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Dispersion Loci of Guided Waves in Viscoelastic Composites of General Anisotropy

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Abstract. Guided waves play an important role in many applications of NDE to structures of flat and cylindrical geometry. In order to develop and optimise the inspection, it is essential to have a good understanding of the wave modes that can propagate in the target structure. These can be complicated, especially in structures composed of multiple layers, anisotropic properties or materials that exhibit damping that absorbs the energy of the waves. Dispersion curves in anisotropic viscoelastic media are presented here. They have been computed by using a pseudospectral collo-cation method, details of its implementation are briefly outlined and references to the relevant literature given.

INTRODUCTION

Non-destructive evaluation applications often use guided waves in order to inspect structures and the dispersion curves of these waves are crucial for understanding their physical properties and select the appropriate mode best suited to the goal of the investigation. The usage of dispersion curves for NDE is well established as the abundant literature and studies on the subject reflect: analytical solutions using potentials or the partial wave decomposition for the isotropic plate and cylinder were found by Mindlin [1] and Gazis [2] for instance. Solutions for anisotropic media in flat, and in cylindrical, geometry can be found in Solie and Auld [3], Nayfeh and Chimenti [4] and, more recently, Li and Thompson [5]. Other solutions based on the Transfer or Global Matrix method are in [6], [7] or [8] and references therein. More recently a Pseudospectral Collocation Method (PSCM) has been successfully used to find dispersion curves of guided waves in isotropic as well anisotropic elastic materials [9, 10].

The majority of the references above assume the materials to be perfectly elastic but a more realistic approach to guided waves is to consider viscoelastic materials where attenuation due to material damping is present. Attenua-tion is accounted for by the imaginary part of the wavenumber and hence complex wavenumbers must be found when viscoelastic materials are considered.

In section II an outline of the Pseudo-Spectral Collocation Method (PSCM) used for the results presented here will be given and the modifications needed in order to find complex roots outlined. Section III presents examples of the results obtained with the PSCM which have been validated both with a Semi-Analytical Finite Element simulation and with the software DISPERSE [11]. Section IV presents an example of three-dimensional dispersion curves obtained by the same PSCM scheme thus showing its potential for further and more complex applications. The paper is closed with section V where a discussion of the results and future work is given.

Pseudospectral Collocation Method for Viscoelastic Materials and Companion Matrix

The main idea behind the PSCM is that one converts a set of Partial Differential Equations (PDEs) into a matrix eigen-value problem. The solution to the PDE are the eigenvalues and eigenvectors. In order to do this, one first discretizes

the physical domain and sets up a grid of points suited for the problem at hand. For the case of bounded domains that will be studied here the *Chebyshev* grid is the best choice. Associated with this grid is a set of polynomials, in this case *Chebyshev polynomials*, out of which *Chebyshev Differentiation Matrices* (CDM) can be built [12]. By substituting each derivative in the PDE by its corresponding CDM (as in Eq. (2)) one obtains the desired eigenvalue form (Eq. (3)) of the acoustic wave equation (Eq. (1)). The reader is referred to the papers by Adamou *et al.* [9] and Hernando *et al.* [10] for a detailed account of this method and its implementation for elastic guided wave problems.

$$\nabla_{jK} c_{KL} \nabla_{Lq}^{sym} u_q = -\rho\, \omega^2\, u_j \tag{1}$$

$$\frac{\partial^{(m)}}{\partial y^{(m)}} \implies \mathbf{D}^{(m)} := [DM_{Cheb}]_{N\times N}^{(m)} \tag{2}$$

$$\mathfrak{L}(k)\, \mathbf{U} = \omega^2\, \mathfrak{M}\, \mathbf{U} \tag{3}$$

One can rewrite Eq. (3) in a different form to make the wavenumber dependence more explicit:

$$\left(\mathbf{Q}_2\, k^2 + \mathbf{Q}_1\, k + \mathbf{Q}_0(\omega^2)\right) \mathbf{U} = 0 \tag{4}$$

where prefactors of the square of the wavenumber $\mathbf{Q}_2$, of the wavenumber $\mathbf{Q}_1$ and independent of the wavenumber $\mathbf{Q}_0$ are built out of the matrices $\mathfrak{L}(k)$ and $\mathfrak{M}$.

For viscoelastic materials, the modes for a given frequency are given by the complex wavenumber whose imaginary part accounts for the attenuation of the mode. However, as can be seen in Eq. (4), the equation is not linear in the wavenumber and hence one needs to linearize it in order to obtain a general eigenvalue form equation like Eq.(3).

The linearization scheme chosen is known as the *Companion Matrix Method*, there are other options but this scheme has proved to be good for our purposes, see Bridges and Morris for further details [13]. One first defines the following companion matrices:

$$\mathbf{M}_1 \equiv \begin{pmatrix} -\mathbf{Q}_1 & -\mathbf{Q}_0 \\ \mathbf{I} & \mathbf{Z} \end{pmatrix} \qquad \mathbf{M}_2 \equiv \begin{pmatrix} \mathbf{Q}_2 & \mathbf{Z} \\ \mathbf{Z} & \mathbf{I} \end{pmatrix} \tag{5}$$

and also the *companion displacement vector field*:

$$\hat{\mathbf{U}} \equiv k\, \mathbf{U} \tag{6}$$

With these definitions, Eq. (4) can be recast as a general eigenvalue problem:

$$\mathbf{M}_1 \begin{pmatrix} \hat{\mathbf{U}} \\ \mathbf{U} \end{pmatrix} = k\, \mathbf{M}_2 \begin{pmatrix} \hat{\mathbf{U}} \\ \mathbf{U} \end{pmatrix} \tag{7}$$

where now, the wavenumber k appears as the eigenvalue. This equation is readily solved for a given real frequency ω by any of the standard libraries available for eigenvalue problems. Further details can be found in Hernando *et al.* [14].

Dispersion Curves for Guided Waves in Viscoelastic Materials: Examples

The first example is a multilayer flat system. It is composed of three Triclinic and Orthorhombic viscoelastic layers. For modelling the viscoelastic behaviour of the materials the *Hysteretic* model has been chosen where the imaginary part of the entries in the stiffness matrix c_{IJ} does not depend on the frequency. The Y axis is perpendicular to the plane of the plates and the propagation takes place along the Z axis.

Figures 1 and 2 show the dispersion curves and attenuation respectively for the aforementioned system. Re-sults given by the PSCM are shown in blue circles and the results obtained with a (Semi-Analytical Finite Element) SAFE (for this method see paper by Marzani *et al.* [15] for instance) simulation are given in red asterisks. As can be seen, the agreement is excellent.
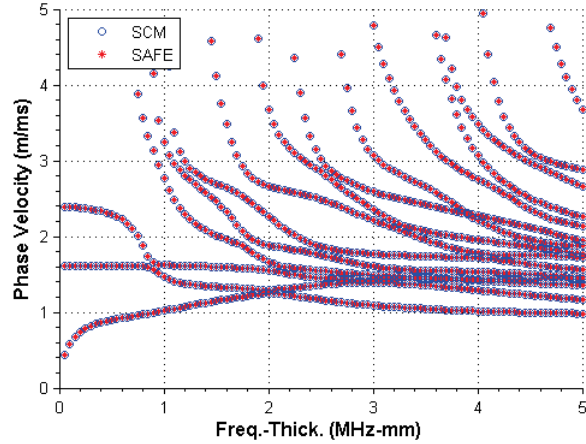
FIGURE 1. Phase velocity for the 3-layered system with hysteretic-type damping: viscoelastic 8 mm thick triclinic layer (top), viscoelastic 5 mm thick orthorhombic layer (middle) and elastic 3 mm thick triclinic layer (bottom). Note that the total thickness has been used for the x axis in the figure. Solutions are plotted as follows: PSCM (blue circles) *vs.* SAFE (red asterisks).
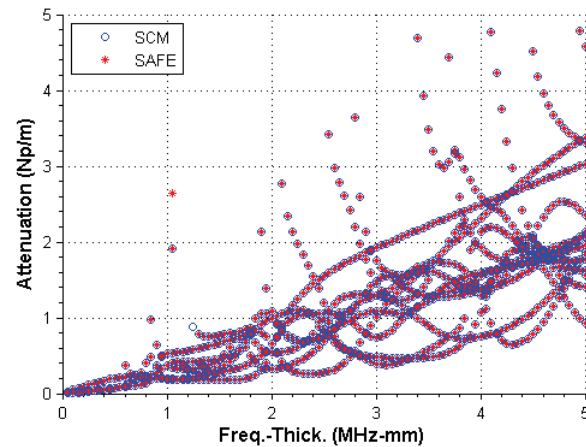

FIGURE 2. Attenuation curves for the 3-layered system of Figure 1.

The next example consists of a cylinder made of a viscoelastic material with hexagonal symmetry (transversely isotropic). In this case the *Kelvin-Voigt* model has been chosen to account for material damping. Unlike the previously used hysteretic model, now the stiffness matrix entries have an imaginary part proportional to the frequency ω.

Figures 3 and 4 display the dispersion curves and attenuation respectively for the first *flexural modes* with *harmonic order n* = 0, 1, 2, 3. The results given by the PSCM are shown in squares of different colours whereas the reference so-lution obtained with a SAFE simulation are given in asterisks. The agreement is again very good as seen in the figures.

The last example illustrates how this scheme can also be used to model viscous fluids. The results given by the PSCM have been compared to those shown in the paper by Ma *et al.* [16] which were obtained with DISPERSE.

The system is composed of an elastic steel pipe filled with glycerol. The attenuation curves and comparison with the results in the paper[16] are shown in Figure 5. Different curves have been plotted for different values of the viscosity. The agreement between the two sets of results is again excellent.
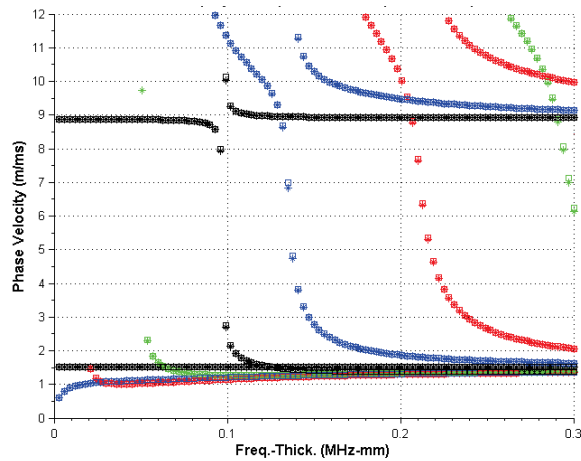
FIGURE 3. Comparison between SCM and SAFE for dispersion curves in a free Kelvin-Voigt-type viscoelastic hexagonal 15 mm thick cylinder of inner radius 50mm. PSCM: n=0 (black squares), n=1 (blue squares), n=2 (red squares) and n=3 (green squares). SAFE: asterisks with the same colour scheme as for the SCM. Fibres and propagation along the {z} axis of the cylinder.
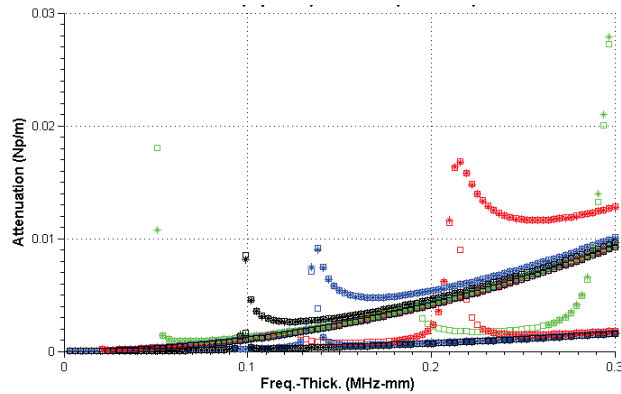

FIGURE 4. Attenuation curves for the viscoelastic hexagonal cylinder of Figure 3.

The reader is referred to the paper by Hernando *et al.* [14] for further details and examples.

Dispersion Loci in 3D

The scheme just described provides much more information than the graphs just shown and by appropriately sort-ing the solution three-dimensional dispersion curves can be easily obtained for elastic as well as viscoelastic materials.

In figure 6 the dispersion curves for the symmetric Lamb modes of an elastic steel plate are shown. This fig-ure can be seen to agree with its analogue first obtained by Mindlin [1] and later reproduced in numerous textbooks on the subject such as those by Auld [17] and Graff [18].

Discussion and Future Work

Solutions obtained with the PSCM and the Companion Matrix Method (section II) for various anisotropic materials with damping have been presented (sections III and IV) and compared to reference solutions obtained from a SAFE simulation and to solutions obtained with DISPERSE in a previous study by Ma *et al.* [16]. This approach shows potential for providing further insight into three-dimensional dispersion curves of guided wave problems as figure 6 shows.
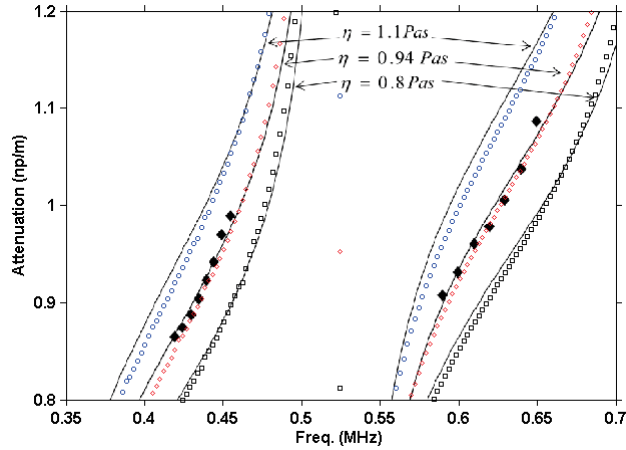
FIGURE 5. Attenuation curves for longitudinal modes in a 0.5 mm thick elastic steel pipe with 4.5 mm inner radius for three different values of the viscosity of glycerol filling the pipe. For the PSCM solution: $\eta = 0.8$ Pas (black squares), $\eta = 0.94$ Pas (red diamonds) and $\eta = 1.1$ Pas (blue circles). In the background Figure 11 of the paper by Ma *et al.*[16] is displayed. The predictions given by DISPERSE are given in solid lines and the black solid squares are the results from the experiment carried out and described in the aforementioned paper.
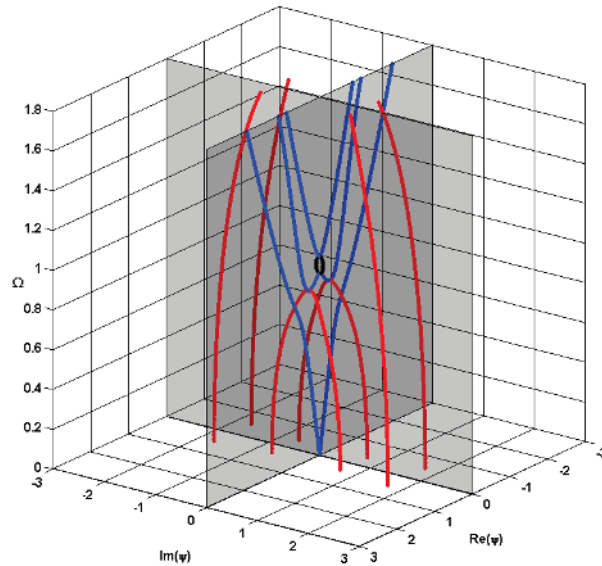

FIGURE 6. Dispersion curves for the first three symmetric Lamb modes in a free elastic steel plate computed with the SCM. Propagating modes in blue. Non-propagating modes in red and black. The non-dimensional axes are, for frequency $\Omega = h\omega / \pi V_{66}$ and for the wavenumber $\Psi = hk / \pi$.

Future work will entail exploiting these possibilities for further study of more complicated three-dimensional dispersion curves in anisotropic materials. This scheme can potentially be extended to handle leaky waves. Finally, optimization of the existing PSCM codes is under development in order to provide with an efficient and robust way of plotting the dispersion curves from the individual roots that have been found by the PSCM.

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