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Letter to the Editors

Displacement at the Centre of the Earth Induced by an Earthquake or an Underground Explosion

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Ben-Menahem & Singh (1968) gave the basic theory for the deformation of a homogeneous, isotropic, non-gravitating, elastic sphere due to a buried tangential dislocation, tensile dislocation or centre of explosion. In general, the displacement field can be evaluated by applying the condition that the stresses vanish at the surface of the sphere. Ben-Menahem and Singh found that the evaluation of the static field for a sphere associated with the Legendre polynomial of the first degree ($l = 1$) poses some problems. In this case one must incorporate additional conditions, namely, that the angular momentum of the sphere about its centre is zero and that the centre of mass of the sphere is not displaced. The aim of the present note is to determine the displacement at the centre of a sphere due to buried sources. It is interesting to note that out of all the values of l (the degree of the Legendre polynomial) only one value, namely $l = 1$, gives non-zero contribution to the angular momentum of the sphere about its centre, the displacement of its centre of mass or the displacement of its centre itself.

We consider a homogeneous elastic sphere of radius a and Poisson ratio σ. Let there be a centre of explosion or a localized tangential dislocation of fault area dS, dip angle δ, slip angle λ and displacement jump U at the point ($r = r_0$, $\theta = 0$) inside the sphere. The origin of a spherical co-ordinate system (r, θ, ϕ) is chosen at the centre of the sphere. Recently Ben-Menahem & Israel (1970) calculated the displacement field at an arbitrary point within the sphere induced by buried sources. To get the residual displacement at the centre of the sphere, we put $r = 0$ in the expressions of Ben-Menahem and Israel and obtain the following results for various sources.

1. *Vertical strike-slip fault* ($\lambda = 0, \delta = 90°$)

$$u_r = u_\theta = u_\phi = 0. \tag{1}$$

2. *Vertical dip-slip fault* ($\lambda = 90°, \delta = 90°$)

$$\begin{pmatrix} u_r \\ u_\theta \\ u_\phi \end{pmatrix} = \Omega U \beta \begin{pmatrix} \sin\theta \sin\phi \\ \cos\theta \sin\phi \\ \cos\phi \end{pmatrix} \tag{2}$$

where

$$\Omega = \frac{dS}{4\pi a^2}, \qquad \beta = \frac{1-2\sigma}{2(1-\sigma)} \cdot \frac{1-t^3}{t^2}, \qquad t = r_0/a. \tag{3}$$

Hence

$$\mathbf{u}_2 = u_r \mathbf{e}_r + u_\theta \mathbf{e}_\theta + u_\phi \mathbf{e}_\phi = \Omega U \beta \mathbf{e}_y. \tag{4}$$

The vector base ($\mathbf{e}_x$, $\mathbf{e}_y$, $\mathbf{e}_z$) is set up at the centre of the sphere with $\mathbf{e}_x$ parallel to the strike-direction and $\mathbf{e}_z$ towards the point source.

3. *Dip-slip on a* 45° *plane* ($\lambda = 90°$, $\delta = 45°$)

$$\begin{pmatrix} u_r \\ u_\theta \\ u_\phi \end{pmatrix} = \Omega U \alpha \begin{pmatrix} -\cos\theta \\ \sin\theta \\ 0 \end{pmatrix} \tag{5}$$

where

$$\alpha = \left[\frac{5-4\sigma}{4(1-\sigma)} - \frac{1-2\sigma}{2(1-\sigma)} t^3\right] \frac{1}{t^2}. \tag{6}$$

Therefore

$$\mathbf{u}_3 = -\Omega U \alpha \mathbf{e}_z. \tag{7}$$

Using equations (1), (4) and (7) and equation (5.14a) of Ben-Menahem & Singh (1968), we get the following expression for the displacement at $r = 0$ caused by an arbitrary tangential dislocation (λ, δ)

$$\mathbf{u} = \Omega U[\beta\{\cos\lambda \cos\delta - \sin\lambda \cos 2\delta\}\, \mathbf{e}_y - \alpha \sin\lambda \sin 2\delta\, \mathbf{e}_z].$$

4. *Centre of explosion*

We obtain

$$\mathbf{u}_0 = -(E_0/a^2)\, \varepsilon \mathbf{e}_z, \tag{8}$$

where

$$\varepsilon = \left[\frac{4+\sigma}{1+\sigma} t^3 - 1\right] \frac{1}{t^2}, \tag{9}$$

and E_0 is a source parameter of dimensions L^3.

For illustration, we take the following numerical values of various parameters occurring in the above equations

$$\left.\begin{array}{ll} \Omega = 5\times 10^{-5} & [dS \simeq 50\times 500\,\text{km}^2] \\ \dfrac{E_0}{a^2} = 5\times 10^{-5} & \\ t = 0{\cdot}99 & [\text{depth of the focus} \simeq 64\,\text{km}] \\ \sigma = 0{\cdot}3 & U = 5\,\text{m}. \end{array}\right\} \tag{10}$$

The above value of E_0 corresponds approximately to a pressure of 100 atms just prior to the explosion [see Ben-Menahem & Israel (1970, p. 388)]. With these numerical values, we obtain the following estimates

$$\left.\begin{array}{l} \mathbf{u}_2 \simeq \quad (2\times 10^{-3}\,\text{mm})\, \mathbf{e}_y, \\ \mathbf{u}_3 \simeq -(2\times 10^{-1}\,\text{mm})\, \mathbf{e}_z, \\ \mathbf{u}_0 \simeq -(10^{-1}\,\text{mm})\, \mathbf{e}_z. \end{array}\right\} \tag{11}$$

Lastly, for arbitrary (λ, δ),

$$\mathbf{u} \simeq (2\times 10^{-3}\,\text{mm})(\cos\lambda \cos\delta - \sin\lambda \cos 2\delta)\, \mathbf{e}_y - (2\times 10^{-1}\,\text{mm}) \sin\lambda \sin 2\delta\, \mathbf{e}_z. \tag{12}$$

We have considered here the static problem. In the case of a dynamic source also it is $l = 1$ which contributes to the motion of the centre of the Earth and, furthermore, there is no motion for a vertical strike-slip fault, the motion is along the y-axis in the case of a vertical dip-slip fault and along z-axis for dip-slip fault on a 45° plane and for a centre of explosion.

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