

DISSIPATION OF MAGNETOHYDRODYNAMIC WAVES ON ENERGETIC PARTICLES: IMPACT ON INTERSTELLAR TURBULENCE AND COSMIC-RAY TRANSPORT

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ABSTRACT

The physical processes involved in diffusion of Galactic cosmic rays in the interstellar medium are addressed. We study the possibility that the nonlinear MHD cascade sets the power-law spectrum of turbulence that scatters charged energetic particles. We find that the dissipation of waves due to the resonant interaction with cosmic-ray particles may terminate the Kraichnan-type cascade below wavelengths 10^{13} cm. The effect of this wave dissipation has been incorporated in the GALPROP numerical propagation code in order to assess the impact on measurable astrophysical data. The energy dependence of the cosmic-ray diffusion coefficient found in the resulting self-consistent model may explain the peaks in the secondary to primary nuclei ratios observed at about $1 \text{ GeV nucleon}^{-1}$.

Subject headings: cosmic rays — diffusion — elementary particles — MHD — turbulence — waves

1. INTRODUCTION

Galactic cosmic rays—the gas of relativistic charged particles with high-energy density—cannot always be treated as test particles moving in given magnetic fields. In particular, the stochastic acceleration of cosmic rays by MHD waves is accompanied by the damping of the waves, since the wave energy is dissipated. The rate of wave damping on cosmic rays through the cyclotron resonance interaction was first estimated by Tidman (1966). If we exclude cold H I regions (where the waves are damped by collisions of ions with neutral atoms) and also perhaps regions of the interstellar gas with very high temperature $T \sim 10^6$ K and weak magnetic field (where Landau damping on thermal particles is high), then we find that this mechanism of dissipation could dominate over other known mechanisms in the interval of wavelengths 10^{11} – 10^{14} cm. The nonresonant interaction of diffusing cosmic rays with magnetosonic waves (Ptuskin 1981) is important only at large wavelengths and is not important for the present investigation. We also do not consider the large body of instabilities in cosmic rays that may arise because of the nonequilibrium distribution of charged energetic particles caused by possible strong anisotropy or large gradients and may amplify waves in the background plasma (see, e.g., Berezhinskii et al. 1990 and

Diehl et al. 2001 for a review of such processes). Cyclotron wave damping on cosmic rays changes the wave spectrum in the interstellar medium, which in turn affects the particle transport, since the cosmic-ray diffusion coefficient is determined by the level of turbulence that scatters charged particles. Thus, in principle the study of cosmic-ray diffusion requires a self-consistent approach. We show below that the effect of cosmic rays on interstellar turbulence should be taken into account at energies below a few gigaelectron volts per nucleon, and this may result in a considerable increase in the particle diffusion coefficient. This picture is also verified by its consistency with observations of interstellar turbulence, as discussed in § 6.

It is remarkable that the interpretation of cosmic-ray data on secondary nuclei may require this effect. The secondary nuclei are produced in cosmic rays in the course of diffusion and nuclear interactions of primary nuclei with interstellar gas. The ^2H , ^3He , Li, Be, and B, as well as many other isotopes and elements, are almost pure secondaries. Their abundance in cosmic-ray sources is negligible, and they result from the fragmentation of heavier nuclei. Cosmic-ray antiprotons and the major fraction of positrons are among the secondary species as well. The ratio of secondary to primary nuclei such as B/C has a peak at about $1 \text{ GeV nucleon}^{-1}$ and decreases both with increasing and decreasing energy (e.g., Engelmann et al. 1990). The high-energy behavior is naturally explained by the effect of particle scattering on interstellar MHD turbulence with a power-law spectrum in wavelength, but the required sharp increase of the diffusion coefficient at small energies has usually been considered improbable, since it requires a drastic, “physically unjustified” bending down

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of the wave spectrum at wavenumbers $k > \frac{1}{3} \times 10^{12} \text{ cm}^{-1}$ (see, e.g., Jones et al. 2001; Moskalenko et al. 2002). Alternative explanations of the peaks in secondary to primary ratios that do not invoke peculiarities in the cosmic-ray diffusion coefficient have been suggested. The two most popular of these involve diffusive-convective particle transport in the hypothetical Galactic wind (Jones 1979; Ptuskin et al. 1997) and the stochastic reacceleration of cosmic rays by interstellar turbulence (Simon et al. 1986; Seo & Ptuskin 1994). These processes are included in the numerical computations of cosmic-ray transport in the Galaxy in the frameworks of the GALPROP code (Strong & Moskalenko 1998; Moskalenko et al. 2002). The situation with the interpretation of the energy dependence of secondaries in cosmic rays is still uncertain. The peaks in the secondary to primary nuclei ratios calculated in diffusion-convection models seem to be too wide. Models with reacceleration reproduce the shape of the peaks, but the absolute flux of antiprotons turns out to be too low compared to observations (Moskalenko et al. 2002). This is why the effect considered in the present work is so important. We investigate a self-consistent model of cosmic-ray diffusion in interstellar turbulence in which the wave damping on energetic particles is taken into account, and we implement this effect in the GALPROP code. To make the first calculations reasonably tractable we keep only the most essential features of cosmic-ray diffusion in random magnetic fields and use the simplest description of the nonlinear wave cascade in the interstellar turbulence.

2. EQUATIONS FOR COSMIC RAYS

The steady state transport equation that describes the diffusion and convective transport of cosmic-ray protons and nuclei in the interstellar medium is of the form (see, e.g., Berezhinskii et al. 1990 for discussion):

$$-\nabla D \nabla \Psi + \nabla(\mathbf{u}\Psi) - \frac{\partial}{\partial p} \left[p^2 K \frac{\partial}{\partial p} (p^{-2} \Psi) \right] - \frac{\partial}{\partial p} \left(p \frac{\nabla \mathbf{u}}{3} \Psi \right) + \frac{\partial}{\partial p} (\dot{p}_{\text{loss}} \Psi) + \frac{\Psi}{\tau} = q. \quad (1)$$

Here $\Psi(p, \mathbf{r})$ is the particle distribution function in momentum p normalized on total cosmic-ray number density as $N_{\text{cr}} = \int dp \Psi$, $D(p, \mathbf{r})$ is the spatial diffusion coefficient, $K(p, \mathbf{r})$ is the diffusion coefficient in momentum, $\mathbf{u}(\mathbf{r})$ is the velocity of large-scale motions of the interstellar medium (e.g., the velocity of a Galactic wind), $\dot{p}_{\text{loss}} = dp/dt < 0$ is the momentum-loss rate for the energetic charged particles moving through the interstellar medium, $\tau(\mathbf{r})$ is the timescale for nuclear fragmentation. If needed, the supplementary term that describes radioactive decay can be added to equation (1). The spatial boundary condition for Ψ is $\Psi|_{\Sigma} = 0$, corresponding to the free exit of cosmic rays from the Galaxy to intergalactic space, where their density is negligible. The region of cosmic-ray diffusion is a cylinder of radius 30 kpc and total width $2H$ ($H \approx 4$ kpc). The source term $q(p, \mathbf{r})$ includes both the direct production of primary energetic particles accelerated from the thermal background in Galactic sources (e.g., supernova remnants) and the contribution to the nuclei considered via the processes of nuclear fragmentation and the radioactive decay of heavier nuclei. The typical value of the diffusion coefficient found from the fit to cosmic-ray and radio astronomical data is $D \sim 3 \times 10^{28} \text{ cm}^2 \text{ s}^{-1}$ at energy $\sim 1 \text{ GeV}$ nucleon $^{-1}$, giving a diffusion mean free path $l = 3D/v \sim 1 \text{ pc}$ ($v \sim c$ is the particle velocity).

On the ‘‘microscopic level’’ the spatial and momentum diffusion of cosmic rays results from the particle scattering on random MHD waves and discontinuities. In the linear approximation, the Alfvén [with the dispersion relation $\omega(\mathbf{k}) = k_{\parallel} V_A$], the fast magnetosonic [$\omega(\mathbf{k}) = k V_A$], and the slow magnetosonic [$\omega(\mathbf{k}) = k_{\parallel} V_s$] waves can propagate in a low- β plasma, $\beta = (V_s/V_A)^2 < 1$, where $V_A = B/(4\pi\rho)^{1/2}$ is the Alfvén velocity and $V_s = (P_g/\rho)^{1/2}$ is the collisionless sound velocity (for MHD conditions it is a factor of a square root of the adiabatic index larger), where B , P_g , and ρ are the magnetic field strength, the pressure, and the mass density of the interstellar gas, respectively. In addition to waves, static entropy variations can exist in the interstellar medium. In a collisionless plasma, the slow magnetosonic waves are not heavily damped only if the plasma is nonisothermal and the electron temperature considerably exceeds the ion temperature. The Alfvén velocity is $1.8 \times 10^5 B_{\mu\text{G}}/\sqrt{n} \text{ cm s}^{-1}$, where n is the number density of hydrogen atoms. The value of the sound velocity is $7.7 \times 10^5 \sqrt{T_4} \text{ cm s}^{-1}$ in neutral gas with temperature $T = 10^4 T_4 \text{ K}$; V_s is larger in the ionized gas because of the free electron contribution to gas pressure P_g . The approximation of low- β plasma is valid in the dominant part of the interstellar medium at $B \approx 5 \mu\text{G}$, since typically $n = 0.002 \text{ cm}^{-3}$ and $T = 10^6 \text{ K}$ in hot H II regions, $n = 0.2 \text{ cm}^{-3}$ and $T = 8 \times 10^3 \text{ K}$ in warm intercloud gas, $n = 30 \text{ cm}^{-3}$ and $T = 100 \text{ K}$ in clouds of atomic hydrogen, $n = 200 \text{ cm}^{-3}$ and $T = 10 \text{ K}$ in molecular clouds, and thus $\beta \approx 0.1-0.3$ everywhere. The effective ‘‘collision integral’’ for energetic charged particles moving in small-amplitude random fields $\delta B \ll B$ can be taken from the standard quasi-linear theory of plasma turbulence (see, e.g., Kennel & Engelmann 1966). The ‘‘collisional integral’’ averaged over fast gyrorotation of particles about the magnetic field \mathbf{B} contains the spectral densities of the effective frequencies of particle collisions with plasmons in the form

$$\nu_{\mu}^{\alpha}(\mathbf{k}, s, p) = \frac{4\pi^2 v}{B^2 |\mu| r_g^2} \left\{ (1 - \delta_{s0}) \delta(k_{\parallel} - s/(r_g |\mu|)) M_{\perp}^{\alpha}(\mathbf{k}) \times (J_{s+1}^2 + J_{s-1}^2) + \frac{\delta_{s0}}{2} \delta[k_{\parallel} - k_{\perp} V_A / (v |\mu|)] J_1^2 M_{\parallel}^{\alpha}(\mathbf{k}) k_{\parallel}^2 k_{\perp}^{-2} \right\}, \quad (2)$$

where $k_{\parallel} > 0$ (as presented by Ptuskin 1989). Here the waves are assumed to be symmetric about the magnetic field direction and to have zero average helicity. The index α characterizes the type of waves, including the direction of their propagation, μ is the cosine of pitch angle, the energy densities of random magnetic fields for perpendicular and parallel to the average field \mathbf{B} components are $M_{\perp}^{\alpha}(\mathbf{k})$ and $M_{\parallel}^{\alpha}(\mathbf{k})$, respectively, and $J_m = J_m[k_{\perp} r_g (1 - \mu^2)^{1/2}]$ is the Bessel function. It is explicitly taken into account that the scattering at $s = 0$ occurs only on the fast magnetosonic waves propagating almost perpendicular to the magnetic field. The Larmor radius is $r_g = pc/(ZeB) = 3.3 \times 10^{12} R_{\text{GV}}/B_{\mu\text{G}} \text{ cm}$, where the particle magnetic rigidity $R = pc/Ze$ is measured in gigavolts and the average magnetic field is measured in microgausses. The wave-particle interaction is of a resonant character, so that an energetic particle is predominantly scattered by the irregularities of magnetic field δB that have the projection of the wavevector direction on the magnetic field equal to $k_{\parallel} = \pm s/(r_g \mu)$. The integers $s = 0, 1, 2, \dots$ correspond to the cyclotron resonances of different orders. The efficiency of particle scattering depends on the polarization of the waves and on their distribution in \mathbf{k} -space. The first-order resonance $s = 1$ is the most important for the isotropic and also for the one-dimensional distribution of random MHD waves

along the average magnetic field, $\mathbf{k} \parallel \mathbf{B}$. In some cases—for the calculation of scattering at small μ and for the calculation of perpendicular diffusion—the broadening of resonances and magnetic mirroring effects should be taken into account.

The evolution of the particle distribution function on time-scales $\Delta t \gg \nu^{-1}$ and distances $\Delta z \gg \nu \nu^{-1}$ can be described in the diffusion approximation, with the following expressions for the spatial diffusion coefficient along the magnetic field D_{\parallel} and the diffusion coefficient in momentum D_{pp} (see Berezhinskii et al. 1990):

$$D_{\parallel}(p) = \frac{v^2}{4} \int_{-1}^{+1} d\mu (1 - \mu^2) \left[\sum_{\alpha, s, k} \nu_{\mu}^{\alpha}(\mathbf{k}, s, p) \right]^{-1}, \quad (3)$$

$$D_{pp}(p) = \frac{p^2}{4} \int_{-1}^{+1} d\mu (1 - \mu^2) \left\{ \sum_{\alpha, s, k} \nu_{\mu}^{\alpha}(\mathbf{k}, s, p) \frac{[V_f^{\alpha}(\mathbf{k})]^2}{v^2} - \left[\sum_{\alpha, s, k} \nu_{\mu}^{\alpha}(\mathbf{k}, s, p) \frac{V_f^{\alpha}(\mathbf{k})}{v} \right]^2 \left[\sum_{\alpha, s, k} \nu_{\mu}^{\alpha}(\mathbf{k}, s, p) \right]^{-1} \right\}, \quad (4)$$

where the analogous equations were derived for one-dimensional turbulence with $k_{\perp} = 0$. Here $V_f^{\alpha}(\mathbf{k}) = \omega^{\alpha}(\mathbf{k})/k_{\parallel}$, the summations contain integrals over \mathbf{k} -space, and the terms that contain $\nu_{\mu}^{\alpha}(\mathbf{k}, s = 0, p)$ should be corrected in equations (3), (4) compared to equation (2): multiplied by $(1 - \mu^2)^2$ in $(\sum \nu_{\mu}^{\alpha})^{-1}$, and multiplied by $(1 - \mu^2)$ in other terms. One can check that the interaction of energetic particles with slow magnetosonic waves is relatively weak (as $V_s^2/V_A^2 \ll 1$) and can be ignored.

Locally, the cosmic-ray diffusion is anisotropic and occurs along the local magnetic field because the particles are strongly magnetized, $r_g \ll l$. The isotropization is accounted for by the presence of strong large-scale (~ 100 pc) fluctuations of the Galactic magnetic field. The problem is not trivial even in the case of relatively weak large-scale random fields, since the field is almost static and the strictly one-dimensional diffusion along the magnetic field lines does not lead to nonzero diffusion perpendicular to \mathbf{B} (see Chuvilgin & Ptuskin 1993; Giacalone & Jokipii 1999; Casse et al. 2001).

Equations (3) and (4) are too cumbersome for our present application. Based on equation (3) and with reference to the detailed treatment of cosmic-ray diffusion by Toptygin (1985) and Berezhinskii et al. (1990), in the rest of this paper we use the following simplified equation for the diffusion coefficient:

$$D = v r_g B^2 / [12\pi k_{\text{res}} W(k_{\text{res}})], \quad (5)$$

where $k_{\text{res}} = 1/r_g$ is the resonant wavenumber, and $W(k)$ is the spectral energy density of waves normalized as $\int dk W(k) = \delta B^2/4\pi$. The random field at the resonance scale is assumed to be weak, $\delta B_{\text{res}} \ll B$.

The cosmic-ray diffusion coefficient is equal to

$$D = \frac{v r_g^a}{3(1-a) k_L^{1-a}} \frac{B^2}{\delta B_L^2} \quad (6)$$

for particles with $r_g < k_L^{-1}$ under the assumption of a power-law spectrum of turbulence $W(k) \propto 1/k^{2-a}$, $k > k_L$. We introduced here the principle wavenumber of the turbulence k_L and

the amplitude of random field δB_L at this scale. The diffusion has scaling $D \propto v(p/Z)^a$.

The diffusion in momentum is described by the following equation which is a simple approximation of equation (4):

$$K = p^2 V_A^2 / (9D). \quad (7)$$

Equations (5)–(7) give estimates of particle diffusion in position and momentum needed in equation (1). They reflect the most essential features of cosmic-ray transport: the frequency of particle scattering on a random magnetic field is determined by the energy density of this field at the resonance wavenumber k_{res} ; the acceleration is produced by the waves moving with typical velocity V_A and is stochastic in nature. Equation (5) implies equal intensities of waves moving along the magnetic field in opposite directions, the imbalanced part of the total wave energy density should not be taken into account when calculating $K(p)$ (see Berezhinskii et al. 1990 for details). Equations (5)–(7) are also valid for small-amplitude nonlinear waves, including weak shocks. The equations can be used for the isotropic distribution of Alfvén and fast magnetosonic waves, and they give correct order-of-magnitude estimates for the wave distribution concentrated around the direction of average magnetic field. It should be pointed out, however, that the isotropization of the diffusion tensor does not occur in the case of a pure parallel propagation of waves ($\mathbf{k} \parallel \mathbf{B}$). Another special case is two-dimensional turbulence with the perpendicular propagation of waves ($\mathbf{k} \perp \mathbf{B}$). In this case, the scattering occurs only for magnetosonic waves through the resonance $s = 0$, which leads to a very large diffusion coefficient, about a factor $(v/V_A)^2$ larger than that given by equations (5) and (6).

Equation (6) shows that the level of interstellar turbulence needed to account for the diffusion of GeV cosmic rays is very small: $\delta B_{\text{res}}^2/B^2 = (1-a)r_g/l \sim 10^{-6}$ at $k_{\text{res}}^{-1} \sim 10^{12}$ cm (if $a \sim 0.5$). An extension to smaller wavenumbers gives $\delta B^2(>k)/B^2 \sim 10^{-6}(10^{12}k)^{1-a}$, where k is in units cm^{-1} .

3. EQUATIONS FOR INTERSTELLAR TURBULENCE

The description of MHD turbulence is a complicated and not completely solved problem even in the case of small-amplitude random fields. Comprehensive reviews of MHD turbulence have been given by Verma (2004) and by Zhou et al. (2004), Elmegreen & Scalo (2004), and Scalo & Elmegreen (2004) with application to interstellar turbulence.

The classic problem is the determination of the wave spectrum in the presence of sources at small wavenumbers $k \sim k_L$ and the strong dissipation at much larger wavenumbers (in some cases the cascade is inverse). Note that the spectrum of interstellar MHD turbulence determines the transport coefficients in equations (5) and (7). According to the Kolmogorov-Obukhov hypothesis (Kolmogorov 1941; Obukhov 1941), the resulting spectrum at intermediate k , i.e., in the inertial range, is characterized by a constant energy flux to higher wavenumbers. The hypothesis was originally suggested for the description of developed hydrodynamic turbulence in incompressible fluids. The Kolmogorov spectrum is of the form $W(k)k^{-5/3}$. The spectrum of weak acoustic turbulence $W(k)k^{-3/2}$ was found by Zakharov & Sagdeev (1970). This result was criticized by Kadomtsev & Petviashvili (1973). They argued that the developed shocks produce an additional dissipation of acoustic wave energy. The shock-dominated turbulence (also called the Burgers turbulence) is characterized by the spectrum k^{-2} irrespective of the nature of dissipation at the shock front.

The presence of magnetic field in MHD turbulence complicates the issue because the turbulence becomes anisotropic, and new types of waves arise in a magnetized medium. Generally, all gradients are larger perpendicular to the field and all perturbations are elongated along the magnetic field direction even with isotropic excitation. Iroshnikov (1964) and Kraichnan (1965) gave the first phenomenological theory of MHD turbulence and obtained the spectrum $W(k)k^{-3/2}$. They assumed that small-scale fluctuations are isotropic, which is contradictory because wave interactions break the isotropy in the presence of an external magnetic field.

Direct observations of MHD turbulence in the solar wind plasma where $\beta \sim 1$ have shown the existence of a Kolmogorov-type turbulence spectrum that contains waves moving both along the magnetic field and in almost perpendicular directions (Saur & Bieber 1999). Such a spectrum was obtained over several decades of wavenumbers in solar wind radio propagation studies (Woo & Armstrong 1979). Numerical simulations of incompressible ($\beta \gg 1$) MHD turbulence favored the Kolmogorov spectrum (Verma et al. 1996).

Over the past decade, there has been a renewed interest in understanding of magnetohydrodynamic turbulence as it applies to interstellar magnetic field and density fluctuations (Goldreich & Sridhar 1995, 1997; Ng & Bhattacharjee 1997; Galtier et al. 2000; see also the earlier work by Shebalin et al. 1983). Goldreich & Sridhar (1995) exploited anisotropy in MHD turbulence and obtained Kolmogorov-like spectrum for the energy density of Alfvén waves. The “elementary interactions” between Alfvén waves satisfies the three-wave resonance conditions. However, there is no exact relation between wavenumber and frequency in this case of strong turbulence. The main part of the energy density in this turbulence is concentrated perpendicular to the local magnetic field wavevectors $k_{\perp} \approx k$, while the parallel wavenumbers are small: $k_{\parallel} \sim [kW(k)/(B_0^2/4\pi)]^{1/2}k_{\perp}$. The cascade is anisotropic with energy confined within the cone $k_{\parallel}k_{\perp}^{2/3}$. Numerical simulations have confirmed this concept (Cho & Vishniac 2000).

Although the formalism has been developed for incompressible MHD turbulence, Lithwick & Goldreich (2001) argued that the compressibility does not essentially alter the results on the Alfvén wave spectrum. The distribution of slow magnetosonic waves passively follows that of Alfvén waves. The fast magnetosonic waves have an independent nonlinear cascade that is isotropic and has a Kraichnan-type spectrum $W(k)k^{-3/2}$. These conclusions were supported by numerical simulations by Cho & Lazarian (2002).

The description of weak MHD turbulence in low- β plasma outlined above is probably not complete and needs further analysis before it is accepted as a standard model of interstellar turbulence. First, there is still the discrepancy between theoretical results of different authors. Thus, considering the scattering of Alfvén waves and fast magnetosonic waves on slow magnetosonic waves, Kuznetsov (2001) found the Kraichnan-type spectra for all these types of waves with their preferentially parallel propagation, which disagrees with Lithwick & Goldreich (2001). Second, the consideration of processes in turbulent collisionless plasmas at the kinetic level involves additional nonlinear processes of induced wave scattering on thermal ions (Livshits & Tsytovich 1970) that may change the spectra (Chashei & Shishov 1985). Third, the real turbulence can be strongly intermittent, imbalanced, etc. (e.g., Lithwick & Goldreich 2003), which may also affect the interstellar MHD spectrum.

Information on the extended interstellar turbulence spectrum has been obtained from radio scintillation and refraction obser-

vations (sensitive to fluctuations of thermal electron density), measurements of the differential Faraday rotation angles from distant sources (mainly produced by fluctuations in the interstellar magnetic field), and the observations of random motions in the interstellar gas. These data are consistent with the assumption that a single close-to-Kolmogorov spectrum extends from scales 10^8 to 10^{20} cm (Lee & Jokipii 1976; see also Armstrong et al. 1981, 1995, and references therein).

In the absence of an easily manageable and commonly accepted exact equation for the energy density $W(k)$, we employ below the simplest steady state phenomenological equation that represents the concept of a wave cascade in the inertial range of wavenumbers (Tu 1988; Norman & Ferrara 1996; this approach goes back to Chandrasekhar 1948 and Heisenberg 1948):

$$\frac{\partial}{\partial k} \left[\frac{kW(k)}{T_{nl}} \right] = 0. \quad (8)$$

Here the approximation of a characteristic time T_{nl} is used to account for the nonlinear wave interactions that provide the transfer of energy in k -space.⁴

Formally, the Kolmogorov-type spectrum $W(k)k^{-5/3}$ follows from equation (8) if $T_{nl} = T_A = \{C_A k [kW(k)/(4\pi\rho)]^{1/2}\}^{-1}$, where the constant $C_A \sim 0.3$ as can be estimated from the simulations of Verma et al. (1996). The Kraichnan spectrum $W(k)k^{-5/3}$ is obtained from equation (8) if $T_{nl} = T_M = [C_M k^2 W(k)/(\rho V_A)]^{-1}$, where $C_M \sim 1$.

The Kolmogorov-type nonlinear rate $1/T_A$ is relatively high, since it is proportional to the amplitude of weak random field, whereas the Kraichnan rate $1/T_M$ is proportional to the amplitude squared. Our preliminary estimates (Moskalenko et al. 2003a; Ptuskin et al. 2003a) showed that the MHD cascade with the Kolmogorov rate is not significantly affected by damping on cosmic rays even if the waves propagate along the magnetic field, which makes the wave-particle interactions the most efficient. In addition, if the concept of Goldreich & Sridhar (1995) works for interstellar turbulence, then the Kolmogorov rate refers to the Alfvén waves, which are distributed almost perpendicular to an external magnetic field and thus do not produce any significant scattering of cosmic rays (see § 2). So we do not consider the damping of cascades with the Kolmogorov rate in what follows.

For a cascade with the Kraichnan rate $1/T_M$, which describes an isotropic or close-to-parallel distribution of MHD waves (according to Goldreich & Sridhar 1995, it refers only to fast magnetosonic waves but not to Alfvén waves), the equation for wave energy density can be written as follows:

$$\frac{\partial}{\partial k} \left[\frac{C_M}{\rho V_A} k^3 W^2(k) \right] = -2\Gamma(k)W(k) + S\delta(k - k_L), \quad (9)$$

where $k \geq k_L$. Here $\Gamma = \Gamma_{cr} + \Gamma_{th}$ is the wave attenuation rate on cosmic-ray particles (Γ_{cr}) and on thermal particles (Γ_{th}), and S characterizes the source strength. The main sources of interstellar turbulence are supernova explosions, winds of massive stars, superbubbles, and differential rotation of the Galactic disk. These sources produce strong random magnetic fields and probably initiate nonlinear wave cascades at the scale $k_L^{-1} \sim 100$ pc.

⁴ See also Cranmer & van Ballegoijen (2003) and references therein for possible modifications of eq. (8) when a diffusive flux of wave energy in k -space is assumed and when the equation is written for a power spectrum $W(k)$ not averaged over direction [here $d^3kW(k) = dkW(k)$].

Below we ignore the dissipation on thermal particles (see discussion below) and set $\Gamma_{\text{th}} = 0$ to isolate the effect of damping on cosmic rays. We actually deal with a small part of the possible global wave spectrum at wavenumbers $k \gtrsim 10^{-14} \text{ cm}^{-1}$, which resonantly scatter cosmic rays with energies less than about 100 GeV nucleon $^{-1}$. The existence of a single interstellar MHD spectrum over 12 orders of magnitude is an unsolved problem in itself and is beyond the scope of the present work. The quantities k_L and $W(k_L)$ are used here only for purposes of normalization.

The equation for wave amplitude attenuation on cosmic rays is (Berezinskii et al. 1990):

$$\Gamma_{\text{cr}}(k) = \frac{\pi Z^2 e^2 V_A^2}{2k c^2} \int_{p_{\text{res}}(k)}^{\infty} \frac{dp}{p} \Psi(p), \quad (10)$$

where $p_{\text{res}}(k) = ZeB/c k$.

The solution of equations (9) and (10) allows us to find the wave spectrum:

$$W(k) = k^{-3/2} \left[k_L^{3/2} W(k_L) - \frac{Z^2 e^2 B^2 V_A}{8 C_M c^2} \int_{k_L}^k dk_1 k_1^{-5/2} \int_{p_{\text{res}}(k_1)}^{\infty} \frac{dp \Psi(p)}{p} \right], \quad (11)$$

where $k > k_L$. Here the spectral wave density at the principal scale is determined by the source strength: $W(k_L) = (\rho V_A S / C_M)^{1/2} k_L^{-3/2}$. The second term in square brackets increases with k . The wave damping on cosmic rays decreases the wave energy density and can even terminate the cascade if the expression in square brackets reduces to zero at some $k = k_*$. The wave energy density should be set to zero at $k > k_*$ in this case.

Now with the use of equations (5) and (11) one can determine the cosmic-ray diffusion coefficient, which is

$$D(p) = \frac{D_0(p)}{1 - g \int_p^{p_L} dp_2 p_2^{1/2} \int_{p_2}^{\infty} dp_1 p_1^{-1} \Psi(p_1)}, \quad (12)$$

$$g = \frac{3\pi V_A p^{1/2} D_0(p)}{2 C_M B^2 r_{\text{gv}}} = \frac{\sqrt{Ze} B^2}{16 \sqrt{\pi \rho c} C_M k_L^{3/2} W(k_L)},$$

where $p < p_L$ and $p_L = p_{\text{res}}(k_L)$. Here $D_0(p) = v r_{\text{g}}^{1/2} B^2 / [12\pi k_L^{3/2} W(k_L)] v (p/Z)^{1/2}$ is the diffusion coefficient calculated for the Kraichnan-type spectrum without considering wave damping. The second term in the denominator of equation (12) describes the modification of the diffusion coefficient due to wave damping. The effect becomes stronger to smaller p . The diffusion coefficient should be formally set to infinity at $p < p_*$ if the square bracket in equation (11) goes to zero at some $p = p_*$. The constant g characterizes the strength of the effect for a given cosmic-ray spectrum $\Psi(p)$.

As the most abundant species, the cosmic-ray protons mainly determine the wave dissipation. Thus, with good precision only the proton component with $Z = 1$ needs to be taken into account to calculate $D(p)$ by the simultaneous solution of equations (1) and (12). The diffusion mean free path for other nuclei of charge Z is $l(p/Z)$ if it is $l(p)$ for protons.

Let us estimate the effect of wave damping. Assuming that the cosmic-ray energy density is about 1 eV cm $^{-3}$, the diffusion coefficient $D = 3 \times 10^{28} \text{ cm}^2 \text{ s}^{-1}$ at 1 GeV, $V_A = 10 \text{ km s}^{-1}$, $B = 5 \mu\text{G}$, and $C_M = 1$, the second term in the denominator in equation (12) is about unity at GeV energies and falls at higher

energies. We conclude that the Kraichnan-type cascade is significantly affected by damping on cosmic rays, and this should lead to the modification of cosmic-ray transport at energies less than about 10 GeV nucleon $^{-1}$.

4. SIMPLE SELF-CONSISTENT MODEL

To demonstrate the effect of wave damping, let us consider a simple case of one-dimensional diffusion with source distribution $q = q_0(p)\delta(z)$ (corresponding to an infinitely thin disk of cosmic-ray sources located at the Galactic midplane $z = 0$) and a flat cosmic-ray halo of height H (see Jones et al. 2001). The source spectrum at $R < 40 \text{ GV}$ is $q_0 \propto p^{-\gamma_s}$, where $\gamma_s = 2.5$ approximately. Considering energetic protons, we ignore energy losses and nuclear fragmentation ($\dot{p}_{\text{loss}} = 0$, $1/\tau = 0$) and assume that a Galactic wind is absent ($u = 0$) in equation (1). Let us also assume that stochastic reacceleration does not significantly change the energies of cosmic-ray particles during the time of their diffusive leakage from the Galaxy and set $K = 0$ in equation (1). The solution of equation (1) in the Galactic disk is then

$$\Psi(p) = \frac{q_0(p)H}{2D(p)}. \quad (13)$$

The escape length (the ‘‘grammage’’), which determines the production of secondaries during the cosmic-ray leakage from the Galaxy, is equal to $X = \mu_{\text{g}} v H / (2D)$, where μ_{g} is the gas surface density of the Galactic disk.

The simultaneous solution of equations (12) and (13) allows us to find $\Psi(p)$ and $D(p)$. Introducing the function $\Psi_0(p) = q_0(p)H / [2D_0(p)]$, which is the cosmic-ray spectrum for an unmodified Kraichnan spectrum, and substituting Ψ from equation (13) into equation (12), one finds after two differentiations the following equation for the ratio $\varphi = D_0(p)/D(p)$ as a function of $x = p^{3/2}$:

$$\frac{d^2 \varphi(x)}{dx^2} = -\frac{4g\Psi_0[p(x)]}{9x} \varphi(x), \quad (14)$$

with the constraint $\varphi(\infty) = 1$. Note that the same equation is valid for the ratio $\Psi(p)/\Psi_0(p)$.

The function Ψ_0/x in the right-hand side of equation (14) can be approximated by a power-law function proportional to x^{-b} , $b = \text{const}$. Thus, for $\gamma_s = 2.5$, one has $b = 3$ for ultrarelativistic ($v = c$) and $b = 11/3$ for nonrelativistic ($v \ll c$) protons, respectively. This allows us to find the solution of equation (14) in an explicit form. The diffusion coefficient $D(p) = D_0(p)/\varphi(p)$ is then

$$D(p) = D_0(p) \frac{w^{1/(b-2)}(p)}{\Gamma[(b-1)/(b-2)] J_{1/(b-2)}[2w(p)]}, \quad (15)$$

$$w(p) = \frac{2}{3(b-2)} \sqrt{gp^{3/2} \Psi_0(p)},$$

where $\Gamma(z)$ is the gamma function and $J_\nu(z)$ is the Bessel function of the first kind. With p decreasing from infinity and the corresponding increase of $w(p)$ from zero, the Bessel function goes to zero at some $w(p_*) = w_*$, where $w_* = 1.92$ at $b = 3$ and $w_* = 1.64$ at $b = 11/3$. This means that $D(p)$ becomes infinite at $p = p_*$ because of complete termination of the cascade; $D(p) = \infty$ at $p \leq p_*$.

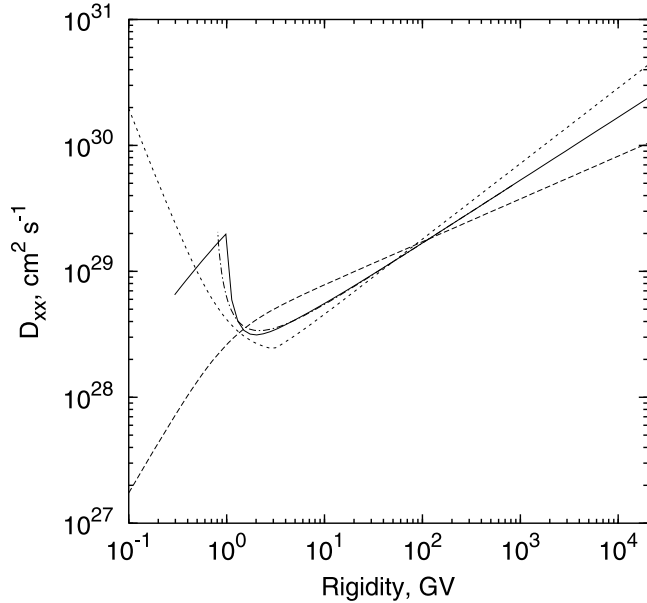


FIG. 1.—Diffusion coefficient D for the models being discussed in this paper: plain diffusion model (PD model, *dotted line*), reacceleration model (RD model, *dashed line*), and diffusive reacceleration with damping model (DRD model, *solid line*), an approximate solution (eq. [15] with $b = 3$) is shown by the dash-dotted line.

The expansion of the Bessel function for small and large arguments results in the following approximations for equation (15):

$$D(p) \approx \frac{D_0(p)}{1 - \{\Gamma[(b-1)/(b-2)]/\Gamma[(2b-3)/(b-2)]\}w^2(p)}, \quad (16)$$

at $w(p) \ll 1$, and

$$D(p) \approx D_0(p) \frac{\sqrt{\pi}w^{b/[2(b-2)]}}{\Gamma[(b-1)/(b-2)] \sin[\frac{\pi}{4}(3b-4)/(b-2) - 2w(p)]} \quad (17)$$

at $1 \lesssim w(p) < w_*$. The last approximate expression for $D(p)$ has a singularity at $w(p) = w_* = (\pi/8)(3b-4)/(b-2)$; $w_* = 1.96$ at $b = 3$ and $w_* = 1.65$ at $b = 11/3$, which is close to the values obtained from equation (15). The corresponding p_* can be found from the equation $p_*^{3/2}\Psi_0(p_*) = [3\pi(3b-4)/16]^2 g^{-1}$.

Equations (15)–(17) exhibit the rapid transition from unmodified to infinite diffusion. Figure 1 demonstrates that equation (15) derived in the considerably simplified model of cosmic-ray propagation (*dash-dotted line*) agrees well with the shape of the diffusion coefficient found in the numerical simulations (*solid line*) described in the § 5. The results of this section can be used for approximate estimates of cosmic-ray diffusion coefficient without invoking the full-scale calculations based on the GALPROP code.

5. NUMERICAL SOLUTION WITH GALPROP CODE

The numerical calculations have been made using the GALPROP code for cosmic-ray propagation. GALPROP is a flexible numerical code written in C++, which incorporates *as much realistic* astrophysical input *as possible*, together with latest theoretical developments. The model is designed to perform cosmic-ray propagation calculations for nuclei (isotopes of H through Ni), antiprotons, electrons, and positrons, and

computes γ -rays and synchrotron emission in the same framework. The current version of the model includes substantial optimizations in comparison to the older model and a full three-dimensional spatial grid for all cosmic-ray species. It explicitly solves the full nuclear reaction network on a spatially resolved grid. GALPROP models have been described in detail elsewhere (Strong & Moskalenko 1998; Moskalenko et al. 2002, 2003b). For the present calculation we use three-dimensional Galactic models with cylindrical symmetry: (R, z, p) spatial variables plus momentum.

GALPROP solves the transport equation (1) for all cosmic-ray species using a Crank-Nicholson implicit second-order scheme. The spatial boundary conditions assume free particle escape. The diffusion coefficient as a function of momentum and the reacceleration or convection parameters are determined by boron-to-carbon (B/C) ratio data. The spatial diffusion coefficient is taken as $D = \kappa\beta(R/R_0)^a$, $\kappa = \text{const}$, and $\beta = v/c$, if necessary with a break ($a = a_1$ below rigidity R_0 and $a = a_2$ above rigidity R_0). The injection spectrum of nucleons is assumed to be a power law in momentum, $q(p) \propto p^{-\gamma_s}$ and may also have breaks. To account for stochastic reacceleration in the interstellar medium, the momentum-space diffusion coefficient K is related to the spatial coefficient D via the Alfvén speed V_A (see eq. [7]).

The interstellar hydrogen distribution (Moskalenko et al. 2002) uses H I and CO surveys and information on the ionized component; the helium fraction of the gas is taken as 0.11 by number. The H₂ and H I gas number densities in the Galactic plane are defined in the form of tables, which are interpolated linearly. The conversion factor is taken as $X_{\text{CO}} \equiv N_{\text{H}_2}/W_{\text{CO}} = 1.9 \times 10^{20} \text{ mol cm}^{-2}/(\text{K km s}^{-1})$ (Strong & Mattox 1996). The extension of the gas distribution to an arbitrary height above the plane is made using analytical approximations. The halo size is assumed to be $H = 4 \text{ kpc}$. The distribution of cosmic-ray sources is chosen to reproduce the cosmic-ray distribution determined by analysis of EGRET γ -ray data (Strong & Mattox 1996; Strong & Moskalenko 1998):

$$q(R, z) = q_0 \left(\frac{R}{R_\odot}\right)^\alpha \exp\left(-\beta \frac{R - R_\odot}{R_\odot} - \frac{|z|}{0.2 \text{ kpc}}\right), \quad (18)$$

where q_0 is a normalization constant, $R_\odot = 8.5 \text{ kpc}$, and $\alpha = 0.5$ and $\beta = 1.0$ are parameters. We note that the adapted source distribution⁵ is flatter than the distribution of supernova remnants (SNR; Case & Bhattacharya 1998) and pulsars (Lorimer 2004).

The code includes cross section measurements and energy-dependent fitting functions (Strong & Moskalenko 2001). The nuclear reaction network is built using the Nuclear Data Sheets. The isotopic cross section database is built using the extensive T16 Los Alamos compilation of the cross sections (Mashnik et al. 1998) and modern nuclear codes CEM2k and LAQGSM (Mashnik et al. 2004). The most important isotopic production cross sections (²H, ³H, ³He, Li, Be, B, Al, Cl, Sc, Ti, V, and Mn) are calculated using our fits to major production channels (e.g., Moskalenko et al. 2001, 2003b; Moskalenko & Mashnik 2003). Other cross sections are calculated using phenomenological approximations by Webber et al. (1990; code WNEWTR.FOR, 1993

⁵ A recent analysis has shown that the apparent discrepancy between the radial gradient in the diffuse Galactic γ -ray emissivity and the distribution of SNR, believed to be the sources of cosmic rays, can be plausibly solved by adopting a conversion factor X_{CO} , which increases with R (see a discussion in Strong et al. 2004).

TABLE 1
PROPAGATION PARAMETER SETS

MODEL	INJECTION INDEX ^a		BREAK RIGIDITY (GV)	DIFFUSION COEFFICIENT AT 3 GV		ALFVÉN SPEED (V_A , km s ⁻¹)	galdef FILE
	Nucleons (γ_s)	Electrons (γ_e)		κ (cm ² s ⁻¹)	Index (α)		
Plain Diffusion (PD).....	2.30/2.15	2.40	40	2.2×10^{28}	0.0/0.60 ^b	...	44_999726
Diffusive Reacceleration (DR).....	1.80/2.40	1.60/2.50	4	5.2×10^{28}	0.34	36	44_599278
Diffusive Reacceleration with Damping (DRD).....	2.40/2.24	2.70	40	2.9×10^{28}	0.50	22	44_999714kr

NOTE.—Adopted halo size $H = 4$ kpc.

^a Index below/above the break rigidity.

^b Index below/above $R_0 = 3$ GV; $D = \beta^{-2}\kappa(R/R_0)^\alpha$.

and 2003 versions) and/or Silberberg and Tsao (Silberberg et al. 1998; code YIELDX_011000.FOR, 2000 version) renormalized to the data where they exists. For pA inelastic cross section we adapted the parameterization by Barashenkov & Polanski (code CROSEC; Barashenkov 1993; Barashenkov & Polanski 1994). The details of proton and antiproton cross sections are given in Moskalenko et al. (2002). Secondary positron and electron production is computed using the formalism described in Moskalenko & Strong (1998), which includes a reevaluation of the secondary π^\pm - and K^\pm -meson decay calculations. Energy losses for nucleons and leptons by ionization, Coulomb scattering, bremsstrahlung, inverse Compton scattering, and synchrotron emission are included in the calculations. The heliospheric modulation is treated using the force-field approximation (Gleeson & Axford 1968).

The reaction network is solved starting at the heaviest nuclei (i.e., ^{64}Ni). The propagation equation is solved, computing all the resulting secondary source functions, and then proceeding to the nuclei with $A - 1$. The procedure is repeated down to $A = 1$. In this way all secondary, tertiary, etc., reactions are automatically accounted for. To be completely accurate for all isotopes, e.g., for some rare cases of β^\pm -decay, the whole loop is repeated twice.

For the present work, a new iterative procedure that includes a self-consistent determination of the cosmic-ray diffusion coefficient was developed. In the first step, cosmic-ray propagation is calculated using the undisturbed diffusion coefficient $D_0(p)$. In the second, we use the propagated proton spectrum at every spatial grid point to recalculate the modified diffusion coefficient according to equation (12). In the third step, cosmic-ray propagation is calculated using the new diffusion coefficient. Steps 2 and 3 are repeated until convergence is obtained. The convergence is fast and the procedure requires only few iterations. In this way, using the secondary to primary ratio B/C, one can derive the damping constant $g = 0.085$, which is considered as an adjustable parameter.

The new self-consistent model of diffusive reacceleration with damping (the DRD model) is compared with two reference models that were earlier found to give good fits to cosmic-ray data: the plain diffusion model (PD model), with an ad hoc break in the diffusion coefficient, and the model with distributed reacceleration (the DR model) and power-law diffusion with no breaks (Moskalenko et al. 2002). The parameters of these models found from the fit to the observed spectra of primary and secondary nuclei are summarized in Table 1. Note that the parameters of the DRD model given in Table 1 are the “seed” values used in the iteration procedure. The diffusion coefficient after iteration calculations is shown in Figure 1.

The diffusion coefficients in all three models are presented in Figure 1. The DR model diffusion coefficient has a weak energy dependence with a single index $\frac{1}{3}$ as dictated by the assumed

Kolmogorov spectrum of interstellar turbulence. The PD model requires a break in the diffusion coefficient and an additional factor of β^{-3} to be able to match the B/C ratio at low energies. The DRD model diffusion coefficient lies between these at high energies and has a sharp increase at 1 GV. Of course, the actual mean free path length cannot be infinite. We put an upper limit of ~ 15 pc, based on the estimate of the streaming instability effect that arises at low rigidities and leads to the generation of additional turbulence, which limits cosmic-ray escape from the Galaxy; see discussion below in § 6. The self-consistent spectrum of waves calculated in the DRD model is shown in Figure 2. The spectrum bends downward at wavenumbers $k > 6 \times 10^{-12}$ cm⁻¹ due to the wave damping on cosmic rays.

The B/C ratio and Carbon spectra (primary) after modulation look almost identical in all three models (Figs. 3 and 4). The modulated proton spectrum (Fig. 5) matches the data in all three cases after some tuning, while the interstellar spectra are different. The most dramatic difference is the reduction of proton flux in the DRD model at low energies, consistent with the steep increase of the diffusion coefficient. The antiproton spectrum (Fig. 6) appears to be very sensitive to the model assumptions and might help to discriminate between the models. The DR model produces too few antiprotons, a well-known effect (Moskalenko et al. 2002). The PD and DRD models are both consistent with antiproton measurements in the heliosphere, but they predict very different spectra in the interstellar medium; the interstellar antiproton flux at low energies (< 600 MeV) in DRD model is an

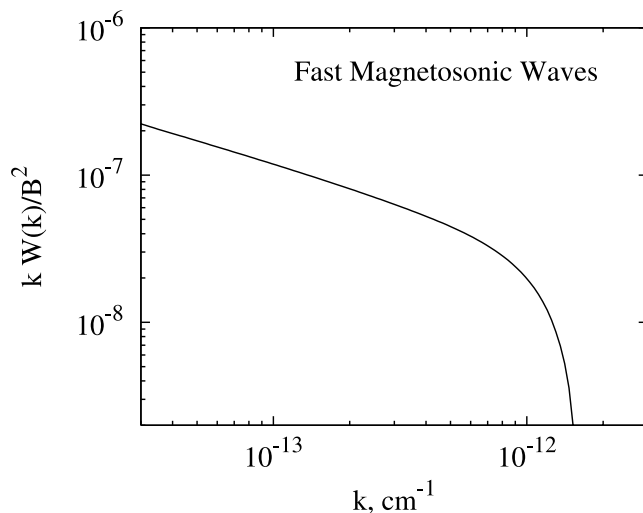


FIG. 2.—Dimensionless spectrum of fast magnetosonic waves $kW(k)/B^2$ modified by interactions with cosmic rays in DRD model.

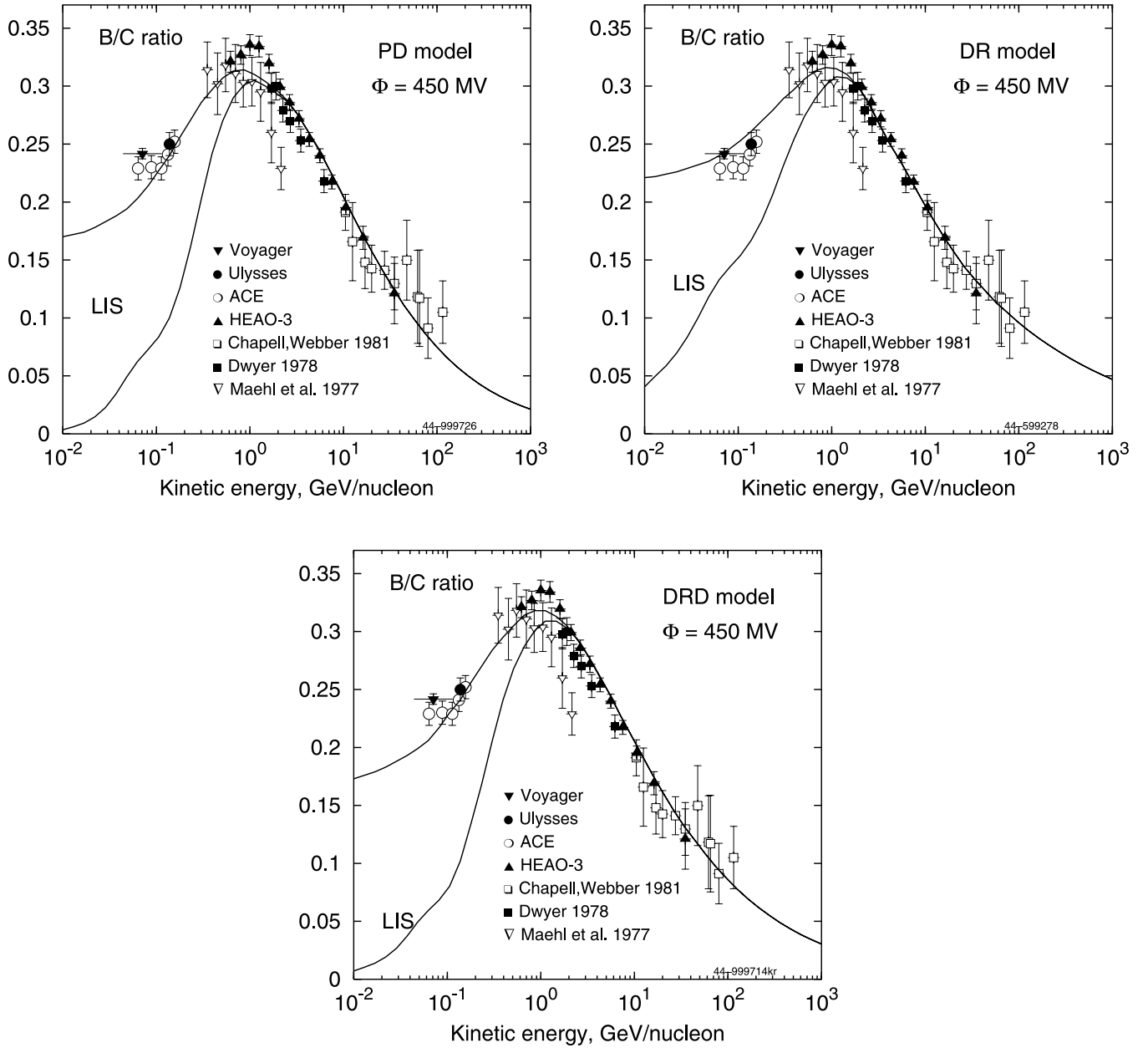


FIG. 3.—B/C ratio as calculated in plain diffusion model (PD model), reacceleration model (RD model), and diffusive reacceleration with damping model (DRD model). *Bottom curve*: LIS; *top curve*: modulated ($\Phi = 450$ MV). Data below $200 \text{ MeV nucleon}^{-1}$: ACE (Davis et al. 2000); Ulysses (DuVernois et al. 1996); Voyager (Lukasiak et al. 1999); high-energy data: HEAO-3 (Engelmann et al. 1990), for other references see Stephens & Streitmatter (1998).

order of magnitude lower than in two other models. In both reacceleration models (DR and DRD), the majority of low-energy antiprotons come from inelastic scattering (so-called “tertiary” antiprotons).

Figures 7 and 8 show secondary positrons and primary plus secondary electrons as calculated in all three models. The spectra are similar in the PD and DR models, while DRD spectra exhibit lower intensities at low energies. This may be an observable effect since the models predict different synchrotron emission spectra (electrons).

6. DISCUSSION

Damping on cosmic rays may terminate the slow Kraichnan-type cascade in the interstellar medium at $k \sim 10^{-12} \text{ cm}^{-1}$. Our estimates were made for the level of MHD turbulence that produces the empirical value of cosmic-ray diffusion coefficient.

This finding suggests a possible explanation for the peaks in secondary/primary nuclei ratios at about $1 \text{ GeV nucleon}^{-1}$ observed in cosmic rays: the amplitude of short waves is small because of damping, and thus the low-energy particles rapidly exit the Galaxy without producing many secondaries. There is no other obvious reason for a sharp cutoff in the wave spectrum. If the concept of MHD turbulence by Goldreich & Sridhar (1995) works for interstellar turbulence, the MHD waves we are dealing with in this context are the fast magnetosonic waves. The Alfvén waves propagate predominantly perpendicular to the magnetic field and because of this they do not significantly scatter cosmic rays. It also explains why radio scintillation observations show no sign of the termination of electron density fluctuations at wavenumbers from 10^{-14} to 10^{-8} cm^{-1} . According to Lithwick & Goldreich (2001) these fluctuations are produced by the slow magnetosonic waves with $k_{\perp} \gg k_{\parallel}$, which are almost not damped

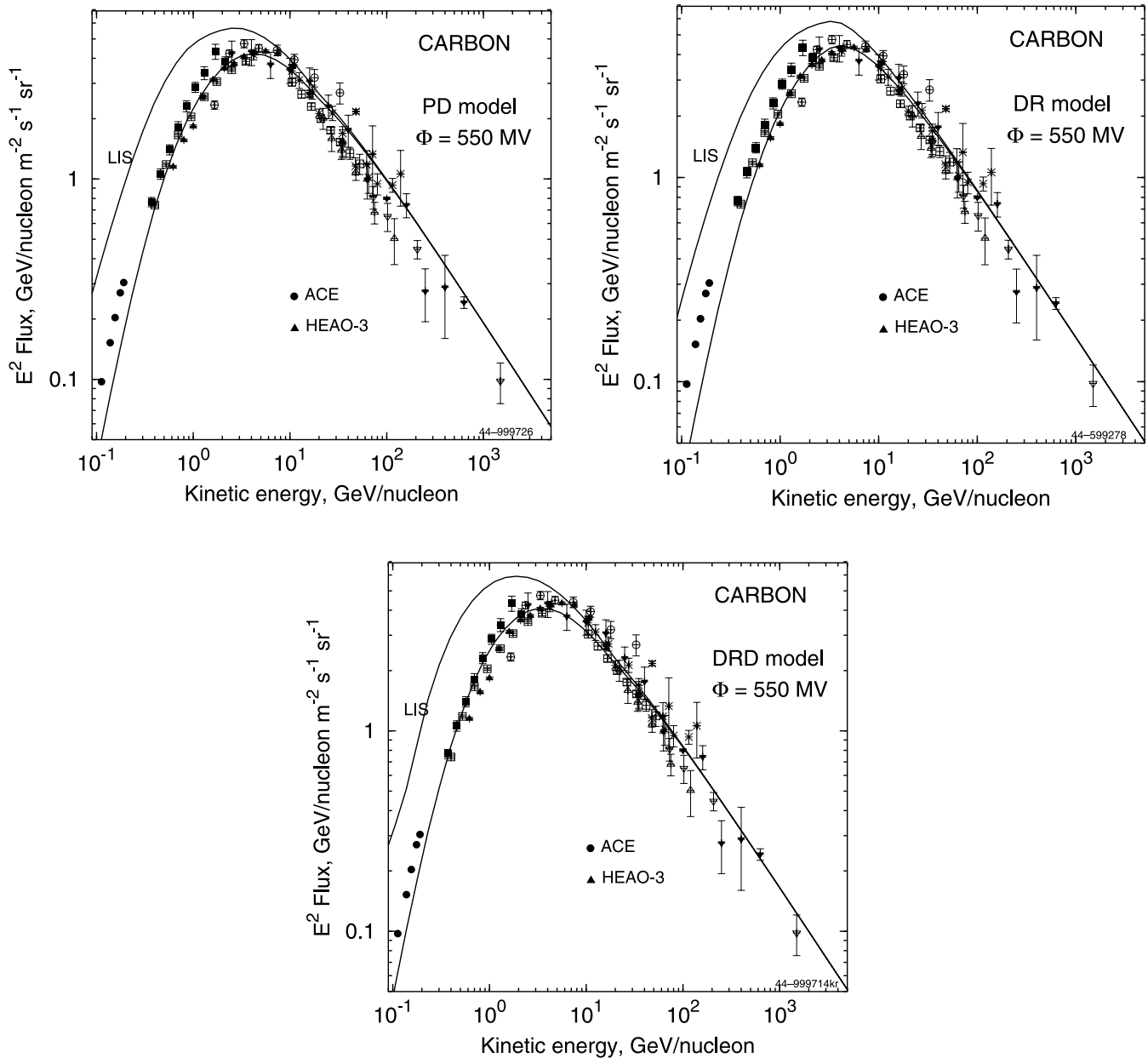


FIG. 4.—Spectrum of carbon calculated in plain diffusion model (PD model), reacceleration model (RD model), and diffusive reacceleration with damping model (DRD model). *Top curve*: LIS; *bottom curve*: modulated using force field approximation ($\Phi = 550 \text{ MV}$). Data: *ACE* (Davis et al. 2000, 2001); *HEAO-3* (Engelmann et al. 1990); for other references see Stephens & Streitmatter (1998) (symbols are changed).

on cosmic rays. An alternative explanation is that the wave damping on cosmic rays and the radio scintillations mainly occur in separate regions of the interstellar medium (see below in the discussion on the “sandwich” model of cosmic-ray propagation in the Galaxy). Some minor contribution to the observed scintillations is possible from fast magnetosonic waves interacting with energetic particles, and in this respect it is of interest that the observations may need an enhancement in the power on large “refractive” scales 10^{13} – 10^{14} cm relative to the power on small “diffractive” scales 10^9 – 10^{10} cm (Lambert & Rickett 2000). This may indicate the cutoff of the spectrum of fast magnetosonic waves due to cosmic-ray action.

While the mere fact of a wave spectrum steepening under the action of damping on cosmic rays can be described by a simple equation (9) with some characteristic time for nonlinear wave interactions $T_{\text{nl}}[W(k), k]$, the exact form of the function $W(k)$ at

large k , where the damping is significant depends critically on the form of the equation for waves. It involves in particular to the vanishing of $W(k)$ at some k_* ($\sim 10^{-12} \text{ cm}^{-1}$) and the corresponding singularity of $D(p)$ at some p_* ($\sim 1 \text{ GV}$) found in our calculations. Less significant in this sense is our approximation of the resonant wavenumber in equation (5), which does not include the particle pitch angle in an explicit form. (If it were included, the term with g in eq. [12] would have a third integration over the pitch angle.) We note, however, that this effect was included in the derivation of equation [10] for the attenuation rate.

In the context of the approximations adopted in the present work, the problem of cosmic-ray transport at $p < p_*$ arises. The free streaming of cosmic rays from the Galaxy leads to an instability and to the growth of waves that scatter particles and thus slow down the streaming (see, e.g., Berezhinskii et al. 1990). We shall consider the processes at low energies in a separate work.

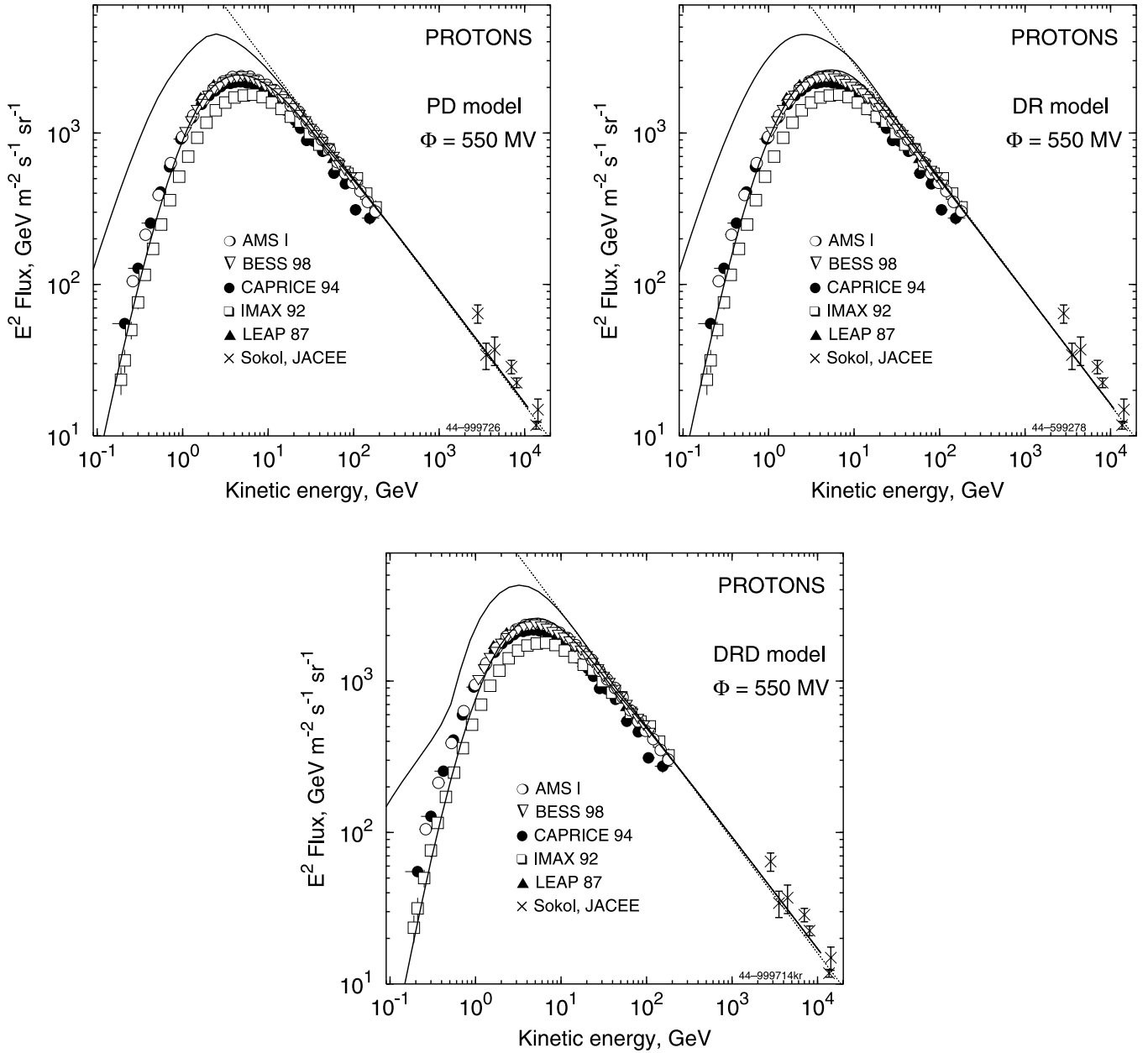


FIG. 5.—Proton spectra as calculated in plain diffusion model (PD model), reacceleration model (RD model), and diffusive reacceleration with damping model (DRD model). *Top curve*: LIS; *bottom curve*: modulated to 550 MV. The thing dotted line shows the LIS spectrum best fitted to the data above 20 GeV (Moskalenko et al. 2002). Data: AMS (Alcaraz et al. 2000b); BESS 98 (Sanuki et al. 2000); CAPRICE 94 (Boezio et al. 1999); IMAX 92 (Menn et al. 2000); LEAP 87 (Seo et al. 1991); Sokol (Ivanenko et al. 1993); JACEE (Asakimori et al. 1998).

Here we note only that the streaming instability develops above the Galactic disk at $D(1 \text{ GV}) \gtrsim V_A H \sim 3 \times 10^{29} \text{ cm}^2 \text{ s}^{-1}$ (if the wave damping is absent) and leads to the diffusive-convective transport of cosmic rays. The given estimate of the diffusion coefficient at 1 GV follows from the condition of cosmic-ray streaming instability $U_{\text{cr}} > V_A$ where U_{cr} is the bulk velocity of cosmic-ray gas. The bulk velocity is $U_{\text{cr}} \approx \delta_{\text{cr}} c$ where the cosmic-ray anisotropy perpendicular to the galactic disk is $\delta_{\text{cr}} \approx D/cH$. The Alfvén velocity in galactic halo is about $2 \times 10^7 \text{ cm s}^{-1}$. It is important that the magnetic rigidity 1 GV corresponds to a kinetic energy 0.43 GeV for protons and 0.13 GeV nucleon $^{-1}$ for nuclei with charge-to-mass ratio $Z/A = 1/2$. The Galactic spectrum of cosmic-ray protons and nuclei at such low energies cannot be derived from direct observations at the Earth because of strong modulation in the solar wind.

Dissipation other than on cosmic rays has been neglected in the present work, although it may completely destroy the MHD cascade or considerably change its angular distribution and thus affect $D(p)$ in a large part of the interstellar medium (see McIvor 1977; Cesarsky 1980; Yan & Lazarian 2004). The region of the cosmic-ray halo is the most “safe” in this sense (Yan & Lazarian 2004). In particular, dissipation on ion-neutral collisions may destroy MHD turbulence in the Galactic disk but not in the halo, where neutrals are absent. We then come to the “sandwich” model for cosmic-ray propagation, with different diffusion coefficients in the Galactic disk, D_g , and in the halo, D_h (it is assumed here that some scattering is present in the Galactic disk and that the diffusion approximation works). In this case, the mean matter thickness traversed by cosmic rays does not depend on diffusion inside the disk even at zero gas density in the halo:

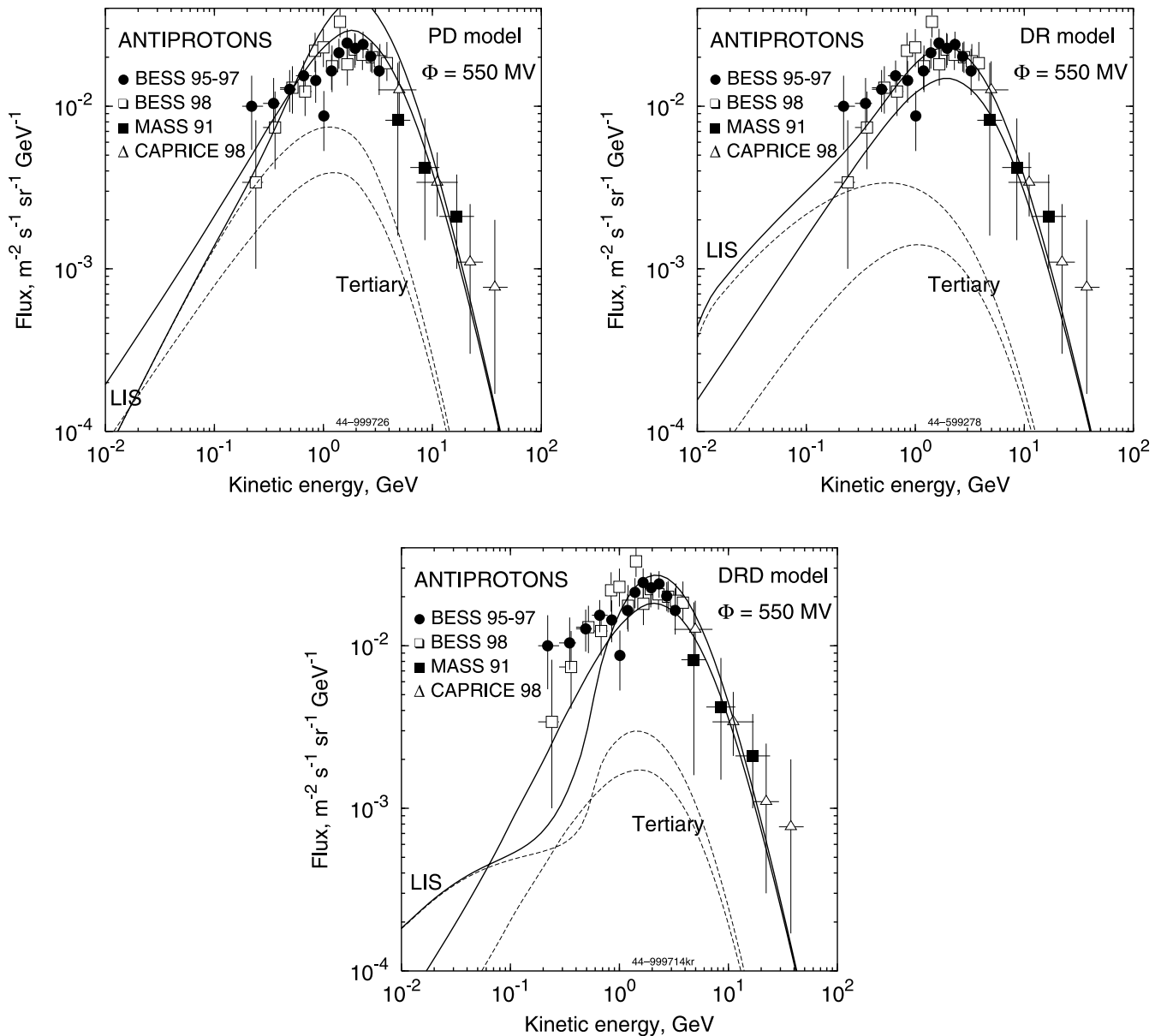


FIG. 6.—Antiproton flux as calculated in plain diffusion model (PD model), reacceleration model (RD model), and diffusive reacceleration with damping model (DRD model). *Top curve:* LIS; *bottom curve:* modulated to 550 MV. Data: BESS 95-97 (Orito et al. 2000); BESS 98 (Asaoka et al. 2002); MASS 91 (Basini et al. 1999); CAPRICE 98 (Boezio et al. 2001).

$X = (\mu_g v/2)[(h/2D_g) + (H/D_h)] \approx \mu_g vH/(2D_h)$ (Ginzburg & Ptuskin 1976). The energy dependence of secondary-to-primary ratios in cosmic rays is determined by the diffusion coefficient in the cosmic-ray halo where the model developed in the present paper is applied.

The estimate based on the empirical value of the diffusion coefficient for GeV particles (see § 1) gives the level of turbulence at the principal scale $\delta B_{\text{tot}}^2/B^2 \sim 0.03$ for a Kraichnan-type spectrum $W(k)k^{-3/2}$, and $\delta B_{\text{tot}}^2/B^2 \sim 1$ for a Kolmogorov-type spectrum $W(k)k^{-5/3}$, if $k_L = 10^{-21} \text{ cm}^{-1}$. At the same time, the data on Faraday rotation angles favor the Kolmogorov spectrum with $\delta B_{\text{tot}}^2/B^2 \sim 1$ and $k_L = 10^{-21} \text{ cm}^{-1}$. The cascades of Alfvén waves (with the scaling $k^{-5/3}$) and the fast magnetosonic waves ($k^{-3/2}$) are independent in the Goldreich & Sridhar (1995) model of MHD turbulence, and the amplitude of Alfvén wave cascade may dominate at the principal scale. Also, in the sandwich model described in the preceding paragraph, the turbulence, which determines the confinement of cosmic rays in the Galaxy, is distributed in the

halo of size $H \sim 4$ kpc, whereas the observations of interstellar turbulence refer to the Galactic disk and the adjacent region, where it can be much stronger.

7. CONCLUSIONS

On the whole, the empirical diffusion model for cosmic rays with energies from 10^8 to 10^{17} eV implies the presence of random magnetic field with an extended power-law spectrum of fluctuations $W(k)k^{-2+a}$, $a \lesssim 0.5$ at wavenumbers from 3×10^{-12} to 10^{-20} cm^{-1} . The existence of such a turbulence spectrum in the interstellar medium seems confirmed by various astronomical observations. It should be emphasized that this does not prove the existence of a spectral cascade or spectral transfer throughout this enormous wavenumber range, however tempting that conclusion may be. Among other puzzles, an oddity is the absence of a spectral feature on spatial scales for which ion-neutral collisional processes should be most pronounced. The two special cases of turbulence spectrum with $a = \frac{1}{3}$ and $\frac{1}{2}$, which correspond to the

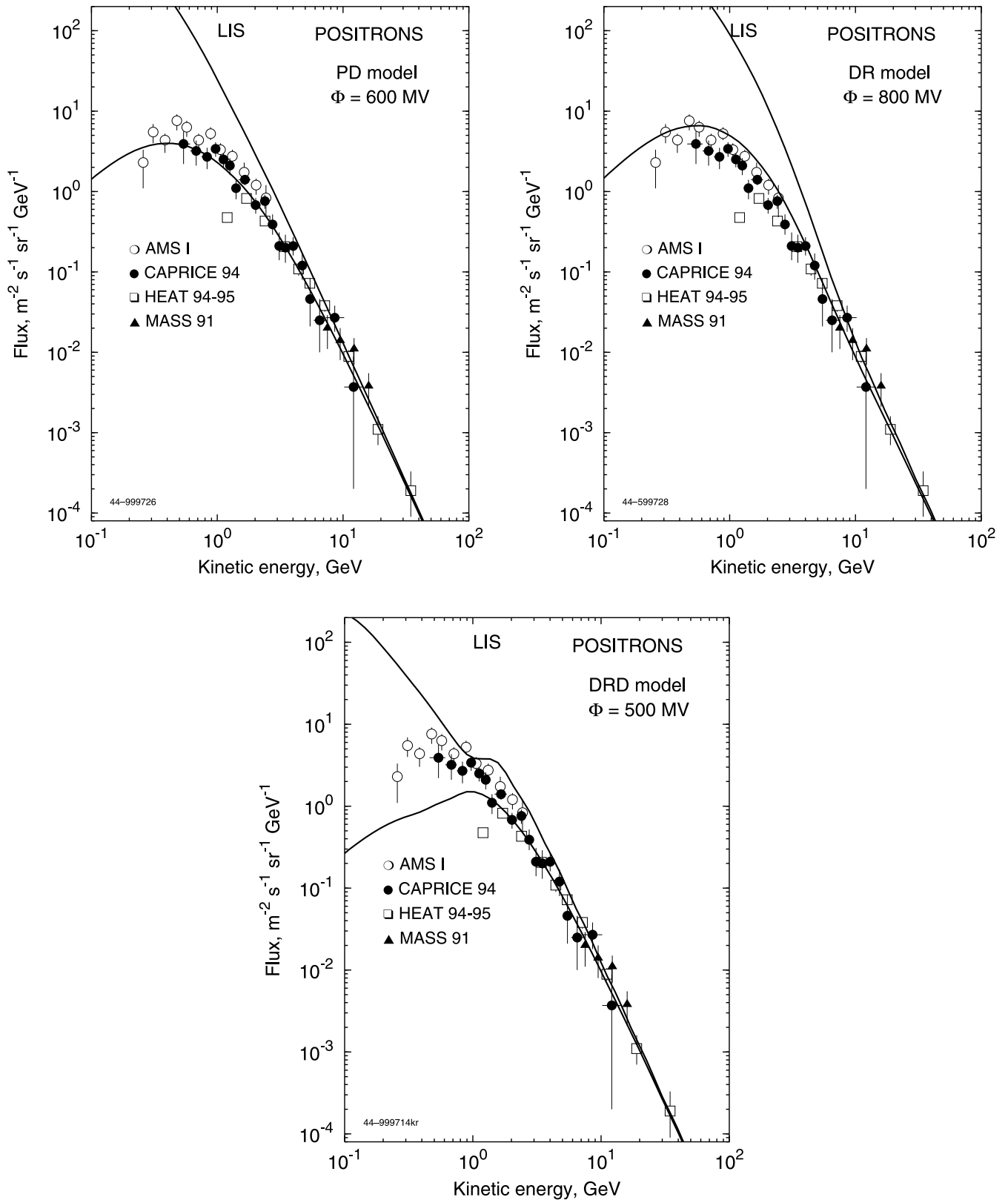


FIG. 7.—Positron flux as calculated in plain diffusion model (PD model), reacceleration model (RD model), and diffusive reacceleration with damping model (DRD model). *Top curve:* LIS; *bottom curve:* modulated. Data: AMS-I (Alcaraz et al. 2000a); CAPRICE 94 (Boezio et al. 2000); HEAT 94-95 (DuVernois et al. 2001); MASS 91 (Grimani et al. 2002).

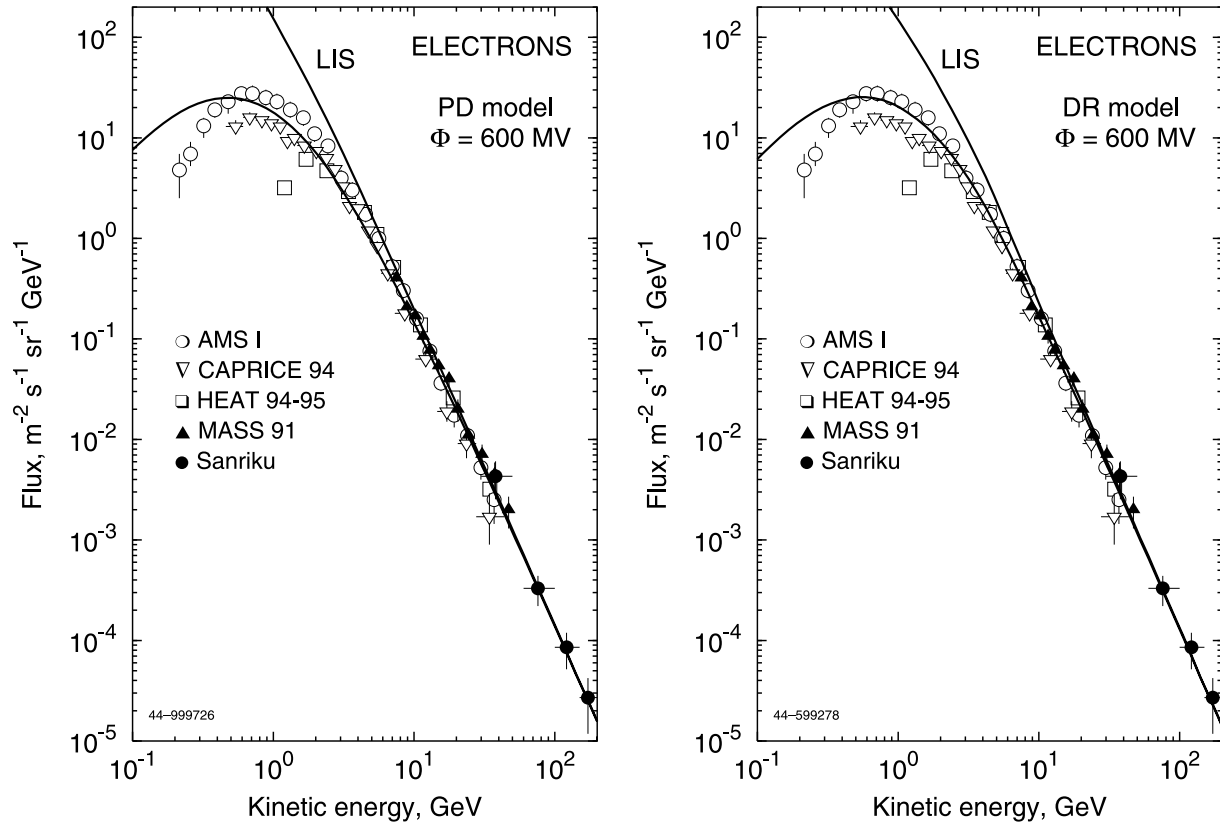


FIG. 8.—Electron flux (primary plus secondary) as calculated in plain diffusion model (PD model), reacceleration model (RD model), and diffusive reacceleration with damping model (DRD model). *Top curve*: LIS; *bottom curve*: modulated to 600 MV. Data: AMS-I (Alcaraz et al. 2000a); CAPRICE 94 (Boezio et al. 2000); HEAT 94-95 (DuVernois et al. 2001); MASS 91 (Grimani et al. 2002); Sanriku (Kobayashi et al. 1999).

Kolmogorov and the Kraichnan spectra, respectively, are used in popular versions of the diffusion model as described in § 5. In both cases, as the present work has demonstrated, one can get a satisfactory fit to the data on energy spectra of secondary and primary nuclei if account is taken for wave damping on cosmic rays for a slow Kraichnan cascade. The fit can be obtained either in the DR model with reacceleration on the Kolmogorov spectrum with no significant effect of cosmic-ray damping (because the Kolmogorov cascade is fast) or in the DRD model with relatively weak reacceleration on the Kraichnan spectrum which is significantly modified by cosmic-ray damping. Some problems still remain to be solved. The main difficulties with the Kolmogorov spectrum are, first, the contradiction with the leading theory of MHD turbulence in which this spectrum is associated with perpendicular propagating Alfvén waves, which almost do not scatter cosmic-ray particles; and, second, the low flux of anti-protons characteristic of models with relatively strong reacceleration. A major concern for diffusion on a Kraichnan spectrum is the relatively strong dependence of diffusion on energy, which leads to an unacceptably large anisotropy of cosmic rays, especially above 10^{14} eV (see Jones et al. 2001; Ptuskin et al. 2003b).

Let us emphasize again that the models of cosmic-ray propagation discussed in the present paper assume that the MHD turbulence required for cosmic-ray scattering is produced by

some external sources. An alternative is the Galactic wind model by Zirakashvili et al. (1996) and Ptuskin et al. (1997). In this model the cosmic-ray pressure drives a wind with a frozen-in regular magnetic field that is shaped into huge spirals (the radius of the Galactic wind cavity is about 300 Kpc). The MHD turbulence in the wind is created by the streaming instability of cosmic rays moving predominantly along the regular magnetic field lines outward from the Galactic disk. The level of turbulence is regulated by the nonlinear Landau damping on thermal ions. The model explains well the cosmic-ray data up to ultra-high energies $\sim 10^{17}$ eV, with the exception of the observed low anisotropy (about the same difficulty with anisotropy as occurs in the diffusion model with a given Kraichnan spectrum). As regards interpretation of cosmic-ray observations, a preference cannot yet be given to any of the models of cosmic-ray transport in the Galaxy discussed above.

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