arm excitatory/inhibitory
 438 sub-population. The parameters τ_d and τ_r denote the decay and rise times, respectively of the sub-
 439 population of neurons. The AA network, and indeed, all networks considered were integrated with a
 440 simple Forward-Euler method and a step size of $dt = 5 \times 10^{-5} s$.

441 FORCE Training the Weights of the Actuator Arm Network

442 The weight matrices $\omega^{AAE,AAE}, \omega^{AAE,AAI}, \omega^{AAI,AAE}, \omega^{AAI,AAI}$ decompose as the sum of a static com-
 443 ponent, and a learned component:

$$\omega^{AAE,AAE} = \omega_0^{AAE,AAE} + (\boldsymbol{\eta}^E)_+(\boldsymbol{\phi}^E)_+ + (\boldsymbol{\eta}^E)_-(\boldsymbol{\phi}^E)_- \quad (6)$$

$$\omega^{AAE,AAI} = \omega_0^{AAE,AAI} + (\boldsymbol{\eta}^E)_-(\boldsymbol{\phi}^I)_+ + (\boldsymbol{\eta}^E)_+(\boldsymbol{\phi}^I)_- \quad (7)$$

$$\omega^{AAI,AAI} = \omega_0^{AAI,AAI} + (\boldsymbol{\eta}^I)_+(\boldsymbol{\phi}^I)_- + (\boldsymbol{\eta}^I)_-(\boldsymbol{\phi}^I)_+ \quad (8)$$

$$\omega^{AAI,AAE} = \omega_0^{AAI,AAE} + (\boldsymbol{\eta}^I)_+(\boldsymbol{\phi}^E)_+ + (\boldsymbol{\eta}^I)_-(\boldsymbol{\phi}^E)_- \quad (9)$$

The functions

$$(x)_+ = \begin{cases} x, & x \geq 0 \\ 0, & x < 0 \end{cases}, \quad (x)_- = \begin{cases} x, & x \leq 0 \\ 0, & x > 0 \end{cases}$$

444 are applied to the components of the matrices $\boldsymbol{\eta} = [\boldsymbol{\eta}^E, \boldsymbol{\eta}^I]$, $\boldsymbol{\phi} = [\boldsymbol{\phi}^E, \boldsymbol{\phi}^I]$ to enforce Dales Law. The
 445 matrices $\boldsymbol{\eta}$ are referred to as the encoders, and help determine the tuning properties of neurons with
 446 respect to the estimated actuator arm position $\hat{\psi}_A$. For each neuron in the AA network, the encoder
 447 for that neuron (a row of $\boldsymbol{\eta}$), is randomly generated and sparse. The encoder is an $N \times k$ matrix where
 448 n_{sup} is the dimension of the supervisor of the network, likewise for the decoder. The encoder is enforced
 449 to be sparse: each neuron has a single element in its encoder that is non zero, and randomly set to
 450 $\pm W$, where $W = 10$ pA. The input weights, $\boldsymbol{\kappa}^{\psi_A}$ and $\boldsymbol{\kappa}^\gamma$ were randomly generated with from a uniform
 451 $[-1, 1]$ distribution, for all neurons in the AA network.

452 The decoders, $\boldsymbol{\phi}$ are learned with FORCE training, which we describe below.

453 The Supervisor and Inputs to the Actuator Arm Network

454 The input to the actuator arm network is a randomly generated signal, $\psi_A(t)$, and its derivative, $\frac{d\psi_A}{dt}$.
 455 The signal is generated with a bounded, double-filtered noisy process. The first filter corresponds to
 456 the acceleration $a(t)$ while the second filter corresponds to the velocity $\psi_A(t)$:

$$\tau_\psi \frac{da}{dt} = -a + \zeta(t) \quad (10)$$

$$\tau_\psi \frac{d\psi_A}{dt} = -\psi_A + a(t) \quad (11)$$

457 where $\psi_A(t) = \pm 1$ causes a reset, as if colliding with a boundary, to the velocity and acceleration of the
 458 actuator arm position $\frac{d\psi_A}{dt} = 0$, $a(t) = 0$. Further, the velocity of the actuator arm is also limited such
 459 that if $|\frac{d\psi_A}{dt}| \geq 1$, the velocity is fixed to ± 1 , to prevent arbitrarily fast motion of the actuator arm.
 460 The variable $\zeta(t)$ is a white noise process with mean 0 and standard deviation of 10^{-3} . The actuator
 461 arm position is unit-less, while the velocity and acceleration are s^{-1} and s^{-2} .

462 The position and velocity inputs ($\psi_A(t), \frac{d\psi_A}{dt}$) are provided to the actuator arm network during
 463 training. However, the position component is dropped stochastically (set to 0) for intervals that are
 464 randomly generated. These intervals are a minimum of 1 second long, and a maximum of 51 seconds
 465 long, with the interval itself drawn from the uniform distribution $U([1, 51])$. Once the position dropping
 466 interval ends, the position is turned back on instantly, for a random period of time. This random interval
 467 with position is also uniformly generated from the distribution $U([1, 51])$. In the intervals where the
 468 position is dropped, the network must rely on velocity and the last known position of the system to
 469 “integrate” and estimate the desired actuator arm position.

470 The supervisor, $\mathbf{s}(t)$ to the actuator arm network is a $n_{sup} = 200$ dimensional vector that is a
471 non-linear transform of the position:

$$s_i(t) = \exp\left(-\frac{(x_i - \psi_A(t))^2}{\sigma_x}\right) \quad (12)$$

472 Each component of the supervisor acts as an activity bump when $\psi_A(t)$ passes near x_i , where x_i is
473 the center of the bump. The centers are uniformly distributed on the interval $[-1, 1]$. The variable σ_x
474 controls the width of the bump, with $\sigma_x = 0.3$ used during training. The velocity component is not
475 contained in the supervisor in any way.

476 Recursive Least Squares

477 The decoders, ϕ are determined dynamically to minimize the squared error between the approximant
478 and intended dynamics, $\mathbf{e}(t) = \hat{\mathbf{s}}(t) - \mathbf{s}(t)$. The Recursive Least Squares (RLS) technique updates the
479 decoders to solve this problem in real-time:

$$\phi(t) = \phi(t - \Delta t) - \frac{\mathbf{e}(t)\mathbf{P}(t - \Delta t)\mathbf{r}(t)}{1 + \mathbf{r}(t)^T\mathbf{P}(t - \Delta t)\mathbf{r}(t)} \quad (13)$$

$$\mathbf{P}(t) = \mathbf{P}(t - \Delta t) - \frac{\mathbf{P}(t - \Delta t)\mathbf{r}(t)\mathbf{r}(t)^T\mathbf{P}(t - \Delta t)}{1 + \mathbf{r}(t)^T\mathbf{P}(t - \Delta t)\mathbf{r}(t)} \quad (14)$$

480 and $\mathbf{r}(t) = (\mathbf{r}^{AAE}(t), \mathbf{r}^{AAI}(t))^T$. The Recursive Least Squares Algorithm and FORCE training is
481 described in greater detail in [80, 81]. The actuator arm network is initialized with $\phi(0) = \mathbf{0}$, $\mathbf{P}(0) =$
482 $\mathbf{I}_N\lambda$, where \mathbf{I}_N is an N -dimensional identity matrix, and λ controls the learning rate of RLS. The value
483 $\lambda = 0.5dt$ was used, where $dt = 5 \times 10^{-5}s$ was the simulation integration step size. To implement
484 Dale's law as in equations (6)-(9), we decompose ϕ into ϕ^E and ϕ^I with $\phi^E = (\phi)_+$ and $\phi^I = (\phi)_-$.
485 The training parameters for the RD network. For the rotating disk network, a value of $\lambda = 0.05dt$ was
486 used.

487 The Rotating Disk Network

488 The rotating disk network is a modification of the so-called "SHOT-CA3" network from [63]. As in the
489 actuator arm network, the rotating disk network consists of coupled leaky integrate-and-fire neurons:

$$\begin{aligned} \tau_m \dot{v}_i^{RD_I} &= -v_i^{RD_I} + RI^{RD_I} + R \sum_{j=1}^{N_{RD_I}} \omega_{ij}^{RD_I, RD_I} r_j^{RD_I}(t) \\ &+ RI^{GABA}(1 + \cos(2\pi\theta_{RW}t)), \quad i = 1, 2, \dots, N_{RD_E} \end{aligned} \quad (15)$$

$$\tau_m \dot{v}_i^{RD_E} = -v_i^{RD_E} + RI^{RD_E} + R \sum_{j=1}^{N_{RD_I}} \omega_{ij}^{RD_E, RD_I} r_j^{RD_I}(t), \quad i = 1, 2, \dots, N_{RD_I} \quad (16)$$

490 where RD_E, RD_I denote the excitatory and inhibitory populations of the rotating disk network. The
491 neurons receive a constant background current I^α for $\alpha = RD_E, RD_I$. The RD_I neurons receive an
492 oscillatory input where θ_{RW} is the input frequency, and $\kappa > 1$ determines the tonic level of inhibitory
493 drive. The INP-MS has amplitude $I^{GABA} = -10$ pA for $i = 1, 2, \dots, N_I$. The network consists of
494 $N_{RD_E} = N_{RD_I}$ spiking neurons.

495 The weight matrices ω^{RD_I, RD_I} , ω^{RD_E, RD_I} , and supervisor used to generate them with FORCE
496 training are described in further detail below, and in the specific methods for individual figures. They
497 decompose similarly to the actuator arm network weights in equations (6)-(9). The supervisor used to
498 train the RD network is a bank of oscillators:

$$s_{RD}(t) = \cos(2\pi\theta_S t + \beta_i^{RD}) \quad (17)$$

499 where ϕ_i is a uniformly distributed $[0, 2\pi]$ phase preference for each oscillator. The phase preferences
500 β_i^{RD} are randomly drawn from a uniform distribution on the interval $[0, 2\pi]$. All weight matrices that
501 we consider are dimensionless with the units of current (pA) carried by the synaptically filtered spike
502 trains $\mathbf{r}(t)$ (see Equation (5)).

503 The Read/Write Head Network

504 The read/write Head network is a population of leaky integrate-and-fire 2000 inhibitory (RW_I) and
505 2000 excitatory (RW_E) leaky-integrate-and-fire neurons:

$$\begin{aligned} \tau_m \dot{v}_i^{RW_I} &= -v_i^{RW_I} + RI^{RW_I} + R \sum_{j=1}^{N_{RW_E}} \omega_{ij}^{RW_I, RW_E} r_j^{RW_E}(t) + R\zeta_i(t), \quad i = 1, 2, \dots, N_{RW_I} \\ \tau_m \dot{v}_i^{RW_E} &= -v_i^{RW_E} + RI^{RW_E} + R \sum_{j=1}^{N_{RW_I}} \omega_{ij}^{RW_E, RW_I} r_j^{RW_I}(t) \\ &+ R \sum_{j=1}^{N_{RD_E}} \omega_{ij}^{RW_E, RD_E} r_j^{RD_E}(t) + RI_{syn,i}^{RW_E, bits}(t), \quad i = 1, 2, \dots, N_{RW_E} \end{aligned}$$

506 where $\zeta_i(t)$ is an independent white noise term with mean 0 and standard deviation $\sigma = 0.2$ pA. This
507 noise term prevents the pathological synchronization of interneurons. The weight matrices ω^{RW_E, RW_I} ,
508 ω^{RW_E, RW_I} are untrained, and described below. The bits sent to the RW_E neurons are expressed as time
509 dependent currents $I_i^{RW_E, bits}(t)$. Finally, the weight matrix ω^{RW_E, RD_E} . The matrix ω^{RW_E, RW_I} is a
510 random matrix with each element drawn from a uniform distribution $[W_{RW_E, RE_I}, 0]$ where $W_{RW_E, RE_I} =$
511 $-\frac{0.04}{N_{RW_I}}$. The matrix ω^{RW_I, RW_E} is also randomly generated, on the interval $[0, W_{RW_I, RW_E}]$ where
512 $W_{RW_I, RW_E} = \frac{25}{N_{RW_E}}$.

513 The weights from the rotating disk excitatory neurons to the read/write head excitatory neurons
514 are learned with a Hebbian-plasticity based learning rule ([63]):

$$\Delta\omega^{RW_E, RD_E} = \epsilon r^{RW_E}(t) (\mathbf{r}^{RD_E}(t))^T \quad (18)$$

515 For efficiency in the numerical simulations, the update rule (18) is applied every 15 dt time steps, rather
516 than every time step. The parameter ϵ acts as a learning rate for the synaptic weight adjustments and
517 controls how rapidly the weights are adjusted.

518 Maximum Likelihood Alignment

519 Identifying Repeating Trajectories

520 To maximally align the spike times, we first selected a trajectory component from the rats navigating
521 the cheeseboard maze from [38], which was restricted to 4 seconds in duration and contained large, linear
522 movements along the maze within those 4 seconds. The initial selected trajectory $\mathbf{p}_0 = (x_0(t), y_0(t))$,
523 for $t \in [t_0, t_0 + 4]$ was then used as a motion template, where other $\mathbf{p}_j = (x_j(t), y_j(t))$, $t \in [t_j, t_j + 4]$ were
524 found by proximity to the initial template via the L_2 norm:

$$d_j = F(t_j) = \|\mathbf{p}_j - \mathbf{p}_0\|^2 \quad (19)$$

525 The trajectory components were found by treating t_j as a continuous variable, τ and floating τ over
526 the entire interval, τ :

$$F(\tau) = \int_0^4 \left[(x(t + \tau) - x_0(\tau))^2 + (y(\tau) - y_0(\tau))^2 \right] dt$$

527 Then, a peak detector algorithm (*findpeaks*, MATLAB 2020a), was used to detect local minima in $F(\tau)$
528 (maxima in $-F(\tau)$). Only the top 70% of these peaks were used, as $F(\tau)$ may contain local minima
529 that are dissimilar from the initial trajectory x_0 . The set of minima of $F(\tau)$, correspond to the discrete
530 times $\tau_1^*, \tau_2^*, \dots, \tau_m^*$.

Parameter Value	RD-E	RD-I	AA-E	AA-I	R/W-E	R/W-I
N	2000	2000	2000	2000	4000	4000
t_{ref}	2 ms	2 ms	2 ms	2 ms	2 ms	2 ms
t_m	10 ms	10 ms	10 ms	10 ms	10 ms	10 ms
I^α		-25 pA/-40 pA	-40 pA	-40 pA	-41 pA/-42 pA	-40 pA
v_{reset}	-65 mV	-65 mV	-65 mV	-65 mV	-65 mV	-65 mV
$v_{threshold}$	-40 mV	-40 mV	-40 mV	-40 mV	-40 mV	-40 mV
τ_d	20 ms	20 ms	20 ms	20 ms	20 ms	20 ms
τ_r	2 ms	2 ms	2 ms	2 ms	2 ms	2 ms

Table 1: The parameters used for the rotating disk (RD), actuator arm (AA) and read/write (R/W) Head networks, unless otherwise specified by the supplementary methods for each figure. The bias currents, I^α , $\alpha = RD, AA, R/W$ vary to change the operational modes of the different sub-networks. Note that the nominal values of the bias currents may differ in specific figures/subfigures.

531 Performing the Maximum Likelihood Alignment

532 With the trajectory alignment times τ_j^* , $j = 1, 2, \dots, m$ determined, a single neuron was selected for
 533 target alignment. The spikes for the m trials of that neuron were selected, and a kernel density
 534 estimator was constructed, $\rho_{ner}(t)$, where $t \in [0, 4]$. The bandwidth of the Gaussian kernel was taken
 535 to be 5 milliseconds. Then, for the m trials, a random $m \times 1$ vector of m time shifts was generation.
 536 Each scalar component of this vector would shift all the spikes within one of the $j = 1, 2, \dots, m$
 537 trials by a constant amount, τ_j^{shift} , $j = 1, 2, \dots, m$. The goal of these random time-shifts is to determine
 538 the time-shift vector $\boldsymbol{\tau}$ which would minimize the following quantity, commonly referred to as the cross
 539 entropy:

$$L(\boldsymbol{\tau}^{shift}) = \int_0^4 \rho_{ner}(t) \log(\rho_{ner}(t)) dt \quad (20)$$

540 Minimizing the cross entropy is mathematically equivalent to maximizing the log-likelihood-function
 541 with the vector of shift times $\boldsymbol{\tau}^{shift}$ serving as the parameters. This procedure is commonly referred to
 542 as Maximum-Likelihood Estimation (MLE) of parameters. We remark that alternative methods can
 543 also be used to align the spike times [113].

544 To minimize the cross entropy, we employed an iterative stochastic gradient descent based algorithm.
 545 At every time point, the best shift vector, $\boldsymbol{\tau}_{opt}^{shift}$ so far is perturbed:

$$\boldsymbol{\tau}_p^{shift} = \boldsymbol{\tau}_{opt}^{shift} + \gamma \cdot \mathbf{p} \quad (21)$$

546 where γ is a variable learning rate, and \mathbf{p}_n is an $m \times 1$ normally distributed random vector, from a
 547 standard normal distribution. The optimal time shift is then updated as:

$$\left[\boldsymbol{\tau}_{opt}^{shift} \right]_{n+1} = \left[\boldsymbol{\tau}_{opt}^{shift} \right]_n - \gamma \left(L(\boldsymbol{\tau}_{opt}^{shift}) - L(\boldsymbol{\tau}_p^{shift}) \right) \mathbf{p} \quad (22)$$

548 which serves to estimate the gradient of L as a function of $\boldsymbol{\tau}^{shift}$ and descend the gradient in one-step.
 549 This stochastic gradient algorithm is run for $n = 10^4$ iterations for all animals, with an initial $\gamma = 10^{-1}$.
 550 Every 10^3 time steps, γ is halved. This slows down the learning rate of this stochastic algorithm for
 551 longer times, and yields more precise solutions to $\boldsymbol{\tau}_{opt}^{shift}$.

552 With the MLE alignment parameters $\boldsymbol{\tau}_{opt}^{shift}$ determined for a single neuron on $[t_j, t_j + 4]$, for the
 553 $j = 1, 2, \dots, m$ trials, the spike times are then shifted by the j the component τ_{opt}^{shift} . The τ_{opt}^{shift}
 554 determined for aligning a single neuron is used for all neurons in the spike-raster plot with spikes son
 555 $[t_j, t_j + 4]$.

556 Specific Methods for Figures/Supplementary Figures

557 Figure 2

558 In Figure 1E-F, the rotating disk network is simulated for a total time of 10 seconds with the θ_{RW}
559 oscillator input off. For Figure 1I-K, the θ_{RW} oscillatory input is turned on for 2 seconds in the interval
560 [3, 5]s. All neurons in the rotating disk network additionally receive a white noise current with mean
561 0 and standard deviation 0.5 pA, to mimic the stochastic firing/SWR initiation observed during slow-
562 wave sleep. The background currents to all neurons, except to the initiators were -40 pA, while the
563 initiator neurons were set to -40.1 pA. Larger or smaller currents to the initiators controls the SWR
564 average rate, while larger/smaller currents to the RD_I neurons controls both the SWR average rate,
565 and the shape of the inter-SWR-interval distributions [63]. For all simulations in Figure 1, the RD_E
566 neurons were split into two tracks. The tracks were assigned by first constructing a random permutation
567 of the rows of ω^{RD_E, RD_I} , splitting the permutation into two sets, and then sorting the permutation
568 with respect to the θ_S phase preference. This is mathematically equivalent to splitting the population
569 of RD_E neurons into two sets, and sorting them on the unit circle. Each track had 50 initiator neurons,
570 and 950 neurons representing the sectors (phases) of the disk. The initiators in each track connect to all
571 other intra-track initiators with a strong recurrent weight ($\omega_i^{RD_{initiator}, RD_{initiator} j} = \frac{1.1}{N_{int}}$), and connect
572 to all other excitatory track neurons, and rotating disk neurons with the weight $\omega^{RD_E, RD_{initiator}} =$
573 $\omega^{RD_I, RD_{initiator}} = \frac{0.1}{N_{int}}$. To implement inter-track competition, the excitatory neurons within a track
574 project to rotating disk interneurons with randomly drawn weights from the interval $[0, 0.2 \frac{1}{N_{AAE} - N_{int}}]$.
575 In Figure 1I, we biased one of the tracks towards being preferentially activated by increasing the bias
576 currents to the track neurons, similarly to the impact the actuator arm network would have. For Figure
577 1I, In the reading mode, all initiators in Track 1 had a bias of -40.3 pA, while those in Track 2 had a
578 bias of -40.5 pA. All remaining track neurons had a bias of $I^{AAE} = -40.25pA$ and $I^{AAI} = -40.5pA$.
579 All excitatory neurons in all tracks and initiators also have a spike-frequency adaptation variable, $u_i(t)$,
580 which increases by an amount of $d = 18$ pA for every spike fired by that neuron and decays with a
581 time constant of 50 ms. The adaptation current is negatively weighted, and serves to slow down or
582 even eliminate repetitive spiking. This adaptation variable can stop a disk-rotation/sharp-wave-ripple.
583 In the writing-mode/theta-oscillation mode, the bias currents for all non-initiator excitatory neurons is
584 increased to $-6pA$. For Figure 1K-L, the total simulation time is 25 seconds, with a non-zero, constant
585 velocity ($\frac{d\psi_A}{dt} = 0.4$ given in the interval [10, 15] seconds. The AA_E in Figure 1K neurons were sorted
586 according to their phase preference, as determined in Supplementary Figure 3. For clarity, only a subset
587 of AA_E neurons (10%) are plotted. In Figure 1N, the R/W head network is simulated for 15 seconds,
588 with 5.1 seconds in the write mode. The information written to the network is a synfire-chain of spikes
589 elicited in the read/write head neurons by an external current. The external current is an additional
590 20 pA applied to each neuron for 40 ms, in sequence. In the read/SWR-mode, the bias currents to the
591 read/write head neurons are $I^{R/W_E} = -41pA$, $I^{R/W_I} = -40pA$. In the write/theta-oscillation mode,
592 the bias currents are unchanged for the R/W neurons, while $I^{RD_E} = -6pA$ and $I^{RD_I} = 15pA$. For
593 both the read and write modes, the bias current for the initiators was -40.3 pA.

594 Figure 3

595 The actuator arm, rotating disk, and read/write head networks were combined into a single network with
596 a 10 second long simulation time. The actuator arm network was identical to all previously considered
597 (Figure 2, supplementary figures) implementations. The signal actuator arm position was generated
598 as the integral of a constant signal, $\psi_A = \frac{2}{dt(t_2 - t_1)}$, over the interval $[t_1, t_2]$ with $\dot{\psi}_A = 0$ outside this
599 interval. The desired actuator arm position is initialized with $\psi_A(t) = -1$ for $t < t_1$. This initialization
600 results in a linearly ramping ψ_A from -1 to 1 on $[t_1, t_2]$, and $\psi_A = 1$ for $t > t_2$. The values $t_1 = 1$,
601 and $t_2 = 2.2$ were used. The rotating disk was as in Figure 1, only with a single track and a single
602 population of initiators. The intra-initiator weight was also stronger, $\omega_i^{AA_{initiator}, AA_{initiator} j} = \frac{1.4}{N_{int}}$,
603 which was necessary to generate replays in the read/write head. The adaptation step size was also
604 increased to $d = 22pA$ in all rotating disk pyramidal neurons, to terminate sharp-wave bursts. The
605 actuator arm projects random, sparse excitatory connections to all non-initiator neurons in the rotating

606 disk network. The weights are drawn from a uniform distribution with mean $[0, \frac{0.1}{\sqrt{N^{AAE}}}]$ with probability
607 $p = 0.1$ and 0 otherwise. The actuator arm also projects to a set of 500 leaky-integrate-and-fire neurons
608 which act to generate the synfire chain of spikes, which serves as the bits to be written in this example.
609 This network has identical parameters as all other neurons, only with a chain of connectivity between
610 these neurons to trigger the synfire chain of spiking. Each neuron in the chain is connected to the
611 next with a synaptic weight of 0.2. These neurons receive a strong hyperpolarizing bias current of
612 $-60.09pA$. A 20 pA current pushes these cells in the superthreshold firing regime where the current is
613 slightly over-threshold (-40.09 pA). All other bias currents are identical to prior implementations in
614 Figure 2 for the isolated networks in reading and writing modes. The learning rate, ϵ , was 1.2×10^{-5} .

615 Figure 4

616 The spikes in figure 4H-K were mapped onto LFP phases, with the LFP taken as be the channel with
617 the most spikes detected. The raw signal was further band-pass filtered with a butterworth filter with
618 a $[4, 12]$ Hz window. The phase from the LFP was computed first via a Hilbert transform, and then the
619 application of the arctangent function (*atan2* in MATLAB 2020a). Only epochs of theta oscillations, as
620 defined in [38] were used in our final analysis. All spikes were then transformed into phases via a linear
621 interpolant (*interp1* in MATLAB 2020a). The spikes were subsequently duplicated from $[0, 2\pi]$ into
622 the intervals $[2\pi, 4\pi]$ and $[4\pi, 6\pi]$, with a simple histogram with binds of width $6\pi/100$ used to compute
623 the spike-base histograms on $[0, 6\pi]$. The neurons were subsequently ordered in sequence according to
624 increasing mean-phase. An identical protocol was applied to the rat data also.

625 Supplementary Figure 2

626 The total simulation time is 400 seconds, with the first second of simulation time used to initialize
627 the chaotic spiking neural network. Recursive Least Squares (RLS) is turned on for the next 350
628 seconds and subsequently turned off. The last 49 seconds of simulation time are used for testing the
629 network performance. The network was initialized in the balanced inhibitory regime: $\omega^{AAE,AAE} =$
630 0 , $\omega^{AAI,AAE} = 0$, that is, all initial excitatory weights were set to 0, and all excitatory weights were
631 learned. This initial state lets us constrain the firing rates of pyramidal cells to arbitrarily low rates. The
632 bias currents were $I^{AAE} = -22pA$, and $I^{AAI} = -20pA$. The initial inhibitory weights were randomly
633 generated with exactly 90% sparsity. Each of the inhibitory neurons made 200 connections, with the
634 connection strength set to $\frac{G}{\sqrt{200 \times 0.1}}$, where $G = -0.1$. We found that this was sufficient to initialize the
635 network into a chaotic spiking (prior to learning), and therefore serve as an adequate reservoir for RLS.
636 RLS was only applied every 20 time steps ($\Delta t = 20$) for efficiency, as the weight updates in equations
637 (13)-(14) are $O(N^2)$ in time complexity.

638 Supplementary Figure 3

639 The trained AA network was simulated for 200 seconds, with a randomly generated actuator arm
640 position $\psi_A(t)$ and $\dot{\psi}_A(t)$, generated similarly to the initial training supervisor. The actuator arm signal,
641 ψ_A remained on for all 200 seconds to establish the neural preference of firing to the ψ_A coordinate.
642 The locations corresponding to each spike times were determined with a linear interpolant (*interp1*,
643 MATLAB 2020a). For each neuron, a histogram was constructed with 41 bins of size 0.05, distributed
644 uniformly over the $[-1, 1]$, which is the operating range of $\psi_A(t)$. For each AAE and AAI neuron,
645 the maximum of the spike- ψ_A histogram was determined. The maxima were sorted in ascending order
646 (Figure SF 3A), and the AA neurons were re-ordered in an identical fashion to expose the weight
647 structures (Figure SF 3B-C). The bias currents were $I^{AAE} = -22pA$, and $I^{AAI} = -20pA$.

648 Supplementary Figure 4

649 The trained AA network was simulated for 50 seconds, with a randomly generated actuator arm position
650 $\psi_A(t)$ and $\dot{\psi}_A(t)$, generated similarly to the initial training supervisor. The bias currents were $I^{AAE} =$
651 $-22pA$, and $I^{AAI} = -20pA$. The position input ψ_A was turned off (set to 0) in Figure SF 4D, while

652 the velocity input was turned off in Figure SF4E after 25 seconds. In all three simulations, the same
 653 random initial seed is used to generate ψ_A and $\dot{\psi}_A$.

654 Supplementary Figure 5

655 For both simulations, the rotating disk network was subdivided into two tracks with 50 initiators each.
 656 In the reading/sharp-wave mode, the bias currents were $I^{AAE} = -22pA$, and $I^{AAI} = -20pA$, and
 657 $I^{RDE} = -40.5pA$, $I^{RDI} = -40pA$. The initiators had a bias current of -40.7 pA. In the writing/theta-
 658 oscillation mode, the BIAS currents were the same for the actuator arm networks, while $I^{RDE} =$
 659 $-9.5pA$, $I^{RDI} = 15pA$. The initiators in the writing mode were kept off with a hyperpolarizing current
 660 ($-60pA$). In the reading mode, the initiators received connections from the AA neurons. The weights
 661 were given by the scaled, FORCE-trained decoder, $\omega^{RD_{initiator},AAE} = 0.5\phi^{AA}$ for the actuator arm
 662 position variables $\hat{s}_i(t)$. The first tracks 50 initiators received 10 duplicates of the first 5 components
 663 of $\hat{s}_i(t)$, while the second tracks initiators received 10 duplicates of the last 5 components of $\hat{s}_i(t)$.
 664 In the writing/theta oscillation mode, the track neurons in the rotating disk network received scaled
 665 decoders as weights, $\omega^{RDE,AAE} = 12\phi^{AA}$. In this case, the 200 components of ϕ were duplicated 5
 666 times each, scaled up, and provided to the rotating disk network as inputs from the actuator arm
 667 excitatory neurons.

668 Supplementary Figure 8

669 The neuron consisted of 20 Poisson neurons which generate spikes stochastically as part of an inhomogeneous
 670 Poisson process. First, in each theta cycle, the individual neurons were modeled as having a
 671 phase preference in generating bursts with the preference given by:

$$pp_j(t) = \frac{1}{2\pi} + \frac{1}{4\pi} \cos(2\pi\theta_{pp}t + \phi_j^{pp}), \quad \phi_j^{pp} = 2\pi j/19, \quad j = 0, 1, 2, \dots, 19, t \in [0, 2\pi] \quad (23)$$

672 where $\theta_{pp} = 8Hz$. In each cycle, a cell is allowed to fire a single burst with a phase-centre drawn from
 673 the probability distribution $pp_j(t)$. The phase centre for the j th neuron on the i th cycle, χ_{ij} is then
 674 used to generate a burst of spikes with probability:

$$\lambda_j(t) = G_{pp} \left(\exp \left(-\frac{(t - \chi_{ij})^2}{\sigma_{pp}} \right) \right), \quad t \in [2\pi(i-1), 2\pi i] \quad (24)$$

675 where $\sigma_{pp} = 0.1$ controls the width of a burst, and $H(x)$ is the Heaviside function, and $G_{pp} = 0.4$ sets
 676 the spikes emitted per burst. The value of G_{pp} was selected so that approximately 10 spikes per burst
 677 were emitted. As measured empirically, the cells fired on average 13.11 spikes per cycle, which was
 678 estimated after 2000 cycles of firing.

679 Supplementary Materials

680 Supplementary Section S1: Disk Drive Dynamics

681 A Hard Disk Drive (HDD) is a computer component dedicated to storing information. The simplest
682 type of HDD consists of two primary sub-components: A rotating disk and an actuator arm (Figure
683 S1A, [47]). The actuator arm moves across the disk to different regions as the disk is spinning at
684 a constant rotational velocity. The disk is spun by a central spindle, which we will refer to as the
685 “rotator” in the main text to avoid confusion with *sleep spindles*. The head of the actuator arm then
686 writes information onto the disk by magnetizing a layer of ferromagnetic material. The magnetic field
687 imprinted on the disk can be oriented in different directions, such as up or down, thereby allowing the
688 encoding of binary information or bits as these magnetic field directions. The head of the actuator arm
689 can then read data from the disk after the writing procedure is complete. For a fixed position of the
690 actuator arm, the region on the disk that the head can access (through disk rotation) is called a track
691 or cylinder (Figure S1A).

692 The physical components of the HDD give rise to 3 parameters that describe its function: the disk
693 rotation speed θ_S , the head writing speed θ_W , and the angle of the actuator arm ψ_A (Figure S1B).
694 The disk rotation speed is the number of revolutions per second that the disk makes, while the head
695 writing speed is the number of bits per second that the disk writes at. Finally, the actuator arm angle
696 dictates which track the head writes information to. Thus, disks operations can be summarized by three
697 parameters: two oscillation frequencies and a position variable for the actuator arm. These variables
698 map nicely onto the dominant models of hippocampal function: dual oscillators and attractor networks.

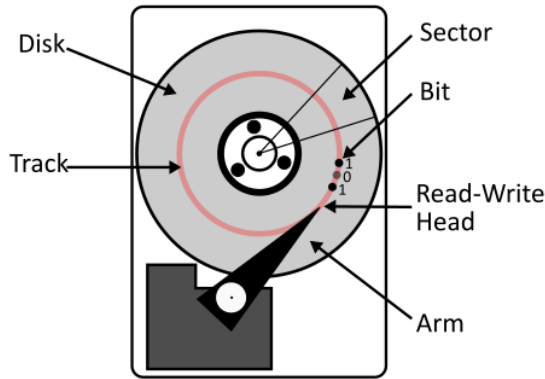
699 For a standard HDD, we expect that the write speed is significantly faster than the disk rotation
700 speed ($\theta_{R/W} \gg \theta_S$, Figure S1C). In this regime, each write cycle of $\theta_{R/W}$ takes place when the disk has
701 advanced to the next sector on the track. Thus, the exact sequence of bits is maintained and encoded
702 onto the disk on the corresponding sequence of sectors. Then, after writing has concluded, the sequence
703 of bits in a single track can be read in a single revolution of the disk (Figure S1D). Thus, hard drives
704 preserve the sequence of bits, and maximize the number of bits written to a track by ensuring that the
705 write speed is significantly faster than the disk rotation speed. We will refer to this as the Hard Drive
706 “nominal” parameter regime, as this is how a typical HDD stores information.

707 However, the sequence of bits can still be encoded with slower writing speeds. In particular when
708 the disk writing speed ($\theta_{R/W}$) is slightly slower than the disk spinning speed (θ_S), $\theta_{R/W} = \theta_S - \epsilon$,
709 sequences of bits can still be written to a track (Figure S1E). In this regime, the disk makes slightly
710 more than a full revolution in between write cycles. This ensures that the head writes to the next
711 sector of the track, even under the constraint that the writing speed and disk rotation speeds are
712 similar. Once again, a single revolution of the disk can subsequently read all the information in a track.
713 We refer to this operating range, when $\theta_{R/W} = \theta_S - \epsilon$ for a small frequency difference ϵ as the Hard
714 Drive “precession” parameter regime. Typical HDDs do not operate in this regime.

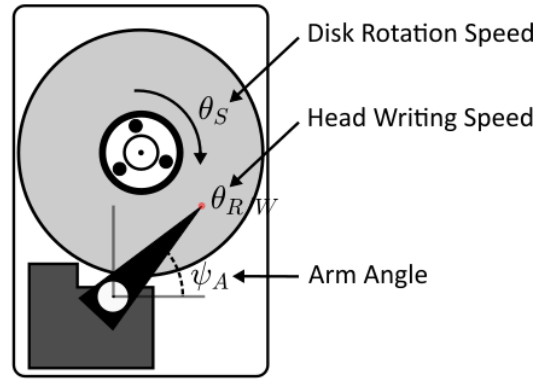
715 When the writing speed is slightly faster than the rotation speed ($\theta_{R/W} \approx \theta_{R/W} + \epsilon$), the disk makes
716 slightly less than a full revolution in between write cycles. This implies that rather than writing to the
717 next sector of the track, the head writes the next bit to the previous sector of the track (Figure S1F).
718 The effect of this operating regime is to encode information in the reverse order that it was observed.
719 A single revolution in a read cycle can then read the entire bit sequence in reverse. We refer to this as
720 the Hard Drive “recession” parameter regime. As in the HDD precession regime, typical HDDs do not
721 operate in this regime.

722 Finally, when the head has saturated a track with a sufficiently long sequence of bits, the only
723 recourse to store more information is to change ψ_A and encode more information onto new tracks
724 (Figure S1G). Thus, multiple read cycles are now necessary to read out sequences that can not fit on
725 a single track (Figure S1H). In particular, the disk must complete multiple full rotations, with the
726 actuator arm switching between tracks in each rotation.

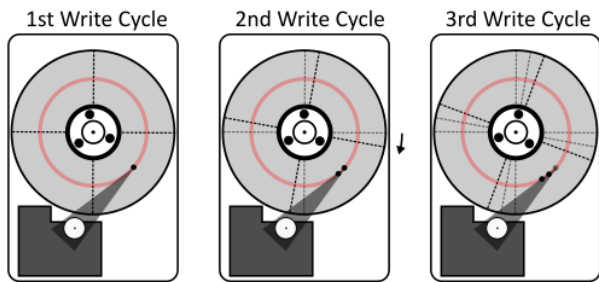
A Hard Disk Drive Components



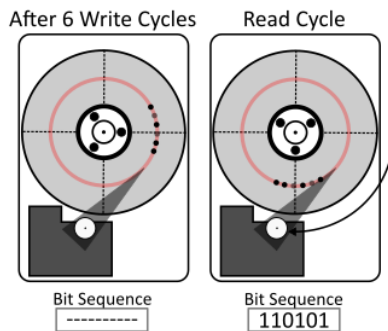
B Functional Parameters



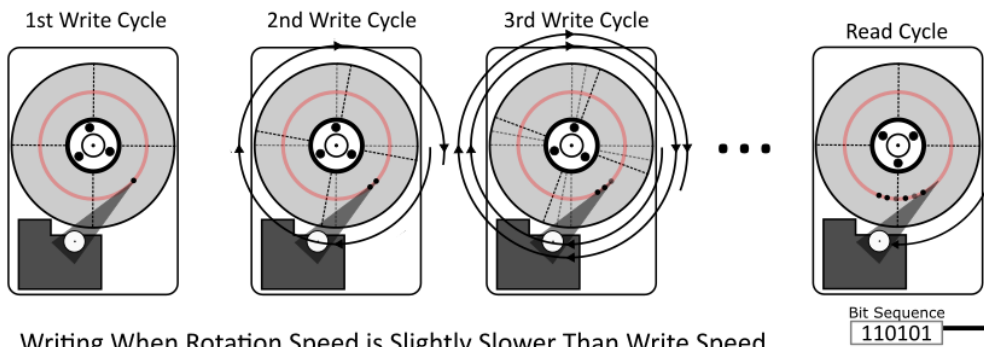
C Writing to the Disk when $\theta_{R/W} \gg \theta_S$



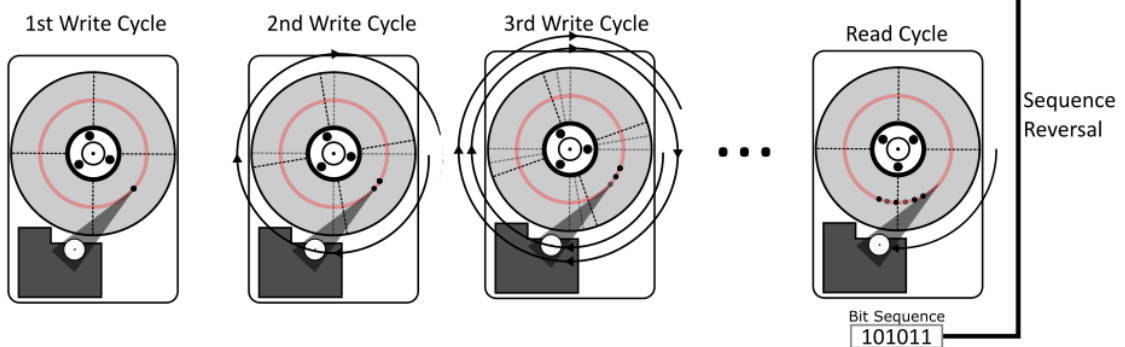
D Reading from the Disk in one Rotation



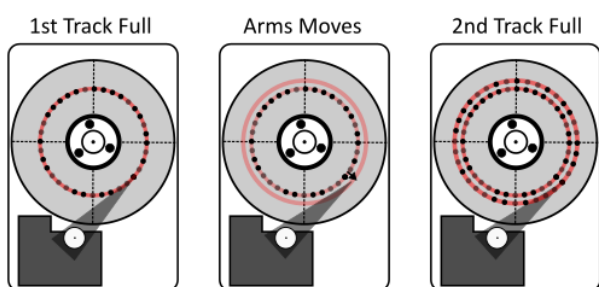
E Writing When Rotation Speed is Slightly Faster Than Write Speed



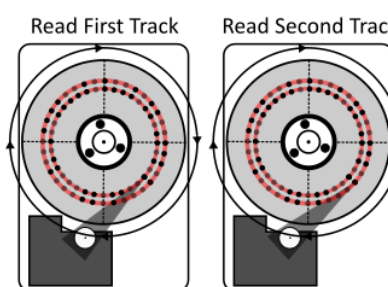
F Writing When Rotation Speed is Slightly Slower Than Write Speed



G Writing to Multiple Tracks on the Disk

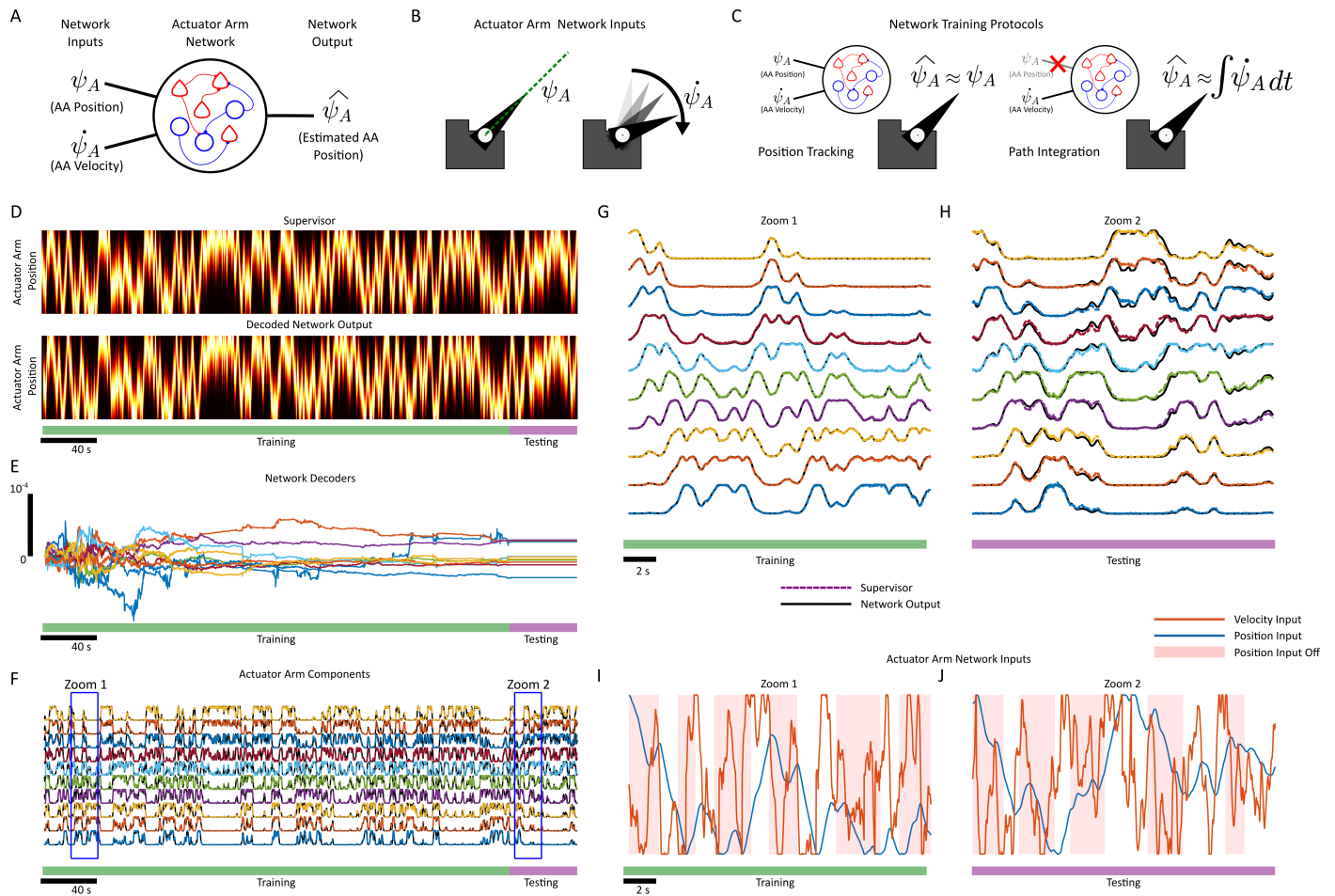


H Reading from the Disk with >1 Spin



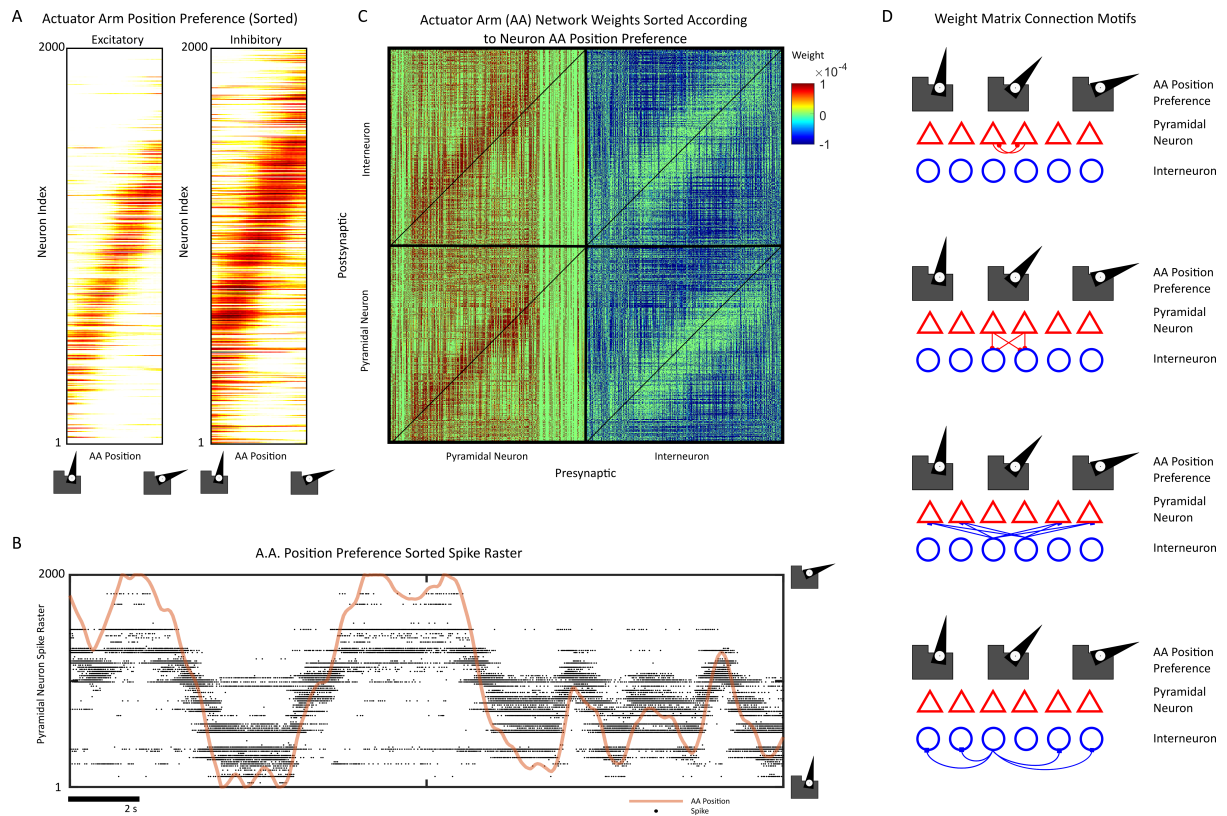
727 **Supplementary Figure 1: The Operations of a Hard Disk Drive**

728 **(A)** A Hard Disk Drive (HDD) schematic with its basic components. The rotating disk is used to store
729 information onto tracks, which correspond to a circular segment of the disk. A sector on the disk is a
730 wedge shaped region formed by any two radii. The actuator arm points to discrete tracks, which are
731 the regions of the disk a stationary actuator arm has access by virtue of disk rotation alone. Bits are
732 encoded onto the rotating disk by a read/write head on the apex of the actuator arm. The bits are
733 encoded by inducing magnetic fields of different directions onto a ferromagnetic material on the disk
734 surface. **(B)** The functional parameters of an HDD that describe its state: the disk rotation speed
735 (θ_S), the angular position of the actuator arm, ψ_A , and the head writing speed, $\theta_{R/W}$. **(C)** When the
736 write speed $\theta_{R/W}$ is substantially faster than the disk rotation speed, bits are written continuously as
737 the disk spins. **(D)** Written bits on a single track can be read in a single revolution of the disk by the
738 read/write head. **(E)** Bit sequences can also be written when $\theta_{R/W} = \theta_S - \epsilon$, where the writing to a
739 track corresponds to slightly greater than a full revolution of the disk. The written information can
740 be read in a single disk rotation, as in **(D)**. **(F)** When the disk rotation speed is slightly slower than
741 the writing speed, the disk has advanced slightly less than a full revolution when a bit is written. This
742 results in the bits being encoded and readout in the reverse order with which they were encoded. **(G)**
743 When a track is saturated with bits, the actuator arm must move to the next track on the disk to write
744 additional information. **(H)** Long sequences of bits must be written to multiple tracks. In order to
745 access this information, multiple disk rotations are required with the actuator arm adjusting between
746 tracks. This results in multiple read cycles/disk rotations required to readout information.



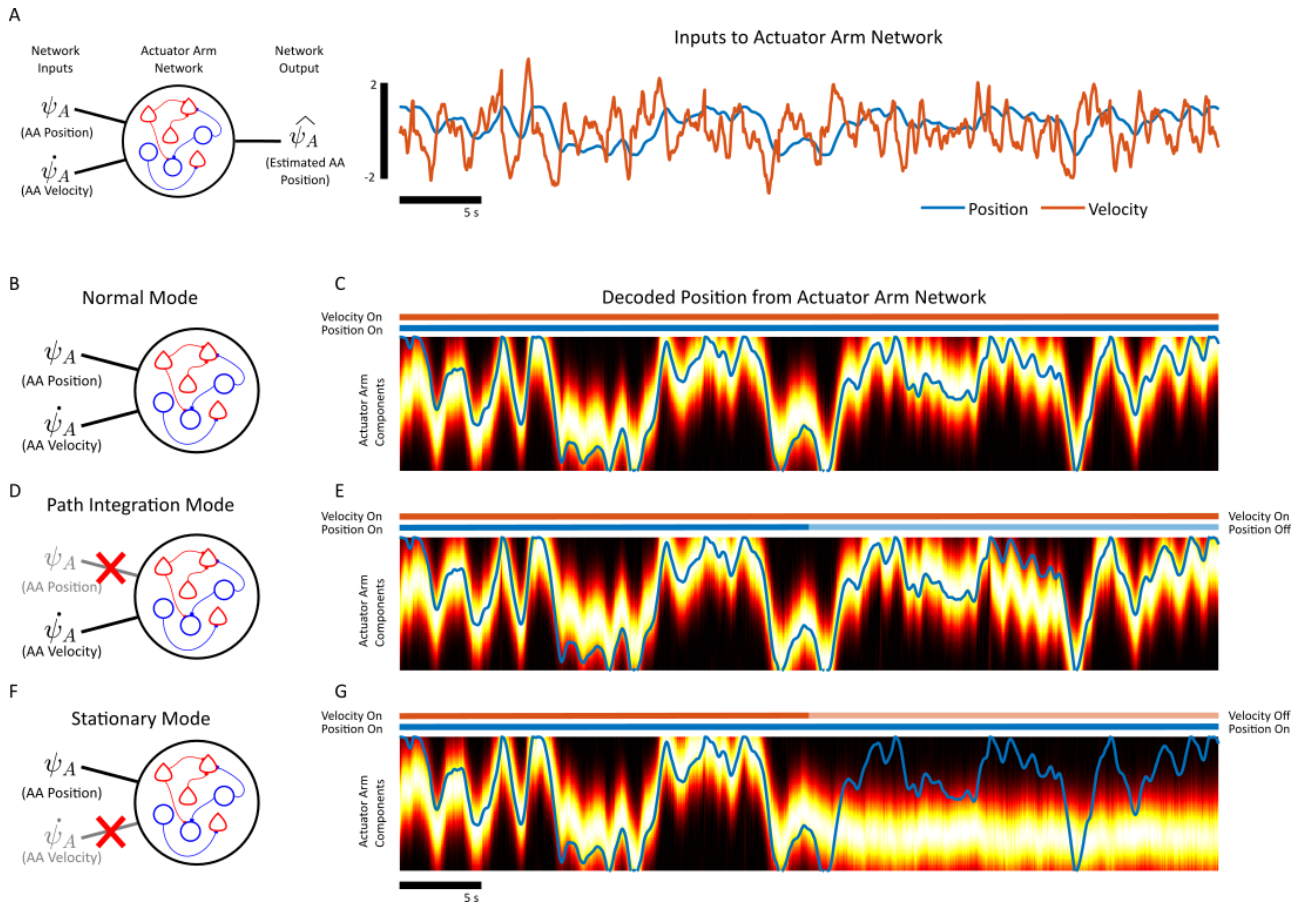
747 **Supplementary Figure 2: FORCE Training the actuator armNetwork**

748 **(A)** A network of 2000 leaky integrate-and-fire pyramidal neurons and 2000 leaky-integrate-and-fire
749 interneurons is collectively trained to estimate the position of the actuator arm, $\hat{\psi}_A$, on a disk drive. **(B)**
750 The network receives two inputs, a position variable ψ_A , and a velocity variable $\dot{\psi}_A$. **(C)** The position
751 variable and velocity variables can set the position of the actuator arm in the actuator armnetwork.
752 When both inputs are on, the network outputs an actuator arm position ($\hat{\psi}_A$) as determined by its
753 position input ψ_A . When the position input is off, the network is trained to integrate the velocity input
754 $\dot{\psi}_A$. **(D)** The supervisor used to train the actuator arm(AA) network consists of 100 components that
755 encode actuator arm position, forming a high-dimensional supervisor [80]. The components are sorted
756 according to the supervisor-encoding preferences with respect to the actuator arm position ψ_A . As a
757 result, the supervisor forms a bump of activity, indicating the actuator arm position $\hat{\psi}_A$. The training
758 signal is randomly generated, with the actuator arm position being randomly dropped during training.
759 FORCE training was as applied for 230 seconds (green), after the first second of simulation time to
760 remove any transients. FORCE training was turned off in the last 19 seconds (purple) of the simulation.
761 The supervisor (top) and decoded network output (bottom) are shown as heat maps. **(E)** A subset of
762 the decoders, ϕ_i , $i = 1, 2, \dots, N$ during (green) and after (purple) FORCE training. **(F)** A subset of
763 the 100 supervisor components from (A)-(B). The network output is plotted as coloured dashed lines,
764 while the supervisor components are in solid black. **(G)** A 10 second zoom of subset of the supervisor
765 and network components while the FORCE training is on. **(H)** A 10 second zoom of the subset of the
766 supervisor and network components while FORCE training is off. **(G)** The inputs (AA velocity in blue,
767 AA position in orange) to the network during training. The red shading denotes the position input
768 was turned off during this time period. **(H)** The inputs (AA velocity in blue, AA position in orange)
769 to the network after training. The red shading denotes the position input was turned off during this
770 time period.



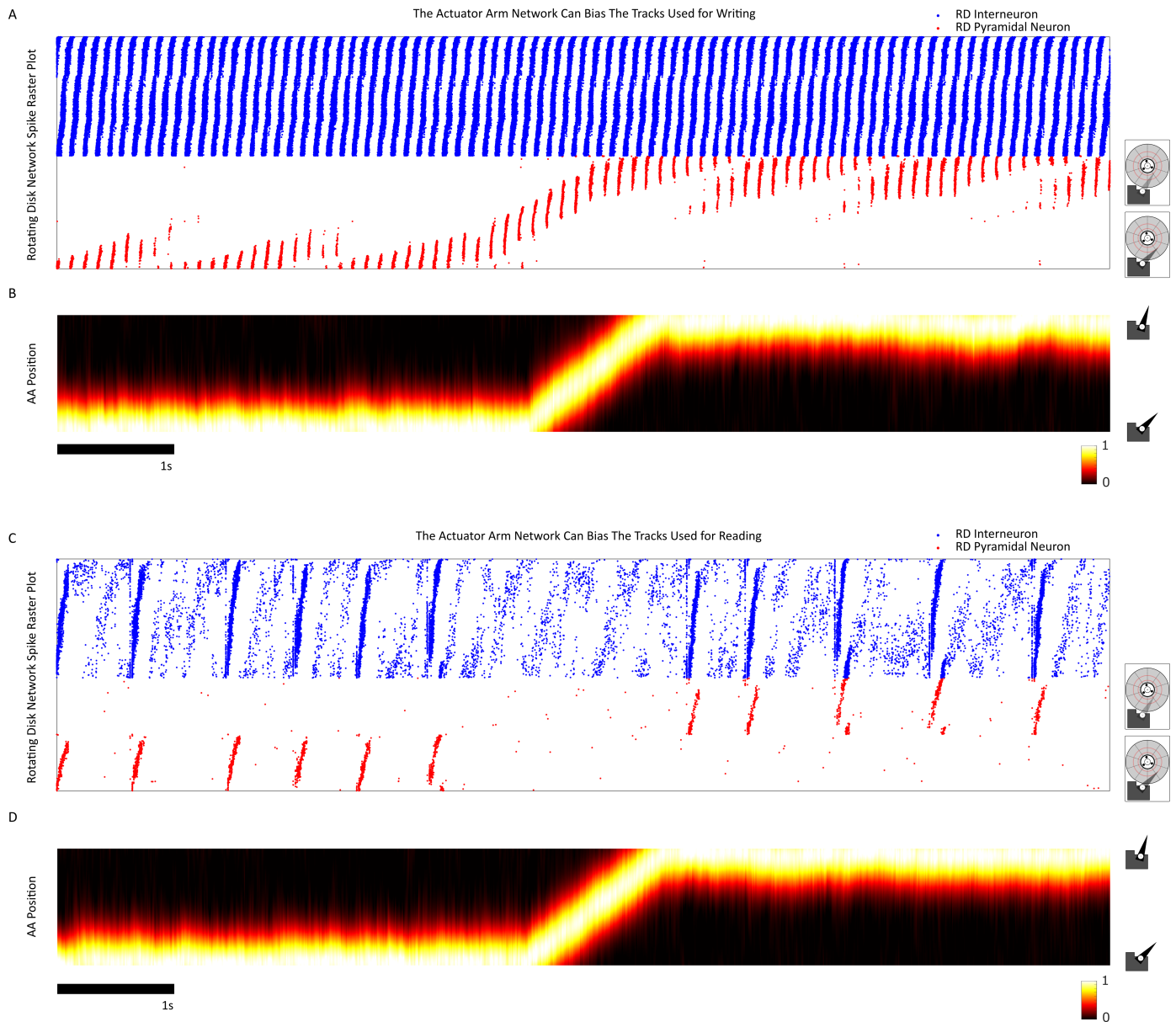
771 **Supplementary Figure 3: Weight Structure of the Force Trained actuator armNetwork**

772 **(A)** The 2000 pyramidal neurons (left) and 2000 interneurons (right) are plotted in order of their firing
773 preference with respect to the actuator arm position ψ_A . The preference is estimated by binning the
774 spikes into a histogram on the actuator arm range $\psi_A \in [-1, 1]$. Both interneurons and pyramidal
775 neurons display firing preferences with respect to ψ_A , although the pyramidal neuron preferences are
776 narrower. **(B)** A spike raster plot of the pyramidal neurons sorted according to the firing preference
777 in ψ_A . The position is scaled and overlaid for comparison. **(C)** The ψ_A -preference sorted weight
778 matrices for the pyramidal and inhibitory neurons. Excitatory connections, which are exclusively made
779 by pyramidal neurons are plotted by red, while inhibitory connections, are plotted in blue. All EE ,
780 EI , II , and IE weight matrices show clear banding along the main diagonal. Note that the green
781 vertical band is caused by a sub-population of pyramidal neurons that did not fire strongly during
782 navigation, and as a result, do not display strong place-preferences and thus continue the banding
783 structure. These neurons were also sorted to the top 400 pyramidal neurons due to the MATLAB
784 *sort* function. **(D)** The four connection motifs created by FORCE training and shown in the banding
785 structure. Pyramidal neurons tend to excite pyramidal neurons with similar ψ_A preference. Pyramidal
786 neurons excite interneurons with similar ψ_A preference. Interneurons inhibit pyramidal neurons with
787 different ψ_A preference. Interneurons inhibit interneurons with different ψ_A preference.



788 **Supplementary Figure 4: Operating Modes of the actuator armNetwork**

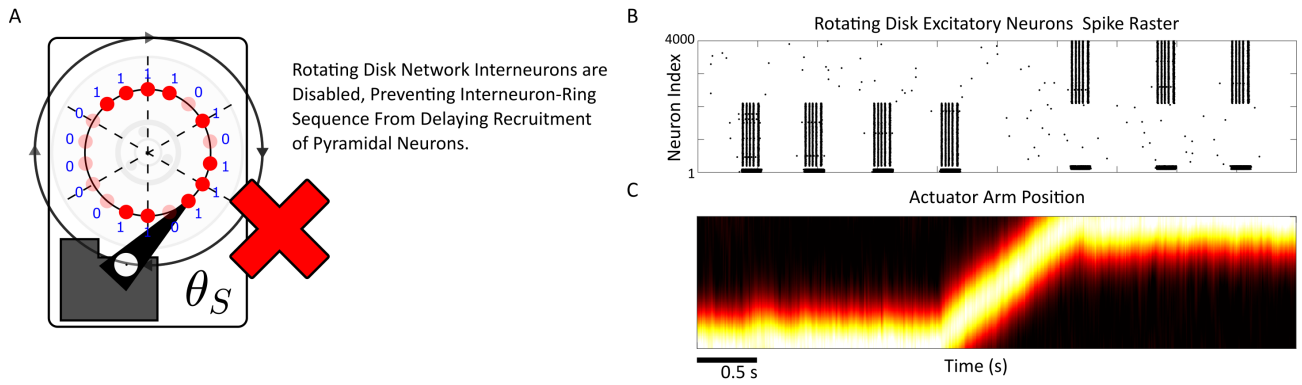
789 **(A)** The actuator arm network receives two signals, a velocity signal $\dot{\psi}_A$ (orange) and a position signal
790 ψ_A (blue). The position signal denotes the desired position of the actuator arm. The velocity signal is
791 randomly generated, while the position signal is the integral of the velocity signal. **(B)** Under “normal
792 operating mode”, the position signal and velocity signal are both applied. **(C)** The position of the
793 actuator arm $\hat{\psi}_A$ closely tracks the desired position ψ_A for a 50 second simulation of the AA network
794 **(D)** In the path integration mode, the position signal ψ_A is not present. **(E)** The position signal is
795 dropped in the last 25 seconds of simulation. The actuator arm network still tracks the desired position
796 of the actuator arm by integrating the velocity signal $\dot{\psi}_A$. **(F)** In the stationary mode, the velocity
797 signal $\dot{\psi}_A$ is dropped while the position signal is still applied. **(G)** The velocity signal is dropped in
798 the last 25 seconds of the simulation. The network interprets this drop as a velocity of 0, and as such,
799 retains an approximation of the last known position of the actuator arm.



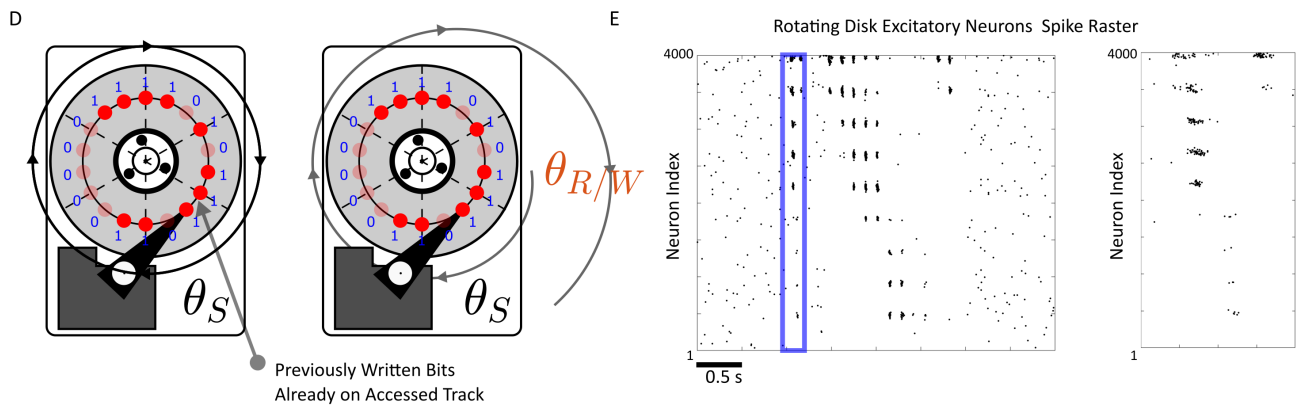
800 **Supplementary Figure 5: The actuator arm network can select tracks on the rotating disk Network**

801 **(A)** The spike raster plot for the excitatory (red) and inhibitory (blue) spikes for the rotating disk
802 network while both oscillations θ_S and $\theta_{R/W}$ are operating. The rotating disk network is separated
803 into two tracks with 950 neurons each. **(B)** The decoded position of the actuator arm. **(C)** The spike
804 raster plot for the rotating disk network when only the θ_S oscillation is transiently induced by noise.
805 **(D)** The position of the actuator arm.

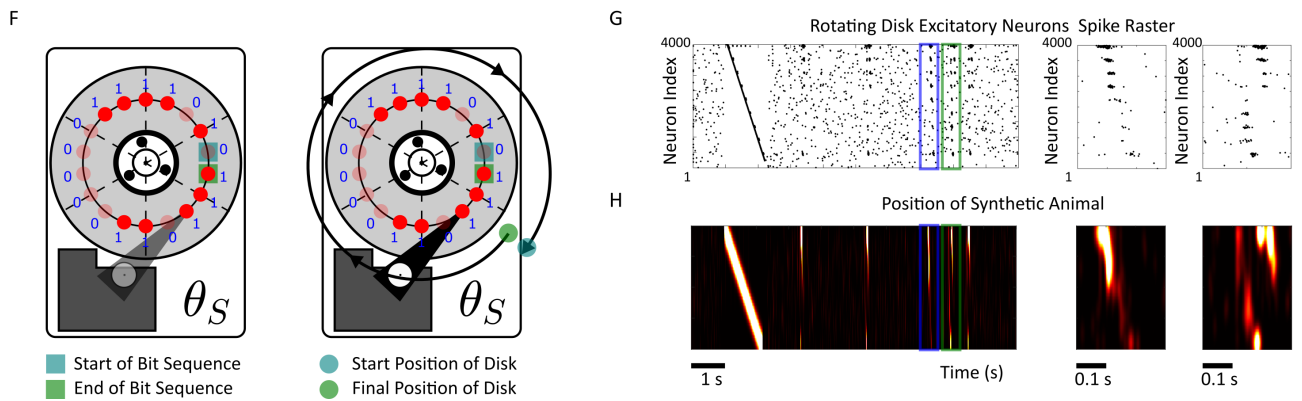
Reactivation: Disabling the Rotating Disk Network



Preplays: Selecting Tracks with Pre-Existing Information



Fragmented Replays: Start of Disk Rotation does not Coincide with Start of Sequence

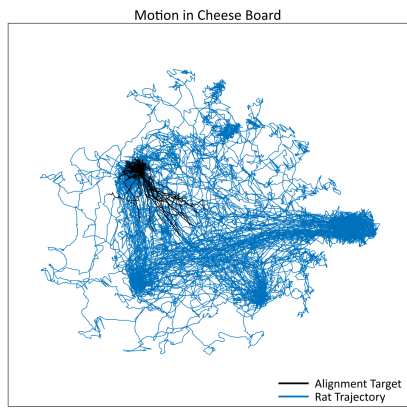


806 **Supplementary Figure 6: The Zoo of Hippocampal Data Access Methods**

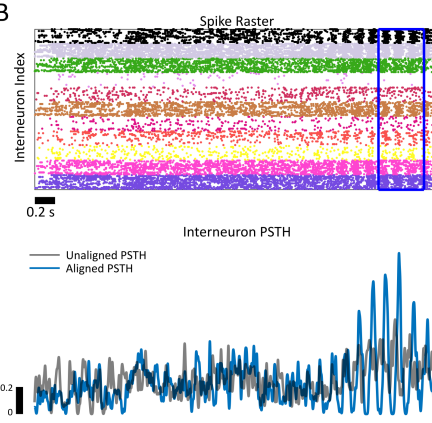
807 **(A)** By disabling the rotating disk network interneurons, the spikes in the pyramidal neurons in the
808 rotating disk network lose sequential content. **(B)** The simulated RD/AA network from Supplementary
809 Figure 5A with the RD interneurons receiving a strong hyperpolarizing current to stop their recruitment.
810 The pyramidal neurons now fire synchronized bursts in an event closer to reactivation rather than replay.
811 **(C)** The position of the actuator arm, which can serve to bias which assembly becomes reactivated. **(D)**
812 If a track already contains pre-written information, then new information can no longer be written to
813 this track without the pre-existing information being deleted. Pre-plays may correspond to this scenario
814 where tracks (SWRs) contain pre-existing information and are accessed during animal navigation. **(E)**
815 In the NDD model, when information is already stored on a track/SWR, a compressed sequence (blue)
816 occurs during a sharp-wave prior to sequential theta sequences during navigation. **(F)** For every
817 sequence written to a track on a disk drive, there is a sequence start and end bit. If the disk rotation
818 does not start with the sequence start bit, the sequence is accessed in a fragmented order. **(G)**
819 Simulation of the NDD model with two separate replay events highlighted in blue and green. The first
820 sequence, blue, corresponds to activating a track when the sequence starts. The second sequence, in
821 green, corresponds to a mismatch between when the learned sequence starts and with which initial
822 phase the rotating disk network is activated. **(H)** The decoded position of a synthetically generated
823 animal. Replays are also decoded by a simple linear decoder, with a normal replay on the left (blue),
824 and a fragmented replay on the right (green).

A

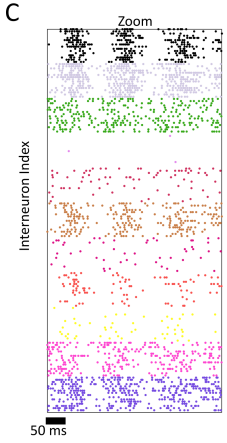
Animal 1



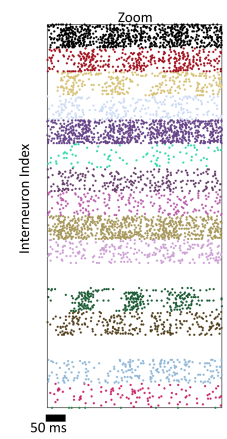
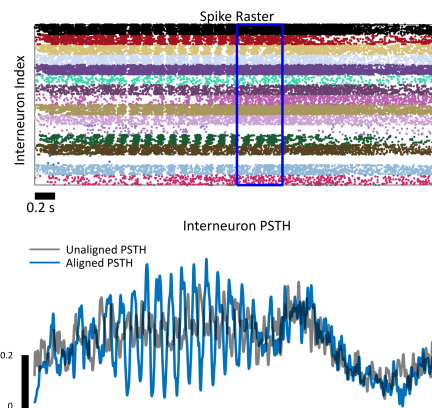
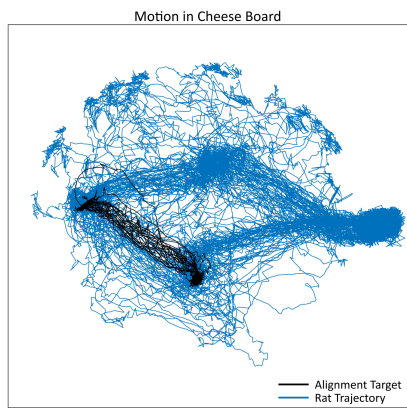
B



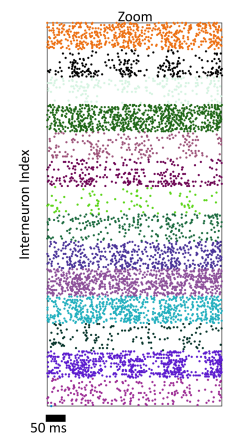
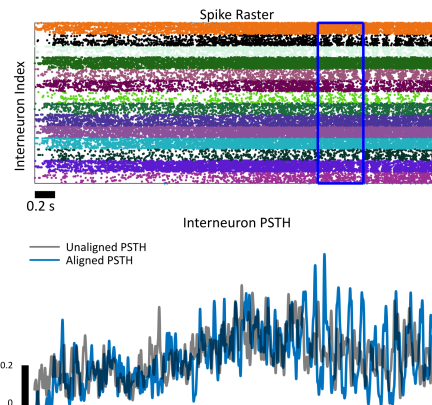
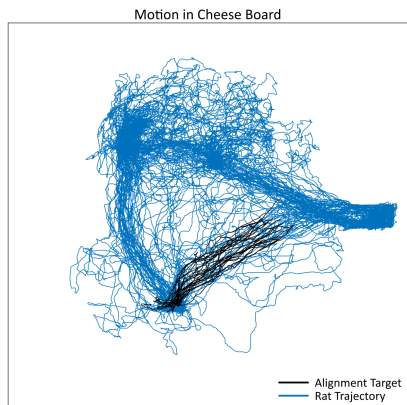
C



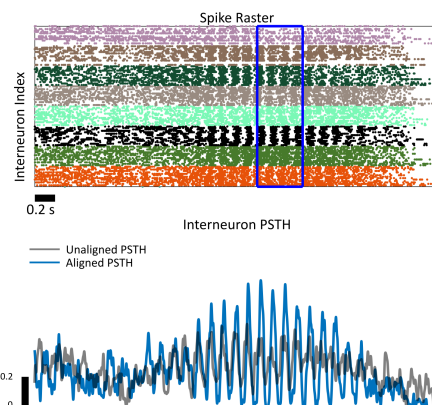
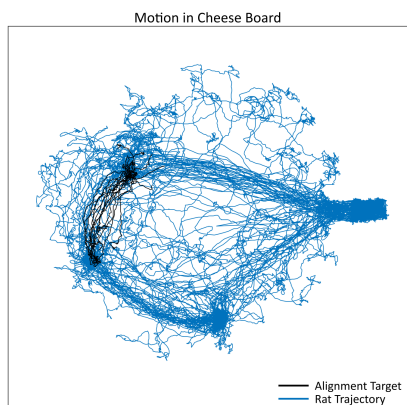
Animal 2



Animal 3



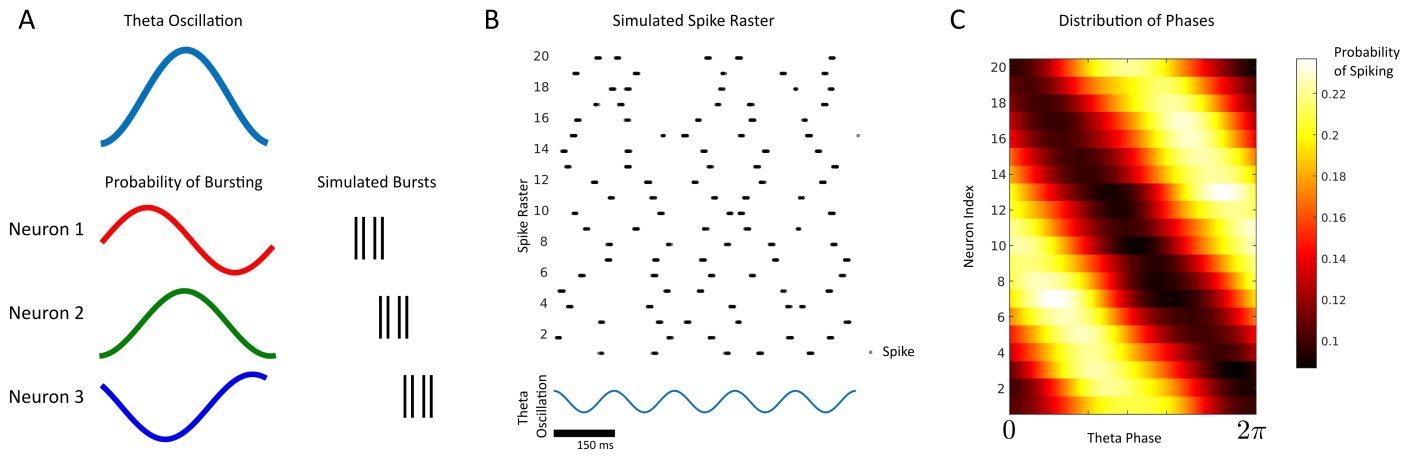
Animal 4



825 **Supplementary Figure 7: Time-Shift Alignment of Interneurons for 4 Different Animals**

826 **(A)** Motion trajectories used for each of the four animals as targets for maximum likelihood estimation
827 based alignment of the spikes. **(B)** The interneurons in the recorded animal. The black interneuron was
828 used as the target for alignment, with its determined time-shifts applied to all other interneurons. The
829 kernel density estimate of the aligned (blue) and un-aligned (black) spike density for all interneurons is
830 show. **(C)** A zoom of the interneurons during periods of high theta-locking in the spike-density. The
831 zoomed segment corresponds to the blue box in (B).

Phase Locking Without Interneuron Ring Sequences



832 **Supplementary Figure 8: Preferred-Phase Firing without Interneuron Ring Sequences**

833 **(A)** In a theta-modulated poisson-spiking model, each neuron fires a burst of spikes with a theta
834 modulated probability. The theta oscillation is used as a clock to force every cell to fire a burst, with
835 the bursts elicited with a specific phase preference. **(B)** Despite phase-preferential firing of the neurons
836 in this simulated system, a ring of bursts is not elicited. **(C)** Despite the lack of a ring of bursting,
837 each neuron in the network exhibits phase locking of their spikes onto the theta oscillation. The phase
838 locking forms a continuous ring, without interneuron-ring-sequences being elicited.

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