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# Dissipation and noise in adiabatic quantum pumps

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We investigate the distribution function, the heat flow and the noise properties of an adiabatic quantum pump for an arbitrary relation of pump frequency  $\omega$  and temperature. To achieve this we start with the scattering matrix approach for ac-transport. This approach leads to expressions for the quantities of interest in terms of the side bands of particles exiting the pump. The side bands correspond to particles which have gained or lost a modulation quantum  $\hbar\omega$ . We find that our results for the pump current, the heat flow and the noise can all be expressed in terms of a parametric emissivity matrix. In particular we find that the current cross-correlations of a multiterminal pump are directly related to a non-diagonal element of the parametric emissivity matrix. The approach allows a description of the quantum statistical correlation properties (noise) of an adiabatic quantum pump.

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## I. INTRODUCTION

A recent experiment by Switkes et al.<sup>1</sup> has stimulated increasing interest in adiabatic quantum charge pumping. Ideally in such an experiment one aims at generating a dc-current by slowly modulating the shape of a mesoscopic conductor with the help of oscillating gate voltages. A single potential oscillating at frequency  $\omega$  does not generate a dc-current, but two potentials oscillating with the same frequency but out of phase can generate a dc-current. The effect is of interest under conditions in which electron motion is phase-coherent and is thus termed *quantum* pumping. The frequency of the potential modulation is small compared to the characteristic times for traversal and reflection of electrons and the pump is thus termed *adiabatic*. Thus carriers traversing the sample see an almost static potential. The last circumstance allows to give an elegant formulation of quantum pumping<sup>2</sup> which is based on the scattering matrix approach to low frequency ac transport in phase coherent mesoscopic systems<sup>3</sup>.

Recently Avron et al.<sup>4</sup> investigated adiabatic quantum pumping with the aim to formulate criteria for an "optimal pump". The term "optimal" means that such a pump is noiseless and transports integer charge in each cycle. To this extent they have investigated not only the dc-current but also the dissipation and the noise generated by a pump. Avron et al. express their results in terms of an energy shift matrix  $i\hbar\partial\hat{s}/\partial t\hat{s}^\dagger$  where  $\hat{s}$  is the time-dependent scattering matrix. This is an elegant formulation which gives a correct description of time-dependent adiabatic currents and dissipation. However, for quantities which invoke correlations at different

times the approach is valid only for pump frequencies  $\hbar\omega \ll k_B T$ .

It is the purpose of this work to investigate the distribution function, heat flow and noise properties of an adiabatic pump for an arbitrary relation of pump frequency and temperature. To achieve this we start with the scattering matrix approach for ac-transport. This approach leads to expressions for the quantities of interest in terms of the side bands of particles exiting the pump. The side bands correspond to particles which have gained or lost a modulation quantum  $\hbar\omega$ . In particular, the approach presented here allows a description of the quantum statistical correlation properties (noise) of an adiabatic quantum pump.

The adiabatic quantum pump<sup>1,2,5-22</sup> of interest here should be distinguished from a variety of other pumping mechanisms. For certain pumps<sup>23,24</sup> the charge transferred in each cycle is quantized. Quantized charge pumping is most easily achieved in devices based on the Coulomb blockade effect<sup>25-30</sup> where the charge on a quantum dot is quantized. This is of considerable metrological interest<sup>31,32</sup>. Other effects which lead to pumping are the photovoltaic effect<sup>33,34</sup> and the acoustoelectric effect<sup>35-40</sup>.

The paper is organized as follows. In Sec.II the essential assumptions we make are described. In Sec.III we calculate the nonequilibrium distribution function for the outgoing particles produced by the pump. In Sec.IV we formulate the condition which is necessary to pump dc-current. In Sec.V we calculate the heat flows produced by an oscillating mesoscopic scatterer. In Sec.VI we consider the shot noise produced by the pump and analyze the noise in terms of uncorrelated movement of nonequilibrium quasi-particles (quasi-electrons and holes) generated by the pump and correlations between them<sup>41,42</sup>. In

Sec.VII we present explicit results for the particular case of a two-leads scatterer with the time-reversal symmetry.

## II. THE MAIN ASSUMPTIONS

To describe the response of a mesoscopic phase coherent sample to slowly oscillating (with a frequency  $\omega$ ) external real parameters  $X_j(t)$  (gate potential, magnetic flux, etc.)

$$X_j(t) = X_j + X_{\omega,j}e^{i(\omega t - \varphi_j)} + X_{\omega,j}e^{-i(\omega t - \varphi_j)}, \quad (1)$$

we will use the scattering matrix approach<sup>43,44,3</sup>. The sample is connected via leads (which we will number via Greek letters  $\alpha, \beta, \gamma$ , etc.) to  $N_r$  reservoirs. The scattering matrix  $\hat{s}$  being a function of parameters  $X_j(t)$  depends on time. Two main assumption will be used. First, we suppose that the external parameter changes so slowly that we can apply an "instant scattering" description using the scattering matrix  $\hat{s}(t)$  frozen at some time  $t$ . Physically this means that the scattering matrix changes only a little while an electron is scattered by the mesoscopic sample (i.e., the frequency  $\omega$  is much smaller than the inverse Wigner time delay<sup>45,46</sup>). In this sense we use the term "adiabatic" pump.

Second, we assume that the amplitude  $X_{\omega,j}$  is small enough to keep only the terms linear in  $X_{\omega,j}$  in an expansion of the scattering matrix

$$\hat{s}(t) \approx \hat{s} + \hat{s}_{-\omega}e^{i\omega t} + \hat{s}_{+\omega}e^{-i\omega t}. \quad (2)$$

In the limit of small frequencies the amplitudes  $\hat{s}_{\pm\omega}$  can be expressed in terms of parametric derivatives of the on-shell scattering matrix  $\hat{s}$ ,

$$\hat{s}_{\pm\omega} = \sum_j X_{\omega,j}e^{\pm i\varphi_j} \partial \hat{s} / \partial X_j. \quad (3)$$

The expansion Eq.(2) is equivalent to the nearest sidebands approximation<sup>47,3</sup> which implies that a scattered electron can absorb or emit only one energy quantum  $\hbar\omega$  before it leaves the scattering region.

The kinetic properties (charge current, heat current, etc.) which are of interest here depend on the values of the scattering matrix within the energy interval of the order of  $\max(k_B T, \hbar\omega)$  near the Fermi energy. In the low frequency ( $\omega \rightarrow 0$ ) and low temperature ( $T \rightarrow 0$ ) limit we assume the scattering matrix to be energy independent.

## III. OUTGOING DISTRIBUTION FUNCTION

In a pump setup the mesoscopic scatterer is coupled to reservoirs  $\alpha = 1, 2, \dots, N_r$  with the same temperatures  $T_\alpha = T$  and electrochemical potentials  $\mu_\alpha = \mu$ . Thus

electrons with the energy  $E$  entering the scatterer are described by the Fermi distribution function

$$f_\alpha^{(in)}(E) = f_0(E) = \frac{1}{1 + e^{\frac{E-\mu}{k_B T}}}.$$

Due to the interaction with an oscillating scatterer an electron can absorb or emit an energy quantum  $\hbar\omega$  that changes the distribution function. Our aim is to find the distribution function for outgoing particles (i.e., for electrons leaving the mesoscopic sample and entering the reservoir) far from the scatterer.

Let us consider a single transverse channel in one of the leads. We introduce two kinds of carriers<sup>44</sup>. First, incoming particles which are going from the reservoir to the scatterer. And, second, outgoing particles which are leaving the scattering region. We can express the operators  $\hat{b}_\alpha$  which annihilate outgoing carriers in the lead  $\alpha$  in terms of operators  $\hat{a}_\beta$  annihilating incoming electrons<sup>44</sup> in lead  $\beta$ . Applying the hypothesis of an instant scattering we can write

$$\hat{b}_\alpha(t) = \sum_\beta s_{\alpha\beta}(t) \hat{a}_\beta(t). \quad (4)$$

Here  $s_{\alpha\beta}$  is an element of the scattering matrix  $\hat{s}$ ; the time dependent operator is  $\hat{a}_\alpha(t) = \int dE \hat{a}_\alpha(E) e^{-iEt/\hbar}$ , and the energy dependent operators obey the following anticommutation relations<sup>44</sup>

$$[\hat{a}_\alpha^\dagger(E), \hat{a}_\beta(E')] = \delta_{\alpha\beta} \delta(E - E').$$

Note that above expressions correspond to single (transverse) channel leads and spinless electrons. For the case of many-channel leads each lead index ( $\alpha, \beta$ , etc.) includes a transverse channel index and any repeating lead index implies implicitly a summation over all the transverse channels in the lead. Similarly an electron spin can be taken into account.

Using Eq.(2) and Eq.(4) we obtain<sup>3</sup>

$$\begin{aligned} \hat{b}_\alpha(E) &= \sum_\beta s_{\alpha\beta} \hat{a}_\beta(E) \\ &+ s_{-\omega, \alpha\beta} \hat{a}_\beta(E + \hbar\omega) + s_{+\omega, \alpha\beta} \hat{a}_\beta(E - \hbar\omega). \end{aligned} \quad (5)$$

The distribution function for electrons leaving the scatterer through the lead  $\alpha$  is  $f_\alpha^{(out)}(E) = \langle \hat{b}_\alpha^\dagger(E) \hat{b}_\alpha(E) \rangle$ , where  $\langle \dots \rangle$  means quantum-mechanical averaging. Substituting Eq.(5) we find

$$\begin{aligned} f_\alpha^{(out)}(E) &= \sum_\beta |s_{\alpha\beta}|^2 f_0(E) \\ &+ |s_{-\omega, \alpha\beta}|^2 f_0(E + \hbar\omega) + |s_{+\omega, \alpha\beta}|^2 f_0(E - \hbar\omega). \end{aligned} \quad (6)$$

Note that the distribution function for outgoing carriers is a nonequilibrium distribution function generated by the nonstationary scatterer. The above expression

gives a simple physical interpretation for the Fourier amplitudes of the scattering matrix.  $|s_{-\omega,\alpha\beta}|^2$  ( $|s_{+\omega,\alpha\beta}|^2$ ) is the probability for an electron entering the scatterer through the lead  $\beta$  and leaving the scatterer through the lead  $\alpha$  to emit (to absorb) an energy quantum<sup>47</sup>  $\hbar\omega$ . Note that  $|s_{\alpha\beta}|^2$  is the probability for the same scattering without the change of an energy. Below we will use Eq.(6) to analyze the kinetics of a pump.

#### IV. DC-CURRENT

To be definite we take currents from the scatterer to the reservoirs to be positive. Using the distribution functions  $f_0(E)$  for incoming electrons and  $f_\alpha^{(out)}(E)$  for outgoing electrons we find for the dc-current  $I_\alpha$  in the lead  $\alpha$  far from the scatterer

$$I_\alpha = \frac{e}{\hbar} \int_0^\infty dE [f_\alpha^{(out)}(E) - f_0(E)]. \quad (7)$$

Substituting Eq.(6) we get

$$I_\alpha = \frac{e\omega}{2\pi} [T_{+\omega,\alpha} - T_{-\omega,\alpha}]. \quad (8)$$

Here we have introduced the total probabilities for electrons scattered into the lead  $\alpha$  (irrespective of the lead through which they entered the scattering region) to absorb  $T_{+\omega,\alpha}$  or to emit  $T_{-\omega,\alpha}$  an energy quantum  $\hbar\omega$

$$T_{\pm\omega,\alpha} = \sum_\beta |s_{\pm\omega,\alpha\beta}|^2. \quad (9)$$

We see that only a scatterer with the property

$$T_{+\omega,\alpha} \neq T_{-\omega,\alpha}, \quad (10)$$

can pump current into the lead  $\alpha$ .

It is useful to express these probabilities in terms of a bare scattering matrix  $\hat{s}$ . To this end we introduce a generalized parametric emissivity matrix  $\hat{\nu}[X]$

$$\hat{\nu}[X] = -\frac{1}{2\pi i} \frac{d\hat{s}}{dX} \hat{s}^\dagger. \quad (11)$$

with matrix elements

$$\nu_{\alpha\beta}[X] = -\frac{1}{2\pi i} \sum_\gamma \frac{ds_{\alpha\gamma}}{dX} s_{\beta\gamma}^*. \quad (12)$$

The diagonal element  $\nu_{\alpha\alpha}[X]$  of the parametric emissivity matrix<sup>2,3,48,49</sup> is the charge that leaves the sample through contact  $\alpha$  in response to a variation of the parameter  $X$ . The non-diagonal element  $\nu_{\alpha\beta}[X]$  ( $\alpha \neq \beta$ ) of the parametric emissivity matrix determines the correlations between current amplitudes generated in the contacts  $\alpha$  and  $\beta$  due to a variation of the parameter  $X$  (see Eq.(26)).

Using Eq.(3) and Eq.(15) we express the probabilities  $T_{\pm\omega,\alpha}$  given by Eq.(9) in terms of the matrix elements of the parametric emissivity matrix (to lowest order in  $X_{\omega,j}$ )

$$T_{\pm\omega,\alpha} = 4\pi^2 \sum_\beta \left| \sum_j X_{\omega,j} e^{\pm i\varphi_j} \nu_{\alpha\beta}[X_j] \right|^2. \quad (13)$$

The quantities  $T_{\pm\omega,\alpha}$  admit a simple interpretation in the quasi-particle picture. Due to scattering the electron system gains an energy from the nonstationary (oscillating) scatterer. Absorption of an energy quantum  $\hbar\omega$  leads to creation of a nonequilibrium (quasi-)electron-hole pair. Note that at any temperature  $T \neq 0$  equilibrium electron-hole pairs exist. This nonequilibrium pair is neutral but transfers an energy  $\hbar\omega$ . From Eq.(6) it follows that  $T_{+(-)\omega,\alpha}$  is proportional to the number of nonequilibrium quasi-electrons (holes) leaving the scattering region through the lead  $\alpha$ . The electron and hole (belonging to the same pair) can be scattered either into one lead (see Fig.1.a) or into different leads (see Fig.1.b). If they are scattered into the same lead they do not contribute to the current. But if they are scattered into different leads they do contribute. In any case they contribute to the heat transfer from the oscillating scatterer into the reservoirs.

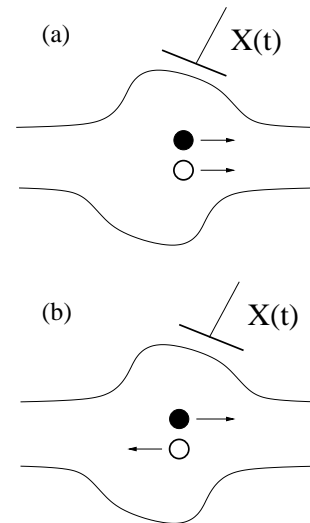


FIG. 1. When the parameter  $X$  changes, the electron system gains energy from the scatterer. Absorption of an energy quantum  $\hbar\omega$  leads to creation of nonequilibrium (quasi-)electron-hole pairs. The electron (black circle) and hole (open circle) belonging to the same pair can be scattered either into one lead (a) or into different leads (b). In the case (a) the quasi-particles do not contribute to the dc current, but in the case (b) they do contribute. The process shown in the panel (b) contributes to the current cross-correlations. In both cases (a) and (b) the quasi-particles carry energy from the scatterer to the reservoirs.

The dc-current  $I_\alpha$  in the lead  $\alpha$  can be represented as a sum of two contributions  $I_\alpha = I_\alpha^{(e)} + I_\alpha^{(h)}$ , where  $I_\alpha^{(e)} = e\omega T_{+\omega,\alpha}/(2\pi)$  and  $I_\alpha^{(h)} = -e\omega T_{-\omega,\alpha}/(2\pi)$  are currents carried by nonequilibrium quasi-electrons and holes, respectively (here e (-e) is an electron (a hole) charge).

Now we will show that current is conserved, i.e.,  $\sum_\alpha I_\alpha = 0$ . To this end we use the fact that the scattering matrix is unitary

$$\hat{s}(t)\hat{s}^\dagger(t) = 1. \quad (14)$$

For the expansion Eq.(2) this leads to the relations

$$\sum_\gamma [s_{\alpha\gamma}s_{\beta\gamma}^* + s_{-\omega,\alpha\gamma}s_{-\omega,\beta\gamma}^* + s_{+\omega,\alpha\gamma}s_{+\omega,\beta\gamma}^*] = \delta_{\alpha\beta}, \quad (15)$$

$$\sum_\gamma s_{\alpha\gamma}s_{-\omega,\beta\gamma}^* = -\sum_\gamma s_{\beta\gamma}^*s_{+\omega,\alpha\gamma}, \quad (16)$$

$$\sum_\gamma s_{\beta\gamma}^*s_{-\omega,\alpha\gamma} = -\sum_\gamma s_{\alpha\gamma}s_{+\omega,\beta\gamma}^*. \quad (17)$$

Multiplying Eq.(16) and Eq.(17) by parts, summing the result over  $\alpha$  and taking into account Eq.(9) we obtain (neglecting the higher powers of  $s_{\pm\omega,\alpha\gamma}$ )

$$\sum_\alpha T_{-\omega,\alpha} = \sum_\alpha T_{+\omega,\alpha}. \quad (18)$$

Using Eqs.(8) and (18) we see that the scatterer does not produce any current  $\sum_\alpha I_\alpha = 0$  but it can only push a current from some reservoir to another reservoir.

An alternative (but equivalent) way to find the dc-current is to average the time-dependent current

$$I_\alpha = \lim_{\Delta t \rightarrow \infty} \frac{1}{\Delta t} \int_0^{\Delta t} dt \langle \hat{I}_\alpha(t) \rangle.$$

The current operator is<sup>44</sup>

$$\hat{I}_\alpha(t) = \frac{e}{h} [\hat{b}_\alpha^\dagger(t)\hat{b}_\alpha(t) - \hat{a}_\alpha^\dagger(t)\hat{a}_\alpha(t)]. \quad (19)$$

Substituting Eqs.(2) and (4) into Eq.(19) and performing quantum mechanical and time averaging we obtain Eq.(8).

Note that in a pump setup where the external reservoirs are at the same macroscopic conditions (electrochemical potential, temperature, etc.) and a periodic in time perturbation is applied directly to the mesoscopic conductor there is no linear regime for dc transport (only ac currents are linear in perturbation). The dc-currents (charge, heat, etc.) are of a quantum mechanical nature and arise because of a nonlinear (quadratic) dependence on the quantum-mechanical (scattering) amplitudes (see Eq.(9)).

## V. HEAT FLOW

Particles traversing the sample absorb energy from a time dependent scatterer and carry it into the reservoirs. We assume that the reservoirs are large enough to absorb this energy and to remain still in thermal equilibrium. In the leads the energy is transferred by electrons only (we neglect any inelastic processes in the leads). Thus to calculate<sup>50-56</sup> an energy flow  $I_{E,\alpha}$  entering the reservoir  $\alpha$  we can use an electron distribution function and write

$$I_{E,\alpha} = \frac{1}{h} \int_0^\infty dE (E - \mu) [f_\alpha^{(out)}(E) - f_0(E)]. \quad (20)$$

Substituting Eq.(6) we obtain

$$I_{E,\alpha} = \frac{\hbar\omega^2}{4\pi} [T_{+\omega,\alpha} + T_{-\omega,\alpha}]. \quad (21)$$

Comparing Eq.(8) and Eq.(21) we see that the time dependent scatterer always generates heat flows (because  $T_{\pm\omega,\alpha}$  are positively defined) and can be considered as a mesoscopic (phase-coherent) heat source which can be useful, for instance, for studying various thermoelectric phenomena in mesoscopic structures. In contrast the existence of a dc-current Eq.(8) requires a special condition (see Eq.(10)). Another difference is that the heat flow is directed (at any lead) from the scatterer to the reservoir (if all the reservoirs are at the same temperature) but the charge flow, if it exists, can be directed either from the reservoir to the scatterer (at some lead) or vice versa (at another lead) because of charge conservation.

The quasi-particle description gives a simple physical interpretation of Eq.(21). We can say that the heat is transported by two kinds of quasi-particles, the quasi-electrons and holes. Each quasi-particle has an energy  $\hbar\omega/2$  (on average). This is because the absorption of any energy quantum  $\hbar\omega$  creates two quasi-particles, a quasi-electron and a hole. Thus the heat (energy) transferred by quasi-electrons and holes is  $I_{E,\alpha}^{(e)} = (\hbar\omega/2)(I_\alpha^{(e)}/e)$  and  $I_{E,\alpha}^{(h)} = (\hbar\omega/2)(I_\alpha^{(h)}/(-e))$ , respectively (the quasi-electron  $I_\alpha^{(e)}$  and hole  $I_\alpha^{(h)}$  currents are defined in the previous section after Eq.(13)). The sum of these contributions gives Eq.(21)

## VI. CURRENT FLUCTUATIONS

The problem of current noise in a quantum pump is closely connected with the problem of quantization of the charge pumped in one cycle<sup>4,6,16,17,29</sup>. On the other hand the noise in mesoscopic phase coherent conductors is interesting in itself<sup>42-44,57</sup> because it is very sensitive

to quantum-mechanical interference effects and can give additional information about the scattering matrix.

To describe the current-current fluctuations we will use the correlation function<sup>42</sup>

$$S_{\alpha\beta}(t, t') = \frac{1}{2} \langle \Delta \hat{I}_\alpha(t) \Delta \hat{I}_\beta(t') + \Delta \hat{I}_\beta(t') \Delta \hat{I}_\alpha(t) \rangle, \quad (22)$$

where  $\Delta \hat{I} = \hat{I} - \langle \hat{I} \rangle$  and  $\hat{I}_\alpha(t)$  is the quantum-mechanical current operator in the lead  $\alpha$  given by Eq.(19). Note that in the case of a time-dependent scatterer the correlation function depends on two times  $t$  and  $t'$ .

Here we are interested in the noise averaged over a long time<sup>58,59</sup> ( $\Delta t \gg 2\pi/\omega$ ) and we investigate

$$S_{\alpha\beta}(t) = \frac{\omega}{2\pi} \int_0^{2\pi/\omega} dt' S_{\alpha\beta}(t, t').$$

In addition we restrict our consideration to the zero-frequency component of the noise spectra  $S_{\alpha\beta} = \int dt S_{\alpha\beta}(t)$ . Substituting the current operator Eq.(19) and taking into account Eq.(2) and Eq.(4) we can write the zero-frequency noise power

$$S_{\alpha\beta} = \frac{2e^2}{h} \int_0^\infty dE \langle \hat{S}_{\alpha\beta}(E, E) + \hat{S}_{\alpha\beta}(E, E - \hbar\omega) + \hat{S}_{\alpha\beta}(E, E + \hbar\omega) \rangle. \quad (23)$$

Here

$$\hat{S}_{\alpha\beta}(E, E') = \frac{1}{2} [\Delta \hat{I}_\alpha(E) \Delta \hat{I}_\beta(E') + \Delta \hat{I}_\beta(E') \Delta \hat{I}_\alpha(E)];$$

$\Delta \hat{I}_\alpha(E) = \hat{I}_\alpha(E) - \langle \hat{I}_\alpha(E) \rangle$  and

$$\hat{I}_\alpha(E) = \hat{b}_\alpha^\dagger(E) \hat{b}_\alpha(E) - \hat{a}_\alpha^\dagger(E) \hat{a}_\alpha(E).$$

For the energy independent scattering matrix in the lowest order in  $\hat{s}_{\pm\omega}$  we obtain  $S_{\alpha\beta} = S_{\alpha\beta}^{(th)} + S_{\alpha\beta}^{(pump)}$ . Here the thermal (or Nyquist-Johnson) noise is<sup>43,42</sup>

$$S_{\alpha\beta}^{(th)} = 2e^2 k_B T / h [2\delta_{\alpha\beta} - |s_{\alpha\beta}|^2 - |s_{\beta\alpha}|^2].$$

The noise power produced by the pump is

$$S_{\alpha\beta}^{(pump)} = \frac{2e^2}{h} F(\hbar\omega, k_B T) \left( \delta_{\alpha\beta} [T_{-\omega, \alpha} + T_{+\omega, \alpha}] - T_{\alpha\beta}^{(cor)} \right), \quad (24)$$

where

$$T_{\alpha\beta}^{(cor)} = \left| \sum_\gamma s_{\beta\gamma} s_{-\omega, \alpha\gamma}^* \right|^2 + \left| \sum_\gamma s_{\beta\gamma} s_{+\omega, \alpha\gamma}^* \right|^2, \quad (25)$$

and  $F(\hbar\omega, k_B T) = \hbar\omega \coth[\hbar\omega/(2k_B T)] - 2k_B T$ .

In addition to the probabilities  $T_{\pm\omega, \alpha}$  which determine the dc current Eq.(8) and heat flow Eq.(21) there appears a third key quantity  $T_{\alpha\beta}^{(cor)}$ , which describes the effect of correlations between (quasi-)particles. Similarly to  $T_{\pm\omega, \alpha}$  (see Eqs.(13)) this probability can be expressed in terms of a generalized emissivity matrix  $\hat{\nu}$  (see Eq.(11))

$$T_{\alpha\beta}^{(cor)} = 4\pi^2 \sum_{\eta=+1, -1} \left| \sum_j X_{\omega, j} e^{i\eta\varphi_j} \nu_{\alpha\beta}[X_j] \right|^2. \quad (26)$$

Note that there is no summation over  $\alpha$  or  $\beta$ : Consequently this probability which determines the current cross-correlation is directly proportional to the off-diagonal element of the emissivity matrix.

Now we will analyze the noise power Eq.(24). We can see that the current cross-correlations  $S_{\alpha\beta}^{(pump)}$  ( $\alpha \neq \beta$ ) produced by the pump are negative: that is quite general for nonequilibrium noise in the system of fermions<sup>44</sup>. The noise generated by the pump obeys the following sum rules<sup>44</sup>  $\sum_\alpha S_{\alpha\beta}^{(pump)} = \sum_\beta S_{\alpha\beta}^{(pump)} = 0$ . This is a straightforward consequence of an instant scattering description applied here. Indeed, using Eqs.(4),(14) and (19) we can see that the conservation law holds not only for a quantum averaged current  $\langle \hat{I}_\alpha(t) \rangle$  but for a current operator as well<sup>44</sup>:  $\sum_\alpha \hat{I}_\alpha(t) = 0$ . Thus the current correlations  $S_{\alpha\beta} \sim \langle \hat{I}_\alpha \hat{I}_\beta \rangle$  must obey the same sum rule.

The function  $F(\hbar\omega, k_B T)$  describes the effect of thermal fluctuation on shot noise and determines the dependence of the noise on the pump frequency  $\omega$ . At sufficiently high temperature  $\hbar\omega \ll k_B T$  the noise Eq.(24) is quadratic in  $\omega$ . This is in agreement with Ref. 4. But at low temperature  $k_B T \ll \hbar\omega$  the noise is linear in  $\omega$  and this is in agreement with the counting statistics calculations of Levitov<sup>16</sup>.

Next consider the three terms in the brackets of the r.h.s. of Eq.(24). Consider the low temperature limit  $k_B T \ll \hbar\omega$  and divide the expression for noise into two parts  $S_{\alpha\beta}^{(pump)} = \delta_{\alpha\beta} S_\alpha^{(pump), (P)} + S_{\alpha\beta}^{(pump), (cor)}$ . The first part

$$S_\alpha^{(pump), (P)} = \frac{e^2 \omega}{\pi} [T_{-\omega, \alpha} + T_{+\omega, \alpha}],$$

is due to an uncorrelated movement of nonequilibrium quasi-electrons and holes. To verify this we apply the Schottky formula<sup>60</sup> for shot noise  $S_{\alpha, q}^{(Sch)} = 2qI_\alpha^{(q)}$  (here  $q$  is a particle charge and the index "q" means that the current is carried by the particles with the charge q). Substituting the current carried by the quasi-electrons  $I_\alpha^{(e)} = e\omega T_{+\omega, \alpha}/(2\pi)$  and by the holes  $I_\alpha^{(h)} = -e\omega T_{+\omega, \alpha}/(2\pi)$  (here e(-e) is an electron (hole) charge) into Schottky's formula we obtain  $S_\alpha^{(pump), (P)} = S_{\alpha, e}^{(Sch)} + S_{\alpha, h}^{(Sch)}$  (in the literature the Schottky result is referred as the Poisson value of shot noise that we indicate by the upper index "P").

The second part

$$S_{\alpha\beta}^{(pump),(cor)} = -\frac{e^2\omega}{\pi}T_{\alpha\beta}^{(cor)},$$

is due to correlations between quasi-electrons and holes. These correlations are a consequence of the common origin of the electron and hole forming a pair and their subsequent scattering into different leads Fig.1.b. Thus we can say that the cross-correlations are exclusively due to dissolving (neutral) electron-hole pairs. Because of charge conservation this gives a simple explanation of a negative sign of cross-correlations in our case Eq.(24). Note that Schottky's result gives no correlations between currents at different leads. Due to  $S_{\alpha\alpha}^{(pump),(cor)}$  the current correlation at the same lead  $S_{\alpha\alpha}^{(pump)}$  is below the Poisson value  $S_{\alpha\alpha}^{(P)}$  and the Fano factor characterizing the deviation of the actual shot noise from the Poisson noise (see, e.g.,<sup>42</sup>)  $F = S_{\alpha\alpha}^{(pump)}/S_{\alpha\alpha}^{(P)}$  is, in general, less than unity. We would like to emphasize that when we calculate the Poisson value of shot noise we do not use the total current  $I_{\alpha}$  in the lead  $\alpha$  but we calculate the sum of the Poisson noises produced by both the quasi-electrons (the current is  $I_{\alpha}^{(e)}$ ) and the holes (the current is  $I_{\alpha}^{(h)}$ ).

## VII. APPLICATIONS

In this section we consider a simple but a quite generic case of a two-terminal mesoscopic conductor with a time-reversal symmetry (without magnetic fluxes). In addition we assume that the external, time-dependent parameters  $X_j(t)$  do not change the total charge on a sample (if this is not the case we need to take into account the self-consistent internal potential<sup>3</sup>). In this case we can choose the scattering matrix as follows (we consider spinless electrons)

$$\hat{s} = \begin{pmatrix} re^{i\theta} & it \\ it & re^{-i\theta} \end{pmatrix}. \quad (27)$$

Here  $r^2$  and  $t^2$  are the reflection and the transmission probability, respectively ( $r^2 + t^2 = 1$ ). We assume the quantities  $r, t, \theta$  to be the functions of external parameters  $X_j$  (see Eq.(1)).

### A. Heat flow and noise in one-parameter "pumps"

If only one external parameter  $X$  is varied then we get from Eq.(3)  $|s_{-\omega,\alpha\beta}|^2 = |s_{+\omega,\alpha\beta}|^2$  for any  $\alpha$  and  $\beta$ . Thus the one-parameter adiabatic "pump" gives the same probability for absorption and emission at both leads ( $\alpha = 1, 2$ )

$$T_{-\omega,\alpha}^{(1)} = T_{+\omega,\alpha}^{(1)} = T_{\omega,\alpha}^{(1)}, \quad (28)$$

$$T_{\omega,\alpha}^{(1)}[X] = X_{\omega}^2 \sum_{\beta} \left| \frac{ds_{\alpha\beta}}{dX} \right|^2,$$

and does not produce a dc-current:  $I_{\alpha} = 0$  (see Eq.(8)). In contrast it does produce the heat flows Eq.(21) and the noise Eq.(24). The contribution to the noise Eq.(25) due to correlations between quasi-particles is

$$T_{\alpha\beta}^{(cor)} = 8\pi^2 X_{\omega}^2 |\nu_{\alpha\beta}[X]|^2. \quad (29)$$

Here  $\nu_{\alpha,\beta}[X]$  is a matrix element of a generalized parametric emissivity matrix  $\hat{\nu}[X]$  Eq.(11). With the scattering matrix Eq.(27) we find a heat flow

$$I_{E,1}^{(1)} = I_{E,2}^{(1)} = \frac{\hbar\omega^2 X_{\omega}^2}{2\pi} \left( r^2 \left( \frac{d\theta}{dX} \right)^2 + \left( \frac{dr}{dX} \right)^2 + \left( \frac{dt}{dX} \right)^2 \right), \quad (30)$$

and a noise  $S_{11}^{(pump)} = S_{22}^{(pump)} = -S_{12}^{(pump)} = -S_{21}^{(pump)} = S^{(1)}$

$$S^{(1)} = \frac{4e^2 X_{\omega}^2}{h} F(\hbar\omega, k_B T) \times \left( r^2 t^2 \left( \frac{d\theta}{dX} \right)^2 + \left( \frac{dr}{dX} \right)^2 + \left( \frac{dt}{dX} \right)^2 \right). \quad (31)$$

We see that the noise produced by the one-parameter "pump" Eq.(31) gives us direct information on the dependence of the scattering matrix on the varying external parameter  $X$ . This dependence  $\hat{s}(X)$  is important for calculating the current produced by the pump if two external parameters are varied Eq.(39). In a real experimental situation the dependence  $\hat{s}(X)$  is unknown and can not be calculated in a simple way. Thus the possibility to obtain this dependence from the experimental data seems useful.

Note that for some particular conditions the noise and the heat flow at the same lead are related by a simple relation. For instance, if the amplitude  $r$  of a reflection coefficient is independent of  $X$  the ratio  $S^{(1)}/I_E^{(1)}$  at low temperature ( $k_B T \ll \hbar\omega$ ) is  $8\pi G/\omega$ , where  $G = e^2 t^2/h$  is the conductance of our mesoscopic sample.

On the other hand if the phase  $\theta$  of a reflection coefficient is independent of the varying parameter  $X$  the contribution of quasi-particle correlations to the current correlations at the same lead vanishes, i.e.,  $T_{\alpha\alpha}^{(cor)} = 0$ . In this case the noise  $S_{\alpha\alpha}^{(pump)}$  reaches the Poisson value (the Fano factor is  $F = 1$ ) and the ratio of the noise to the heat flow is a universal function of the temperature and the pump frequency

$$S^{(1)}/I_E^{(1)} = F^{(1)}(\hbar\omega, k_B T) = 4e^2 F(\hbar\omega, k_B T)/(\hbar\omega)^2,$$

which is independent of individual features of a scatterer. Comparing Eq.(21) and Eq.(24) we see that this conclusion is a quite general feature of a weak amplitude pump: if  $T_{\alpha\alpha}^{(cor)} = 0$  the ratio  $S_{\alpha\alpha}^{(pump)}/I_{E,\alpha} = F^{(1)}(\hbar\omega, k_B T)$ . Note that even if the phase  $\theta$  is independent of  $X$ ,  $T_{\alpha\beta}^{(cor)} \neq 0$  ( $\alpha \neq \beta$ ) and the quasi-particle correlations are important for the current cross-correlations. To illustrate this fact in the next subsection we consider a particular case of a multiterminal (three-terminal) conductor. We investigate a case when the phase of the transmission (reflection) amplitude is unimportant but the current cross-correlations are present.

## B. Noise and heat flow of an oscillating wave splitter

Let us consider a wave splitter in which one lead  $\alpha = 1$  couples via a tunnel barrier with transparency  $\epsilon$  symmetrically to two leads  $\alpha = 2, 3$ . We assume that this three lead structure is described by the single parameter scattering matrix<sup>61</sup>

$$\hat{s} = \begin{pmatrix} -(a+b) & \sqrt{\epsilon} & \sqrt{\epsilon} \\ \sqrt{\epsilon} & a & b \\ \sqrt{\epsilon} & b & a \end{pmatrix}. \quad (32)$$

where  $a = (\sqrt{1-2\epsilon} - 1)/2$  and  $b = (\sqrt{1-2\epsilon} + 1)/2$ . For  $\epsilon = 0$  carriers incident from lead 1 are completely reflected, for  $\epsilon = 1/2$  carriers incident from lead 1 are transmitted (without reflection) with equal probability into leads 2 and 3.

We choose the transparency  $\epsilon$  as an external parameter and assume that it is subject to small amplitude oscillations  $\epsilon(t) = \epsilon + 2\epsilon_\omega \cos(\omega t)$  ( $\epsilon_\omega \ll \epsilon$ ). With the scattering matrix Eq.(32) the parametric emissivity matrix  $\hat{\nu}[\epsilon]$  Eq.(11) is

$$\hat{\nu}[\epsilon] = \frac{1}{4\pi\sqrt{(1-2\epsilon)\epsilon}} \begin{pmatrix} 0 & i & i \\ -i & 0 & 0 \\ -i & 0 & 0 \end{pmatrix}. \quad (33)$$

Using Eqs.(13) and (26) we can calculate  $T_{\pm\omega,\alpha}$  and  $T_{\alpha\beta}^{(cor)}$ . Substituting these probabilities into Eqs.(8), (21) and (24) we obtain the quantities of interest here. We see that the dc charge current  $I_\alpha$  is zero at all leads (as it is expected because only one parameter is varied). However the heat flow  $I_{E,\alpha}$  and zero-frequency noise power show interesting features.

Despite the fact that lead  $\alpha = 1$  is coupled only weakly, the heat flow  $I_{E,\alpha}$  (for any  $\epsilon > 0$ ) and the shot noise power  $S_{\alpha\alpha}^{(pump)}$  are the same in all three leads

$$I_{E,1} = I_{E,2} = I_{E,3} = \frac{\hbar\omega^2\epsilon_\omega^2}{4\pi\epsilon(1-2\epsilon)}. \quad (34)$$

The heat flow is related to the noise in the simple way discussed at the end of the previous subsection  $S_{\alpha\alpha}^{(pump)}/I_{E,\alpha} = 4e^2 F(\hbar\omega, k_B T)/(\hbar\omega)^2$ .

The asymmetry between leads appears in cross-correlations  $S_{\alpha\beta}^{(pump)}$  ( $\alpha \neq \beta$ ). The cross-correlation of current fluctuations at the two symmetrically coupled leads vanishes  $S_{23}^{(pump)} = 0$  but the cross-correlations invoking the fluctuating current of the weakly coupled lead 1 are non-vanishing,  $S_{12}^{(pump)} = S_{13}^{(pump)} = -\frac{1}{2}S_{\alpha\alpha}^{(pump)}$ .

## C. Heat flow and noise in two parameter pumps

Now we return to the scattering matrix Eq.(27) and assume that it depends on two parameters  $X_1(t)$  and  $X_2(t)$ . In this case we can represent the probabilities for absorption and emission in terms of a symmetric and antisymmetric contribution

$$T_{\pm\omega,\alpha}^{(2)} = T_{\omega,\alpha}^{(s)} \pm T_{\omega,\alpha}^{(a)}. \quad (35)$$

Here the symmetric (with respect to absorption and emission of a modulation quantum  $\hbar\omega$ )  $T_{\omega,\alpha}^{(s)}$  and antisymmetric  $T_{\omega,\alpha}^{(a)}$  parts which determine the heat flow Eq.(21) and the dc-current Eq.(8), respectively, are

$$T_{\omega,\alpha}^{(s)} = T_{\omega,\alpha}^{(1)}[X_1] + T_{\omega,\alpha}^{(1)}[X_2] + 2X_{\omega,1}X_{\omega,2} \cos(\varphi_2 - \varphi_1) \text{Re}[\Pi_\alpha(X_1, X_2)], \quad (36)$$

$$T_{\omega,\alpha}^{(a)} = 2X_{\omega,1}X_{\omega,2} \sin(\varphi_2 - \varphi_1) \text{Im}[\Pi_\alpha(X_1, X_2)], \quad (37)$$

with the quantity  $\Pi_\alpha(X_1, X_2)$  being

$$\begin{aligned} \Pi_\alpha(X_1, X_2) &= \sum_\beta \left[ \frac{\partial s_{\alpha\beta}^*}{\partial X_1} \frac{\partial s_{\alpha\beta}}{\partial X_2} \right] \\ &= 4\pi^2 \sum_\beta \nu_{\alpha\beta}^*[X_1] \nu_{\alpha\beta}[X_2]. \end{aligned} \quad (38)$$

In Eq.(36) the quantity  $T_{\omega,\alpha}^{(1)}[X]$  is given by Eq.(28).

Substituting Eq.(35) into Eq.(8) we immediately reproduce the result obtained by Brouwer<sup>2</sup> for the pumped current (at small amplitudes  $X_{\omega,j}$ )

$$I_\alpha = \frac{2e\omega}{\pi} X_{\omega,1} X_{\omega,2} \sin(\varphi_2 - \varphi_1) \text{Im}[\Pi_\alpha(X_1, X_2)]. \quad (39)$$

We see that varying only one (no matter which) parameter we can not generate a dc-current. But if at least two parameters  $X_1$  and  $X_2$  are varied periodically but out of phase  $\varphi_1 \neq \varphi_2$  then the mesoscopic sample can continuously pump charge between reservoirs with the same chemical potentials. This is a consequence of quantum-mechanical interference effects. These effects manifest themselves not only in the dc-current but also in the heat flow and in the noise.



To characterize the contribution of interference effects to the heat flow we consider the difference  $\Delta I_{E,\alpha}$  between the heat flow  $I_E^{(2)}$  when the two parameters are varied simultaneously and the sum of heat flows when only one parameter oscillates  $I_{E,\alpha}^{(1)}[X_1] + I_{E,\alpha}^{(1)}[X_2]$ . This difference is

$$\Delta I_{E,\alpha} = \frac{\hbar\omega^2}{\pi} X_{\omega,1} X_{\omega,2} \cos(\varphi_2 - \varphi_1) \text{Re}[\Pi_\alpha(X_1, X_2)]. \quad (40)$$

We see that the additional heat production  $\Delta I_{E,\alpha}$  and the dc-current give a full description of the quantity  $\Pi_\alpha(X_1, X_2)$ , i.e., they determine the real and the imaginary parts.

For the scattering matrix Eq.(27) we get

$$\begin{aligned} \text{Re}[\Pi_1(X_1, X_2)] &= \text{Re}[\Pi_2(X_1, X_2)] \\ &= r^2 \frac{\partial\theta}{\partial X_1} \frac{\partial\theta}{\partial X_2} + \frac{\partial r}{\partial X_1} \frac{\partial r}{\partial X_2} + \frac{\partial t}{\partial X_1} \frac{\partial t}{\partial X_2}, \end{aligned} \quad (41)$$

$$\begin{aligned} \text{Im}[\Pi_1(X_1, X_2)] &= -\text{Im}[\Pi_2(X_1, X_2)] \\ &= \frac{1}{2} \left( \frac{\partial r^2}{\partial X_1} \frac{\partial\theta}{\partial X_2} - \frac{\partial\theta}{\partial X_1} \frac{\partial r^2}{\partial X_2} \right). \end{aligned} \quad (42)$$

As we did for the heat production we calculate an additional noise  $\Delta S_{\alpha\beta}^{(pump)}$  generated by two simultaneously oscillating parameters  $X_1$  and  $X_2$  over the sum of noises produced by each of them separately,

$$\begin{aligned} \Delta S_{\alpha\beta}^{(pump)} &= \frac{8e^2}{h} X_{\omega,1} X_{\omega,2} F(\hbar\omega, k_B T) \\ &\quad \times \cos(\varphi_2 - \varphi_1) \text{Re}[N_{\alpha\beta}(X_1, X_2)], \end{aligned} \quad (43)$$

where

$$N_{\alpha\alpha}(X_1, X_2) = 4\pi^2 \sum_{\gamma \neq \alpha} \nu_{\alpha\gamma}^*[X_1] \nu_{\alpha\gamma}[X_2], \quad (44)$$

$$N_{\alpha\beta}(X_1, X_2) = -4\pi^2 \nu_{\alpha\beta}^*[X_1] \nu_{\alpha\beta}[X_2], \quad \alpha \neq \beta. \quad (45)$$

Note the cosine dependence of the additional heat and noise on the phase difference  $\Delta\varphi = \varphi_2 - \varphi_1$ . In contrast the pumped current Eq.(39) is determined by the sine of the phase difference  $\Delta\varphi$ . As a consequence if the pumped current is large (as a function of  $\Delta\varphi$ ) the additional noise and heat flow are small.

For the scattering matrix Eq.(27) we have  $\text{Re}[N_{11}] = \text{Re}[N_{22}] = -\text{Re}[N_{12}] = -\text{Re}[N_{21}] = \text{Re}[N^{(2)}]$

$$\begin{aligned} \text{Re}[N^{(2)}(X_1, X_2)] &= r^2 t^2 \frac{\partial\theta}{\partial X_1} \frac{\partial\theta}{\partial X_2} \\ &\quad + \frac{\partial r}{\partial X_1} \frac{\partial r}{\partial X_2} + \frac{\partial t}{\partial X_1} \frac{\partial t}{\partial X_2}. \end{aligned} \quad (46)$$

From Eq.(41) and Eq.(46) we can see that the additional heat production and the additional noise vanish if

the amplitude of the reflection coefficient  $r$  and its phase  $\theta$  depend on a single parameter only (not necessary the same). However there remains a heat production Eq.(30) and noise Eq.(31) owing to independently oscillating parameters. As it is evident from Eq.(28) this unavoidable heat production (see Eq.(21)) and noise (see Eq.(24)) are present always if only the "pump" is working. On the other hand if the phase  $\theta$  of the reflection coefficient depends on only one varying parameter the ratio of additional noise Eq.(43) to additional heat production Eq.(40) does not depend on the scattering matrix and is equal to  $4e^2 F(\hbar\omega, k_B T)/(\hbar\omega)^2$ .

Under some conditions the additional noise  $\Delta S_{\alpha\alpha}^{(pump)}$  can be related to the dc-current  $I_\alpha$  at the same lead. If  $r = r(X_1)$  (or  $r = r(X_2)$ ) then their ratio

$$\frac{\Delta S_{\alpha\alpha}^{(pump)}}{I_\alpha} = (-) 4et^2 \cot(\varphi_2 - \varphi_1) \frac{\partial\theta}{\partial r^2} \frac{F(\hbar\omega, k_B T)}{\hbar\omega},$$

is independent of the varying parameters. On the other hand if  $\theta = \theta(X_2)$  (or  $\theta = \theta(X_1)$ ) then we get

$$\frac{\Delta S_{\alpha\alpha}^{(pump)}}{I_\alpha} = (-) \frac{e}{t^2 r^2} \cot(\varphi_2 - \varphi_1) \frac{\partial r^2}{\partial\theta} \frac{F(\hbar\omega, k_B T)}{\hbar\omega}.$$

In the low temperature limit this ratio becomes independent of frequency.

## VIII. CONCLUSION

In this work we have developed the approach to the kinetics of an adiabatic quantum pump for an arbitrary relation of pump frequency and temperature. Our consideration is based on the scattering matrix approach for ac-transport. This approach takes into account the existence of the side bands of particles exiting the pump and thus allows a description of the quantum statistical correlation properties (e.g., noise) of an adiabatic quantum pump. The side bands correspond to particles which have gained or lost a modulation quantum  $\hbar\omega$ . We find that our results for the pump current, the heat flow and the noise can all be expressed in terms of a parametric emissivity matrix. In particular we find that the current cross-correlations of a multiterminal pump are directly related to a non-diagonal element of the parametric emissivity matrix.

Using the quasi-particle picture we have given a simple physical interpretations of processes leading to charge and energy transfer in the system. Due to the oscillations of the scatterer the electron system gains energy (the side bands arise). Absorption of an energy quantum  $\hbar\omega$  leads to the creation of a nonequilibrium (quasi-)electron-hole pair. These quasi-particles carry energy from the scatterer to the reservoirs. On average the electron-hole pair is neutral thus the pump is not a source of a charge current but under some conditions<sup>2</sup> can only push charge from some reservoirs to others. These conditions can

be realized if the quasi-electron and hole (belonging to the same pair) leave the scatterer through different leads (say,  $\alpha$  and  $\beta$ ). In this case these quasi-particles contribute to the charge transfer between the reservoirs  $\alpha$  and  $\beta$ . These quasiparticles are correlated since they are created in the same event. This is also the source of the correlations between the currents at the leads  $\alpha$  and  $\beta$  (cross-correlations). Thus we conclude that the existence of the dc-current in a weak amplitude pump is always accompanied by current correlations (shot noise). This type of a pump can not be optimal (in particular, noiseless) in the sense of Ref.<sup>4</sup>.

To assess the possibility of an optimal adiabatic quantum pump further investigations are necessary. In particular large amplitude variations of the external parameter have to be considered. We hope that by taking into account many photon processes (see, e.g.,<sup>59</sup>) the approach developed in the present paper can be generalized to the case of a strong amplitude adiabatic quantum pump.

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