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Dissipation of contestable rents by small numbers of contenders

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1. Introduction

1.1 *Rent dissipation*

The theory of rent seeking with its origins in the observations of Gordon Tullock (1967) – or to use Jagdish Bhagwati's (1982) proposed term, the theory of directly unproductive profit-seeking activities – is concerned with the potentially adverse effects on resource allocation of incentives to capture and defend artificially-contrived rents and transfers. The scope for social loss proposed by the theory derives from the relation between the value of a contestable prize and the value of the resources attracted into the contest to determine the beneficiary of the prize. Underlying this social loss is a specification of how rational behavior by optimizing agents links the value of the prize sought to the resources expended.

It has been traditional to assume competitive behavior in describing the activities of lobbying and influence seeking. Then, if some further conditions are satisfied,[1] the total value of the resources expended precisely equals the value of the prize sought, so dissipation is complete.[2] Consequently, the social cost associated with contestability of a rent can be inferred from the value of the rent itself, and the detailed and hard-to-come-by information on individual outlays made in the course of the contest becomes unnecessary. By basing their analyses on competitive dissipation, contributors to the rent seeking literature (see the review by Robert Tollison, 1982) have been able to presume that the observed value of a contested rent is an exact measure of the associated social cost of monopoly power or regulation. Similarly, in the trade-theoretic literature where the rights contested are to quota premia or revenues from trade taxes (Krueger, 1974; Bhagwati

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and Srinivasan, 1980, 1982, 1983) rent and revenue seeking have been portrayed as competitive activities.[3]

Competition may however well be absent from contests to secure rights to rents and transfers, just as it is absent from the regulated, monopolized and protected markets where the rents arise. Special knowledge, connections, prior positioning in relationships within bureaucratic structures, political advantage, or a general historically-based advantage of incumbent placement, may all restrict participation in a contest to only a small number of participants. The assumption of a perfectly competitive environment therefore makes the claim of social loss due to contestability of rents or transfers vulnerable to the observation that contests may be restricted to small numbers of participants. Absence of conditions assuring competitive rent seeking is for example an element in Franklin Fisher's critique (1985) of the Tullock/Krueger/Posner presumption that an observed rent reflects a social cost of equal value.[4]

In this paper we present a theoretical basis for a presumption of complete dissipation of indivisible artificially-contrived contestable rents or transfers which does not rely on a competitive environment. Rational equilibrium behavior is investigated in small-numbers contests wherein the successful contender will have made the greatest outlay in seeking to influence the outcome in his or her favor.[5] Our basic result is that with risk-neutrality and no minimum outlay required for participation, equilibrium behavior results in the sum of all outlays made being equal to, in an expected sense, the value of the prize secured by the successful participant. Such expectationally complete dissipation arises for any number of participants in a contest. Hence, the number of rival contenders does not influence the expected magnitude of social loss due to contestability of rents or transfers. Dissipation is quite simply complete on average independently of the number of individuals placed to participate in a contest.

A theoretical foundation other than limiting competitive behavior is therefore provided for basing estimates of the social costs of contestability of rents and transfers on observed values of the prizes contested. With appropriate measurement (see Franklin Fisher and John McGowan, 1983; Harold Demsetz, 1985), averaging on an economy-wide basis, any number of potential monopolists is consistent with an association of the full value of a contested rent with an equivalent social cost. Or any number of individuals competing for a position in a bureaucratic hierarchy is consistent with outlays by contenders equal on average to the rent associated with incumbency. In the trade-theoretic settings, there may not be free entry into contests where the objective is to influence the transfer policy of governments, but dissipation of rents arising from trade restrictions can nevertheless be proposed to be expectationally complete even if the number of contenders is small.

1.2. *Equilibrium behavior*

In the contests with which we are concerned – where all outlays made are irretrievably lost and the largest outlay determines the successful contender – it is well known that there can exist no *pure* strategy equilibrium. An equilibrium requires that no contender have an incentive to change his behavior, given the behavior of rivals. However, if the contender making the greatest outlay wins, the incentive is ever present to expend marginally more than the previous highest outlay. This had led to conjectures that *no* equilibrium exists. We shall show that there does however exist a rational pattern of equilibrium behavior for rival contenders characterized by *mixed* strategies.

Let the value of the indivisible prize being contested be given by V, and let there be n risk-neutral agents with the requisite information and positioning to compete symmetrically as potential beneficiaries. Denote by $x \geq 0$ the outlay made by a potential beneficiary, where there is no minimum outlay required. Then there is a unique symmetric Nash equilibrium which is described by contenders' randomizing choice of outlays according to the continuous function

$$F^*(x) = \left(\frac{x}{V}\right)^{1/(n-1)}, \quad 0 \leq x \leq V$$

where $F^*(x)$ is the probability that a contender's outlay does not exceed x.

Thus, when there are but two contenders, equilibrium behavior entails the choice of an outlay from the uniform distribution over (0, V). That the equilibrium is the uniform distribution for the two-contender case has been previously noted by Hirshleifer and Riley (1978).

When the number of contenders increases to three, equilibrium behavior entails randomization of outlays according to $(x/V)^{1/2}$; and so on. As the number of contenders further increases, the equilibrium distribution places increasingly greater weight on smaller outlays, to compensate for the reduced likelihood of any particular contender winning.

When individuals adopt the above equilibrium pattern of behavior, the expected value of an outlay is given by

$$Ex = \int_0^V x \, d\left(\frac{x}{V}\right)^{1/(n-1)}.$$

As we shall show, this can be readily expanded to reveal that

$$\int_0^V x \, d\left(\frac{x}{V}\right)^{1/(n-1)} = V/n$$

and hence

$$nEx = V.$$

That is, the expected value of the sum of the outlays made in quest of the rent or transfer V is precisely equal to V itself. Moreover, this relation holds for any value of the number of contenders n.

Ex-ante equilibrium behavior is symmetric since each contender chooses his outlay according to F*. Ex-post behavior will however be observed to be diverse since realized outlays will differ. Diversity in ex-post behavior (i.e., outlays) is of course necessary given that no symmetric equilibrium in pure strategies (i.e., all participants making the same bid) exists. Individuals can hope to be successful in a contest only by differentiating their ex-post behavior from that of other participants. The distribution F*(x) describes the common ex-ante equilibrium strategy for differentiation of ex-post behavior. Randomization is consistent with reported observations of behavior. Robert Tollison reports in his survey (1982: 585) that in experiments in contests in which the highest outlays wins 'the results have been all over the board'.

The equilibrium pattern of behavior need not be symmetric, even though all contenders have the same information and place identical valuations on the prize sought. Nonsymmetric equilibria exist and can take but one form, which is that some potential participants choose to make zero outlays. But then those individuals who do actively participate still completely dissipate the rent or transfer sought. Rent dissipation remains expectationally complete without regard for the number of contenders, active or inactive.

1.3 *Qualifications*

The result of expectationally complete rent dissipation requires qualification in the presence of a minimum outlay requirement for participation. A minimal outlay leads the expected value of outlays to fall short of the value of the rent or transfer being contested. However, expectationally complete dissipation nevertheless obtains, in the sense that the expected value of outlays equals the *expected* value of the prize, EV, associated with the contest.

The equilibrium F*(x) is continuous; that is, contenders do not place positive weight on any particular outlay. Any outlay with concentrated probability would be dominated by a marginally greater outlay – no pure-strategy equilibrium exists. A minimum outlay requirement however rules out marginal departures from a bid of zero and gives rise to a unique symmetric equilibrium in mixed strategies which assigns concentrated probability to a zero outlay. There is consequently some positive probability that no positive outlay will be realized (the joint probability that all contenders make zero outlays). Hence, if a positive outlay is required for the rent to be assigned, the *expected* value of the prize in the contest falls below V.

In effect, the minimum outlay acts as a barrier to entry. The greater the minimum outlay relative to the value of the rent contested, the fewer the number of individuals who on average actively participate by making strictly positive outlays, and overall the smaller the expected value of the resources allocated to the quest for the rent. In the competitive limit the expected value of the resources expended falls short of the value of the rent by precisely the required minimum outlay.

Incomplete dissipation also occurs when contenders are risk-averse. Given the uncertainty in a contest (there is but one winner) risk aversion reduces the outlays which contenders are prepared to make in quest of a given prize.

2. Equilibria in contests where the largest outlay wins

We now proceed to present the formal detailed proofs of our rent dissipation propositions. Then, subsequently, in the final section, we compare rent dissipation in our contests with outcomes in Gordon Tullock's (1980) 'efficient rent seeking' contests and the adaptation of 'efficient rent seeking' by Higgins, Shughart and Tollison (1985). We also briefly describe the relationship between our contests and 'wars of attrition' and then finally distinguish between circumstances of lobbying and influence seeking to which our representation of contests might and might not apply.

2.1 *Preliminaries*

Let an indivisible rent or transfer of value V be contested by n symmetrically placed individuals. Contenders know that the rights to the rent or transfer will be conferred on the individual who expends the most in lobbying activity.

In the *basic game* which we now proceed to describe, contenders are risk neutral and there is no initial minimum outlay required for entering a contest. In the event of no active participation (i.e., nobody making a positive bid) the rent is not assigned, and in the event of tied positive bids, the rent is shared.[6]

A Nash equilibrium *in pure strategies* for the contest is a set of outlays by the n contenders $(x_1^*, \ldots, x_n^*)$ such that x_i^* is the best response of participant i when that individual assumes that each rival contender $j \neq i$ chooses to outlay x_j^*. It can be readily demonstrated that no pure-strategy equilibrium exists. Suppose that $(x_1^*, \ldots, x_n^*)$ were an equilibrium. These outlays cannot contain a positive x_i^* which is strictly less than the maximal outlay, since anyone who chose such an outlay knowingly incurs a loss and would be better off (given what others are doing) by outlaying zero. Hence, in the equilibrium $(x_1^*, \ldots, x_n^*)$, all the positive x_i^*'s are the same. If there is but one positive x_i^*, then the individual who has made this bid does better (given everyone else's outlays) by spending marginally less – with the smaller outlay he still wins. And if there are a number of such x_i^*'s, then any one individual is better off marginally increasing his bid and thereby winning the contest. Finally, if there are no positive x_i^*'s at all, then someone can win by making a small positive bid; hence a set of outlays with no positive x_i^*'s can also not be an equilibrium. No equilibrium in pure strategies therefore exists.

Let the participants now consider adopting *mixed strategies*. A participant's decision variable is then the probability distribution from which outlays will be drawn. Let $F_i(x)$ denote the probability distribution for participant i's choice of x. That is, $F_i(x)$ is the probability that participant i chooses an outlay no greater than x. Denote by $T_i(x_1, \ldots, x_n)$ the payoff function for individual i when the outlays chosen are $(x_1, \ldots, x_n)$. Then, when the n participants choose the mixed strategies $(F_1, \ldots, F_n)$, individual i's expected payoff is given by

$$T_i(F_1, \ldots, F_n) = \int \ldots \int T_i(x_1, \ldots, x_n)\, dF_1(x_1) \ldots dF_n(x_n). \qquad (1)$$

Alternatively, (1) can be expressed as

$$T_i(F_1, \ldots, F_n) = \int T_i(F_1, \ldots, F_{i-1}, x, F_{i+1}, \ldots, F_n)\, dF_i(x), \qquad (1')$$

where the integrand T_i is the expected payoff to participant i when all participants except i use the mixed strategies F_j and participant i outlays x with probability 1.

The mixed strategies $(F_1^*, \ldots, F_n^*)$ constitute a Nash equilibrium when F_i^* is the best response of participant i to the choice of strategies of the other (n – 1) participants; that is, for each participant i

$$T_i(F_1^*, \ldots, F_i^*, \ldots, F_n^*) = \max_{F_i} T_i\, (F_1^*, \ldots, F_i, \ldots, F_n^*). \qquad (2)$$

Denote by $(\pi_1^*, \ldots, \pi_n^*)$ the associated values of individuals' expected payoffs for the set of strategies $(F_1^*, \ldots, F_n^*)$. That is,

$$\pi_i^* = T_i(F_1^*, \ldots, F_i^*, \ldots, F_n^*) \qquad (3)$$

The following proposition characterizes a Nash equilibrium and is very helpful in computing equilibria.

Proposition 1: The set of strategies $(F_1^*, \ldots, F_i^*, \ldots, F_n^*)$ is a Nash equilibrium if and only if for each individual i and each pure strategy x_i of i

$$T_i(F_1^*, \ldots, F_{i-1}^*, x_i, F_{i+1}^*, \ldots, F_n^*) \leqq \pi_i^*. \tag{4}$$

Clearly if $(F_1^*, \ldots, F_n^*)$ is an equilibrium then $F_i^*$is a best response for i and in particular no pure strategy can provide i with a payoff higher than π_i^*. Conversely, if (4) is satisfied then no mixed strategy F_i yields more than $\pi_i^*$to i, since F_i is just a mixing of pure strategies. More formally, by (1′) and (4),

$$\begin{aligned} T_i\,(F_1^*, \ldots, F_i, \ldots, F_n^*) &= \int T_i\,(F_1^*, \ldots, x, \ldots, F_n^*)\, dF_i(x) \\ &\leqq \int \pi_i^* dF_i^*(x) = \pi_i^* \end{aligned}$$

Hence when $(F_1^*, \ldots, F_n^*)$ is an equilibrium, no pure strategy x_i for player i yields, by Proposition 1, more than π_i^*. But since F_i^* yields exactly π_i^*, some pure strategies must also yield π_i^*. The following proposition characterizes outlays which have in common that they yield the expected return π_i^*. We designate x an *increasing point* of a distribution function F if F is not constant in a neighborhood of x – in other words, if for each $\epsilon > 0$ the probability of having a value in $(x-\epsilon, x+\epsilon)$ is positive.

Proposition 2: If $(F_1^*, \ldots, F_n^*)$ is an equilibrium with the associated payoffs $(\pi_1^*, \ldots \pi_n^*)$, then for each i and for each increasing point x_i of F_i such that $T_i(F_1^*, \ldots, x_i, \ldots, F_n^*)$ is continuous at x_i:

$$T_i(F_1^*, \ldots, x_i, \ldots, F_n^*) = \pi_i^*.$$

Moreover (5) holds for each x_i at which F_i^* is discontinuous.

2.2 *The symmetric equilibrium*

In a symmetric equilibrium, all participants choose the same mixed strategy $F^*(x)$ and have the same expected payoff π^*. In the following proposition we characterize the unique symmetric equilibrium of the basic game described above.

Proposition 3: (i) There is a unique symmetric equilibrium in the basic game which is given by the distribution function

$$F^*(x) = \begin{cases} 0 & x \leqq 0 \\ \left(\dfrac{x}{V}\right)^{1/(n-1)} & 0 \leqq x \leqq V \\ 1 & V \leqq x \end{cases}$$

(ii) The associated equilibrium payoff π^* is zero.

Proof: We begin by showing that F^* in (6) is an equilibrium: When player i outlays x, his probability of winning is given by $F^*(x)^{n-1}$ (i.e., the joint probability that the other (n – 1) contenders outlay less than x). Taking participant 1 as representative, the expected payoff from outlaying x is

$$T_1(x, F^*, \ldots, F^*) = F^*(x)^{n-1}V - x. \tag{7}$$

Thus, substituting for $F^*(x)$ from (6) in the range [0,V],

$$F^*(x)^{n-1}V - x = (x/V)V - x = T_1(x, F^*, \ldots, F^*) = 0. \tag{7'}$$

That is, each pure strategy in [0,V] yields an expected return of zero, and therefore so does the *equilibrium* strategy F^* which mixes these pure strategies. π^* is therefore zero. Clearly outlays greater than V yield less than 0. This with (7′) guarantees by Proposition 1 that F^* is an equilibrium.

We now proceed to demonstrate that F^* in (6) is the *unique* symmetric equilibrium. As a first step to showing that F^* must be given by (6), substitute $x = 0$ into (4) and, noting that $T_1(0, F^*, \ldots, F^*) \geqq 0$, it follows that $\pi^* \geqq 0$. That is, the equilibrium payoff cannot be negative. The option always exists of outlaying zero and receiving a return (with certainty) of zero, making zero a lower bound to π^*.

Next, we observe that the equilibrium distribution F^* must be continuous.[7] Since F^* does not place positive weight on points, the probability of ties is zero. An individual's expected gain π^* is then given by the probability $F^*(x)^{n-1}$ that x exceeds the maximal bid of the other (n – 1) contenders, multiplied by the rent V, minus the outlay x. $T_i(F^*, \ldots, x, \ldots, F^*)$ is continuous in x and hence, by Proposition (2), for each increasing point x of F^*,

$$F^*(x)^{n-1} \cdot V - x = \pi^*. \tag{10}$$

In particular $x = 0$ is an increasing point[8] and so yields π^*. Hence the expected return at $x = 0$ can be used to establish π^*. Substituting $x = 0$ into (10), we obtain $F^*(0)^{n-1}V - 0 = \pi^*$ which reveals that $\pi^* = 0$.

It therefore follows from (10) that at each increasing point of F^*,

$$F^*(x)^{n-1} = x/V. \tag{11}$$

Since F^* is continuous and increasing, (10) holds for all x in the interval [0,V]. That is, the set of all increasing points coincides with this interval. (6) follows directly from (11). The unique symmetric equilibrium is therefore given by F^* in (6).

2.3 *Rent dissipation*

Now consider rent dissipation. In the symmetric equilibrium described by F*, the expected outlay of a participant is

$$Ex = \int_0^V x \, dF^*(x) \tag{12}$$

$$= \int_0^V x \, d(\frac{x}{V})^{1/(n-1)}$$

$$= \int_0^V x \, F^{*\prime}(x)dx$$

$$= \frac{1}{(n-1)} V^{-1/(n-1)} \int_0^V x^{1/(n-1)} \, dx$$

$$= \frac{1}{(n-1)} V^{-1/(n-1)} [\frac{(n-1)}{n} x^{n/(n-1)}]_0^V$$

$$= \frac{V}{n}$$

and hence

$$nEx = V. \tag{12'}$$

Thus the expected value of outlays for any number of participants n is equal to the value of the rent V.

2.4 *The nonsymmetric equilibria*

There also exist nonsymmetric equilibria. Of n potential participants in a contest, let $(n-m)$ be inactive in the sense that they outlay zero with probability one. It is evident that the number of inactive individuals in equilibrium is not n, for we have already shown that in the unique symmetric equilibrium all potential participants are active and behave according to the mixed strategy given by (6). There cannot be a lone active individual, since such an individual would place zero mass on each interval $[a,V]$ with $a > 0$ since he would always do better outlaying $(a-\epsilon)$. Hence, there are at least two active individuals.

Proposition 4: A set of strategies in which $(n-m)$ players are inactive

($2 \leqq m \leqq n$) and the remaining players choose the mixed strategy given by

$$F^*(x) = \begin{cases} 0 & x \leqq 0 \\ (\frac{x}{V})^{1/(m-1)} & 0 \leqq x \leqq V \\ 1 & V \leqq x \end{cases}$$

is an equilibrium. Moreover all equilibria of the basic game (risk neutrality and no minimum outlay) take this form.[9]

To confirm that the set of strategies described in Proposition 4 is an equilibrium, observe that each active player views the game as one in which there are m participants. Each outlay yields an active player an expected payoff of zero, which shows that F^* is a best response for an active player. Inactive individuals outlay zero with probability one and have a return of zero, and cannot improve upon this outcome, for if an inactive individual chooses to participate by outlaying $0 < x \leqq V$, his expected return is negative, given by

$$F^*(x)^m V - x = (\frac{x}{V})^{m/(n-1)} V - x < 0. \tag{13}$$

The asymmetry between inactive and active players enters via the number of rival contenders that each type of player confronts when making a positive outlay. Active players confront $(m-1)$ rivals making positive outlays and their expected return is zero. Inactive players, should they decide to change their behavior and make a positive outlay, find themselves confronting m contenders, which via (13) yields a negative expected return. Hence, zero is the best response for an inactive individual, and therefore the strategies of Proposition 4 are an equilibrium.

One might observe that an inactive individual could outlay V which in (13) yields a return of zero. However, if one individual outlays V, this can be an equilibrium only if that individual then wins the rent with probability one. Then all other participants outlay zero, in which case there is no point in the individual who bid V maintaining that bid. Hence a bid of V cannot be an equilibrium.

Observe that for purposes of rent dissipation it does not matter how many individuals are active and how many are inactive in the asymmetric equilibrium. Active individuals, whatever their number, completely dissipate the rent on average.

2.5 *Minimum outlays for participation*

We now introduce the requirement of a minimum outlay for participation in a contest. Denote the minimum outlay by c (less than V). The set of pure strategies available to a player then consists of the outlay zero and outlays in the interval (c,V]. A mixed strategy is a probability distribution over the restricted set of pure strategies and is described by a cumulative distribution function F* which assigns probability zero to positive expenditures less than c. That is, F* is constant on [0,c).

The argument that prohibits F* from placing positive weight on any one particular point remains valid for the interval (c,V]. That is, F* is continuous on this interval. Similarly our previous argument that $\pi^* \geqq 0$ also applies.

Since F* is continuous at c and is constant on [0,c), it follows that F*(0) = F*(c). It therefore cannot be the case that F*(0) = 0; if this were so, the first increasing point of F* would be positive, which would then imply that a participant's expected payoff π^* is negative. The probability of that participant's expending zero, F*(0), is therefore strictly positive. It follows that the pure strategy of outlaying zero yields the payoff π^* to a participant when all other individuals choose the mixed strategy F*.

The value of π^* can be determined from the payoffs when participants make the common bid of zero. In that case the rent is not assigned, so an individual who is not active and outlays zero receives a return of zero independently of the outlays of other participants. Hence $\pi^* = 0$.

But since π^* is common to all increasing points, for each $x \geqq c$ in which F* increases,

$$F^*(x)^{n-1} V - x = \pi^* = 0. \tag{14}$$

Hence

$$F^*(x) = \left(\frac{x}{V}\right)^{1/(n-1)}, \quad x \geqq c. \tag{6''}$$

Since F* is continuous on the interval (c,V] and is an increasing function, (6″) holds for all x in the interval (c,V]. On this interval, F* therefore coincides with the equilibrium which obtains in the absence of a minimum outlay.

Now consider the interval [0,c). At x = c,

$$F^*(c) = \left(\frac{c}{V}\right)^{1/(n-1)}. \tag{15}$$

Since F* is constant over [0,c),

$$F^*(0) = F^*(c) = \left(\frac{c}{V}\right)^{1/(n-1)} \tag{15'}$$

which is the probability of expending zero. This probability increases as c increases. In the limit as $c \to V$, $F^*(0) \to 1$.

If a symmetric equilibrium exists, then that equilibrium is necessarily F* as given by (6″) and (15′).

To confirm that F* is indeed an equilibrium, suppose that one player makes an outlay x between c and V and all others choose F*. Since F*(x) has the same value for $x > c$ as in the game without minimum outlays, the deviating player's expected payoff is zero. Alternatively, let the deviating player's outlay be zero. But then his payoff is also zero. All pure strategies therefore yield zero (when everybody else is playing F*). Therefore every mixed strategy must yield zero. In particular, no player can derive any benefit by deviating from F* F* is therefore an equilibrium, and it is the only symmetric equilibrium.

Now consider rent dissipation. The expected outlay of a participant is

$$E(x) = \int_0^V x dF^*(x) = 0. \left(\frac{c}{V}\right)^{1/(n-1)} + \int_c^V x d\left(\frac{x}{V}\right)^{1/(n-1)}$$

$$= \frac{1}{n} V^{-1/(n-1)} [x^{n/(n-1)}]_c^V$$

$$= \frac{1}{n} [V - (c^n/V)^{1/(n-1)}].$$

Total expected expenditure is thus less than the value of the rent V, since

$$nE(x) = [V - (c^n)^{1/(n-1)}] < V.$$

We also observe that

$$\lim_{n \to \infty} nE(x) = (V - c) < V. \tag{17'}$$

So, in the competitive limit the shortfall of the expected value of outlays from V is precisely the minimum outlay required of an active participant, c.

V is the value of the prize if there is at least one active participant who makes a positively valued outlay. But we have observed that there is a positive probability of an outcome with no positive outlay. The expected value of the prize for the contest is therefore the probability that all individuals outlay zero multiplied by the corresponding zero prize plus the probability that at least one individual is active and makes a bid in excess of the minimum outlay multiplied by the prize V. That is, denoting Pr(nobody bids $x > 0$) by P,

$$
\begin{aligned}
EV &= 0P + V(1-P) \\
&= V[1-F^*(0)^n] \\
&= V[1-(c/V)^{n/(n-1)}] \\
&= n\,Ex.
\end{aligned}
$$

Rent dissipation therefore remains on average complete, in the sense that the expected value of the prize equals the expected value of resources used in the contest.

2.6 *Risk aversion*

Now depart from the assumption of risk neutrality. Let all individuals have the same continuous increasing strictly concave utility function $U(\cdot)$ reflecting risk aversion, normalized so that $U(0) = 0$, $U(V) = 1$. The same arguments as applied in the case of risk neutrality continue to ensure that F^* is continuous and that π^* (in utility terms) is zero. An individual outlaying x which is an increasing point of F^* has expected utility from participation

$$F^*(x)^{n-1}\, U(V-x) + [1-F^*(x)^{n-1}]\, U(-x) = \pi^* = 0 \tag{19}$$

and solving for points x for which $F^*(x)$ is increasing yields

$$F^*(x) = \left[\frac{-U(-x)}{U(V-x)-U(-x)}\right]^{1/(n-1)}. \tag{20}$$

With $U(\cdot)$ continuous and increasing and with $U(0) = 0$, it can readily be confirmed that $F^*(x)$ in (20) is a cumulative distribution which holds for all x in $[0,V]$. $F^*(x)$ is the unique symmetric equilibrium when participants are risk averse.

The expected outlay of a participant in the mixed-strategy equilibrium is given by:

$$
\begin{aligned}
Ex &= \int_0^V x\,dF^*(x) \qquad\qquad (21)\\
&= F^*(x)\Big|_0^V - \int_0^V F^*(x)\,dx \\
&= V - \int_0^V F^*(x)\,dx.
\end{aligned}
$$

Since U is concave, $U(t)/t$ is a decreasing function of t, and therefore for $x \geqq 0$

$$\frac{U(-x)}{-x} \geqq \frac{U(V-x)}{V-x} \tag{22}$$

which implies

$$g(x) = \frac{-U(-x)}{U(V-x)-U(-x)} \geqq \frac{x}{V}. \tag{23}$$

Thus

$$Ex = V - \int_0^V F^*(x)dx \leqq V - \int_0^V (x/V)^{1/(n-1)}\, dx = V/n. \tag{24}$$

In particular, when U is strictly concave,

$$nEx < V. \tag{25}$$

Thus, in the presence of risk aversion rent dissipation is incomplete.

To compute total expected rent seeking outlays as n becomes large, use (21) and (24) to obtain

$$nE(x) = \frac{[V - \int_0^V g^{1/(n-1)}\,(x)dx]}{1/n}. \tag{26}$$

Replacing n by a continuous parameter t and applying L'Hopital's rule yields

$$\lim_{t \to \infty} nEx = \int_0^V \ln g^{-1}(x)\, dx. \tag{27}$$

Then, as a consequence of (23),

$$\int_0^V \ln g^{-1}(x)\, dx \leqq \int_0^V \ln(V/x)\, dx = V. \tag{28}$$

If U is strictly concave, (28) holds with strict inequality. So, no matter how many participants enter a contest, if participants are risk averse the value of a rent overstates the value of resources expended in quest of the rent.

3. Comparisons

3.1 *Efficient rent seeking*

The proposition that rent dissipation will be expectationally complete independently of the number of contenders contrasts with the outcomes which obtain (under the same conditions of risk neutrality and no minimum outlay) in Gordon Tullock's (1980) small-numbers rent-seeking contests. In Tullock's contests, which he describes as 'efficient rent seeking', strategic behavior by small numbers of participants results in general in underdissipation of a rent, and the extent of dissipation depends upon the number of participants. Yet our contests yield *complete* dissipation on average for *any* number of participants.[10]

The different conclusions can be readily traced to the rule for designating the successful contender in a contest. Tullock assumes that individuals' outlays determine probabilities of success. Rational equilibrium behavior is described by a Nash equilibrium in pure strategies (the pure strategy consisting of the outlay made). In a symmetric equilibrium all contenders make the same outlays and confront the same probabilities of success. Some random mechanism (a lottery) must then be called upon to choose the successful contender.[11]

Richard Higgins, William Shughart II and Robert Tollison (1985) have extended Tullock's 'efficient rent seeking' contests to two-stage games which, as do our contests, have mixed strategy equilibria. In the first stage individuals use mixed strategies to choose whether to incur a sunk cost which is a prerequisite for participation in stage two. Then, for those individuals continuing on in the contest, the second stage of the game takes on the basic character of Tullock's contests. The stage-one probability that an individual will proceed to participate in stage two is endogenously determined to ensure that the number of participants in stage two is on average consistent with complete rent dissipation in an 'efficient rent seeking' contest. If the number of active participants realized as the consequence of stage one of the game gives the stage-two game a negative expected value, then individuals further randomize and drop out, incurring the sunk cost, until the expected value of the stage-two game is no longer negative. In stage-two, outlays are chosen as pure strategies. In a symmetric Nash equilibrium a common outlay is chosen by stage-two participants. However, individuals' effective outlays are subject to differing random influences (suggestively, observation error). The individual with the greatest stochastic addition to the common pure-strategy outlay is the winner of the contest.

This two-stage contest, which adds a preliminary bout to Tullock's 'efficient rent seeking' pure strategy contests, clearly differs in nature from our formulation where the highest outlay wins. In particular, rent dissipation in the Higgins/Shughart/Tollison game is associated with the stochastic entry decisions made in the first stage of the contest, whereas in our game stochastic entry decisions affecting the number of actively participating contenders play no role in establishing the result that rent dissipation will be expectationally complete. Our contests are characterized by expectationally complete rent dissipation for any number of active contenders without the need for preliminary elimination bouts.[12]

3.2 *Wars of attrition*

In a war of attrition the winning contender perseveres the longest.[13] A stopping rule is chosen ex ante by participants, who determine their outlays stochastically and drop out of a contest when their maximal outlay is surpassed by rivals. When the second to last contender has dropped out of contention, the remaining contender claims the prize. The cost to the winner is determined by the resources expended by the last rival to drop out, whereas in a rent-seeking contest each participant loses the value of his *own* bid.

It is in principle possible to distinguish our contests observationally from a war of attrition via the values of the outlays made. No participant in our contests makes a realized outlay in excess of the value of the prize for the contest, whereas in a war of attrition realized outlays may exceed the value of the prize.

One may wish to suggest that, at least in certain instances, the quest for a rent or transfer may take the form of a war of attrition rather than conform to the specifications of our contests. That is, perseverance may matter. In that case well-known results can be called upon to establish the extent of rent dissipation. These results reinforce our own conclusions. In a war of attrition with symmetric information and evaluation of the prize,[14] rent dissipation is expectationally complete for any number of participants. Hence, if perseverance does determine the winner of a rent or transfer seeking contest, so that the contest takes on the character of a war of attrition, our conclusions regarding the association between the value of the resources expended and the observed value of a rent or transfer apply. Independently of conditions of entry the value of a contestable rent or transfer reflects, on average, the total value of outlays made by rival contenders.

3.3 *Contests with countervailing influence*

The contests which we have portrayed reflect circumstances where politicians or regulators engage in decision processes which give rise to contestable rents or revenues which are awarded *indivisibly*. Rents may be associated with monopoly power or regulation of an industry, with protection, or the prize may for example be the gains which accrue from the location of a government facility in a particular geographical area. Given the prize contested, politically-allocated transfers are indivisibly secured by the individual or group which has expended the most in lobbying.

Such contests for prizes secured in full by the ultimate winner differ from the perceptions of competition in analyses of political-market allocation which stress the compromise nature of political outcomes – where political self-interest of governments gives rise to a trade-off between the interests of gainers and losers from intervention (as in Peltzman, 1976, in the context of regulation, or Hillman, 1982, in the context of protection) or where the circumstances of competition among pressure groups limit the transfer gainers can obtain from losers (Becker, 1983, 1985). In these formulations conditions describing marginal changes in political influence or support determine the distribution of gains and losses among competing groups – the highest bidder does not secure an indivisible prize, but rather countervailing power influences the distribution of gains.

NOTES

1. Sufficient conditions are that outlays made to influence the likelihood of success in a contest yield constant returns, there is no advantage of incumbency, and contenders are risk-neutral – on the respective consequences of departures from these assumptions, see Gordon Tullock (1980), William Rogerson (1982), and Arye L. Hillman and Eliakim Katz (1984).
2. The resources expended by rival contenders in these contests are irretrievably lost. The characteristics of total irretrievable loss of all contenders' outlays distinguishes such contests from the various forms of auctions in which unsuccessful contenders retain (at least part of) their bids or in which the successful bidder pays an amount other than his own bid: on the design and characteristics of auctions wherein some bids are retained, see John Riley and William Samuelson (1981), Paul Milgrom and Robert Weber (1982).
3. In the trade-theoretic literature Jagdish Bhagwati has emphasized that in initially distorted equilibria the shadow price of resources used in lobbying and influence-seeking activities may well be negative and hence such activities may be welfare improving. This reflects the converse of immiserizing growth. If additional resources make a country worse off, withdrawing these same resources from production can conversely be beneficial. We shall assume positive shadow prices for domestic resources used in contesting rents and transfers. But whether shadow prices are positive is not an important issue in this paper. Our basic question concerns the relation between the value at market prices of resources used in lobbying and the value of the rent contested.
4. Fisher also makes the quite valid point that the mere observation of a rent does not imply an associated social cost. Some rents were not (or are not) contestable, and some rents are not artificially contrived via government regulation or protection but rather reflect superior efficiency. On this distinction see Harold Demsetz (1974, 1976). Thomas DiLorenzo (1984) develops a similar theme emphasizing the essential role of government in the creation of contestable rents to which social loss can be attributed.
5. Robert Tollison (1982: 585) in his survey of the rent seeking literature associates such contests with the name of Geoffrey Brennan.
6. Neither of these two characteristics substantively affects the analysis of the basic game. In equilibrium the probability of tied outlays (including a tied outlay of zero) will be shown to be zero.
7. Suppose on the contrary that there were to exist an outlay x_0 assigned positive probability α by F^*, and compare the outlays x_0 and $(x_0+\epsilon)$, $\epsilon > 0$. The increase in outlay entails loss with certainty of ϵ. The expected gain from increasing the outlay is at least $\alpha^{n-1}(V-V/n)$: α^{n-1} is the probability that the other $(n-1)$ contenders will outlay x_0, and outlaying $(x_0+\epsilon)$ yields V as opposed to the share V/n if all outlays equal x_0. The expected gain

$\alpha^{n-1}(V - V/n)$ is however independent of ϵ, and hence if ϵ is chosen to be small enough, $(x_0 + \epsilon)$ is expectationally preferred to x_0. But by Proposition 2, since F* is discontinuous at x_0, $T_1(x_0, F^*, \ldots, F^*) = \pi^*$, which implies $T_1(x_0 + \epsilon, F^*, \ldots, F^*) > \pi^*$ and contradicts (4). Hence F*(x) must be a continuous function. For a similar argument in a related setting see Eric Maskin and John Riley (1984, pp. 1484–85). (Note the link to the absence of an equilibrium in pure strategies. If x_0 has positive weight, it always pays to increase the outlay marginally above x_0, since this yields a non-marginal expected gain.)

8. Suppose that $x = 0$ were not an increasing point of F*. Then F* would be constant in the neighborhood of $x = 0$. Let $\underline{x} > 0$ be the first point at which F* increases, so $F^*(\underline{x}) = 0$ since F* is continuous. Via (10) $F^*(\underline{x})^{n-1} V - \underline{x} = \pi^*$. But then $\pi^* = -\underline{x} < 0$ which contradicts our previous observation that the equilibrium payoff is non-negative.
9. We omit the proof of the last statement of the proposition.
10. Tullock provides numerical examples where the sum of contenders' outlays exceeds the value of the rent (Tullock, 1980: Tables 6.1 and 6.2: 102), although, as Tullock observed, such outcomes are inconsistent with the existence of equilibrium. Complete rent dissipation is a possibility in Tullock's contests; a necessary (but not sufficient) condition is that the scale parameter r take on a value between unity and two (see Richard Higgins, William Shughart II and Robert Tollison, 1985). The more general conclusion from Tullock's contests is that, if an equilibrium exists, combinations of the number of participants and r yield underdissipation of the rent sought. On the other hand, in an extension of Tullock's contests which assumes that the rival contenders are themselves the source of the payment to the successful contender, Elie Appelbaum and Eliakim Katz (1986) obtain an overdissipation result; this outcome arises because unsuccessful contenders incur a dead-weight-loss in addition to the transfer and make outlays with a rent-avoiding as well as rent seeking motive. The role of dead-weight-losses incurred in the distributive process is stressed in Gary Becker's analysis (1983, 1985) of political influence and transfers.
11. This scheme has been maintained in various extensions of efficient rent seeking, for example, Hillman and Katz (1984, 1986), Corcoran (1984), Tullock (1984), Corcoran and Karels (1985), Higgins, Shugart and Tollisons (1985), Appelbaum and Katz (1986, forthcoming), Long and Vousden (1986). Rogerson's (1982) format has similar characteristics.
12. See also Tullock (1985) on the Higgins/Shugart/Tollison contest.
13. On wars of attrition, see for example Jack Hirshleifer and John Riley (1978), Bradford Cornell and Richard Roll (1981) and John Maynard Smith (1982).
14. The symmetric information and evaluation case of a war of attrition corresponds to the conditions of our contests. On wars on attrition when this symmetry is not present, see Nalebuff and Riley (1985).

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