K.-E. Peiponen E.M. Vartiainen T. Asakura

## Dispersion, Complex Analysis and Optical Spectroscopy

**Classical Theory** 

With 46 Figures



## Contents

1.	Classical Dispersion Theory				
	1.1	Equation of Motion	1		
	1.2	Maxwell's Equations and Medium Properties	3		
	1.3	Lorentz and Drude Models for Linear Susceptibility	4		
	1.4	Wave Equation and the Complex Refractive Index	8		
	1.5	Complex Reflectivity	10		
2.	Dispersion Relations in Linear Optics 1				
	2.1	Causality	17		
	2.2	Hilbert Transforms	21		
	2.3	Kramers-Kronig Relations in Transmission Spectroscopy	25		
	2.4	Multiply-Subtractive Kramers-Kronig Relations	29		
	2.5	Imaginary Angular Frequencies	31		
	2.6	Kramers-Kronig Relations in Reflection Spectroscopy	34		
	2.7	Kramers-Kronig Relations for the Effective Optical			
		Constants of Two-Phase Nanocomposites	39		
	2.8	Dispersion Relations in Magneto-Optics	42		
3.	Dispersion Relations in Nonlinear Optics 4				
	3.1	Hyperpolarizability	47		
	3.2	Anharmonic Lorentz Oscillator Model	49		
	3.3	Nonlinear Susceptibilities and Causality	52		
	3.4	Dispersion Relations for Holomorphic Nonlinear Susceptibilities	52		
	3.5	Examples of Meromorphic Nonlinear and Total Susceptibilities	55		
	3.6	Dispersion Theory of Meromorphic Susceptibilities	58		
4.	Cor	nformal Mappings in Analysis of Optical Spectra	61		
	4.1	Conformal Mappings	61		
	4.2	Laurent Series Expansion of Complex Refractive Index			
		in Unit Disk	63		
	4.3	Phase Retrieval Using Laurent Series Expansion			
		of the Complex Derivative of Normal Reflectance	66		
	4.4	Conformal Mapping in Description			
		of Oblique-Angle Reflectivity	68		

## VIII Contents

	4.5	Conformal Mapping of Nonlinear Susceptibilities	69	
	4.6	Conformal Mapping in Analysis of the Modulus		
		of Nonlinear Susceptibility	73	
5.	Ma	ximum Entropy Method	79	
	5.1	Maximum Entropy Model	79	
	5.2	Phase Retrieval Procedure	85	
	5.3	Applications in Reflection Spectroscopy	88	
	5.4	Applications in Nonlinear Optical Spectroscopy	90	
6.	Sur	n Rules	97	
	6.1	f-Sum Rule and Average-Optical Constant Sum Rules	97	
	6.2	Sum Rules for the Powers of the Complex Refractive Index	100	
	6.3	Sum Rules for the Powers of the Complex Reflectivity	102	
	6.4	Sum Rules in Magneto-Optics	106	
	6.5	Sum Rules in Nonlinear Optics	107	
	6.6	Poles and Zeros of Meromorphic Susceptibility	111	
Ar	peno	dices	113	
_	Â	Cauchy's Integral Theorem	113	
	В	Theorem of Residues		
	C	Jordan's Lemma		
	D	Phase Retrieval for Oblique-Angle Reflectance: s-Polarization	115	
	$\mathbf{E}$	Complex Analysis with Several Variables		
	$\mathbf{F}$	Surface Integral of Generalized Cauchy Formula		
	$\mathbf{G}$	Derivation of (4.40)		
	$\mathbf{H}$	Derivation of (5.21)	120	
	Ι	Superconvergence Theorem [33]		
References				
т	J		100	