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Dissipative solitons for mode-locked lasers

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Dissipative solitons are localized formations of an electromagnetic field that are balanced through an energy exchange with the environment in presence of nonlinearity, dispersion and/or diffraction. Their growing use in the area of passively mode-locked lasers is remarkable: the concept of a dissipative soliton provides an excellent framework for understanding complex pulse dynamics and stimulates innovative cavity designs. Reciprocally, the field of mode-locked lasers serves as an ideal playground for testing the concept of dissipative solitons and revealing their unusual dynamics. This Review provides basic definitions of dissipative solitons, summarizes their implications for the design of high-energy mode-locked fibre laser cavities, highlights striking emerging dynamics such as dissipative soliton molecules, pulsations, explosions and rain, and finally provides an outlook for dissipative light bullets.

Introduced in 1965 by Zabusky and Kruskal, the term 'soliton' was originally used to refer to localized solutions of integrable nonlinear systems. Such solutions are remarkable because they maintain their shape and velocity after colliding with each other, and remain intact when interacting with radiation waves. In optics, integrable conventional solitons result from the single balance between nonlinearity and dispersion/diffraction. Over the years, scientists have gradually adopted the term to cover localized solutions in nonlinear science, despite some resistance from the adepts of strict mathematical definitions[1]. A dramatic turning point occurred at the beginning of the 1990s, when physicists realized that solitary waves did exist in a wide range of non-integrable and non-conservative systems. New terminology needed to be introduced, and thus solitary waves in nonlinear optical systems with nonlinear gain or loss mechanisms were thereafter referred to as dissipative solitons[2,3]. The term 'autosoliton', developed for describing reaction–diffusion systems in the field of plasma physics[4], was also used early on in an optical context[5]. Reaction–diffusion systems involve the supply and decay of matter[6], whereas in optics we consider the balance of energy. Dissipative optical solitons in a nonlinear medium are confined wave packets of light whose existence and stability depend crucially on the energy balance. In 2005, the major principles of dissipative solitons were unified and presented in a single volume[7]. Although the initial contributions of dissipative solitions mostly related to phenomena observed or predicted in nonlinear optics, the new concept seemed sufficiently general to be applied across other disciplines, such as biology[8].

The concept of dissipative solitons, although principally a fundamental extension of conventional soliton theory, incorporates powerful notions developed in the theory of nonlinear dynamics, such as attractors and bifurcations. The concept also stems from the seminal ideas of Prigogine on open systems far from equilibrium[9]; 'dissipative systems', through the perpetual and bidirectional exchange of energy with their environment, can manifest conditions for self-organization and become a fascinating area of research. Solitons, nonlinear dynamics and dissipative systems therefore essentially constitute the three sources and component parts of the dissipative soliton concept[8]. In practice, this extension of the conventional soliton concept implies that the single balance between nonlinearity and dispersion is replaced by a composite balance between several effects, as shown in Fig. 1a. An important point here is that the balance between gain and loss, which should be exact in order to produce stationary localized solutions, plays a dominant role in the dynamics. As a consequence, and compared with the case of conventional (conservative) solitons, the impact of dissipative effects reveals unusual situations in which solitons can be found, such as bright dissipative solitons in the normal dispersion regime[10], or stable multisoliton complexes, which are also known as soliton molecules[11–13].

The stability of a dissipative soliton is directly linked to the stability of its associated fixed point, in the terminology of nonlinear dynamics. A strong attractor naturally provides a high degree of stability. In contrast with the conservative case, in which a given set of parameters for the governing propagation equation generally leads to an infinite number of soliton solutions, the attractor in a dissipative system is a fixed localized solution. Specifically, the energy, profile and chirp of a dissipative soliton are predetermined by the equation parameters, rather than by the initial conditions. These features of dissipative solitons offer highly desirable properties for applications, such as the generation of stable trains of laser pulses by mode-locked cavities, or the in-line regeneration of optical data streams.

Although the profile of a dissipative soliton is indeed fixed for a given set of equation parameters, a wide range of profiles can be obtained by tuning these parameters. This way, one can witness various transformations of dissipative solitons, either smooth or in the form of bifurcations. One well-known type of bifurcation transforms a fixed point into a limit cycle, which is an oscillatory attracting state. Thus, a previously stationary soliton becomes a pulsating localized formation[14]. Further transformations in the parameter space may produce irregular and chaotic soliton dynamics. Because the optical field is a function of continuous spatial variables, the related dynamical system possesses infinite degrees of freedom. Hence, an extended exploration in the parameter space is propitious to reveal large numbers of dissipative soliton varieties and bifurcations. This constitutes a fertile ground for investigations, as is currently the case for the exploration of mode-locked laser dynamics. The motivation for the latter is twofold. The field of mode-locked lasers can be considered as an ideal playground for exploring nonlinear dissipative dynamics. Varieties of accessible laser configurations, generated pulse shapes and their dynamics have expanded very quickly over the past few years, particularly within fibre laser

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technology. Another motivation is at the applied level, for which the implementation of dissipative soliton concepts would be highly useful in the development of compact, efficient and reliable sources of short and ultrashort light pulses with unprecedented pulse energy or repetition rate and improved output stability.

Before developing these ideas further, it is useful to highlight one significant difference between conventional (conservative) solitons and dissipative solitons. In order to be stationary, solitons in dissipative systems must have distinct internal regions that extract energy from an external source, as well as other internal regions through which energy is lost to the environment. A stationary dissipative soliton therefore results from the continuous exchange of energy with the environment along with the live redistribution of energy between various parts of the soliton. This internal energy flow corresponds to a non-uniform phase across the dissipative soliton profile, which is in marked contrast with the constant nonlinear phase profile of a stationary conservative soliton. At any given part of the pulse, the energy flow is proportional to the first temporal derivative of the phase, whereas the density of energy generation is proportional to the second temporal derivative of the phase. Interestingly, pulsating dissipative solitons that undergo a periodic evolution along their direction of propagation generalize this idea further to the spatial domain; locations where the soliton mostly extracts energy are followed by locations that are dominated by losses, thus providing an average balance inside a period of evolution. This typical situation is met in all laser cavities.

Producing dissipative solitons

Here we consider passive mode-locking, which exploits nonlinearity to promote a saturable absorber effect and allows the generation of ultrashort optical pulses. Although the concept of dissipative solitons can be applied in the context of active mode-locking[15], this relationship has not yet been thoroughly investigated. A given laser system can be modelled to varying degrees of complexity. Because lasers are made of several components, an accurate model should involve consecutive sets of propagation equations. Models can be vectorial when the polarization nature of light is involved, and can also include the delayed response of the saturable absorber and gain medium. However, dealing with too many parameters and lengthy numerical calculations hampers the exploration and classification of original dynamics. Furthermore, all parameters are not always precisely known or are indeed even relevant. For these reasons, and in an attempt to investigate an underlying 'mother of all mode-locking theories', it is beneficial to start the exploration of dissipative soliton dynamics with a master equation that incorporates the main physical ingredients at play in mode-locked lasers, in a distributed way. This initiative is inspired by the work of H. Haus, who, from the mid-1970s, developed the master equation approach to mode-locking[16,17]. It turns out that the main master equation studied by Haus is similar to the cubic complex Ginzburg–Landau equation, a universal equation used to describe systems in the vicinity of bifurcations[18–20]. However, the inclusion of a quintic saturating term in the Ginzburg–Landau equation was shown to be essential for the stability of pulsed solutions[21–23]. From another point of view, the complex cubic–quintic Ginzburg–Landau equation (CGLE) is the extension of the nonlinear Schrödinger equation to higher-order and dissipative terms. The CGLE has been used to describe a wide range of nonlinear optical systems, such as passively mode-locked lasers with fast saturable absorbers, parametric oscillators, wide-aperture lasers and nonlinear optical transmission lines[1,8]. Its normalized form, in the (1 + 1)-dimensional case, reads:

$$i\psi_z + D\psi_{tt}/2 + |\psi|^2\psi + \nu|\psi|^4\psi = i\delta\psi + i\varepsilon|\psi|^2\psi + i\beta\psi_{tt} + i\mu|\psi|^4\psi$$

where the optical field envelope ψ is a function of t, the retarded time in the frame moving with the pulse, and z, the propagation distance. ψ_z is the first-order z-derivative and ψ_{tt} is the second-order t-derivative. The left-hand side of the equation contains the conservative terms, where D is the anomalous (normal) dispersion propagation regime if positive (negative), and ν applies to a quintic Kerr effect. The right-hand side of the equation includes all dissipative terms: δ, ε, β and μ are the coefficients for linear loss (if negative), nonlinear gain (if positive), spectral filtering and saturation of the nonlinear gain (if negative), respectively. These are the basic physical effects required to build a mode-locked laser, with the expected signs of their contributions as suggested above. In particular, linear loss and nonlinear gain account for the required saturable absorber effect. Using an averaging method such as multiscale analysis, it is possible to relate, albeit in an approximate way, all the coefficients of the CGLE to the physical parameters of the laser cavity[24–26]. This procedure reveals surprising features: for instance, although the quintic saturation of the Kerr effect is not reached in most laser materials under practical operating conditions, an effective quintic Kerr term arises because the lumped laser propagation model is reduced to a distributed model[25]. Numerical comparisons show that the generally good qualitative picture provided by the distributed CGLE model becomes quantitative when the pulse alterations inside each cavity roundtrip are not too large[26]. Naturally, gain saturation delayed dynamics is outside the scope of the CGLE model, which assumes an instantaneous response[27,28].

The CGLE is non-integrable, which means that general analytical solutions are not available. Selected analytical solutions can only be found for specific relations between the equation parameters. Approximate techniques such as the Lagrangian approach or the method of moments can be used to provide simple localized solutions[29–31]. By projecting the initial evolution problem into a low-dimensional subspace of collective variables such as pulse width, amplitude and chirp, one can obtain approximate relations between the soliton solution and the equation parameters. Although the search for a unified picture of mode-locking regimes presents a grand challenge, direct comparisons with numerical simulations remain essential for ensuring the validity of approximate methods.

Generally, the variety of stable soliton profiles is significantly wider than we can imagine and approximate, as illustrated in Fig. 1b. Here, varying the nonlinear gain parameter ε transforms the nearly hyperbolic–secant pulse shape into a flat-top profile, which can be thought of as two mirror-imaged interacting fronts. At certain values of the CGLE parameters, the combination of two fronts and a bell-shaped pulse, known as a composite pulse, also becomes stable (Fig. 1c). It follows from the phase profile that energy flows from the centre of the structure to the wings of the composite pulse. For lasers, this implies that varying the amount of nonlinear dissipative effects strongly influences the pulse shape, even for given amounts of dispersion and self-phase modulation.

The large and multiple regions of parameter domains of the CGLE in which stable dissipative solitons can be found is a valuable gift for laser physicists, as it demonstrates substantial room for exploring innovative cavity designs. For instance, the fact that highly stable bright dissipative solitons have been found within the normal dispersion regime[32,33] means that the goal of achieving high-energy stable pulses can be gradually released from the constraint of dispersion compensation. In fact, playing with dispersion compensation — also known as dispersion management — gradually shifted the interest of laser physicists to the net normal dispersion regime. This was stimulated by the dispersion-managed fibre laser configurations tested by Haus and co-workers in the mid-1990s[34]. Whereas the mode-locked fibre laser configurations tested in the early 1990s favoured conventional 'Schrödinger soliton' pulse shaping by using predominant anomalous fibre segments, the stretched-pulse mode-locked fibre laser combines fibres of opposite dispersion signs, thereby introducing huge intracavity temporal breathing dynamics as soon as there is approximate compensation between anomalous

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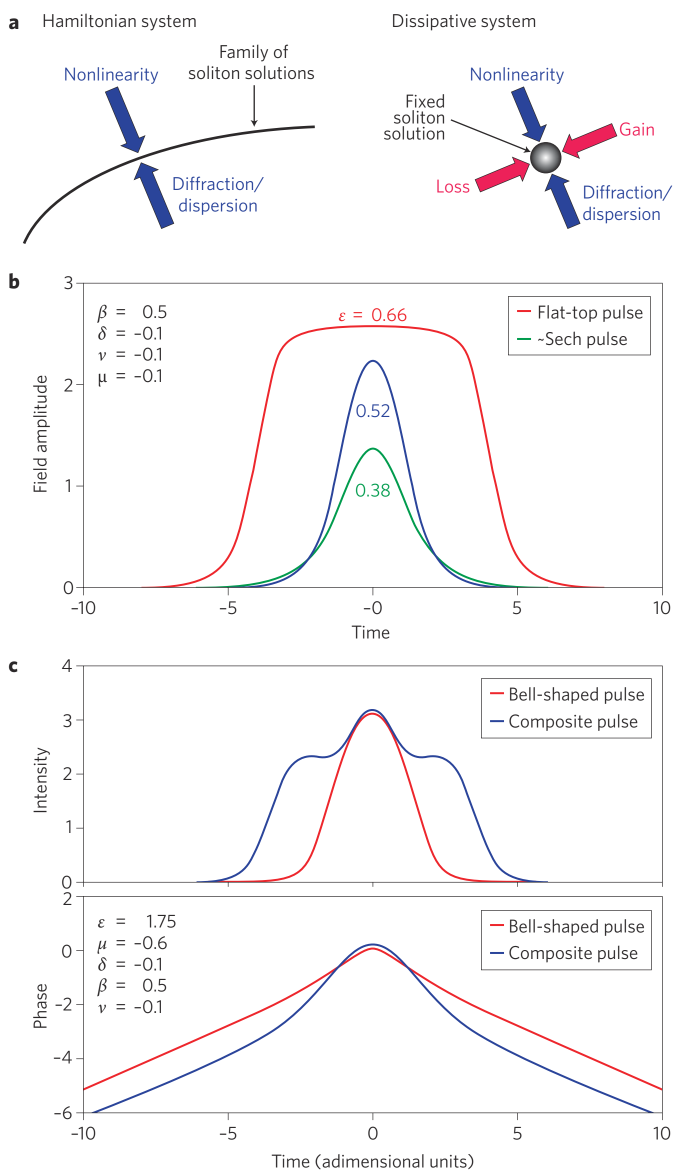

Figure 1 | The concept of dissipative solitons. a, Qualitative differences between the solitons in Hamiltonian and dissipative systems, after setting the equation parameters. **b**,**c**, Examples of dissipative solitons that are stable attractors of the CGLE master equation. They can take wide range of profiles, including roughly hyperbolic–secant, flat-top (**b**) and composite (**c**). The system parameters are shown inside each figure part. Figure **b**,**c** reproduced with permission from ref. 1, © 1997 Chapman & Hall.

and normal dispersions. Such circulation of highly stretched pulses with reduced amplitude allows researchers to reach pulse energies approaching the nanojoule range — an order of magnitude higher than in all-anomalous-dispersion lasers.

To understand the challenge of producing high-energy ultrashort pulses from a laser oscillator, we remind the reader that the combined action of the Kerr nonlinearity and chromatic dispersion generally leads to pulse break-up after the accumulated nonlinear phase has exceeded a certain level. Overdriving the ultrafast saturable absorber effect, as performed in popular nonlinear-polarization-evolution mode-locking, is also a directly related issue. These effects can be circumvented by employing significant temporal stretching of the pulse inside the cavity, thus reducing the peak power while maintaining a large spectral content. This strategy essentially incorporates chirped-pulse amplification inside the laser oscillator[35–37].

At the beginning of the 2000s, laser physicists found alternative cavity designs for achieving stable stretched-pulse operation with higher pulse energies. In 2004, the group led by F. Wise successfully implemented the self-similar propagation of chirped parabolic pulses in the passive part of the laser cavity — a normally dispersive fibre. This led to an increased pulse-energy performance of nearly 10 nJ per pulse[38]. The implementation of self-similarity inside a cavity relies on a delicate balance: normal dispersion and nonlinearity, which act together in a passive fibre, must be balanced by spectral filtering, nonlinear gain and anomalous dispersion in the subsequent portion of the laser cavity, thus performing a precise self-consistent pulse reshaping process prior to re-injection into the passive fibre. The use of self-similarity is currently being revisited in the light of the dissipative soliton concept[39,40]. Indeed, the acceptance of unusually large intracavity gain and losses, through spectral filtering in particular, seems to be the key for entering specific pulse regimes such as self-similarity. Self-similar pulses, also known as similaritons, are forever-expanding pulses that do not match the notion of a soliton. However, they possess a useful attracting asymptotic state that can be exploited — albeit partially, as the propagation distance is limited — in the design of a mode-locked laser cavity to provide improved stability, as was recently demonstrated with the report of a soliton–similariton fibre laser[39]. This laser features active self-similarity (obtained in the gain fibre) followed by conventional soliton reshaping in the passive anomalous fibre. Interestingly, such laser operation brings about the 'management of dynamics' within a single cavity roundtrip, thus further confirming that the balance between strong physical effects can be achieved at the scale of the entire cavity, and providing original dissipative solitons dynamics that are difficult to access in distributed models. The transition from similariton to soliton dynamics is performed by a spectral filter, which truncates the pulse exiting the gain fibre in both the spectral and temporal domains. Spectrally filtering a highly chirped pulse has a significant clipping effect on the temporal pulse wings; this clipping effect was also used a few years ago to allow mode-locked operation in all-normal-dispersion fibre lasers[41–43].

In all-normal-dispersion fibre lasers, the pulse duration does not oscillate strongly; namely, highly chirped pulses do not 'breathe'. After the pulsed operation has been promoted by the saturable absorber effect, the cavity dynamics become dominated by the composite balance between spectral filtering on one side, and nonlinearity, dispersion and gain on the other side, thus tending to produce M-shaped optical spectra. An example of such a laser cavity and its output features is illustrated in Fig. 2a,b. Ytterbium-doped mode-locked fibre lasers employing such a design can produce pulses with energies above 20 nJ using standard fibre technology[41–43]. Many researchers refer to these devices as 'dissipative-soliton lasers'[43–45]. Such a name is justified in the sense that strongly dissipative effects per roundtrip are at the heart of pulse formation and stability. However, this term could also cause confusion because it suggests that dissipative solitons are found exclusively in all-normal-dispersion lasers. As we shall see in the following sections, dissipative solitons are also found in lasers with anomalous dispersion, revealing dynamics that are unusual to Schrödinger solitons.

There are currently several trends in the development of high-pulse-energy mode-locked fibre laser oscillators. Perhaps the most efficient of these relies on the use of advanced technology, in which microjoule-level pulse energies have been achieved by developing large-mode-area gain fibres through microstructuring[46]. Another trend is to shift mode-locking operation further into the normal dispersion regime, thereby increasing the pulse chirping effect[35,47]. In order to remain compatible with today's available pumping powers and heat-management systems, such strategies are usually supplemented with an increased cavity length to reduce the pulse repetition rate.

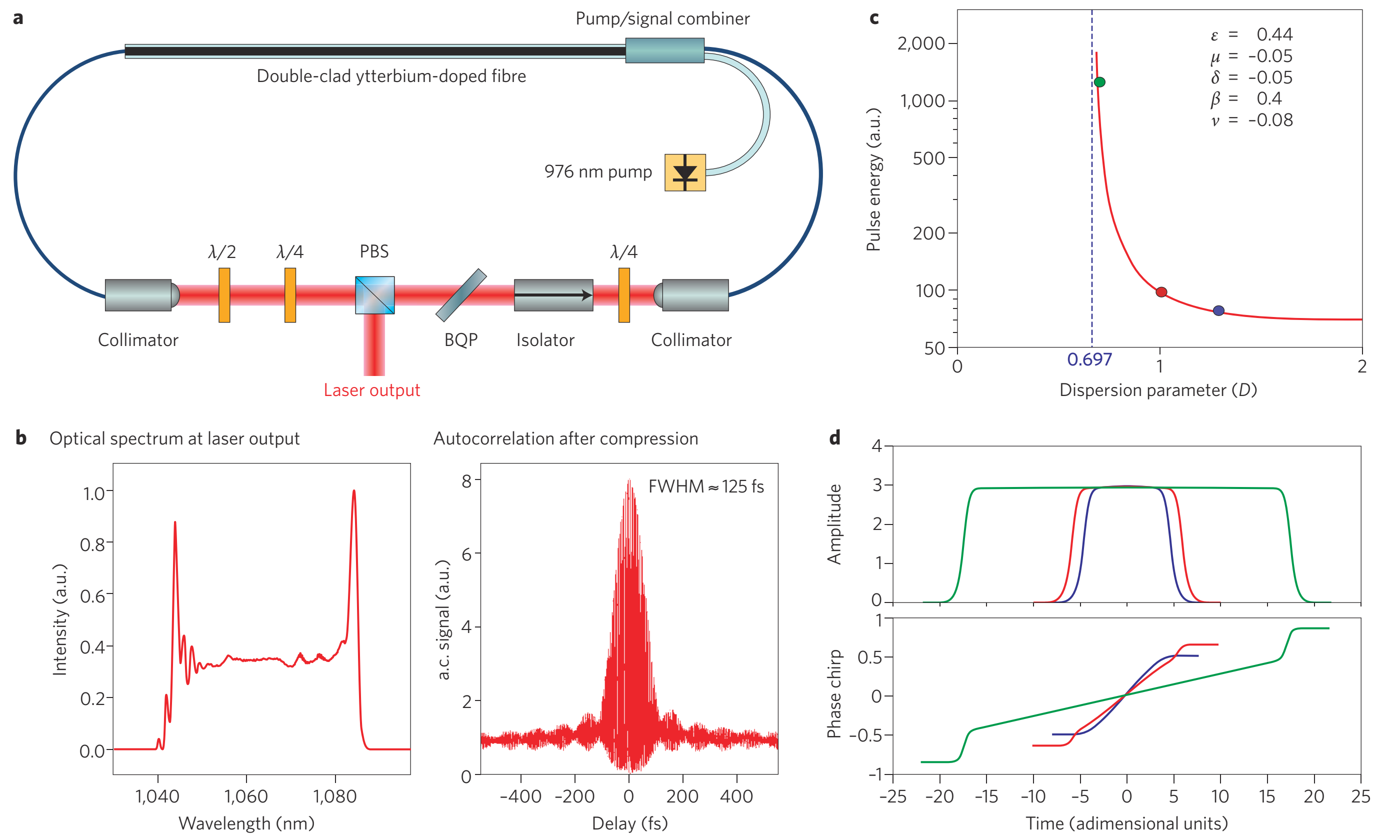

Figure 2 | High-energy dissipative solitons. a, Experimental set-up of an all-normal, dispersion-compensation-free fibre laser cavity operating at a 70 MHz repetition rate and generating 31 nJ pulses. PBS, polarizing beamsplitter; BQP, birefringent quartz plate; $\lambda/4$, quarter waveplate, $\lambda/2$, half waveplate. **b**, Characterization of the output pulses. Autocorrelation is performed after dechirping outside the laser cavity, achieving 80 fs pulses with 200 kW of peak power. FWHM, full width at half-maximum. **c**, Illustration of the DSR phenomenon. The CGLE model (with parameters shown inside the plot) predicts that DSR occurs when the dispersion parameter approaches 0.697. **d**, Amplitude and phase profiles of the stable pulse solutions for the values of *D* indicated in **c** by dots of corresponding colours. Figure reproduced with permission from: **b**, ref. 43, © 2009 OSA; **d**, ref. 49, © 2010 OSA.

Recently, an alternative conceptual approach known as dissipative soliton resonance (DSR) was proposed for increasing the pulse energy[48–50]. Found initially in the frame of the CGLE master equation, DSR describes dissipative solitons that acquire theoretically infinite energy when the equation parameters approach a specific hyper-surface in the parameter space. In practice, this implies that mode-locked laser set-ups could be designed in which the pulse energy would increase linearly with the pumping power without suffering pulse break-up, within a narrow range of laser parameters. An illustration of the DSR phenomenon is presented in Fig. 2c, where the pulse energy increases by orders of magnitude in response to tiny changes in the dispersion parameter. Pulses close to DSR share common features, such as a clamped peak power and a finite bandwidth. Thus, the boost in energy causes the duration of the chirped pulse to increase (Fig. 2d). A similar approach utilizes a trial function to reduce the CGLE dynamics to a master diagram, which allows a scalable pulse energy to be found[51,52]. A numerical analysis of the DSR phenomenon in a realistic fibre laser set-up, taking into account gain saturation, provides guidelines for testing the concept experimentally[50], although in such cases high-order physical effects might flatten the resonance effect. Experimental indications for the existence of the DSR phenomenon have been reported[53,54].

Dissipative soliton molecules

Soliton molecules in dissipative systems and higher-order soliton solutions of the nonlinear Schrödinger equation are different formations[11]. Higher-order nonlinear Schrödinger equation solitons are nonlinear superpositions of fundamental solitons, which oscillate periodically in both the temporal and spectral domains. They are easily destroyed by external perturbations or additional effects such as third-order dispersion or Raman scattering. The *N*th-order soliton requires a total energy proportional to N^2. In contrast, dissipative soliton molecules are robust multisoliton structures that can be perfectly stationary and whose energy remains proportional to the number of single-soliton constituents. In analogy with the physics of matter, soliton molecules can be formed by the interaction of initially separate single solitons. Because the energy of a soliton molecule is slightly different from the sum of the energies of its soliton constituents, forming a soliton molecule requires dissipative processes and implies the existence of a binding energy. Self-phase-locking of these pulses can produce soliton molecules because the interaction of optical pulses through their tails is phase-sensitive[55,56]. Stable soliton pairs, which are characterized by a quadrature-phase difference, were predicted in 1997 using the CGLE model[57] and demonstrated in a fibre laser experiment a few years later[58]. These results were obtained in the case of anomalous dispersion. Stable soliton pairs have also been found in diverse laser experimental configurations, dispersion regimes and emission wavelengths, and have been confirmed in a series of numerical simulations using various models[59–63]. Soliton pairs therefore represent the central feature of the collective dynamics of dissipative solitons[7,11]. They also establish a frame for understanding some of the early experimental observations of multiple pulsing, particularly in the case of pulse bunching[64–67]. The potential to form a stationary soliton pair is linked to the existence of a point attractor in the dynamical system (Fig. 3a).

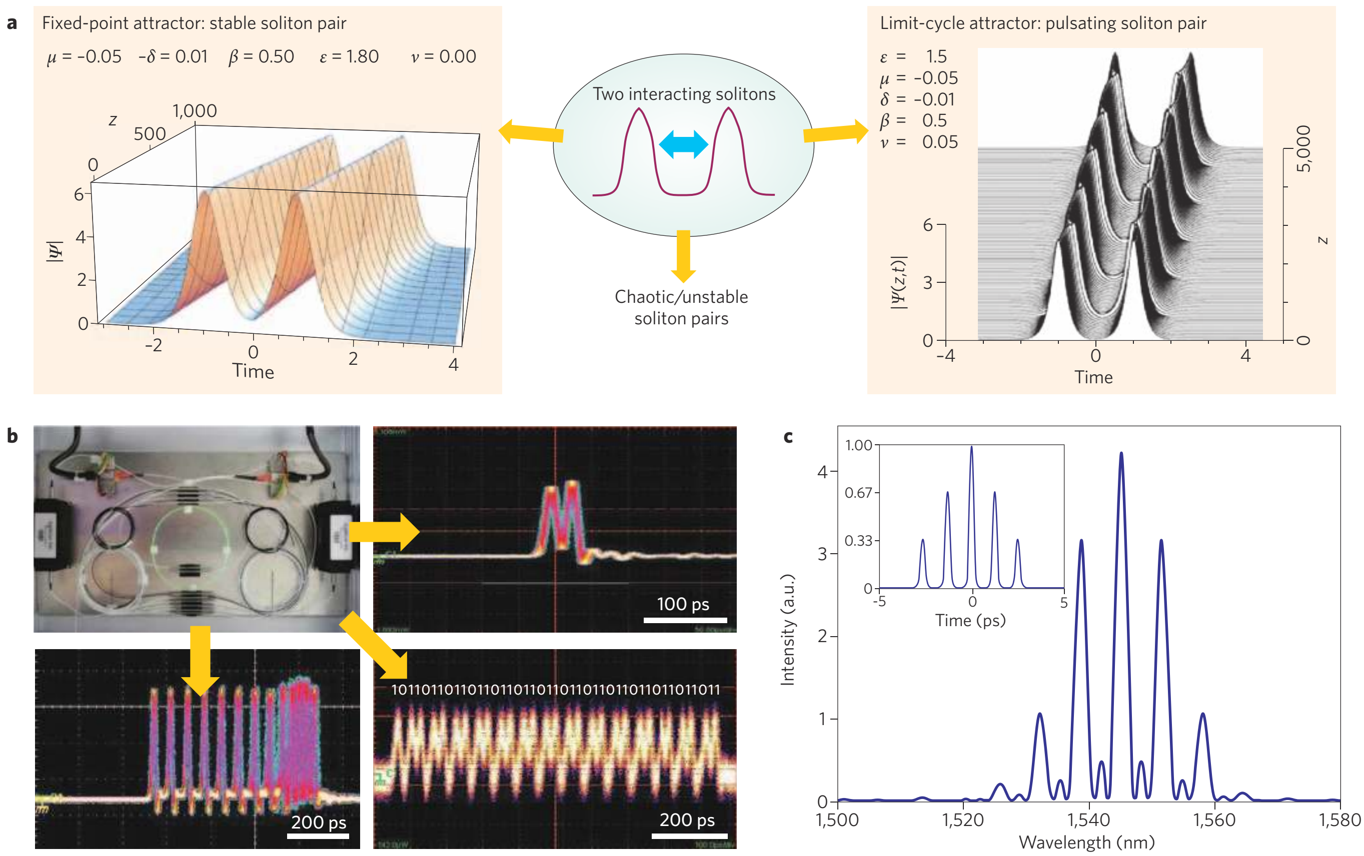

Figure 3 | Dissipative soliton molecules. a, Direct interaction between two dissipative solitons may lead to various types of soliton pairs in the CGLE model, depending on the values of the equation parameters. **b**, Direct recording of stable self-assembled soliton molecules in a fibre laser experiment (top left): a soliton pair (top right), an irregularly spaced 13-soliton molecule (bottom left) and a $1(011)_{N=14}$ macromolecule (bottom right). The experiment employed a 30 GHz sampling oscilloscope. **c**, The optical spectrum and intensity autocorrelation (inset) of a stable soliton triplet. Stable spectral fringes are a direct consequence of self-phase locking of the three pulses with +π/2 and −π/2 consecutive phase differences, at a relative separation of 1.2 ps. Figure reproduced with permission from: **a**, ref. 11, © 2008 Springer; **c**, ref. 76, © 2005 Elsevier.

The properties of the attractor vary with the model and its parameters. For instance, bound pulses tend to become out-of-phase in the normal dispersion regime[59,61,63,68], whereas cavity discreetness in the anomalous regime implies the existence of a large chain of attractors and the subsequent quantization of allowed temporal separations[69]. Experimentally, the direct observation of soliton molecules on an oscilloscope trace, as shown on Fig. 3b, challenges the bandwidth of fast electronics when the temporal separation falls below 20 ps. In addition, full characterization requires proof of self-phase locking. For these reasons, soliton molecules are better characterized through the analysis of spectral fringes and an optical autocorrelation, as illustrated in Fig. 3c. From studies in nonlinear dynamics, we know that Hopf-type bifurcations can occur when the system parameters are altered, thus transforming a fixed-point attractor into a limit cycle. In the case of infinite-dimensional systems, these bifurcations provide pulsating dissipative solitons[12,70,71] and vibrating soliton pairs[72,73]. Chaotic bound states are a further generalization of the notion of dissipative soliton pairs[74].

Another way to introduce complexity is by increasing the number of interacting solitons. 'Isomers' can be considered when dealing with three pulses[75], although in this case the complexity of the dynamics becomes a significant challenge. Periodic pulse collisions resulting from the mutual exchange between a soliton pair and a third interacting soliton have been reported experimentally and analysed theoretically. These phase-sensitive interactions provide the ability to control the synthesis and dissociation of soliton molecules[76,77]. In the anomalous dispersion regime, macromolecules comprising several hundred bound dissipative solitons can be formed. Although various types of soliton patterns can also be self-assembled (Fig. 3b), regular and stable large soliton molecules are not generic. The soliton crystal is the most ordered self-arrangement observed so far[78]. There may be a fascinating conceptual bridge between soliton macromolecules and crystals, and modulational-instability laser dynamics reinterpreted within dissipative four-wave mixing[79].

Complex dynamics: pulsations, explosions and soliton rain

Robust pulsations require a stable limit-cycle attractor, which exists only in nonlinear dissipative systems. Because laser cavities all possess an internal structure, their overall dynamics converges to a single point attractor as soon as, for any given location within the cavity, the pulse parameters recover the same values after every roundtrip. When the pulse is observed through a given output port, the experimentalist witnesses, in a stroboscopic manner, the pulse macro-evolution at multiples of the roundtrip time. Pulsations can be divided into two categories: short-period pulsations, whose periodicity remains comparable to the roundtrip time, and long-period pulsations. Both have been predicted and observed in mode-locked lasers[70]. Pulsations are usually found, in the parameter space, close to the boundary between the domains of stationary solitons and unstable non-vanishing solutions. Aside from the archetypal cascade of period-doubling bifurcations leading to chaotic pulse dynamics, other types of bifurcation, such as period-tripling, for instance, can also be found. Combining short- and long-period pulsations is another possibility.

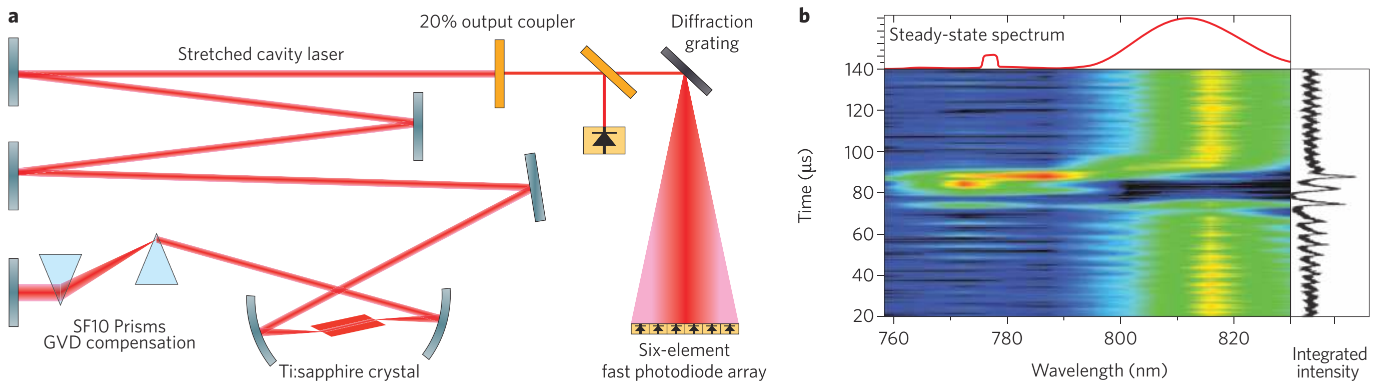

Figure 4 | Soliton explosions in a mode-locked Ti:sapphire laser. a, The experimental set-up. GVD, group-velocity dispersion. **b**, Fast spectral dynamics displaying a soliton explosion event. Figure **b** reproduced with permission from ref. 80, © 2002 APS.

Pulsating solitons can become chaotic at certain values of the cavity parameters; the macro-evolution of their profiles never repeats, although they remain localized. When such dynamics occurs for a wide range of localized initial conditions, the term 'strange soliton attractor' becomes appropriate. One of the most striking species of strange soliton attractors is that of soliton explosions[80,81] — soliton solutions that intermittently suffer explosive instabilities but return to their original shapes after 'bursting' in both the temporal and spectral domains. Such a soliton has extended intervals of almost stationary propagation, during which an instability develops and explosion occurs, followed by a recovery of the pseudo-stationary profile. All successive explosions have similar features, but, being chaotic, they are not identical. The dynamics of a strange attractor naturally attracts neighbouring trajectories in phase space. Figure 4, a scheme based on a Kerr-lens mode-locked Ti:sapphire laser, illustrates the concept of a soliton explosion. To record the explosions, which take place on the microsecond timescale, the laser output is dispersed by a diffraction grating across an array of six fast detectors, thus providing a real-time spectrogram. The spectral signature of one soliton explosion is prominent in the spectrogram of Fig. 4b. Researchers have used numerical simulations based on a parameter-managed CGLE model to corroborate this event[80].

Chaotic pulsations are another evolution of multisoliton complexes, although they may be even more difficult to detect than single-soliton explosions. One example of a chaotic pulsation is shaking soliton pairs, which have many similarities with soliton explosions. Chaotic pulsations also exhibit intermittent periods of instability followed by recovery[73].

Among the complex dynamics associated with large sets of dissipative solitons, 'soliton rain' is one particularly spectacular illustration of self-organization[82]. Soliton rain was discovered in an erbium-doped mode-locked fibre laser experiment, in which the formation of numerous intracavity pulses was favoured by operating in the anomalous dispersion regime at pumping powers of less than 1 W. The existence of a relatively intense noisy background, together with tens of soliton pulses aggregated in a condensed soliton phase, constitutes a necessary condition for the appearance of the effect. The temporal dynamics, recorded in real time (Fig. 5), display new soliton pulses that form spontaneously from background fluctuations and drift at a constant velocity until they reach the main peak component of the temporal intensity. This peak represents a compact ('condensed') soliton phase. Indeed, zooming in with a 30 GHz sampling oscilloscope reveals a substructure of bound jittering solitons. The term 'soliton rain' originates from a compelling analogy with the evaporative cycle of water. First, the condensed soliton phase emits large amounts of radiation that moves forward to shorter times. This radiation, which is superimposed on pre-existing continuous-wave modes, produces a noisy inhomogeneous background. Second, at random times and relative locations, fluctuations exceed a threshold over which a new dissipative soliton can be formed. Third, solitons randomly appearing ('raindrops') then drift back to the condensed phase. The whole scenario repeats endlessly as a quasi-stationary process that can also be viewed as an illustration of self-excitability. The dynamics of soliton rain address the largely unexplored fields of nonlinear dissipative dynamics in the case of large soliton numbers[83] and that of dissipative soliton interactions in the presence of a fluctuating background[84].

Spatiotemporal dynamics: towards dissipative light bullets

The generalization of stable dissipative solitons to higher space–time dimensionality can be readily achieved from both theoretical and numerical points of view[85–87]. Dissipative solitons remain stable in more than two dimensions, in contrast with Kerr solitons[88,89] whose instability is known to result in either expansion or collapse. One of the major challenges in nonlinear optics is the experimental demonstration of light bullets — beautiful objects of fundamental science that could be useful for applications such as parallel optical data processing[90,91]. A light bullet is a complete spatiotemporal soliton for which confinement in the three spatial dimensions, as well as localization in the temporal domain, are achieved by balancing the focusing nonlinearity and the spreading due to both chromatic dispersion and angular diffraction, while propagating in a homogeneous medium (Fig. 6a). The use of dissipative nonlinearities is widely considered to be the most promising strategy for the experimental implementation of stable light bullets[91–94], although other approaches are also possible[95–97]. In practice, the experiment could take the form of a wide-aperture laser cavity, whereby saturable absorption would provide both transverse localization and mode-locking simultaneously[86,92]. Semiconductor devices offer many advantages for such a prospect because semiconductors are based on highly nonlinear materials and most of their geometrical and physical properties can be engineered and therefore scaled for integration. One important step in the realization of light bullets has been the experimental observation and manipulation of two-dimensional spatial solitons in optically driven semiconductor microcavities. Dissipative solitons in driven nonlinear optical cavities have been referred to as 'cavity solitons'[93,98]. Recently, devices capable of generating cavity solitons without the need for permanent external driving fields — namely cavity soliton lasers — have been demonstrated[99,100]. One example is provided in Fig. 6b: the device consists of two vertical-cavity surface-emitting semiconductor lasers coupled in a self-imaging configuration. One of the lasers is below threshold to act as a saturable absorber. A transient writing beam enables the cavity solitons to be switched on and off

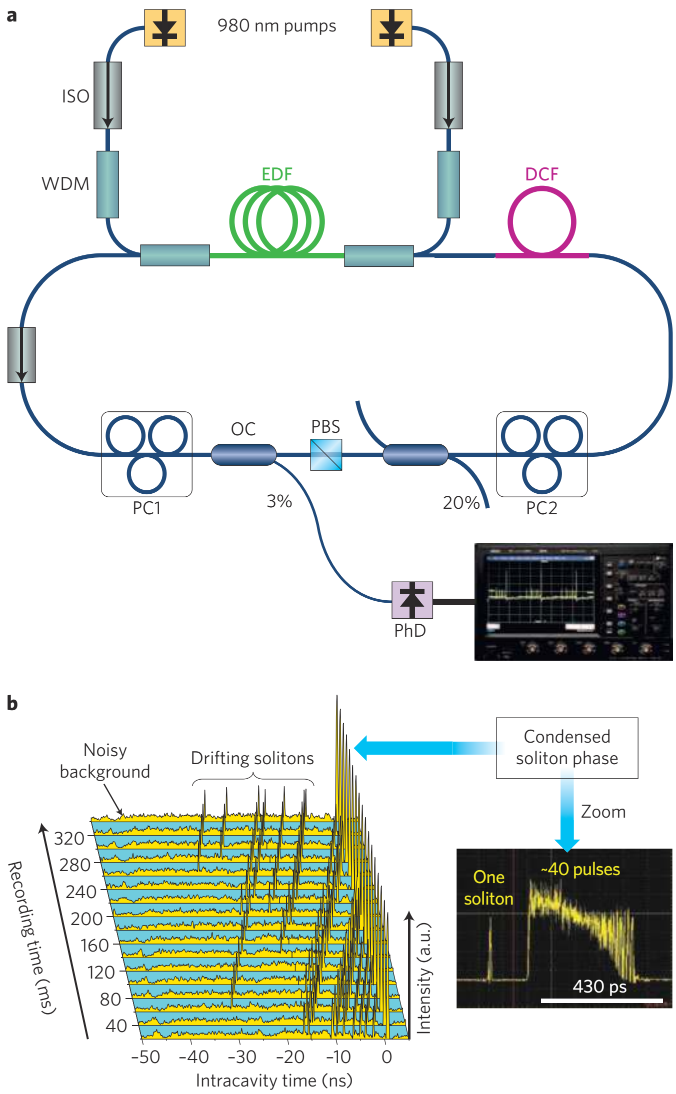

Figure 5 | Soliton rain in a mode-locked fibre laser. a, Sketch of an all-fibre erbium-doped laser set-up. EDF, erbium-doped fibre; WDM, 980/1,550 nm wavelength multiplexer; ISO, optical isolator; PC, polarization controller; DCF, dispersion-compensating fibre; OC, output coupler; PBS, integrated polarizing beamsplitter; PhD, 5.6 GHz photodiode. **b**, Stroboscopic recording of the output optical intensity trace using a 6 GHz real-time oscilloscope, showing three main field components: a noisy background, drifting solitons and a condensed phase of bound solitons (highest peak). Temporal magnification of the condensed phase is performed with a 30 GHz sampling oscilloscope. Figure **b** reproduced with permission from ref. 82, © 2010 APS.

(Fig. 6c). Implementing mode-locking in addition to spatial localization in a semiconductor device remains a practical challenge. In terms of fabrication, the preparation of uniform samples to allow homogeneous injection of the electrical current density is complicated for devices whose transverse section is of the order of 100 μm or larger. Composite cavities, which have separate sections for gain and saturation, must be adequately long with respect to the available gain bandwidth and the relaxation time of electrical carriers. Despite such challenges, the observation of transient and pulsating cavity soliton dynamics[101] provides reasonable hope that mode-locked cavity soliton lasers could be realized in the near future, opening up new prospects for fundamental studies, technological breakthroughs and all-optical data processing applications.

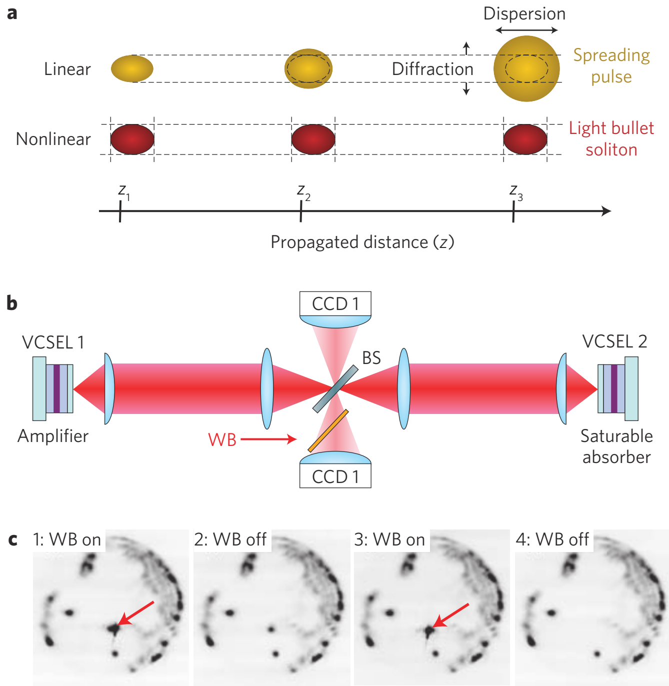

Figure 6 | Towards a mode-locked cavity soliton laser. a, The concept of light bullets. **b**, Illustration of a cavity soliton laser experiment, with VCSELs coupled in a self-imaging configuration. BS, beamsplitter; WB, writing beam. **c**. Cavity solitons switched on and off by the transient application of the external WB at a specific location (indicated by the arrow). Figure **c** reproduced with permission from ref. 100, © 2008 APS.

Conclusion: the bright future of dissipative solitons

The entire field of passively mode-locked laser dynamics is being influenced by the ideas of dissipative solitons, to the extent that some systems are now referred to as 'dissipative soliton lasers'. We therefore felt that a timely clarification and dissemination of the underlying principles governing dissipative solitons would be of interest to the wider optics community. The potentially enormous variety of dissipative soliton dynamics is not critically limited by particular physical properties such as the sign of chromatic dispersion, the type of mode-locking mechanism or the pumping power. In addition, large areas of knowledge in this field remain unexplored. The notions of attractors and their transformations in the space of laser parameters have been central in making the dissipative soliton concept a functional one. The stability of these formations is an important attribute for potential applications. The development of original interdisciplinary tools using the principles of statistical physics and nonlinear science will be needed to tackle the behaviour of large groups of dissipative solitons, and such views are likely to create a fascinating new chapter of dissipative soliton dynamics. The appearance of rogue waves in dissipative nonlinear systems may also attract significant attention[102]. Although here we have concentrated on the close interplay between dissipative solitons and mode-locked laser dynamics, one should keep in mind the broader applications of the concept to nonlinear optics in general, as well to many other scientific fields.

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Additional information
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