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Abstract

Transport coefficients of small chemical potential quark gluon plasma are estimated and dissipative corrections to the scaling hydrodynamic equations for ultrarelativistic nuclear collisions are studied. The absence of heat conduction phenomena is clarified. Lower and upper bounds on the shear viscosity coefficient are derived. The importance of non-perturbative antiscreening effects for transport properties is emphasized. Bulk viscosity associated with the plasma to hadron transition is considered. Finally, effects of dissipative phenomena on the relation between initial energy density and final rapidity density are estimated.

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1. Introduction

Ultra-relativistic collision of heavy nuclei offer the possibility of creating a new state of matter - the quark gluon plasma. Current estimates[1] indicate that this plasma state could be reached by increasing the energy density of hadronic matter by an order of magnitude over that found in nuclei ($\epsilon_{Nuc} \sim 0.15$ GeV/fm^3). Unfortunately, such high energy densities are expected to be reached only for short times, $\Delta\tau \sim$ few fm/c, due to the rapid longitudinal expansion of the plasma[1]. However, if the expansion proceeds hydrodynamically[2-5], then information about the interesting early stages of the collisions can be extracted from the final rapidity density, dN/dy, of hadrons. Specifically, the initial energy density, ϵ_0, can be related to dN/dy via[6] $\epsilon_0 \propto (dN/dy)^{1+c^2}$, where c_0^2 is the speed of sound. The above relation is obtained assuming the validity of scaling hydrodynamics[5,7-9] and the absence of dissipative effects.

In this paper we estimate the magnitude of the transport coefficients of an ideal quark-gluon plasma with SU(3) color and two flavors. We concentrate on the mid-rapidity plasma where the baryon chemical potential, μ_B, can be ignored in comparison to the temperature, T. Dimensional considerations dictate[10-15] that the viscosity coefficient, η, must be found proportional to T^3. By imposing the physical constrains that the momentum degradation mean free path, λ, must be larger than both the interparticle spacing and the thermal Compton wavelength, we obtain an approximate lower bound, $\eta \gtrsim 2T^3$. We also derive a practical upper bound on $\eta \lesssim \epsilon\tau/4 \sim 3T^3(T\tau)$ that is necessary for the applicability of the Navier-Stokes equation. We study how standard many body perturbation theory estimates of η fails near the transition temperature and consider the possible importance of anti-screening effects. The absence of heat conduction, κ, in baryon free plasmas is emphasized, and we show why the Landau definition of κ is preferred over the Eckart choice for small chemical potential. A possible source of bulk viscosity, ξ, is considered due to the finite relaxation time of the plasma to hadron phase transition. Finally, we solve the Navier-Stokes equation with the scaling boundary condition to estimate the magnitude of entropy production due to dissipative process in ultra-relativistic nuclear collisions. We find that dissipative effects could reduce the estimated initial energy density by a few GeV/fm^3 relative to ideal hydrodynamics estimates.

2. Relativistic Hydrodynamics and Heat Conduction

We review first the hydrodynamic formulation to help clarify some confusion in the literature concerning the heat conduction[3,5,9-11]. Relativistic hydrodynamics is based on the local conservation laws,

$$\frac{\partial T^{\alpha\beta}}{\partial x^\alpha} = 0 \quad , \qquad (2.1)$$

$$\frac{\partial n^\alpha}{\partial x^\alpha} = 0 \quad , \qquad (2.2)$$

where $T^{\alpha\beta}$ is the energy-momentum flux tensor, and n^α is the 4-flux of a quantum number. The equations are closed through the assumption of local thermodynamic equilibrium. In the local rest frame of an ideal fluid[16]

$$T^{\alpha\beta} = \mathrm{diag}\,(\varepsilon,p,p,p)\ , \quad n^{\alpha} = (n,0,0,0), \tag{2.3}$$

and the volume energy density ε, pressure p, and densities of quantum numbers n are related through the equation of state of the medium. If the local rest frame is boosted to the velocity $\underset{\sim}{v}$, then with an introduction of the 4-velocity

$$u^{\alpha} = (\gamma,\ \gamma\underset{\sim}{v}), \tag{2.4}$$

and the projection operator onto the local frame 3-space

$$\Delta^{\alpha}{}_{\beta} = g^{\alpha}{}_{\beta} - u^{\alpha}u_{\beta}, \tag{2.5}$$

the fluxes can be written as

$$\begin{aligned} T^{\alpha\beta} &= \varepsilon u^{\alpha}u^{\beta} - p\Delta^{\alpha\beta} = (\varepsilon + p)\ u^{\alpha}u^{\beta} - pg^{\alpha\beta}, \\ n^{\alpha} &= nu^{\alpha}. \end{aligned} \tag{2.6}$$

It is straightforward to demonstrate that Eqs. (2.1-4,6), yield the usual nonrelativistic hydrodynamic equations of an ideal fluid.

The dissipative corrections $\tau^{\alpha\beta}$, and ν^{α},

$$\begin{aligned} T^{\alpha\beta} &= \varepsilon u^{\alpha}u^{\beta} - p\Delta^{\alpha\beta} + \tau^{\alpha\beta} \\ n^{\alpha} &= nu^{\alpha} + \nu^{\alpha}, \end{aligned} \tag{2.7}$$

account for the fact that it is not possible to maintain the local equilibrium down to the arbitrarily small space and time scales. The form of the corrections may be found by demanding that they are of the first order in gradients and that the law of increase of entropy be satisfied[16]. The power of the method is that it does not require specific microscopic considerations at this point. One identifies the entropy 4-flux

$$\sigma^\alpha = \sigma u^\alpha - \mu_k \nu_k^\alpha / T, \tag{2.8}$$

with σ the rest-frame entropy-density, T the temperature, and k indexing the quantum numbers with the associated chemical potentials μ_k. For the dissipative fluxes one finds [16]

$$\tau^{\alpha\beta} = \eta \left(\nabla^\alpha u^\beta + \nabla^\beta u^\alpha - \frac{2}{3}\Delta^{\alpha\beta} \nabla_\rho u^\rho\right) + \xi \Delta^{\alpha\beta} \nabla_\rho u^\rho , \tag{2.9}$$

and

$$\nu_k^\alpha \propto \nabla^\alpha (\mu_k / T) , \tag{2.10}$$

with

$$\nabla^\alpha = \Delta^{\alpha\beta} \frac{\partial}{\partial x^\beta}$$

and with the coefficients η and ξ in (2.9) and the coefficients in (2.10) being positive. The second term in (2.9) constitutes a correction to the pressure: $p \rightarrow p - \xi \nabla_\rho u^\rho$. The transport coefficients may be identified by going to the nonrelativistic limit of the equations. There η and ξ are found to be the shear and bulk viscosity coefficients, respectively. In the case of a single conserved quantum number[16]

$$\nu^{\alpha} = \kappa\left(\frac{nT}{\epsilon + p}\right)^2 \nabla^{\alpha}(\mu/T) \qquad (2.11)$$

with κ being the heat conduction coefficient.

Equations (2.8–11) correspond to the Landau-Lifshitz definition of hydrodynamic velocity in which the local rest-frame is determined by the vanishing of the energy 3-flux ($u_{\alpha}\, \tau^{\alpha\beta} = 0$). This definition of the local rest frame, however, is not unique when there exist conserved quantum numbers. Another natural choice of the rest frame, proposed by Eckart[17], is the frame where the 3-flux of the conserved current vanishes ($\Delta^{\alpha\beta}\nu_{\beta} = 0$). If several conserved currents exist, then the rest frame could also be chosen as that in which the 3-flux of some linear combination of those currents vanishes.

This freedom to adopt different definitions of the local fluid velocity leads to different ways in which heat conduction enters the problem. In the Landau-Liftshitz definition, having chosen the frame in which $T^{oi} = 0$, heat conduction arises in (2.11) as a correction to the spacial current of the conserved quantum number. In the Eckart definition, having chosen the frame in which $n^i = 0$, heat conduction arises as a correction to T^{oi}. For several conserved currents, $\partial_{\mu} n_k^{\mu} = 0$, having chosen the rest frame for example by $n_1^i = 0$, there would arise a first order correction in gradients to T^{oi} as well as to the other currents n_k^i, $k > 1$. The corrections would be identified with diffusion effects.[18]

In the symmetric quark-gluon plasma, the fluxes of the conserved quantum numbers are identically zero – we have only one equation of motion (2.1). We are bound to adopt the Landau-Lifshitz definition of hydrodynamic velocity. Therefore, the notion of the heat conduction does not arise. The fact that in a symmetric system the heat conduction is absent has been noted already long ago by Emel'yanov[13]. Even if the baryon flux were finite but small compared with particle fluxes, it is still preferable to adopt the Landau-Lifshitz definition of the hydrodynamic velocity as we now show. The simplest way to see this is to note that for finite $\nabla_i(\mu/T)$ we expect in the Landau convention a finite correction, ν_i, in (2.11) even in the limit $n \to 0$ (see eq. (2.19)). In that limit $n_i \approx \nu_i$. If we now adopt the Eckart definition of the velocity, u_E^μ, then by definition $n^i = nu_E^i \approx \nu^i$ and hence

$$u_E^i \approx \frac{\nu^i}{n} \xrightarrow[n \to 0]{} \infty \ . \tag{2.12}$$

This shows that for all chemical potentials with finite gradients, the Eckart fluid velocity would lead to arbitrarily large corrections to T^{oi}. A more detailed understanding of how the Eckart frame breaks down for small μ can be obtained by calculating the thermal conductivity in the relaxation time approximation.[11] The correction $\delta n(x,p)$ to the distribution function $n_o(x,p)$ in equilibrium is then given by

$$p^\mu \partial_\mu n_o(x,p) \approx - \frac{(pu)}{\tau_c} \delta n(x,p) \quad . \tag{2.13}$$

With (2.13) we can calculate the corrections to $T^{\mu\nu}$ and n^μ as

$$\delta T^{\mu\nu}(x) = \sum_i \int \frac{d^3p}{p_o} p^\mu p^\nu \delta n_i(x,p)$$

$$\delta n^\mu(x) = \sum_i \pm \int \frac{d^3p}{p_o} p^\mu \delta n_i(x,p) \quad , \tag{2.14}$$

where the sum in $\delta T^{\mu\nu}$ runs over quark, antiquark, and gluon distributions, and in δn^{μ} only over quark and antiquark distributions with the respective ± signs. The equilibrium distributions are

$$n_i^o = [\exp\{[(p_\mu u^\mu(x)) \pm \mu(x)]/T(x)\} \pm 1]^{-1} \qquad (2.15)$$

where ±1 is for quarks (gluons) and $\pm\mu$ is the chemical potential only for quarks and antiquarks. In Ref. (11), (2.13-2.14) were used to show that

$$\delta T_g^{oi} = \kappa_g \left(\frac{nT^2}{\varepsilon + p}\right) \nabla_i \left(\frac{\mu}{T}\right) \qquad (2.16)$$

with $\kappa_g \propto \tau_c T^3$ being the gluon contribution to heat conduction. The source of trouble are however the quark and antiquark contributions which were not calculated in Ref. (11). Following the same steps as in Ref. (11), we find that quarks-antiquarks contribute

$$\delta T_q^{oi} = A_q \nabla_i \left(\frac{\mu}{T}\right)$$

$$\delta n^i = B_q \nabla_i \left(\frac{\mu}{T}\right) \qquad (2.17)$$

where A_q, B_q are finite integrals over the equilibrium distributions in the frame $u^\mu = (1,0)$. Note first that the 3-flux of baryons does not vanish in the frame $u^\mu = (1,0)$ in which $\delta T^{\mu\nu} = \delta T_g^{\mu\nu} + \delta T_q^{\mu\nu}$ were calculated. Therefore, δT^{o1} contains energy flux associated with translational motion as well as heat flux. Consequently, the choice $u^\mu = (1,0)$ neither correspond to the Eckart choice nor the Landau choice. The Eckart choice would be the frame where $\delta n^i = 0$. The boost velocity $\delta u^\mu = (1, \delta u^i)$ to the Eckart frame would thus be determined as $\delta n^i - n\delta u^i = 0$ and therefore

$$\delta u^i = \frac{1}{n} B_q \nabla_i \left(\frac{\mu}{T}\right) \xrightarrow[n \to 0]{} \infty, \tag{2.18}$$

The δT^{0i} in that frame would also obviously diverge in the small n limit as well. We are therefore forced to use the Landau choice for small baryon density problems. In the small baryon density limit, the relaxation time approximation leads in that case to the following dissipative baryon flux

$$\nu_B^\alpha \simeq \frac{1}{36}(n_q + n_{\bar{q}})\tau_c \nabla^\alpha \left(\frac{\mu_B}{T}\right) . \tag{2.19}$$

With a small baryon number flux, the heat conduction representing a correction to a small quantity, could be discarded (unless one were specifically interested in the baryon number evolution).

3. Viscosity Coefficients

3.1 Bounds on Shear Viscosity

Familiar kinetic theory arguments[19] lead to the following estimate of the shear viscosity coefficient

$$\eta \approx \frac{1}{3}\sum_i (n\langle p\rangle\lambda)_i , \tag{3.1}$$

where n is the local density of quanta i transporting an average momentum $\langle p\rangle_i$ over a momentum degradation mean free path λ_i. More detailed kinetic theory derivations[18,20] replace 1/3 by 4/15 in the ultra-relativistic ($T \gg m$) domain and 1/3 by 0.21 in the non-relativistic domain[19].

Furthermore, λ_i can be related to the differential cross sections, $d\sigma^{ij}/d\Omega$, via

$$1/\lambda_i = \sum n_j \int d\Omega \frac{d\sigma^{ij}}{d\Omega} \sin^2\theta \quad , \tag{3.2}$$

The $\sin^2\theta$ weight in eq. (3.2) arises because large angle scatterings are most effective in momentum degradation.

The above relations are valid only in gases where (1) the mean free paths are small compared to the size of the system $\lambda_i \ll L$ and (2) correlations among the particles can be neglected. For $\lambda_i \gtrsim L$, one body dissipation dominates and λ_i are replaced[19] by L. In fluids or crystals, involving strong correlations, particles are confined in local field minima and momentum transport is enhanced by mean field phenomena. For a quark-gluon plasma, the gas description should apply at very high energy densities because of asymptotic freedom. In contrast, a hadronic medium can be considered a gas at low energy densities because of the short range nature of the forces. Current Bag model and QCD Lattice calculations[1] suggest that the gas approximation should hold for $\varepsilon < \varepsilon_H \sim 0.5$ GeV/fm^3 and $\varepsilon > \varepsilon_Q \sim 2$ GeV/fm^3. In the transition region, the properties of matter are very uncertain. We will simply interpolate linearly between $\eta_H \cong \eta(\varepsilon_H)$ and $\eta_Q \cong \eta(\varepsilon_Q)$ as a function of ε, as would be appropriate for a first order transition.

Before estimating λ_i via eq. (3.2) we note several physical constrains on λ_i. First, the uncertainty principle implies that λ_i cannot be arbitrarily small for finite <p>. In fact, $\lambda_i \gtrsim 1/\langle p \rangle$, and consequently

$$\eta \gtrsim \frac{1}{3} n \quad , \tag{3.3}$$

where $n = \sum n_i$ is the total density of quanta. No matter how large is the free space cross section, the collision rate in many body systems is bounded by the typical energy per particle[21,22]. Second, in a gas λ_i must exceed the interparticle distance, $\lambda_i \gtrsim n^{-1/3}$. This leads to another lower bound

$$\eta \gtrsim \frac{1}{3} \langle p \rangle \, n^{2/3} \tag{3.4}$$

A violation of (3.4) would mean that it is possible to maintain local equilibrium on distance scales involving only one particle. This is only possible in fluids and crystals, where, however, gas kinetic estimates for η tend to grossly under estimate η in any case. Note that for a fixed energy density $\varepsilon \approx \langle p \rangle n$, the two lower bounds are equal if $n = \varepsilon^{3/4}$. Consequently, we can combine them to obtain

$$\eta \gtrsim \frac{1}{3} \varepsilon^{3/4} \tag{3.5}$$

For $\mu = 0$ quark-gluon plasmas, $\varepsilon = 12.2\ T^4$, $n_g = 1.95\ T^3$, $n_q = 2.2 T^3$, $n = 4.15\ T^3$. In this case eqs. (3.3 - 3.5) given $\eta \gtrsim 1.4\ T^3$, $2.6\ T^3$, $2.2\ T^3$ respectively. Clearly, eq. (3.4) imposes the most severe constraint because the average distance between quanta, $n^{-1/3} \sim 0.6/T$, exceeds the thermal compton wavelength, $1/\langle p \rangle \sim 0.3/T$.

In summary, a reasonable lower bound on the shear viscosity coefficients for $\mu << T$ quark gluon plasmas is

$$\eta \gtrsim 2 T^3 \quad . \qquad (3.6)$$

Finite chemical potentials would tend to raise η due to Pauli blocking effects.

In addition to the physical lower bound (3.6), there is a practical upper bound on η necessary for the applicability of the Navier-Stokes equation. The derivation of the dissipative corrections to $T^{\alpha\beta}$ and n^{α} in eq. (7) from transport theory relies on the smallness of the mean free path in comparison to gradients of field quantities. For the scaling hydordynamic problem this requires

$$\lambda |\partial \ell n\varepsilon/\partial\tau| < 1 \quad . \qquad (3.7)$$

Therefore, in order to apply the Navier-Stokes theory we must have

$$\eta < \frac{1}{3} \varepsilon/|\partial \ell n\varepsilon/\partial\tau| \quad . \qquad (3.8)$$

For Lorentz invariant initial conditions Ref. (5, 7-9, 11) and zero chemical potential the hydrodynamic equations (2.1,2.2) reduce to

$$\frac{d\varepsilon}{d\tau} + \frac{1}{\tau} (\varepsilon + p) = \frac{1}{\tau^2} (\frac{4}{3} \eta + \xi) \quad . \qquad (3.9)$$

To apply (3.9) to (3.8) we must neglect the right hand side since it is higher order in λ. This leads to $\varepsilon(\tau) = \varepsilon_0(\tau_0/\tau)^{4/3}$ and hence

$$\eta < \frac{1}{4} \varepsilon\tau \quad . \qquad (3.10)$$

Inserting this upper bound into (3.9), we see that the plasma cools more slowly than with $\eta = 0$:

$$\epsilon(\tau) = \epsilon_0 (\tau_0/\tau) \qquad . \tag{3.11}$$

This limit just corresponds to constant energy rather than isentropic expansion. It also coincides with the maximum entropy expansion considered in Ref. (6). The rate of energy density loss, p/τ, due to pdV work done on expansion is exactly compensated for by viscous reheating, $4\eta/3\tau^2$, in this limit. This reheating arises by the conversion of longitudinal flow energy into local excitation energy.

Because $\epsilon\tau$ is approximately a constant of motion for η near the upper bound, we can evaluate the right hand side of (3.10) at the initial time τ_0 marking the onset of final state hydrodynamic expansion. Together with eq. (3.6) this leads to

$$2T^3 \lesssim \eta \lesssim \epsilon_0\tau_0/4 \approx 3T_0^3(\tau_0 T_0) \tag{3.12}$$

From the derivation, it is clear that there is on the order of a factor of two uncertainty on both bounds. Nevertheless, it is surprising that the range of acceptable η is so "narrow." Only for high $T_0 >> 200$ MeV and/or late times $\tau_0 >> 1$ fm/c does the acceptable range open up.

So far we have considered η only in the plasma phase. In the hadronic phase, typical transport cross sections are $\sigma_\eta \approx 10 - 20$ mb. In this case (3.1) yields

$$\eta_H \approx \frac{T}{\sigma_\eta} \sim \left(\frac{T}{200\ \text{MeV}}\right) \frac{(0.5 - 1)}{\text{fm}^3} \quad . \tag{3.13}$$

In order to compare (3.13) to the lower bounds (3.3, 3.4), we must adopt a model of the hadronic phase. A simple yet flexible model is that of a Shuryak resonance gas,[4,6] for which $p_h = c_H^2\epsilon_h$, $\epsilon_h = \epsilon_H(T/T_c)^{(1+c_H^2)/c_H^2}$, $\sigma_h = \sigma_H(T/T_c)^{1/c_H^2}$ and the density of hadrons is

$$n_h = \sigma_h/z(c_H^2) = n_c\left(T/T_c\right)^{1/c_H^2} \quad , \qquad (3.14)$$

where $n_c = \sigma_H/z(c_H^2)$ and $z(c_H^2) = 2.2,\ 3.6.\ 6.9$ for $c_H^2 = 1/2,\ 1/3,\ 1/6$ respectively.[6] For illustration two sets of parameters were considered in Ref. (6) that cover a plausible range of equations of state. The first set (I) corresponds to a strong first order transition at $T_c = 200$ MeV with ϵ_H, $\epsilon_Q = 0.7,\ 3.3$ GeV/fm^3, $c_H^2 = 1/6$, and $n_c = 0.6$ fm^{-3}. The second set (II) correspods to a weak first order transition at $T_c = 140$ MeV with ϵ_H, $\epsilon_Q = 0.45,\ 0.67$, $c_H^2 = 1/3$, and $n_c = 1.2$ fm^{-3}. For these equations of state (3.3) gives

$$n_H \gtrsim \begin{cases} 0.2(T/T_c)^6 \mathrm{fm}^{-3} & : \quad \mathrm{I} \\ 0.4(T/T_c)^3 \mathrm{fm}^{-3} & : \quad \mathrm{II} \ . \end{cases} \qquad (3.15)$$

This is satisfied near T_c for $\sigma_\eta < 50$ mb, 18 mb respectively. Equation (3.4) gives, on the other hand,

$$n_H \gtrsim \begin{cases} 0.7(T/T_c)^5 \mathrm{fm}^{-3} & : \quad \mathrm{I} \\ 0.8(T/T_c)^3 \mathrm{fm}^{-3} & : \quad \mathrm{II} \end{cases} \qquad (3.16)$$

This more severe constraint is satisfied by (3.13) only if $\sigma_\eta < 14$ mb, 9 mb respectively. Since (3.13) tends to fall below the lower bound in (3.16), we expect that in fact the shear viscosity coefficient will be as small as it can be in the hadron phase. In numerical estimates we shall therefore use (3.16).

It is interesting to compare (3.16) with the maximal practical value of η, (3.10), consistent with scaling Navier-Stokes theory. For $p = c_H^2\varepsilon$, $|\partial \ell n\ \varepsilon/\partial\tau| = (1 + c_H^2)/\tau$, and so the coefficient in front of $\varepsilon\tau$ in (3.10) remains close to 1/4 for both sets of parameters. Furthermore, using the maximal value $\sim\varepsilon\tau/4$ in (3.9) leads to $\varepsilon\tau \approx \varepsilon_o\tau_o \approx \varepsilon_H\tau_o\ (\varepsilon_o/\varepsilon_H)$, with ε approximately independent of τ. Therefore, we can compare (3.16) to

$$\eta_H < \frac{1}{4}\,\varepsilon\tau \approx \frac{1}{4}\,\varepsilon_H\tau_o\left(\frac{\varepsilon_o}{\varepsilon_H}\right) \approx (2-3)\left(\frac{T}{T_c}\right)^{1+1/c_H^2} \text{fm}^{-3} \quad . \qquad (3.17)$$

For both sets of parameters the upper bound exceeds the lower bound for the relevant range of temperatures $T_c/2 < T < T_c$. Consequently, Navier-Stokes should apply to the expansion of the hadronic phase.

4. Perturbation Theory Estimates

It must be emphasized that the upper bound in (3.12) is only a practical constraint. It is entirely possible that η in QCD violates that bound near the critical temperature. In that case, we must (1) abandon the scaling boundary conditions that lead to the enormous velocity gradients and/or (2) abandon the Navier Stokes description of the final state expansion phase at high energy densities. This possibility can only be assessed after reliable lattice calculations of η using the Kubo formulas[10] become available.

As an intermediate step, we turn to perturbation theory for possible guidance. Consider the Born approximation for the elastic cross sections[1]

$$\frac{d\sigma^{ij}}{dt} = c_{ij} \frac{\pi\alpha_s^2}{s^2} \frac{s^2 + u^2}{t^2} \quad , \tag{4.1}$$

where c_{ij} = 9/4, 1, 4/9 is the color factor for ij = gg, gq, qq scattering respectively. Strictly speaking. eq. (4.1) holds only for qq scattering with different flavors. For forward scattering, $|t| \ll s$, (4.1) is nevertheless a good approximation to the Born cross sections in all channels. For 90° scattering, $|t| = |u| = s/2$, it underestimates the qg cross section by only 20%, while for gg it is a factor of three too small. Since the longer quark mean free paths set the scale for dissipative effects, (4.1) will be adequate for our purposes.

At finite temperatures the renormalization group analysis of the running color electric coupling constant leads to the approximate form[23-26]

$$\alpha_s(t/\Lambda^2, T/\Lambda) \quad \frac{4\pi}{(11 - \frac{2}{3} N_f)\ell n(|t|/\Lambda^2) + 16\pi^2(1 + N_f/6)T^2/|t|} \; . \quad (4.2)$$

For $t \gg T^2$, i.e. small distance scales, $\alpha_s(t,0)$ reduces to the free space running coupling constant $4\pi/(11 - 2N_f/3)\ell n(|t|/\Lambda^2)$ with N_f flavors. This is intuitively clear because at distance scales much smaller than the interparticle spacing the effects of the many body medium should be unimportant. The parameter Λ is therefore to be identified with its value determined from deep inelastic eN, charmonium spectra, etc., $\Lambda \sim 200 - 600$ MeV. For $|t| \ll T^2$ the plasma acts to shield color electric fields. The scale at which screening becomes important is $|t| \sim k_D^2$ where $k_D^2 = \Pi_{oo}(q = 0) \approx 4\pi\alpha_s(1 + N_f/6)T^2$ is the Debye wave vector or electric mass squared. We note that for $q = (0, \underset{\sim}{k}) \neq 0$, k_D^2 decreases to $3/8\ k_D^2$ for $k/T \gg 1$. That k_D^2 does not go to zero for $k \gg T$ is due to the contribution of the point like four gluon vertex to Π_{oo}.

With (4.2) $d\sigma/dt$ does not diverge at small t. The maximum value of $d\sigma/dt$ occurs near $\ell n|t|/\Lambda^2 \doteq -1$. That maximum value, however, diverges as T is lowered to $T \approx 0.13\ \Lambda$. Therefore, perturbation theory must break down for $T/\Lambda < 1$, marking presumably the onset of confinement.

With (3.1, 3.2, 4.1, 4.2), we can estimate η for $n_q \approx n_g$ plasmas as

$$\eta = \frac{n_g T}{n_g \sigma_{gg} + n_q \sigma_{qg}} + \frac{n_q T}{n_q \sigma_{qq} + n_g \sigma_{qg}} = \frac{T}{\sigma_\eta} \quad , \qquad (4.3)$$

where the transport cross section is $(x = |t|/s)$

$$\sigma_\eta = \frac{4\pi}{s} \int_0^1 dx \; (2 - 2x + x^2) \frac{(1-x)}{x} \alpha_s^2(xs/\Lambda^2, T/\Lambda) \quad . \tag{4.4}$$

Note that due to color factors the gluon contribution to (4.3) is only 4/9 as large as the quark contribution. Also, σ_η remains finite because of color screening for small x.

The average square of the cm energy is $s = \langle (p_1 + p_2)^2 \rangle \approx 17\, T^2$ in the plasma. Therefore, for $T \to \infty$ we can approximate

$$\alpha_s(xs/\Lambda^2, T/\Lambda) \xrightarrow[T\to\infty]{} \alpha(T/\Lambda) \frac{x}{x + x_D(T)} \tag{4.5}$$

Where the Debye x_D is given by

$$x_D(T) = k_D^2(T)/s \approx 4\pi\alpha(T/\Lambda)(1 + N_f/6)/17 \tag{4.6}$$

and the effective temperature dependent running coupling is

$$\alpha(T/\Lambda) = 4\pi/[(11 - 2N_f/3)\ell n(17T^2/\Lambda^2)] \quad . \tag{4.7}$$

With (4.5), the integral in (4.4) is elementary

$$\sigma_\eta = \frac{4\pi\alpha^2(T/\Lambda)}{17\, T^2}\{(2 + 8x_D + 9x_D^2 + 4x_D^3)\ell n(1 + 1/x_D) - (29/6 + 7x_D + 4x_D^2)\} \; . \tag{4.8}$$

In the <u>extreme</u> asymptotic limit $T \to \infty$, $x_D \to 0$, the logarithmic approximation would hold and

$$\eta/T^3 \underset{T\to\infty}{\longrightarrow} [\tfrac{8\pi}{17}\, \alpha^2(T/\Lambda)\ln(1/\alpha(T/\Lambda))]^{-1} \qquad (4.9)$$

In practice, because of the 29/6 in (4.8), the logarithmic approximation underestimates η by more than factor of 3 for $T/\Lambda < 10^6$. Even in the extreme limit (4.9) is a factor of three larger than that estimated in Ref. (11) because of the smaller cross sections for quarks ($\sigma_{qg} \sim \sigma_{gg}/2$) included in (4.9) and the use of the large 90° gg cross section in Ref. (11). For $T/\Lambda > 10$ (4.8) overestimates (4.4) by only 20%. Numerically evaluating (4.4), we find $\eta/T^3 = 4.9, 8.8, 15, 42$ for $T/\Lambda = 0.3, 0.5, 1, 10$ and $\alpha(T/\Lambda) = 3.1, 0.9, 0.46, 0.17$ respectively.

We can now assess, at least within the limited context of perturbation theory, under what conditions, (3.12), scaling Navier-Stokes theory may apply to the expansion stage of the plasma. Using the numerical values of η/T^3, (3.12) implies that Navier-Stokes applies only for proper times later than $\tau_0 \approx (5 - 6)\Lambda^{-1}$ for $T/\Lambda \lesssim 1$ and $\tau_0 < \tau^{-1}$ for $T/\Lambda > 10$. Since $\Lambda \sim 200 - 600$ MeV perturbation theory assures us that Navier Stokes applies for the interesting first few fm/c region only for T > 1 GeV. Unfortunately, for relevant temperatures T ~ 200 - 300 MeV these estimates suggest that the first few fm/c evolution could be far from local equilibrium. However (for temperatures of interest), $\alpha_s \sim 0.5$ is not small and higher order corrections may be important.

One indication of that higher order effects must become important is that the Debye length $r_D = 1/k_D$ becomes smaller than the interparticle spacing $n^{-1/3} \approx 0.6\ T^{-1}$ when $\alpha_s > 0.15$, i.e. for $T/\Lambda < 10$.

Physically, however, we expect screening only on distance scales, $r_D \gtrsim n^{-1/3}$. This corresponds to $nr_D^3 > 1$, as usual in plasmas. In terms of x_D, $nr_D^3 > 1$ implies that $x_D \lesssim n^{2/3}/s \approx 0.15$. In that case the expression in the brackets in (4.8) is close to unity and $\eta/T^3 = 1.8$, 6.9, for $T/\Lambda = 0.5$, 1 respectively. This is a very substantial reduction of η as compared with using (4.6) in (4.8). Therefore higher order effects on the screening mechanism for $T \sim \Lambda$ could play a major role in keeping η within the bounds of (3.12).

Further support for the above can be found from the self consistent solution for the electric mass in Ref. (26). Using the Schwinger-Dyson equations to sum higher order (loops within loops) corrections it was found that the correction to order g^3 was large and negative. This correction reduced k_D^2 to $k_D^2 (1 - \sqrt{\gamma\alpha_s})$ with $\gamma \sim 1$. This physically appealing result indicates that for $\alpha \gg 1$ antiscreening and presumably confinement could set in. With this reduction factor, not only does $nr_D^3 > 1$ remain satisfied but in fact nr_D^3 increases rapidly near the transition temperature. Taken face value k_D^2 would be three times as small for $T/\Lambda \sim 1$ and (4.4) would yield $\sigma_n \approx 3$ mb. This value is reasonable from the point of view of the additive quark model. In that case $\eta/T^3 \approx 3$ also satisfies the criteria (3.12) for applicability of Navier-Stokes theory. Clearly the problem of higher order corrections is subtle and will need much more study before more quantitative conclusions can be reached.

5. Bulk Vicosity in the Mixed Phase

It is known that bulk viscosity, ξ, vanishes both in the non-relativistic and ultrarelativistic limits for a gas with a conserved number of particles.[11] However, for processes occurring with a finite relaxation time τ^* the variation of the sound velocity gives rise to finite bulk viscosity.[16] We consider here a possible source of bulk viscosity due to the finite relaxation time involved in the quark-gluon plasma to hadronic matter transition. For illustration, we consider the first order transition within the context of the Bag model.[1,6] For the hadronic phase we take ϵ_h, p_h, σ_h for a Shuryak gas[4,6] as in (3.14). For the quark phase, we take

$$\epsilon_q = (\epsilon_Q - B)\left(\frac{T}{T_c}\right)^4 + B \quad ; \quad p_q = \frac{1}{3}(\epsilon_q - 4B) \quad , \tag{5.1}$$

where $\epsilon_Q = \epsilon_q(T_c)$ and the condition that $p_q = p_h = p_c$ at T_c gives $4B = \epsilon_Q - 3c_H^2\epsilon_H$. The latent heat in the transition is $\Delta\epsilon = \epsilon_Q - \epsilon_H \approx 4B$. In chemical equilibrium for energy densities ϵ between $\epsilon_H \leq \epsilon \leq \epsilon_Q$, the system would be in a mixed phase with a fraction

$$\lambda_{eq}(\epsilon) = \frac{\epsilon - \epsilon_H}{\epsilon_Q - \epsilon_H} \tag{5.2}$$

of the volume occupied by the plasma.

In a dynamically evolving system where $\epsilon(\tau)$ varies as in (3.9), the concentration $\lambda(\tau)$ may differ from $\lambda_{eq}(\epsilon(\tau))$ because it takes a finite time to convert the plasma into hadrons. In that case the pressure $p(\varepsilon,\lambda)$ in (3.9) is a function of both ε and λ. In equilibrium with $\varepsilon_H \leq \varepsilon \leq \varepsilon_Q$ the pressure is constant

$$p = p_{eq}(\varepsilon) = p(\varepsilon,\lambda_{eq}(\varepsilon)) = p_c \quad . \tag{5.3}$$

Therefore for small deviations from equilibrium, $\lambda = \lambda_{eq} + \delta\lambda$,

$$p(\varepsilon,\lambda) \approx p_c + \delta\lambda\left(\frac{\partial p}{\partial\lambda}\right)_\varepsilon \quad , \tag{5.4}$$

with the right hand side evaluated at $\lambda = \lambda_{eq}$. Noting that the speed of sound in the mixed phase is zero

$$c_0^2 = \frac{\partial p(\varepsilon,\lambda^{eq}(\varepsilon))}{\partial\varepsilon} = \left(\frac{\partial p}{\partial\varepsilon}\right)_{\lambda=\lambda_{eq}} + \left(\frac{\partial p}{\partial\lambda}\right)_\varepsilon \left(\frac{\partial\lambda^{eq}}{\partial\varepsilon}\right) = 0 \quad , \tag{5.5}$$

(5.2, 5.5) yield

$$\left(\frac{\partial p}{\partial\lambda}\right)_\varepsilon = - c_\lambda^2 \Delta\varepsilon \quad , \tag{5.6}$$

where $c_\lambda^2(\varepsilon) = (\partial p/\partial\varepsilon)_{\lambda^{eq}}$ is the speed of sound squared in the system when the plasma fraction is held fixed. For $\varepsilon \lesssim \varepsilon_Q$, $c_\lambda^2 \approx c_Q^2 = 1/3$, where c_Q^2 is the speed of sound squared in the plasma. For $\varepsilon \gtrsim \varepsilon_H$, $c_\lambda^2 \gtrsim c_H^2 \sim 1/6 - 1/3$ where c_H^2 is the speed of sound in hadronic matter.

To estimate $\delta\lambda$ in (5.4) we use the relaxation time approximation[16]

$$\frac{d\lambda}{d\tau} = -\frac{1}{\tau^*}(\lambda - \lambda_{eq}) \quad , \tag{5.7}$$

where $\tau^* \sim \Lambda^{-1} \sim 1$ fm/c is the expected order of magnitude of the transition or nucleation time. Solving for $\lambda = \lambda_{eq} - \tau^* d\lambda/d\tau$, we find to lowest order that

$$\delta\lambda = -\tau^* \frac{d\lambda^{eq}}{d\tau} = -\frac{\tau^*}{\Delta\varepsilon}\frac{d\varepsilon}{d\tau} = \frac{\tau^*}{\Delta\varepsilon}\frac{\varepsilon + p}{\tau} \quad . \tag{5.8}$$

Substituting into (5.4), the pressure is reduced relative to its equilibrium value as

$$p = p_c - \tau^* c_\lambda^2 \frac{(e + p)}{\tau} \quad . \tag{5.9}$$

Comparing with (3.9) we identify

$$\xi = \tau^* c_\lambda^2 (\varepsilon + p_c) \quad . \tag{5.10}$$

Near the top of the transition $\varepsilon \approx \varepsilon_Q$, $c_\gamma^2 \approx 1/3$, and $\xi = \tau^* 4(\varepsilon_Q - B)/9 \sim$ GeV/fm^2 $\sim 5T_c^3$. Note that ξ is comparable to η near the transition temperature. In order that Navier-Stokes applies $(p_c - p)/p$ should be small. Obviously this requires that the transition occurs when the gradients in time have become sufficiently small.

Including ξ in the estimate of the practical upper bound for η in (3.10) we get

$$2T_c^3 \lesssim \eta \lesssim \frac{1}{4}\,\varepsilon_Q\tau - \frac{4}{9}\,\varepsilon_Q\tau^* \tag{5.11}$$

In order for the upper bound to exceed the lower bound in (5.11),we must start with an initial energy density $\varepsilon_0 \approx \varepsilon(\tau)(\tau/\tau_0)$ such that

$$\varepsilon_0 > \varepsilon_0^{crit} = \frac{16}{9}\,\varepsilon_Q\left(\frac{\tau^*}{\tau_0}\right) + \frac{8T_c^3}{\tau_0} \approx 2.5\,\varepsilon_Q \quad . \tag{5.12}$$

Starting with lower energy densities can lead to supercooling[27] and violent non-equilibrium phenomena such as deflagrations or detonations. Even (5.12) may not be enough if the shear viscosity is not close to the lower bound in (5.1). For $\eta = (4-6)T_c^3$, $\varepsilon_0 > (3-4)\varepsilon_Q$ would be required for dissipative effects to remain manageable through the transition region.

To calculate the dependence of c_λ^2 on ε we need to further specify a relation between the temperatures T_q, T_h of the quarks and hadrons in the mixture. Since thermal equilibrium usually has the shortest relaxation time we will consider $T_q = T_h = T$. When $\lambda \neq \lambda_{eq}(\varepsilon)$, then $T \neq T_c$ either. The temperature and pressure are determined by

$$\begin{aligned} \varepsilon &= \lambda\varepsilon_q(T) + (1-\lambda)\varepsilon_h(T) \\ p &= \lambda p_q(T) + (1-\lambda)p_h(T) \quad . \end{aligned} \tag{5.13}$$

Using $\partial p_i/\partial T = \sigma_i$ and $\partial p_i/\partial\varepsilon_i = c_i^2$ for $i = q$ or h we find that

$$c_\lambda^2 = \frac{\lambda\sigma_q + (1-\lambda)\sigma_h}{\lambda\sigma_q/c_Q^2 + (1-\lambda)\sigma_h/c_H^2} \quad , \qquad (5.14)$$

Thus c_λ^2 smoothly interpolates beeween c_Q^2 at $\varepsilon = \varepsilon_Q$ and c_H^2 at $\varepsilon = \varepsilon_H$. If we would relax the assumption that $T_q = T_h$, then the interpolation formula would change. However for $c_Q^2 = c_H^2$, $c_\lambda^2 = c_Q^2$ is independent of ε as it should be. In the numerical examples in the next section we will use (5.14).

Finally, we comment on the bulk viscosity in the hadronic phase. As noted in Ref. (11), it is difficult to maintain chemical equilibrium in the hadronic phase because of the smallness of HH $\to$ HHH, inelastic rate at temperatures $T \lesssim 200$ MeV. The inability of the system to maintain chemical equilibrium brings about a change of the speed of sound and hence leads to bulk viscosity. As shown in Ref. (11), the magnitude of is then likely to be comparable with η in the hadronic phase. Only in the quark-gluon plasma is bulk viscosity likely to be negligible.

6. Results and Summary

We now apply the previous estimates to the problem of relating final observed rapidity densities to initial energy densities.[5,6] For that purpose we recall[6] that in the scaling regime

$$\frac{dN}{dy} = \frac{1}{4} A_\perp \tau_f \sigma(\tau_f) \quad , \qquad (6.1)$$

where $\sigma(\tau) = (\varepsilon + p)/T$ is the entropy density, τ_f is the breakup time and

$A_\perp \approx \pi r_0^2 A^{2/3}$ ($r_0 \approx 1.18$ fm) is the transverse area of the beam. Since $d\varepsilon = T d\sigma$, from (3.9), the entropy evolves according to

$$\frac{d(\tau\sigma)}{d\tau} = \frac{1}{\tau T}\left(\frac{4}{3}\,\eta + \xi\right) \quad . \tag{6.2}$$

In the absence of dissipative effects $\tau\sigma$ is a constant of motion, we can replace $\tau_f\sigma(\tau_f)$ by $\tau_0\sigma(\tau_0) \propto \varepsilon_0^{c_0^2/(1+c_0^2)}$, and obtain via (5.1) the relation[6] $\varepsilon_0 \propto (dN/dy)^{1+c_0^2}$. However, for nonvanishing η, ξ, we must solve (6.2). For the plasma phase we use (5.1) and $\eta = bT^3$, $b \gtrsim 2$ and $\xi = 0$. In that case (6.2) is easily solved[11,13] giving

$$T(\tau) = (T_0 + \delta T_0)\left(\frac{\tau_0}{\tau}\right)^{1/3} - \delta T_0\left(\frac{\tau_0}{\tau}\right) \quad , \tag{6.3}$$

with

$$\delta T_0 = \frac{1}{2K\tau_0}\left(\frac{\eta}{T^3}\right) \quad , \tag{6.4}$$

with $K = (\varepsilon_Q - B)T_c^4 \approx 12$. If we ignore possible entropy production in the transition region, then the rapidity density becomes for $\tau_f >> \tau_0$

$$\frac{1}{A_\perp}\frac{dN}{dy} = \frac{1}{4}\,\tau_f\sigma(\tau_f) \approx \frac{1}{4}\,\tau_0\sigma(\tau_0)\left(1 + \frac{\delta T_0}{T_0}\right)^3 \tag{6.5}$$

Therefore, dissipative effects enhance the rapidity density by a factor $(1 + \delta T_0/T_0)^3$. If η is close to the minimum value $2T^3$, then that enhancement factor is 1.3. For $\eta = 4T^3$ it becomes 1.6. In the extreme case, when dissipative effects are as large as they could get, $\varepsilon\tau$ is approximately constant and $\sigma\tau \approx \sigma_0\tau_0(\tau/\tau_0)^{1/4}$ increases with τ. In that limit the rest energy per unit rapidity is approximately constant and we recover Bjorken's estimate[5] $\epsilon_0 \propto dN/dy$.

In Fig. 1 we show the evolution of the energy density, entropy, and entropy production rates for the case $\epsilon = 12.2\ T^4$, $\sigma = 16.3\ T^3$. Curve 1 corresponds to ideal non-viscous expansion. Curves 2, 3, 4 have $\eta/T^3 = 2, 6, 14$. Observe that for extreme η the energy density could even rise initially. As noted before for such large η Navier-Stokes theory does not apply. Nevertheless curves 3 and 4 illustrate an interesting point. There is a very large energy reservoir stored in the form of kinetic energy of the nuclear fragments. In the central region considered here only $e^{-y_0} \sim 10^{-2}$ of the total energy available remains after the two nuclei pass through one another. It is that 1% residue of the collision whose subsequent final state expansion we are considering here. If the dynamics is simply parton-parton scattering and radiation, then that reservoir of energy cannot be tapped, and $\epsilon(\tau)$ must be a monotonically decreasing function. However, it is not ruled out that the confinement mechanism produces color fields or strings that connect the partons in the central region to the high rapidity partons. In terms of kinetic theory such effects would have to be included in Vlasov terms. Those color fields or strings could accelerate the quarks and gluons and even lead to an increasing internal energy $\epsilon(\tau)$ as shown mimicked by curve 4. However, this type of behavior would be quite exotic. Our expectation is that the energy density will decrease with in the region bounded by curve 1 corresponding to isentropic expansion and the dotted line corresponding to isoergic expansion. From Fig. 1 we also see that most of entropy produced is in the first few fm/c. The reason again is that velocity gradients become small at later times. The asymptotic value of $\tau\sigma$, (6.5), is thus approached rather quickly.

Next we solve (6.2) for an assumed first order transition within the Bag model (3.14, 5.1). To cover a broad range of possibilities we employ the two sets of parameters[6] discussed above (3.15). In Fig. 2, part (a) is for a strong first order transition at T_c = 200 MeV, and part (b) is for a weak transition at T_c = 140 MeV. In both parts the curve $\eta = 0$ corresponds to isentropic expanson as computed in Ref. (6). The dashed curve in each is appropriate for the isoergic $(d(\epsilon\tau)/d\tau = 0)$ expansion considered in Refs. (5,6). The curve η_{min} was calculated using $\eta = 2T^3$ for $\epsilon > \epsilon_Q$ and (3.16) for $\epsilon < \epsilon_H$. For $\epsilon_H < \epsilon < \epsilon_Q$ we linearly interpolate, $\eta = \lambda\eta_Q + (1 - \lambda)\eta_H$, with λ given by (5.2). For these curves $\xi = 0$. The curves labeled ξ still include not only η_{min} but also bulk viscosity (5.10, 5.14) in the transition region $\epsilon_H < \epsilon < \epsilon_Q$.

For both sets of parameters, the inclusion of minimal shear viscosity lowers the initial energy density by ~1 GeV/fm^3. For large shear viscosity $\eta = 3\ \eta_{min}$ the curves (not shown) fall below the isoergic line. For the range of energy densities and rapidity densities considered, the shear viscosity must therefore be less than 3 η_{min} in order for Navier-Stokes to apply. While in the hadronic phase this condition appears to be satisfied, we recall from section 4 that there is considerable uncertainty on the value of η in the plasma phase. The inclusion of bulk viscosity associated with the transition obviously has greater effect in part (a) for the strong transition. If the latent heat is several Gev/fm^3, then a large amount of time is spent in the transition region, and considerable entropy is produced. In the extreme case (a) inclusion of ξ reduced ε_0 by another GeV/fm^3 for fixed dN/dy. In case (b) ξ had negligible effect because of the smallness of the latent heat.

We conclude from this study that finite mean free paths and relaxation times are likely to lead to a dynamical path intermediate between the idealized isentropic[7-9] and isoergic[5] ones. In terms of entropy production at least 20% and up to 100% enhancement of the entropy were found. While we were not able to rule out anomalously large dissipative effects in the plasma near the transition temperature, the qualitative arguments pointed to values of η satisfying (3.12). Antiscreening was identified as potentially important effect in keeping η down.

Our results are in accord with previous estimates[5-7] with regard to the range initial energy densities that can be expected in ultra-relativistic nuclear collisions. With rapidity densities as already observed in several cosmic ray events,[28] $\varepsilon_0 \gtrsim$ few GeV/fm^3 can be expected for energies $E_{lab} > 1$ TeV/A. The most important consequences of dissipative effects are likely to be on the signatures[1,29] of the plasma phase.

The energy density and temperature generally decrease slower with the inclusion of dissipative effects. This would lead to greater yields of direct probes such as photons and dileptons, which are sensitive to the thermal history of the reaction. On the other hand, larger transverse momentum associated with hydrodynamic expansion would be reduced as collective flow velocities are dissipated into heat. In general dissipative effects would also dampen fluctuations that could otherwise serve as signatures of unusual phenomena.[27] Also any rapid variations of quantities such as the K or $\overline{\Lambda}$ multiplicity with increasing A number marking the transition from non-equilibrium to equilibrium dynamics would be smeared out by dissipative effects. Since most proposed[1,29] observables of the plasma phase are sensitive to the full space-time history of the reaction, dissipative phenomena must be taken into account if quantitatitive predictions are to be made. To that end, QCD lattice studies of $T^{\mu\nu}$ correlation functions and a better understanding of the reaction mechanism in ultra relativistic nuclear collisions are needed. The main theoretical challenge will be to understand how the rapidly expanding plasma converts into hadrons in the final state. We must keep in mind that the problem of confinement in a dynamical environment may lead to completely unexpected transport phenomena.

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Figure Captions

1. Evolution of the quark-gluon plasma in the proper time τ: a) energy density ε, b) transverse entropy-density $\sigma\tau$, c) entropy production. Curve 1 corresponds to the ideal non-viscous dynamics and curves 2, 3, 4 correspond to $\eta/T^3 = 2, 6, 14$ respectively. The dotted line corresponds to isoergic expansion.[5,6]
2. Initial energy density ε_0 at onset of the hydrodynamic expansion $\tau \sim 1$ fm/c versus pion rapidity density reduced by $A^{2/3}$ for central A + A collisions. Parts (a) and (b) correspond to two models of the equation of state (see above (3.15)). The isentropic and isoergic curves from Ref. (6) are included. Curves labelled η_{min} include minimal viscous effects as dictated by finite interparticle spacing and the uncertainty principle. Curves labelled ξ incorporate bulk viscosity (5.10) as well as η_{min}. For reference the average reduced density in the Si + Ag JACEE event[28] is indicated.

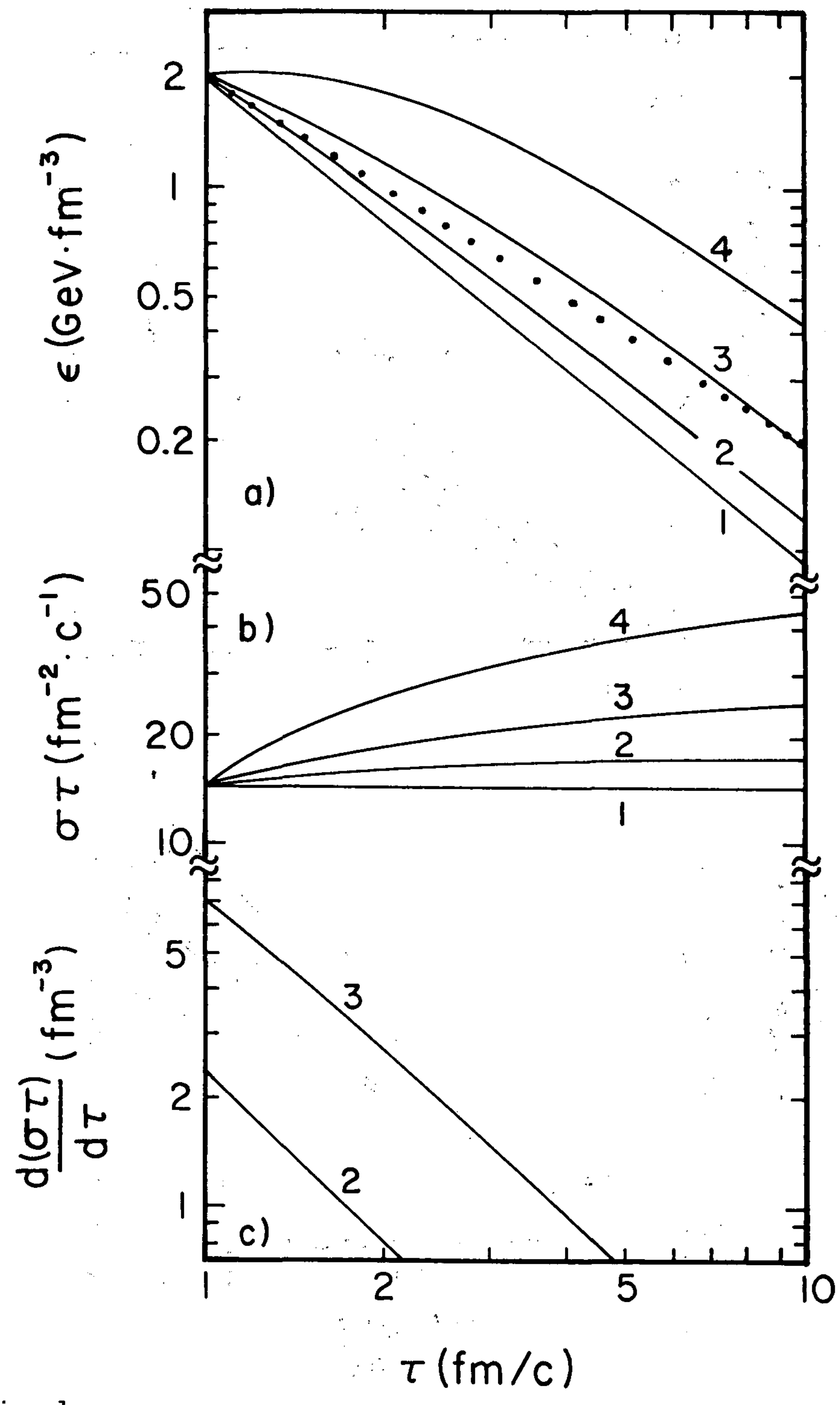

Fig. 1

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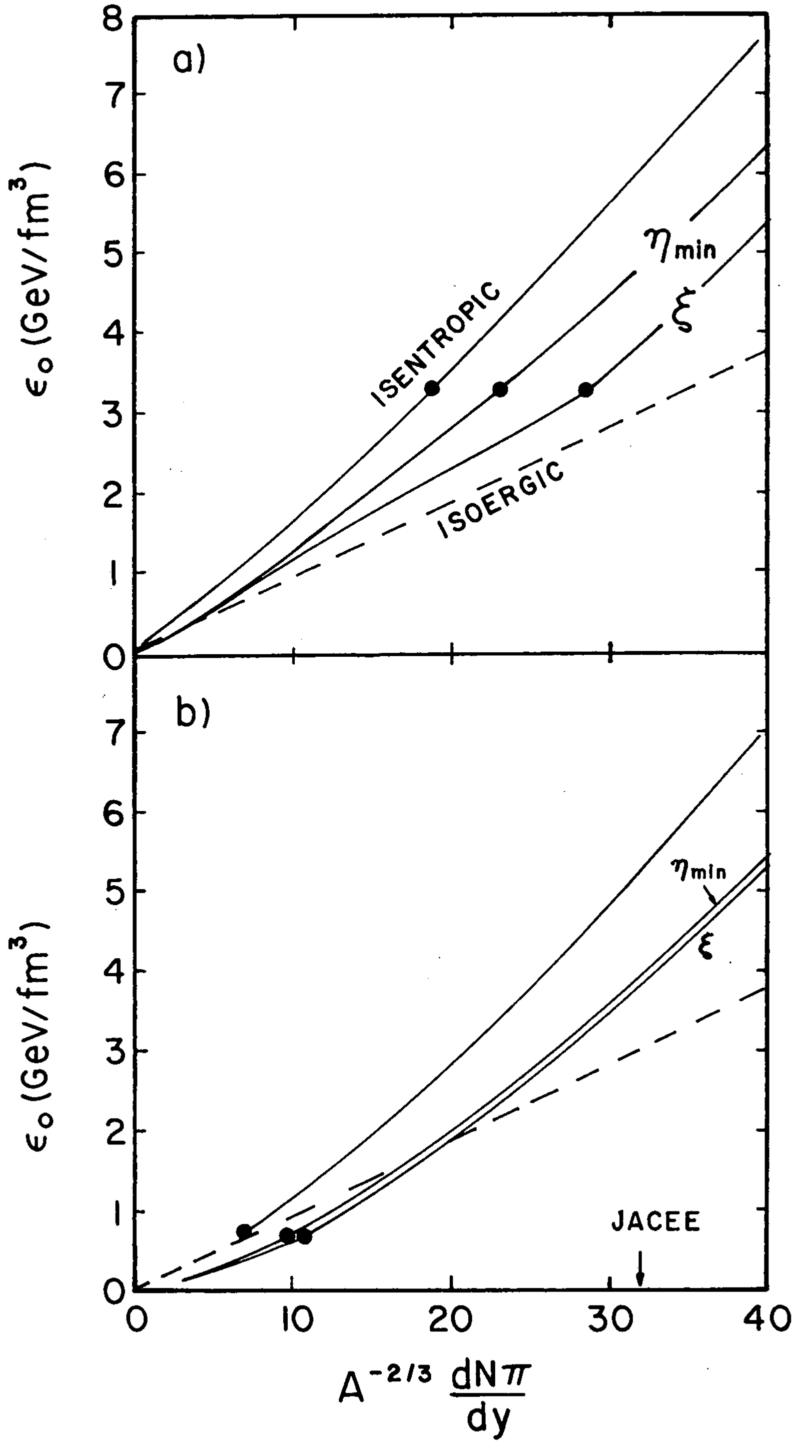

Fig. 2

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