Dispersion Relation of Electromagnetic Waves in One-Dimensional Plasma Photonic Crystals

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The dispersion relation of electromagnetic waves in one-dimensional plasma photonic crystals is studied. The plasma photonic crystal is a periodic array composed of alternating thin plasma and dielectric material. The dispersion relation is obtained by solving a Maxwell wave equation using a method analogous to Kronig-Penny's problem in quantum mechanics, and it is found that the frequency gap and cut-off appear in the dispersion relation. The frequency gap is shown to become larger with the increase of the plasma density as well as plasma width.

Keywords:

electromagnetic wave, dispersion relation, plasma photonic crystal, frequency gap, micro-plasma

Photonic crystals, which are known to exhibit many unique features, have been recently gaining attention in the fields of solid-state and optical physics [1,2]. The technological applications of photonic crystals are expanding widely as, for example, in frequency filters, frequency converters, etc.

In this paper, we study the dispersion relation of electromagnetic waves in one-dimensional(1-d) plasma photonic crystals newly proposed in Ref.3. The schematic of one-dimensional plasma photonic crystals is shown in Fig.1. The plasma photonic crystal is a periodic array composed of alternating thin plasma and dielectric material. The calculation of the dispersion relation in 1-d plasma photonic crystals is analogous to that of the energy band in photonic crystals, and we expect that band structures can appear in the dispersion relation of electromagnetic waves in 1-d plasma photonic crystals.

Our starting point is a 1-d stationary Maxwell wave equation given by

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}z^2} + k_0^2\,\varepsilon(z)\right]E(z) = 0\,, \tag{1}$$

$$\varepsilon(z) = \begin{cases} 1 - \left(\dfrac{\omega_{\mathrm{pe}}}{\omega}\right)^2, & -Ld \le z \le 0 \\ \varepsilon_{\mathrm{m}}\,, & 0 < z < L \end{cases} \tag{2}$$

$$\varepsilon[z \pm L(1+d)] = \varepsilon[z]\,, \tag{3}$$

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where $k_0 = \omega/c$, ω is wave frequency, c is the speed of light, $\omega_{\mathrm{pe}} = (e^2 n_{\mathrm{p}}/\varepsilon_0 m)^{1/2}$ is the electron plasma frequency with a density n_{p}, and ε_{m} is the dielectric constant of a dielectric material.

We can solve eqs.(1)-(3) using a method analogous to Kronig-Penny's problem with a periodic potential in quantum mechanics. Thus, the solution of eq.(1) is given by, for $\omega > \omega_{\mathrm{pe}}$,

$$E(z) = \begin{cases} A\exp(\mathrm{i}\,k_{\mathrm{m}}z) + B\exp(-\mathrm{i}\,k_{\mathrm{m}}z)\,, & 0 < z < L \\ C\exp(\mathrm{i}\,k_{\mathrm{p}}z) + D\exp(-\mathrm{i}\,k_{\mathrm{p}}z)\,, & -Ld \le z \le 0 \end{cases} \tag{4}$$

and for $\omega < \omega_{\mathrm{pe}}$,

$$E(z) = \begin{cases} A\exp(\mathrm{i}\,k_{\mathrm{m}}z) + B\exp(-\mathrm{i}\,k_{\mathrm{m}}z)\,, & 0 < z < L \\ C\exp(\kappa z) + D\exp(-\kappa z)\,, & -Ld \le z \le 0\,, \end{cases} \tag{5}$$

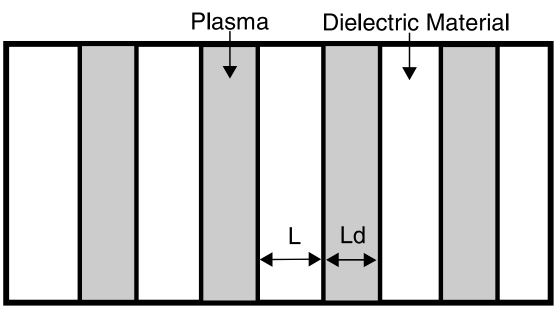

Fig.1 Schematic of 1-d plasma photonic crystal.

where $k_p = k_0\sqrt{1-(\omega_{pe}/\omega)^2}$, $\kappa = k_0\sqrt{(\omega_{pe}/\omega)^2-1}$, and $k_m = k_0\sqrt{\varepsilon_m}$. The four coefficients A, B, C and D are determined as follows: We first impose the continuity conditions of E and dE/dz at z = 0. Secondly, from the periodicity of $\varepsilon(z)$ shown by eq.(3) and also of $E(z)$ given by

$$E[z + L(1 + d)] = E[z] , \tag{6}$$

we obtain

$$E(L) = \lambda E(-Ld) \text{ , and } E'(L) = \lambda E'(-Ld) , \tag{7}$$

$$\lambda = \exp[ikL(1 + d)] , \tag{8}$$

where E' denotes the derivative of E. From these conditions and by rather lengthy calculations, we obtain the dispersion equation of electromagnetic waves in 1-d plasma photonic crystals given by, for $\omega < \omega_{pe}$,

$$\begin{aligned}\cos[kL(1+d)] &= \frac{\kappa^2 - k_m^2}{2\kappa k_m}\sin(k_m L)\sinh(\kappa Ld) \\ &\quad + \cos(k_m L)\cosh(\kappa Ld) ,\end{aligned} \tag{9}$$

and for $\omega > \omega_{pe}$,

$$\begin{aligned}\cos[kL(1+d)] &= \frac{k_p^2 + k_m^2}{2k_p k_m}\sin(k_m L)\sinh(k_p Ld) \\ &\quad + \cos(k_m L)\cosh(k_p Ld) ,\end{aligned} \tag{10}$$

Hereafter, we show the results of numerical calculations for eq.(9) or (10). We note that there are three selective parameters, $\omega_{pe}L/c$, d, and ε_m, in the numerical calculations. We first show the dispersion relation of electromagnetic waves for $\omega_{pe}L/c$ = 1, d = 1, and ε_m = 1 and 5 in Fig.2. We see that the dispersion relation becomes a band structure with frequency gaps, and a cup-off frequency exists. It is clear that the phase velocity decreases for the larger value of ε_m. We next show the dispersion relation for $\omega_{pe}L/c$ = 4, d = 1, and ε_m = 1 and 5 in Fig.3. We see that the frequency gap becomes larger with the increase of the plasma density or plasma width. For a micro-plasma such as PDP plasma, we have $\omega_{pe}L/c \sim 1$ for $n_p = 1 \times 10^{14}$cm^{-3} and L = 0.5mm. Figure 4 shows the dispersion relation for $\omega_{pe}L/c$ = 2, ε_m = 5, and d = 0.1 and 1. This figure shows that the phase velocity becomes slower and the frequency gap is smaller for the narrower plasma width when the width of a dielectric material is fixed.

Finally, we consider that the plasma photonic crystal can be applied to new plasma-functional devices, for example, frequency filters in the millimeter-wave range. This work was partly supported by Effective Promotion of Joint Research with Industry, Academia, and Government in Special Coordination Funds for Promoting Science and Technology, MEXT.

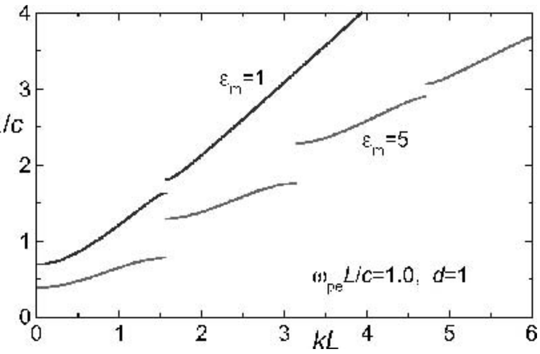

Fig. 2 Dispersion relation for $\omega_{pe}L/c$ = 1, d = 1 and ε_m = 1 and 5.0.

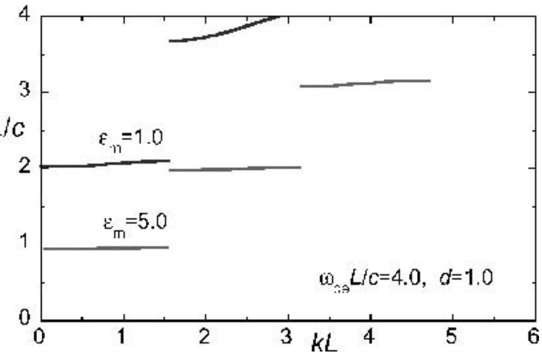

Fig. 3 Dispersion relation for $\omega_{pe}L/c$ = 4, d = 1 and ε_m = 1 and 5.

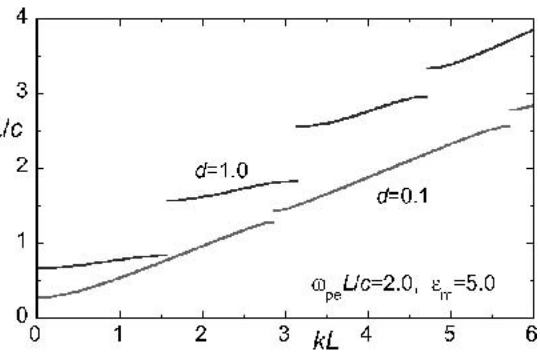

Fig. 4 Dispersion relation for $\omega_{pe}L/c$ = 2, ε_m = 5 and d = 1 and 0.1.

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