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***Abstract*—This paper is concerned with the dissipativity analysis and design of discrete Markovian jumping neural networks with sector-bounded nonlinear activation functions and time-varying delays represented by Takagi-Sugeno fuzzy model. The augmented fuzzy neural networks with Markovian jumps are firstly constructed based on estimator of Luenberger observer type. Then, applying piecewise Lyapunov-Krasovskii functional approach and stochastic analysis technique, a sufficient condition is provided to guarantee that the augmented fuzzy jump neural networks are stochastically dissipative. Moreover, a less conservative criterion is established to solve the dissipative state estimation problem by using matrix decomposition approach. Furthermore, to reduce the computational complexity of the algorithm, a dissipative estimator is designed to ensure stochastic dissipativity of the error fuzzy jump neural networks. As a special case, we have also considered the mixed H_∞ and passive analysis of fuzzy jump neural networks. All criteria can be formulated in terms of linear matrix inequalities. Finally, two examples are given to show the effectiveness and potential of the new design techniques.**

***Index Terms*—Fuzzy neural networks; dissipativity; Markovian jump parameters; stochastic state estimation; time-varying delays.**

I. Introduction

In the past decades, neural networks have received increasing interest due to their extensive applications in a variety of areas, such as pattern recognition, signal processing, solving optimization problems, associative memories, and target tracking, and so forth [1, 2]. It has been recognized that the existence of time delay can render the instability and poor performance of network dynamics in the signal transmission between neurons, and much work of time-delay neural networks has been reported in the literature, such as exponential stability, the existence of an equivalent point, global asymptotic stability, passivity analysis, and synchronization [3-8]. In reality, neural networks often accompany the information latching. In other words, a neural network can have finite modes switching from one to another at different time, which can lead to the switching between different modes being governed by a Markov chain. Therefore, neural networks with Markovian jump parameters have become the focus of increasing research attention in control communities and mathematics, and many appealing results have been investigated by utilizing linear matrix inequality (LMI) approach. For example, the exponential stability criteria were investigated for time-delay recurrent neural networks with Markovian jumps in [9]. The authors in [10] discussed the stability and synchronization of discrete Markovian jumping neural networks with mixed mode-dependent time delays. In [11], the passivity conditions were conducted for discrete jump neural networks with mixed time delays, and the case of Markov chain with partially unknown transition probabilities was also considered. More results related to Markovian jump neural networks involving time delays can also be found in [12, 13] and the references therein.

On the other hand, Takagi-Sugeno (T-S) fuzzy model has been recognized as a popular and effective approach in analyzing, synthesizing and approximating complex nonlinear systems in [14]. Consequently, stability analysis, H_∞ control, reliable control, filter design, and adaptive control problems for T-S fuzzy systems have attracted considerable attention, and many important results have been reported in the references [15-18]. Recently, T-S fuzzy model has been successfully used to represent complex nonlinear neural networks, see [19-22]. Based on the Lyapunov stability theory and the stochastic analysis technique, the authors in [23] addressed the robust stability analysis of continuous-time uncertain fuzzy Hopfield neural networks with Markovian jumps and time-delays. In [24], the robust stochastic stability criteria were presented for uncertain fuzzy jump discrete-time neural networks with various activation functions and mixed time delay. It is necessary to point out that the states of neural networks can generally be not completely available in the network outputs. Thus, in real applications, we need to estimate the neuron states through available measurements and then exploit the estimated neuron states to obtain the design requirements. Much work has been investigated for the state estimation of the neural networks, see the references [25, 26]. It should noted that state estimation problems were also studied for continuous-time T-S fuzzy neural networks in [27-29]. Therefore, as an analogue of the continuous-time case, it is essential to deal with state estimation of discrete-time T-S fuzzy neural

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networks due to both theoretical and practical importance to study the dynamics of discrete-time neural networks.

Another area that has been well studied is dissipative theory, which was originally introduced in [30] and subsequently developed in [31, 32]. The dissipativity has played an important role in system analysis and synthesis. The dissipativity theorem extends many basic system theorems, such as bounded real theorem, passivity theorem, and sector-bounded nonlinearity. For example, by utilizing the delay partitioning technique, the authors in [33] tackled dissipativity analysis of stochastic neural networks with time delays. In [34], the problems of dissipativity analysis and synthesis were investigated for discrete delayed T-S fuzzy systems with stochastic perturbation. By applying the novel model transformation approach combined with the Lyapunov-Krasovskii technique, sufficient criteria of dissipativity were established in the form of LMIs and a fuzzy controller was also designed to ensure the dissipative performance of the closed-loop system. This work is the first attempt, to the best of the authors' knowledge, to explore the dissipativity analysis and synthesis for discrete delayed fuzzy neural networks with Markovian jumps and sector-bounded nonlinear activation functions.

In this paper, the dissipativity analysis and design are tackled for discrete time-delay neural networks with Markovian jump parameters and sector-bounded nonlinear activation functions represented by T-S fuzzy model. Using Lyapunov stability theory and matrix decomposition approach, a sufficient condition is initially provided to ensure that the augmented fuzzy jump neural networks are stochastically dissipative. Then, a less conservative criterion is established to solve the dissipative state estimation problem. Moreover, to reduce the computing complexity of the algorithm, a dissipative estimator is also designed to guarantee that the error fuzzy neural networks with Markovian jumps are stochastically dissipative. As a special case, the mixed H_∞ and passive analysis are also tackled for the class of fuzzy jump neural networks. These criteria can be characterized in terms of LMIs. Finally, two numerical examples are presented to show the effectiveness of the proposed results. The major contributions of this paper are as follows: (i) by applying piecewise Lyapunov-Krasovskii functional approach and stochastic analysis technique, the dissipativity analysis and design are presented for discrete fuzzy jump neural networks; (ii) the mixed H_∞ and passive analysis and design of the class of fuzzy jump neural networks are derived; (iii) the obtained criteria are characterized in terms of LMIs by applying matrix decomposition techniques. Therefore, the main aim of this paper is to make the first attempt to tackle the listed contributions.

The rest of this paper is organized as follows. Section II is problem statement and preliminaries. The results on dissipativity analysis and design are provided for discrete T-S fuzzy jump neural networks with sector-bounded nonlinear activation functions in Section III. Section IV gives simulation results to demonstrate the validity of the proposed methods and, finally, the conclusions are drawn in Section V.

Notations. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ represent the sets of n component real vectors and $n \times m$ real matrices, respectively. $(\Omega, \mathbb{F}, \mathbb{P})$ is probability space, Ω is the sample space, $\mathbb{F}$ is the σ-algebra of subsets of the sample space and $\mathbb{P}$ is the probability measure on $\mathbb{F}$. I_n denotes the identity of n dimension, $\mathbf{E}\{.\}$ represents the expectation operator with some probability measure $\mathbb{P}$. $A > 0$ (or $A < 0$) denotes a symmetric positive (or negative) matrix. The symbol $*$ refers to the term of a matrix which can be inferred by symmetry and diag$\{\cdots\}$ denotes a block-diagonal matrix. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations. A^T and A^{-1} stand for the matrix transpose and matrix inverse, respectively.

II. Problem formulation

Fix a probability space $(\Omega, \mathbb{F}, \mathbb{P})$ and consider the following discrete Markovian jump neural networks (DMJNNs) with time-varying delays which could be described by a T-S fuzzy model:

Plant Rules $i, i = 1, 2, \cdots, f$: IF θ_1 is μ_{i1}, θ_2 is μ_{i2}, $\cdots$, θ_g is μ_{ig}, THEN

$$\begin{array}{rcl} x(k+1) & = & A_i(r_k)x(k) + B_i(r_k)f(x(k)) \\ & & +C_i(r_k)g(x(k-d(k))) \\ & & +B_{\omega i}(r_k)\omega(k), \\ y(k) & = & C_{1i}(r_k)x(k) + C_{2i}(r_k)x(k-d(k)), \\ z(k) & = & E_i(r_k)x(k), \\ x(j) & = & \phi(j), \quad j \in \{-d_2, \cdots, -1, 0\}, \end{array} \tag{1}$$

where $x(k) \in \mathbb{R}^n, y(k) \in \mathbb{R}^p$, $z(k) \in \mathbb{R}^q$ and $\phi(j)$ are the neuron state vector, the network output measured vector, the signal to be estimated and the compatible vector-valued initial condition, respectively. The external disturbance input $\omega(k) \in \mathbb{R}^m$ belongs to $l_2[0, +\infty)$. The stochastic jump process $\{r_k, k \geq 0\}$ is a discrete-time, discrete-state Markov chain taking values in a finite set $\mathcal{L} = \{1, 2, \cdots, s\}$ with transition probabilities π_{lm} given by $\sum_{m=1}^{s} \pi_{lm} = 1, \pi_{lm} > 0, l \in \mathcal{L}$. The mode-dependent matrices $B_i(r_k)$, $C_i(r_k)$, $B_{\omega i}(r_k)$, $C_{1i}(r_k)$, $C_{2i}(r_k)$ and $E_i(r_k)$ are known the real matrices of appropriate dimensions. $d(k)$ denotes the transmission delay satisfying $0 < d_1 \leq d(k) \leq d_2$, which d_1 and d_2 are prescribed positive integers representing the lower and upper bounds of the delay, respectively. $f(x(k))$ and $g(x(k-d(k)))$ are the neuron activation functions. $A_i(r_k) = \text{diag}(a_{1i}(r_k), a_{2i}(r_k), \cdots, a_{ni}(r_k))$ is the known mode-dependent diagonal matrix. θ_j and μ_{ij} $(i = 1, \cdots, f, j = 1, \cdots, g)$ are respectively the premise variables and the fuzzy sets, f is the number of IF-THEN rules. The fuzzy basis functions are given by $h_i(\theta(k)) = \prod_{j=1}^{g} \mu_{ij}(\theta_j(k)) / \sum_{i=1}^{f} \prod_{j=1}^{g} \mu_{ij}(\theta_j(k))$, in which $\mu_{ij}(\theta_j(k))$ represents the grade of membership of $\theta_j(k)$ in μ_{ij}. It follows that $\sum_{i=1}^{f} h_i(\theta(k)) = 1$ with $h_i(\theta(k)) > 0$.

To simplify the presentation of this paper, in the sequel, for each possible $r_k = l, l \in \mathcal{L}$, matrix $M_i(r_k)$ will be denoted by $M_{i,l}$; for instance, $A_i(r_k)$ will be denoted by $A_{i,l}$, $A_{di}(r_k)$ by $A_{di,l}$, and so on. In addition, $h_i(k)$, Λ and $\check{P}_l$ represent $h_i(\theta(k))$, $\{1, \cdots, f\}$ and $\sum_{j=1}^{s} \pi_{lj} \bar{P}_j$, respectively.

By applying the fuzzy blending method, the overall fuzzy

DMJNNs could be rewritten as follows:

$$\begin{array}{rcl} x(k+1) &=& \sum_{i=1}^{f} h_i(k)[A_{i,l}x(k)+B_{i,l}f(x(k)) \\ && +C_{i,l}g(x(k-d(k))) \\ && +B_{\omega i,l}\omega(k)], \\ y(k) &=& \sum_{i=1}^{f} h_i(k)[C_{1i,l}x(k) \\ && +C_{2i,l}x(k-d(k))], \\ z(k) &=& \sum_{i=1}^{f} h_i(k)[E_{i,l}x(k)], \\ x(j) &=& \phi(j), \quad j\in\{-d_2,\cdots,0\}. \end{array} \tag{2}$$

The estimator of a Luenberger observer type in this paper is performed through the parallel distributed compensation and the overall estimator is thus inferred as follows

$$\begin{array}{rcl} \tilde{x}(k+1) &=& \sum_{i=1}^{f} h_i(k)[A_{i,l}\tilde{x}(k)+B_{i,l}f(\tilde{x}(k)) \\ && +C_{i,l}g(\tilde{x}(k-d(k))) \\ && +H_{i,l}(y(k)-\tilde{y}(k)], \\ \tilde{y}(k) &=& \sum_{i=1}^{f} h_i(k)[C_{1i,l}\tilde{x}(k) \\ && +C_{2i,l}\tilde{x}(k-d(k))], \\ \tilde{z}(k) &=& \sum_{i=1}^{f} h_i(k)[E_{i,l}\tilde{x}(k)], \\ \tilde{x}(j) &=& \tilde{\phi}(j), \quad j\in\{-d_2,\cdots,0\}. \end{array} \tag{3}$$

where $\tilde{x}(k)$, $\tilde{y}(k)$, and $\tilde{z}(k)$ are, respectively, the estimated state, the estimated output and the estimated signal, $\tilde{\phi}(j)$ is a compatible state estimation vector-valued initial condition, $H_{i,l}$ is to be designed the parameter matrices of appropriate dimensions. Define $e(k) = x(k)-\tilde{x}(k)$, $\bar{x}^T(k) = \left[\begin{array}{cc} x^T(k) & e^T(k)\end{array}\right]$ and $\bar{z}^T(k) = \left[\begin{array}{cc} z^T(k) & z^T(k)-\tilde{z}^T(k)\end{array}\right]$. Then, the resulting augmented fuzzy DMJNNs can be obtained as:

$$\begin{array}{rcl} \bar{x}(k+1) &=& \bar{A}_l\bar{x}(k)+\bar{A}_{d,l}\bar{x}(k-d(k)) \\ && +\bar{B}_lF(\bar{x}(k))+\bar{C}_lG(\bar{x}(k-d(k))) \\ && +\bar{B}_{\omega,l}\omega(k), \\ \bar{z}(k) &=& \bar{E}_l\bar{x}(k), \end{array} \tag{4}$$

where

$$\begin{array}{l} \bar{A}_l = \sum_{i=1}^{f}\sum_{j=1}^{f} h_i(k)h_j(k)\left[\begin{array}{cc} A_{i,l} & 0 \\ 0 & A_{i,l}-H_{i,l}C_{1j,l}\end{array}\right], \\ \bar{A}_{d,l} = \sum_{i=1}^{f}\sum_{j=1}^{f} h_i(k)h_j(k)\left[\begin{array}{cc} 0 & 0 \\ 0 & -H_{i,l}C_{2j,l}\end{array}\right], \\ \bar{B}_l = \sum_{i=1}^{f} h_i(k)\left[\begin{array}{cc} B_{i,l} & 0 \\ 0 & B_{i,l}\end{array}\right], \\ \bar{B}_{\omega,l} = \sum_{i=1}^{f} h_i(k)\left[\begin{array}{c} B_{\omega i,l} \\ B_{\omega i,l}\end{array}\right], \\ \bar{C}_l = \sum_{i=1}^{f} h_i(k)\left[\begin{array}{cc} C_{i,l} & 0 \\ 0 & C_{i,l}\end{array}\right], \\ \bar{E}_l = \sum_{i=1}^{f} h_i(k)\left[\begin{array}{cc} E_{i,l} & 0 \\ 0 & E_{i,l}\end{array}\right], \\ F^T(\bar{x}(k)) = \left[\begin{array}{cc} f^T(x(k)) & f^T(x(k))-f^T(\tilde{x}(k))\end{array}\right], \\ G^T(\bar{x}(k)) = \left[\begin{array}{cc} g^T(x(k)) & \hat{G}^T(e(k-d(k)))\end{array}\right], \\ \hat{G}(e(k-d(k))) = g(x(k-d(k)))-g(\tilde{x}(k-d(k))). \end{array}$$

For the neuron activation functions, the following assumption is required.

Assumption 1 (Sector-bounded conditions, see [26, 35]). The neuron state-based nonlinear functions $f(\cdot)$ and $g(\cdot)$ in (1) are continuous and satisfy $f(0)=0$, $g(0)=0$, and there exist real matrices U_1, U_2, V_1 and V_2 with appropriate dimensions such that

$$f^T(x,y,U_1)f(x,y,U_2)\leq 0, \tag{5a}$$

$$g^T(x,y,V_1)g(x,y,V_2)\leq 0 \tag{5b}$$

with $f(x,y,U)=f(x)-f(y)-U(x-y)$ and $g(x,y,V)=g(x)-g(y)-V(x-y)$.

Remark 1. Observe that, when $U_1=-U_2=U$ and $V_1=-V_2=V$, conditions (5a) and (5b) are respectively reduced to

$$\begin{array}{l} [f(x)-f(y)]^T[f(x)-f(y)]\leq [x-y]^TU^TU[x-y], \\ [g(x)-g(y)]^T[g(x)-g(y)]\leq [x-y]^TV^TV[x-y]. \end{array}$$

Furthermore, $|f(x)-f(y)|\leq U|x-y|$ and $|g(x)-g(y)|\leq V|x-y|$ hold when $U>0$ and $V>0$. Therefore, the neuron activation functions under Assumption 1 are more general than those usual or analogous Lipschitz-type conditions by [9-13, 22-24, 27, 28]. Moreover, the nonlinear functions $f(x)$ and $g(x)$ are said to belong to sectors $[U_1,U_2]$ and $[V_1,V_2]$, respectively (see Ref. [35]). The systems with sector-bounded nonlinearity have been intensively studied (see Refs. [36-38]).

Before ending this section, we recall the following definition and lemma, which will be used in the proof of our main results.

The "energy supply function" of the augmented fuzzy DMJNNs (4) is defined as

$$\bar{J}_l^*(\omega,\bar{z},N^*)=\sum_{k=0}^{N^*}\psi_l(\omega(k),\bar{z}(k)), \forall N^*\geq 0, \tag{6}$$

where $\psi_l(\omega,\bar{z})=\bar{z}^T\mathcal{X}_l\bar{z}+2\bar{z}^T\mathcal{S}_l\omega+\omega^T\mathcal{R}_l\omega$ with $\mathcal{X}_l^T=\mathcal{X}_l$ and $\mathcal{R}_l^T=\mathcal{R}_l$.

Definition 1 (Strictly stochastic dissipativity). Under zero initial state, the augmented fuzzy DMJNNs (4) are said to be strictly stochastically $(\mathcal{X}_l,\mathcal{S}_l,\mathcal{R}_l)$-$\alpha$-dissipative, if for some sufficiently small scalar $\alpha>0$, for all $l\in\mathcal{L}$ and $\omega(k)\in l_2[0,+\infty)$, the energy supply function satisfies:

$$\mathbf{E}\{\bar{J}_l^*(\omega,\bar{z},N^*)\}\geq\alpha\sum_{k=0}^{N^*}\omega^T(k)\omega(k). \tag{7}$$

In addition, the system (4) is called stochastically $(\mathcal{X}_l,\mathcal{S}_l,\mathcal{R}_l)$-dissipative when $\alpha=0$. In many cases, it is assumed that $\mathcal{X}_l<0$.

Remark 2. In [30], the dissipative notion of continuous-time systems was first given by Willems, and then Goodwin and Sin extended it to discrete-time case in [32]. The dissipative systems including continuous- and discrete- time cases satisfy a time-based property that relates an input-output energy supply function to a state based storage function. The original notions of dissipativity are defined for deterministic systems. Until recently, the authors in [34] expended the original definition into stochastic dissipativity for T-S fuzzy systems. In this paper, the system (4) are said to be strictly stochastically $(\mathcal{X}_l,\mathcal{S}_l,\mathcal{R}_l)$-$\alpha$-dissipative, if the energy supply function $\bar{J}_l^*(\omega,\bar{z},N^*)$ satisfies the condition (7) for every mode $l\in\mathcal{L}$, which is similar to the definition in [34].

Remark 3. It should be noted that the dissipative performance analysis is a generalized form of the bounded real lemma, passivity and mixed H_∞ and passive performance. If dimensions of $\bar{z}(k)$ and $\omega(k)$ are assumed to be compatible, that is to say that m and q satisfy $m = 2q$. Then, it follows that (7) is reduced to the standard H_∞ performance when $\mathcal{R}_l = \gamma^2 I, \gamma > 0$, $S_l = 0$, $\mathcal{X}_l = -I$ and $\alpha = 0$; (7) is reduced to the strictly positive real performance when $\mathcal{R}_l = 0$, $S_l = I$ and $\mathcal{X}_l = 0$; and (7) is reduced to the strictly mixed H_∞ and passive performance when $\mathcal{R}_l = \delta\gamma^2 I, \gamma > 0$, $S_l = (1-\delta)I, \delta \in [0, 1]$ and $\mathcal{X}_l = -\delta I$.

Lemma 1 (Discrete Jensen Inequality, see [11, 13, 34]). For any constant positive definite symmetric matrix $Z \in \mathbb{R}^{n\times n}$, positive integers d_1 and d_2 satisfying $d_2 \geq d_1$, the following inequality holds:

$$(d_2 - d_1 + 1)\sum_{k=d_1}^{d_2} \xi^T(k)Z\xi(k) \geq (\sum_{k=d_1}^{d_2} \xi(k))^T Z(\sum_{k=d_1}^{d_2} \xi(k)). \tag{8}$$

III. MAIN RESULTS

In this section, we firstly consider estimator design of the form (3) which can guarantee that the augmented fuzzy DMJNNs (4) are strictly stochastically $(\mathcal{X}_l, \mathcal{S}_l, \mathcal{R}_l)$-$\alpha$-dissipative, and when $\omega(k) \equiv 0$, the model in (4) is exponentially stable in mean square sense. We introduce the following denotation for presentation convenience:

$$\begin{aligned}
&F_1 = \mathrm{diag}\{(U_1^T U_2 + U_1 U_2^T)/2, (U_1^T U_2 + U_1 U_2^T)/2\},\\
&F_2 = \mathrm{diag}\{-(U_1 + U_2)/2, -(U_1 + U_2)/2\},\\
&G_1 = \mathrm{diag}\{(V_1^T V_2 + V_1 V_2^T)/2, (V_1^T V_2 + V_1 V_2^T)/2\},\\
&G_2 = \mathrm{diag}\{-(V_1 + V_2)/2, -(V_1 + V_2)/2\},\\
&\bar{\eta}_1^T(k) = \left[\begin{array}{ccc} \bar{x}^T(k) & \bar{x}^T(k-d_1) & \bar{x}^T(k-d(k)) \end{array}\right],\\
&\bar{\eta}_2^T(k) = \left[\begin{array}{ccc} \bar{x}^T(k-d_2) & F^T(\bar{x}(k)) & G^T(\bar{x}(k-d(k)) \end{array}\right],\\
&\bar{\eta}^T(k) = \left[\begin{array}{cc} \bar{\eta}_1^T(k) & \bar{\eta}_2^T(k) \end{array}\right];
\end{aligned}$$

In addition, $\xi(k) = x(k+1) - x(k)$, $\Phi = \left[\begin{array}{cc} I_n & 0_{n\times n} \end{array}\right]$ and $\bar{e}_\nu = \left[\begin{array}{ccc} 0_{2n\times 2(\nu-1)n} & I_{2n} & 0_{2n\times 2(6-\nu)n} \end{array}\right]$ $(\nu = 1, 2, \cdots, 6)$. Then, the fuzzy DMJNNs (4) can be rewritten as

$$\bar{x}(k+1) = \bar{\Gamma}_l \bar{\eta}(k) + \bar{B}_{\omega,l}\omega(k), \tag{9a}$$

$$\bar{z}(k) = \bar{E}_l \bar{e}_1 \bar{\eta}(k), \tag{9b}$$

where $\bar{\Gamma}_l = \bar{A}_l\bar{e}_1 + \bar{A}_{d,l}\bar{e}_3 + \bar{B}_l\bar{e}_5 + \bar{C}_l\bar{e}_6$.

Theorem 1. Under Assumption 1, the augmented fuzzy DMJNNs (4) are strictly stochastically $(\mathcal{X}_l, \mathcal{S}_l, \mathcal{R}_l)$-$\alpha$-dissipative if there exist a scalar $\alpha > 0$, sets of scalars $\{\theta_{1,l} > 0, l \in \mathcal{L}\}$ and $\{\theta_{2,l} > 0, l \in \mathcal{L}\}$, symmetric positive-definite matrices Q_1, Q_2, Q_3, Z_1, Z_2, a set of symmetric positive-definite matrices $\{\bar{P}_l, l \in \mathcal{L}\}$, a set of matrices $\{H_{i,l}, i \in \Lambda, l \in \mathcal{L}\}$, for all $l \in \mathcal{L}$ and $i, j \in \Lambda$, such that

$$\bar{\Xi}_{ij,l} + \bar{\Xi}_{ji,l} < 0, \quad i < j, \tag{10a}$$

$$\bar{\Xi}_{ii,l} < 0, \tag{10b}$$

where

$$\bar{\Xi}_{ij,l} = \left[\begin{array}{ccccc} \bar{\Xi}_{11i,l} & * & * & * & * \\ \hat{P}_l\bar{L}_{1ij,l} & -\bar{\Xi}_{22,l} & * & * & * \\ \bar{L}_{2i,l} & 0 & -Z_1 & * & * \\ \bar{L}_{3i,l} & 0 & 0 & -Z_2 & * \\ \bar{L}_{4i,l} & 0 & 0 & 0 & \mathcal{X}_l \end{array}\right],$$

$$\bar{\Xi}_{11i,l} = \left[\begin{array}{cc} \bar{\Xi}_{11,l}^{11} & * \\ -\mathcal{S}_l^T\bar{E}_{i,l}\bar{e}_1 & -(\mathcal{R}_l - \alpha I_m) \end{array}\right],$$

$$\begin{aligned}
\bar{\Xi}_{11,l}^{11} &= \bar{e}_1^T(Q_1 + Q_2 + (d_{12}+1)Q_3 - \bar{P}_l)\bar{e}_1\\
&\quad -\bar{e}_2^T Q_1\bar{e}_2 - \bar{e}_3^T Q_3\bar{e}_3 - \bar{e}_4^T Q_2\bar{e}_4\\
&\quad -(\bar{e}_1 - \bar{e}_2)^T\Phi^T Z_1\Phi(\bar{e}_1 - \bar{e}_2)\\
&\quad -(\bar{e}_2 - \bar{e}_4)^T\Phi^T Z_2\Phi(\bar{e}_2 - \bar{e}_4)\\
&\quad -\theta_{1,l}\left[\begin{array}{c} \bar{e}_1 \\ \bar{e}_5 \end{array}\right]^T \left[\begin{array}{cc} F_1 & * \\ F_2 & I_{2n} \end{array}\right]\left[\begin{array}{c} \bar{e}_1 \\ \bar{e}_5 \end{array}\right]\\
&\quad -\theta_{2,l}\left[\begin{array}{c} \bar{e}_3 \\ \bar{e}_6 \end{array}\right]^T \left[\begin{array}{cc} G_1 & * \\ G_2 & I_{2n} \end{array}\right]\left[\begin{array}{c} \bar{e}_3 \\ \bar{e}_6 \end{array}\right],
\end{aligned}$$

$$\begin{aligned}
&\bar{L}_{1ij,l} = \bar{\pi}_l^T\left[\begin{array}{cc} \bar{\Gamma}_{ij,l} & \bar{B}_{\omega i,l} \end{array}\right],\\
&\bar{\pi}_l = \left[\begin{array}{cccc} \sqrt{\pi_{l1}}I_{2n} & \sqrt{\pi_{l2}}I_{2n} & \cdots & \sqrt{\pi_{ls}}I_{2n} \end{array}\right],\\
&\bar{L}_{2i,l} = d_1 Z_1\Phi\left[\begin{array}{cc} \bar{\Gamma}_{ij,l} - \bar{e}_1 & \bar{B}_{\omega i,l} \end{array}\right],\\
&\bar{L}_{3i,l} = d_{12} Z_2\Phi\left[\begin{array}{cc} \bar{\Gamma}_{ij,l} - \bar{e}_1 & \bar{B}_{\omega i,l} \end{array}\right],\\
&\bar{L}_{4i,l} = \left[\begin{array}{cc} \mathcal{X}_l\bar{E}_{i,l}\bar{e}_1 & 0_{2q\times m} \end{array}\right], d_{12} = d_2 - d_1,\\
&\bar{\Gamma}_{ij,l} = \bar{A}_{ij,l}\bar{e}_1 + \bar{A}_{dij,l}\bar{e}_3 + \bar{B}_{i,l}\bar{e}_5 + \bar{C}_{i,l}\bar{e}_6,\\
&\bar{A}_{ij,l} = \left[\begin{array}{cc} A_{i,l} & 0 \\ 0 & A_{i,l} - H_{i,l}C_{1j,l} \end{array}\right],\\
&\bar{A}_{dij,l} = \left[\begin{array}{cc} 0 & 0 \\ 0 & -H_{i,l}C_{2j,l} \end{array}\right], \bar{B}_{\omega i,l} = \left[\begin{array}{c} B_{\omega i,l} \\ B_{\omega i,l} \end{array}\right],\\
&\bar{B}_{i,l} = \left[\begin{array}{cc} B_{i,l} & 0 \\ 0 & B_{i,l} \end{array}\right], \bar{C}_{i,l} = \left[\begin{array}{cc} C_{i,l} & 0 \\ 0 & C_{i,l} \end{array}\right],\\
&\bar{E}_{i,l} = \left[\begin{array}{cc} E_{i,l} & 0 \\ 0 & E_{i,l} \end{array}\right], \hat{P}_l = \mathrm{diag}\{\bar{P}_l, \bar{P}_l, \cdots, \bar{P}_l\}\\
&\bar{\Xi}_{22,l} = \mathrm{diag}\{2\bar{P}_l - \bar{P}_1, 2\bar{P}_l - \bar{P}_2, \cdots, 2\bar{P}_l - \bar{P}_s\}.
\end{aligned}$$

Proof. Consider the following piecewise stochastic Lyapunov-Krasovskii functional:

$$V(\bar{x}(k), r_k = l, k) = \textstyle\sum_{j=1}^5 V_j(\bar{x}(k), r_k = l, k), \tag{11}$$

where

$$\begin{aligned}
V_1(\bar{x}(k), r_k = l, k) &= \bar{x}^T(k)\bar{P}_l\bar{x}(k),\\
V_2(\bar{x}(k), r_k = l, k) &= \textstyle\sum_{m=k-d_1}^{k-1}\bar{x}^T(m)Q_1\bar{x}(m)\\
&\quad + \textstyle\sum_{m=k-d_2}^{k-1}\bar{x}^T(m)Q_2\bar{x}(m),\\
V_3(\bar{x}(k), r_k = l, k) &= \textstyle\sum_{m=k-d(k)}^{k-1}\bar{x}^T(m)Q_3\bar{x}(m)\\
&\quad + \textstyle\sum_{j=-d_2+1}^{-d_1}\sum_{m=k+j}^{k-1}\bar{x}^T(m)Q_3\bar{x}(m),\\
V_4(\bar{x}(k), r_k = l, k) &= d_1\textstyle\sum_{j=-d_1}^{-1}\sum_{m=k+j}^{k-1}\xi^T(m)Z_1\xi(m),\\
V_5(\bar{x}(k), r_k = l, k) &= d_{12}\textstyle\sum_{j=-d_2}^{-d_1-1}\sum_{m=k+j}^{k-1}\xi^T(m)Z_2\xi(m).
\end{aligned}$$

Define $\mathbf{E}\{\Delta V(k)\} = \mathbf{E}\{V(\bar{x}(k+1), r_{k+1} = j, k+1|r_k = l)\} - V(\bar{x}(k), r_k = l, k)$, and note that $x(k) = \Phi\bar{x}(k)$ and $\xi(k) = (\Phi\bar{\Gamma}_{\omega,l})\left[\begin{array}{cc} \bar{\eta}^T(k) & \omega^T(k) \end{array}\right]^T$ with $\bar{\Gamma}_{\omega,l} = \left[\begin{array}{cc} \bar{\Gamma}_l - \bar{e}_1 & \bar{B}_{\omega,l} \end{array}\right]$. Then, calculating the value $\mathbf{E}\{\Delta V(k)\}$ along the solution of (4), we have

$$\begin{aligned}
&\mathbf{E}\{\Delta V_1(k)\}\\
&= \textstyle\sum_{j=1}^s \pi_{lj}\bar{x}^T(k+1)\bar{P}_j\bar{x}(k+1) - \bar{x}^T(k)\bar{P}_l\bar{x}(k)\\
&= \left[\begin{array}{c} \bar{\eta}(k) \\ \omega(k) \end{array}\right]^T \left[\begin{array}{cc} \bar{\Gamma}_l & \bar{B}_{\omega,l} \end{array}\right]^T \check{P}_l \left[\begin{array}{cc} \bar{\Gamma}_l & \bar{B}_{\omega,l} \end{array}\right]\left[\begin{array}{c} \bar{\eta}(k) \\ \omega(k) \end{array}\right]\\
&\quad -\bar{\eta}^T(k)\bar{e}_1^T\bar{P}_l\bar{e}_1\bar{\eta}(k),
\end{aligned} \tag{12}$$

$$\begin{aligned}
&\mathbf{E}\{\Delta V_2(k)\}\\
&= \bar{\eta}^T(k)(\bar{e}_1^T(Q_1 + Q_2)\bar{e}_1 - \bar{e}_2^T Q_1\bar{e}_2 - \bar{e}_4^T Q_2\bar{e}_4)\bar{\eta}(k),
\end{aligned} \tag{13}$$

$$\begin{aligned}
&\mathbf{E}\{\Delta V_3(k)\}\\
&\leq \bar{\eta}^T(k)((d_{12}+1)\bar{e}_1^T Q_3\bar{e}_1 - \bar{e}_3^T Q_3\bar{e}_3)\bar{\eta}(k),
\end{aligned} \tag{14}$$

$$
\begin{aligned}
&\mathbf{E}\{\Delta V_4(k)\}\\
&= d_1^2\xi^T(k)Z_1\xi(k) - d_1\sum_{m=k-d_1}^{k-1}\xi^T(m)Z_1\xi(m),\\
&\le d_1^2\xi^T(k)Z_1\xi(k) - (\sum_{m=k-d_1}^{k-1}\xi(m))^T Z_1(\sum_{m=k-d_1}^{k-1}\xi(m))\\
&= d_1^2\xi^T(k)Z_1\xi(k)\\
&\quad -(x(k)-x(k-d_1))^T Z_1(x(k)-x(k-d_1))\\
&= d_1^2\begin{bmatrix}\bar{\eta}(k)\\ \omega(k)\end{bmatrix}^T \bar{\Gamma}_{\omega,l}^T\Phi^T Z_1\Phi\bar{\Gamma}_{\omega,l}\begin{bmatrix}\bar{\eta}(k)\\ \omega(k)\end{bmatrix}\\
&\quad -\bar{\eta}^T(k)(\bar{e}_1-\bar{e}_2)^T\Phi^T Z_1\Phi(\bar{e}_1-\bar{e}_2)\bar{\eta}(k),
\end{aligned}
\tag{15}
$$

$$
\begin{aligned}
&\mathbf{E}\{\Delta V_5(k)\}\\
&= d_{12}^2\xi^T(k)Z_2\xi(k) - d_{12}\sum_{m=k-d_2}^{k-d_1-1}\xi^T(m)Z_2\xi(m),\\
&\le d_{12}^2\xi^T(k)Z_2\xi(k) - (\sum_{m=k-d_2}^{k-d_1-1}\xi(m))^T Z_2(\sum_{m=k-d_2}^{k-d_1-1}\xi(m))\\
&= d_{12}^2\xi^T(k)Z_2\xi(k)\\
&\quad -(x(k-d_1)-x(k-d_2))^T Z_2(x(k-d_1)-x(k-d_2))\\
&= d_{12}^2\begin{bmatrix}\bar{\eta}(k)\\ \omega(k)\end{bmatrix}^T \bar{\Gamma}_{\omega,l}^T\Phi^T Z_2\Phi\bar{\Gamma}_{\omega,l}\begin{bmatrix}\bar{\eta}(k)\\ \omega(k)\end{bmatrix}\\
&\quad -\bar{\eta}^T(k)(\bar{e}_2-\bar{e}_4)^T\Phi^T Z_2\Phi(\bar{e}_2-\bar{e}_4)\bar{\eta}(k),
\end{aligned}
\tag{16}
$$

where Lemma 1 is applied in (15) and (16). From Assumption 1, we can obtain that

$$
\begin{bmatrix}\bar{x}(k)\\ F(\bar{x}(k))\end{bmatrix}^T\begin{bmatrix}F_1 & *\\ F_2 & I_{2n}\end{bmatrix}\begin{bmatrix}\bar{x}(k)\\ F(\bar{x}(k))\end{bmatrix}\le 0, \tag{17}
$$

$$
\begin{bmatrix}\bar{x}(k-d(k))\\ G(\bar{x}(k-d(k)))\end{bmatrix}^T\begin{bmatrix}G_1 & *\\ G_2 & I_{2n}\end{bmatrix}\begin{bmatrix}\bar{x}(k-d(k))\\ G(\bar{x}(k-d(k)))\end{bmatrix}\le 0 \tag{18}
$$

which are, respectively, equivalent to

$$
\bar{\eta}^T(k)\begin{bmatrix}\bar{e}_1\\ \bar{e}_5\end{bmatrix}^T\begin{bmatrix}F_1 & *\\ F_2 & I_{2n}\end{bmatrix}\begin{bmatrix}\bar{e}_1\\ \bar{e}_5\end{bmatrix}\bar{\eta}(k) \le 0, \tag{19}
$$

$$
\bar{\eta}^T(k)\begin{bmatrix}\bar{e}_3\\ \bar{e}_6\end{bmatrix}^T\begin{bmatrix}G_1 & *\\ G_2 & I_{2n}\end{bmatrix}\begin{bmatrix}\bar{e}_3\\ \bar{e}_6\end{bmatrix}\bar{\eta}(k) \le 0. \tag{20}
$$

Given that $\theta_{1,l}>0$ and $\theta_{2,l}>0$, it follows from (19) and (20) that

$$
\begin{aligned}
&\bar{J}_l(k)\\
\triangleq\ &\mathbf{E}\{\Delta V(k) - \bar{z}^T(k)\mathcal{X}_l\bar{z}(k) - 2\bar{z}^T(k)\mathcal{S}_l\omega(k)\\
&-\omega^T(k)(\mathcal{R}_l-\alpha I_m)\omega(k)\}\\
\le\ &\mathbf{E}\{\Delta V(k)\} - \mathbf{E}\{\bar{z}^T(k)\mathcal{X}_l\bar{z}(k) + 2\bar{z}^T(k)\mathcal{S}_l\omega(k)\\
&+\omega^T(k)(\mathcal{R}_l-\alpha I_m)\omega(k)\}\\
&-\theta_{1,l}\bar{\eta}^T(k)\begin{bmatrix}\bar{e}_1\\ \bar{e}_5\end{bmatrix}^T\begin{bmatrix}F_1 & *\\ F_2 & I_{2n}\end{bmatrix}\begin{bmatrix}\bar{e}_1\\ \bar{e}_5\end{bmatrix}\bar{\eta}(k)\\
&-\theta_{2,l}\bar{\eta}^T(k)\begin{bmatrix}\bar{e}_3\\ \bar{e}_6\end{bmatrix}^T\begin{bmatrix}G_1 & *\\ G_2 & I_{2n}\end{bmatrix}\begin{bmatrix}\bar{e}_3\\ \bar{e}_6\end{bmatrix}\bar{\eta}(k).
\end{aligned}
\tag{21}
$$

Let

$$
\begin{aligned}
&\bar{\Xi}_{11,l} = \begin{bmatrix}\bar{\Xi}_{11,l}^{11} & *\\ -\mathcal{S}^T\bar{E}_l\bar{e}_1 & -(\mathcal{R}_l-\alpha I_m)\end{bmatrix},\\
&\bar{L}_{1,l} = \bar{\pi}_l^T\begin{bmatrix}\bar{\Gamma}_l & \bar{B}_{\omega,l}\end{bmatrix}, \bar{L}_{2,l} = d_1Z_1\Phi\bar{\Gamma}_{\omega,l},\\
&\bar{L}_{3,l} = d_{12}Z_2\Phi\bar{\Gamma}_{\omega,l}, \bar{L}_{4,l} = \begin{bmatrix}\mathcal{X}_l\bar{E}_l\bar{e}_1 & 0_{2q\times m}\end{bmatrix},\\
&\tilde{P} = \mathrm{diag}\{\bar{P}_1,\bar{P}_2,\cdots,\bar{P}_s\}, \hat{P}_l = \mathrm{diag}\{\bar{P}_l,\bar{P}_l,\cdots,\bar{P}_l\}.
\end{aligned}
$$

Then, it follows from (11)-(16) that (21) is equivalent to

$$
\bar{J}_l(k) \le \begin{bmatrix}\bar{\eta}(k)\\ \omega(k)\end{bmatrix}^T\bar{\Theta}_l\begin{bmatrix}\bar{\eta}(k)\\ \omega(k)\end{bmatrix}, \tag{22}
$$

where

$$
\begin{aligned}
\bar{\Theta}_l &= \bar{\Xi}_{11,l} + \bar{L}_{1,l}^T\tilde{P}\bar{L}_{1,l} - \bar{L}_{2,l}^T(-Z_1)^{-1}\bar{L}_{2,l}\\
&\quad -\bar{L}_{3,l}^T(-Z_2)^{-1}\bar{L}_{3,l} - \bar{L}_{4,l}^T\mathcal{X}_l^{-1}\bar{L}_{4,l}\\
&= \bar{\Xi}_{11,l} - (\hat{P}_l\bar{L}_{1,l})^T(-\hat{P}_l\tilde{P}^{-1}\hat{P}_l)^{-1}(\hat{P}_l\bar{L}_{1,l})\\
&\quad -\bar{L}_{2,l}^T(-Z_1)^{-1}\bar{L}_{2,l} - \bar{L}_{3,l}^T(-Z_2)^{-1}\bar{L}_{3,l}\\
&\quad -\bar{L}_{4,l}^T\mathcal{X}_l^{-1}\bar{L}_{4,l}.
\end{aligned}
\tag{23}
$$

Taking into account that $(\bar{P}_m-\bar{P}_l)^T\bar{P}_m^{-1}(\bar{P}_m-\bar{P}_l)\ge 0$, which is equivalent to

$$
\bar{P}_m - 2\bar{P}_l \ge -\bar{P}_l\bar{P}_m^{-1}\bar{P}_l, \quad \forall l,m=1,2,\cdots,s. \tag{24}
$$

Denote $\bar{\Xi}_{22,l}=\mathrm{diag}\{2\bar{P}_l-\bar{P}_1, 2\bar{P}_l-\bar{P}_2,\cdots,2\bar{P}_l-\bar{P}_s\}$. Hence, $\bar{\Theta}_l<0$ if the following condition holds:

$$
\begin{aligned}
&\bar{\Xi}_{11,l} - (\hat{P}_l\bar{L}_{1,l})^T(-\bar{\Xi}_{22,l})^{-1}(\hat{P}_l\bar{L}_{1,l})\\
&-\bar{L}_{2,l}^T(-Z_1)^{-1}\bar{L}_{2,l} - \bar{L}_{3,l}^T(-Z_2)^{-1}\bar{L}_{3,l}\\
&-\bar{L}_{4,l}^T\mathcal{X}_l^{-1}\bar{L}_{4,l} < 0.
\end{aligned}
\tag{25}
$$

Using the Schur complement, condition (25) holds if and only if the following inequality holds:

$$
\begin{bmatrix}
\bar{\Xi}_{11,l} & * & * & * & *\\
\hat{P}_l\bar{L}_{1,l} & -\bar{\Xi}_{22,l} & * & * & *\\
\bar{L}_{2,l} & 0 & -Z_1 & * & *\\
\bar{L}_{3,l} & 0 & 0 & -Z_2 & *\\
\bar{L}_{4,l} & 0 & 0 & 0 & \mathcal{X}_l
\end{bmatrix} < 0. \tag{26}
$$

Condition (26) can be rewritten as

$$
\sum_{i=1}^{f}\sum_{j=1}^{f} h_i(k)h_j(k)\bar{\Xi}_{ij}(l) < 0. \tag{27}
$$

Thus, we can deduce that condition (27) is equivalent to the following inequality:

$$
\begin{aligned}
&\textstyle\sum_{i=1}^{f}\sum_{j>i}^{f} h_i(k)h_j(k)[\bar{\Xi}_{ij}(l)+\bar{\Xi}_{ji}(l)]\\
&\quad +\textstyle\sum_{i=1}^{f} h_i^2(k)\bar{\Xi}_{ii}(l) < 0.
\end{aligned}
\tag{28}
$$

Note that (10a) and (10b) can guarantee that (28) holds. Thus, we have

$$
\begin{aligned}
&\bar{J}_l(k)\\
=\ &\mathbf{E}\{\Delta V(k) - \bar{z}^T(k)\mathcal{X}_l\bar{z}(k) - 2\bar{z}^T(k)\mathcal{S}_l\omega(k)\\
&-w^T(k)(\mathcal{R}_l-\alpha I_m)\omega(k)\} < 0.
\end{aligned}
\tag{29}
$$

Summating both sides of the above inequality from 0 to N^* leads to

$$
\begin{aligned}
&V(\bar{x}^*(N^*+1)) - V(\bar{x}(0))\\
<\ &\mathbf{E}\{\bar{J}_l^*(\omega,\bar{z},N^*)\} - \alpha\textstyle\sum_{k=0}^{N^*}\omega^T(k)\omega(k).
\end{aligned}
\tag{30}
$$

Thus, $\mathbf{E}\{\bar{J}_l^*(\omega,\bar{z},N^*)\} > \alpha\sum_{k=0}^{N^*}\omega^T(k)\omega(k)$ because of $V(0)=0$ under zero initial condition and $V(\bar{x}^*(N^*+1))\ge 0$. Therefore, the augmented fuzzy DMJNNs (4) are strictly stochastically $(\mathcal{X}_l,\mathcal{S}_l,\mathcal{R}_l)$-$\alpha$-dissipative. This completes this proof.

Next, LMI criterion of dissipativity-based stochastic state estimation can be obtained for the augmented fuzzy DMJNNs (4) as follows.

Theorem 2. Under Assumption 1, the augmented fuzzy DMJNNs (4) are strictly stochastically $(\mathcal{X}_l, \mathcal{S}_l, \mathcal{R}_l)$-$\alpha$-dissipative if there exist a scalar $\alpha > 0$, sets of scalars $\{\theta_{1,l} > 0, l \in \mathcal{L}\}$ and $\{\theta_{2,l} > 0, l \in \mathcal{L}\}$, symmetric positive-definite matrices Q_1, Q_2, Q_3, Z_1, Z_2, sets of symmetric positive-definite matrices $\{P_{1,l}, l \in \mathcal{L}\}$ and $\{P_{2,l}, l \in \mathcal{L}\}$, a set of matrices $\{W_{i,l}, i \in \Lambda, l \in \mathcal{L}\}$, for all $l \in \mathcal{L}$ and $i, j \in \Lambda$, such that

$$\Xi_{ij,l} + \Xi_{ji,l} < 0, \quad i < j, \tag{31a}$$
$$\Xi_{ii,l} < 0, \tag{31b}$$

where

$$\Xi_{ij,l} = \begin{bmatrix} \bar{\Xi}_{11i,l} & * & * & * & * \\ \tilde{L}_{1ij,l} & -\bar{\Xi}_{22,l} & * & * & * \\ \bar{L}_{2i,l} & 0 & -Z_1 & * & * \\ \bar{L}_{3i,l} & 0 & 0 & -Z_2 & * \\ \bar{L}_{4i,l} & 0 & 0 & 0 & \mathcal{X}_l \end{bmatrix},$$
$$\tilde{L}_{1ij,l} = \bar{\pi}_l^T \begin{bmatrix} \tilde{\Gamma}_{ij,l} & \tilde{B}_{\omega i,l} \end{bmatrix}, \bar{P}_l = \mathrm{diag}\{P_{1,l}, P_{2,l}\},$$
$$\tilde{\Gamma}_{ij,l} = \tilde{A}_{ij,l}\bar{e}_1 + \tilde{A}_{dij,l}\bar{e}_3 + \tilde{B}_{i,l}\bar{e}_5 + \tilde{C}_{i,l}\bar{e}_6,$$
$$\tilde{A}_{ij,l} = \begin{bmatrix} P_{1,l}A_{i,l} & 0 \\ 0 & P_{2,l}A_{i,l} - W_{i,l}C_{1j,l} \end{bmatrix},$$
$$\tilde{A}_{dij,l} = \begin{bmatrix} 0 & 0 \\ 0 & -W_{i,l}C_{2j,l} \end{bmatrix},$$
$$\tilde{B}_{i,l} = \begin{bmatrix} P_{1,l}B_{i,l} & 0 \\ 0 & P_{2,l}B_{i,l} \end{bmatrix},$$
$$\tilde{C}_{i,l} = \begin{bmatrix} P_{1,l}C_{i,l} & 0 \\ 0 & P_{2i,l}C_{i,l} \end{bmatrix}, \tilde{B}_{\omega i,l} = \begin{bmatrix} P_{1,l}B_{\omega i,l} \\ P_{2,l}B_{\omega i,l} \end{bmatrix},$$

and other matrix parameters are defined in Theorem 1. Moreover, the desired state estimator is given by

$$H_{i,l} = P_{2,l}^{-1}W_{i,l}. \tag{32}$$

Proof. Note that $\Phi = \begin{bmatrix} I_n & 0_{n\times n} \end{bmatrix}$, it follows that $\bar{L}_{2i,l}$ and $\bar{L}_{3i,l}$ are relevant to Z_1 and Z_2, respectively. Therefore, to solve Theorem 1, taking account into the special forms of $\bar{A}_{ij,l}, \bar{A}_{dij,l}, \bar{B}_{i,l}, \bar{C}_{ij,l}, \bar{B}_{\omega i,l}$, letting $\bar{P}_l = \mathrm{diag}\{P_{1,l}, P_{2,l}\}$ and $W_{i,l} = P_{2,l}H_{i,l}$. Then, (10a) and (10b) are equivalent to standard LMIs (31a) and (31b), respectively. This completes the proof.

Remark 4. Theorem 2 gives the less conservative LMI conditions on strictly stochastic $(\mathcal{X}_l, \mathcal{S}_l, \mathcal{R}_l)$-$\alpha$-dissipativity of the augmented fuzzy DMJNNs (4). It can be seen from the proof of Theorem 1 that, when $\omega(k) \equiv 0$, we can deduce from (29) that that $\mathbf{E}\{\Delta V(k)\} < 0$ holds, which imply that there exists a scalar $\epsilon > 0$ such that

$$\mathbf{E}\{\Delta V(k)\} < -\epsilon\|\bar{x}(k)\|^2. \tag{33}$$

Taking into account that the specific forms of (11) and $\mathbf{E}\{\Delta V(k)\}$, we can be easily verify that the augmented fuzzy DMJNNs (4) are exponentially stable in mean square sense.

Assuming $m = 2q$, and choosing $\mathcal{R}_l = \delta\gamma^2 I, \gamma > 0$, $S_l = (1-\delta)I, \delta \in [0,1]$ and $\mathcal{X}_l = -\delta I$, we give the mixed H_∞ and passive performance of the augmented DMJNNs (4) as follows.

Corollary 1. Given scalar $\delta \in [0,1]$, under Assumption 1, the augmented DMJNNs (4) are with a strictly mixed H_∞ and passive performance, if there exist scalars $\alpha > 0, \gamma > 0$, sets of scalars $\{\theta_{1,l} > 0, l \in \mathcal{L}\}$ and $\{\theta_{2,l} > 0, l \in \mathcal{L}\}$, symmetric positive-definite matrices Q_1, Q_2, Q_3, Z_1, Z_2, sets of symmetric positive-definite matrices $\{P_{1,l}, l \in \mathcal{L}\}$ and $\{P_{2,l}, l \in \mathcal{L}\}$, a set of matrices $\{W_{i,l}, i \in \Lambda, l \in \mathcal{L}\}$, for all $l \in \mathcal{L}$ and $i, j \in \Lambda$, such that

$$\tilde{\Xi}_{ij,l} + \tilde{\Xi}_{ji,l} < 0, \quad i < j, \tag{34a}$$
$$\tilde{\Xi}_{ii,l} < 0, \tag{34b}$$

where

$$\tilde{\Xi}_{ij,l} = \begin{bmatrix} \tilde{\Xi}_{11i,l} & * & * & * & * \\ \tilde{L}_{1ij,l} & -\bar{\Xi}_{22,l} & * & * & * \\ \bar{L}_{2i,l} & 0 & -Z_1 & * & * \\ \bar{L}_{3i,l} & 0 & 0 & -Z_2 & * \\ \tilde{L}_{4i,l} & 0 & 0 & 0 & -\delta I_m \end{bmatrix},$$
$$\tilde{\Xi}_{11i,l} = \begin{bmatrix} \Xi_{11,l}^{11} & * \\ -(1-\delta)\bar{E}_{i,l}\bar{e}_1 & -(\delta\gamma^2 - \alpha)I_m \end{bmatrix},$$
$$\tilde{L}_{4i,l} = \begin{bmatrix} -\delta\bar{E}_{i,l}\bar{e}_1 & 0_{m\times m} \end{bmatrix},$$

and other matrix parameters are showed in Theorem 2. Moreover, the desired state estimator can be chosen by (32).

In the following, to reduce the dimensions of LMIs in Theorem 2, we consider the dissipativity analysis of the error dynamics of (2) and (3). Let $\hat{z}(k) = z(k) - \tilde{z}(k)$. Then, the resulting error fuzzy DMJNNs can be described as

$$\begin{aligned} e(k+1) &= \hat{A}_l e(k) + \hat{A}_{d,l}e(k-d(k)) \\ &\quad + \hat{B}_l\hat{F}(e(k)) + \hat{C}_l\hat{G}(e(k-d(k))) \\ &\quad + \hat{B}_{\omega,l}\omega(k), \\ \hat{z}(k) &= \hat{E}_l e(k), \end{aligned} \tag{35}$$

where

$$\hat{A}_l = \textstyle\sum_{i=1}^{f}\sum_{j=1}^{f} h_i(k)h_j(k)(A_{i,l} - H_{i,l}C_{1j,l}),$$
$$\hat{A}_{d,l} = \textstyle\sum_{i=1}^{f}\sum_{j=1}^{f} h_i(k)h_j(k)(-H_{i,l}C_{2j,l}),$$
$$\hat{B}_l = \textstyle\sum_{i=1}^{f} h_i(k)B_{i,l}, \hat{B}_{\omega,l} = \sum_{i=1}^{f} h_i(k)B_{\omega i,l},$$
$$\hat{C}_l = \textstyle\sum_{i=1}^{f} h_i(k)C_{i,l}, \hat{E}_l = \sum_{i=1}^{f} h_i(k)E_{i,l},$$
$$\hat{F}(e(k)) = f(x(k)) - f(\tilde{x}(k)),$$
$$\hat{G}(e(k-d(k))) = g(x(k-d(k))) - g(\tilde{x}(k-d(k))).$$

We choose the following stochastic Lyapunov functional:

$$\hat{V}(x(k), e(k), r_k = l, k) = \textstyle\sum_{j=1}^{3}\hat{V}_j(e(k), r_k = l, k), \tag{36}$$

where

$$\hat{V}_1(e(k), r_k = l, k) = e^T(k)P_l e(k),$$
$$\hat{V}_2(e(k), r_k = l, k) = \textstyle\sum_{m=k-d_1}^{k-1} e^T(m)\hat{Q}_1 e(m) + \sum_{m=k-d_2}^{k-1} e^T(m)\hat{Q}_2 e(m),$$
$$\hat{V}_3(e(k), r_k = l, k) = \textstyle\sum_{m=k-d(k)}^{k-1} e^T(m)\hat{Q}_3 e(m) + \sum_{j=-d_2+1}^{-d_1}\sum_{m=k+j}^{k-1} e^T(m)\hat{Q}_3 e(m).$$

Then, by using the similar approach in Theorems 1 and 2, we can obtain the dissipative result of the error fuzzy DMJNNs (35) as follows.

Theorem 3. Under Assumption 1, the error fuzzy DMJNNs (35) are strictly stochastically $(\mathcal{X}_l, \mathcal{S}_l, \mathcal{R}_l)$-$\alpha$-dissipative if there exist a scalar $\alpha > 0$, sets of scalars $\{\theta_{1,l} > 0, l \in \mathcal{L}\}$ and $\{\theta_{2,l} > 0, l \in \mathcal{L}\}$, symmetric positive-definite matrices $\hat{Q}_1, \hat{Q}_2, \hat{Q}_3$, a set of symmetric positive-definite matrices

$\{P_l, l \in \mathcal{L}\}$, a set of matrices $\{W_{i,l}, i \in \Lambda, l \in \mathcal{L}\}$, for all $l \in \mathcal{L}$ and $i, j \in \Lambda$, such that

$$\Lambda_{ij,l} + \Lambda_{ji,l} < 0, \quad i < j, \tag{37a}$$
$$\Lambda_{ii,l} < 0, \tag{37b}$$

where

$$\begin{aligned}
&\Lambda_{ij,l} = \begin{bmatrix} \Lambda_{11i,l} & * & * \\ \Lambda_{21ij,l} & -\Lambda_{22,l} & * \\ \Lambda_{31i,l} & 0 & \mathcal{X}_l \end{bmatrix},\\
&\Lambda_{11i,l} = \begin{bmatrix} \Lambda_{11,l}^{11} & * \\ -\mathcal{S}_l^T E_{i,l} e_1 & -(\mathcal{R}_l - \alpha I_m) \end{bmatrix},\\
&\Lambda_{11,l}^{11} = e_1^T(\hat{Q}_1 + \hat{Q}_2 + (d_{12}+1)\hat{Q}_3 - P_l)e_1 \\
&\quad -e_2^T \hat{Q}_1 e_2 - e_3^T \hat{Q}_3 e_3 - e_4^T \hat{Q}_2 e_4 \\
&\quad -\theta_{1,l}\begin{bmatrix} e_1 \\ e_5 \end{bmatrix}^T \begin{bmatrix} \hat{F}_1 & * \\ \hat{F}_2 & I_n \end{bmatrix} \begin{bmatrix} e_1 \\ e_5 \end{bmatrix} \\
&\quad -\theta_{2,l}\begin{bmatrix} e_3 \\ e_6 \end{bmatrix}^T \begin{bmatrix} \hat{G}_1 & * \\ \hat{G}_2 & I_n \end{bmatrix} \begin{bmatrix} e_3 \\ e_6 \end{bmatrix},\\
&\hat{F}_1 = (U_1^T U_2 + U_1 U_2^T)/2, \hat{F}_2 = -(U_1 + U_2)/2,\\
&\hat{G}_1 = (V_1^T V_2 + V_1 V_2^T)/2, \hat{G}_2 = -(V_1 + V_2)/2,\\
&\Lambda_{21ij,l} = \pi_l^T \begin{bmatrix} \Upsilon_{ij,l} & P_l B_{\omega i,l} \end{bmatrix},\\
&\pi_l = \begin{bmatrix} \sqrt{\pi_{l1}} I_n & \sqrt{\pi_{l2}} I_n & \cdots & \sqrt{\pi_{ls}} I_n \end{bmatrix},\\
&\Lambda_{31i,l} = \begin{bmatrix} \mathcal{X}_l E_{i,l} e_1 & 0_{q\times m} \end{bmatrix},\\
&\Lambda_{22,l} = \mathrm{diag}\{2P_l - P_1, 2P_l - P_2, \cdots, 2P_l - P_s\},\\
&\Upsilon_{ij,l} = (P_l A_{i,l} - W_{i,l} C_{1j,l})e_1 - W_{i,l} C_{2j,l} e_3 \\
&\quad + P_l B_{i,l} e_5 + P_l C_{i,l} e_6,\\
&e_\nu = \begin{bmatrix} 0_{n\times(\nu-1)n} & I_n & 0_{n\times(6-\nu)n} \end{bmatrix} (\nu = 1, 2, \cdots, 6).
\end{aligned}$$

In addition, the desired state estimator is constructed by

$$H_{i,l} = P_l^{-1} W_{i,l}. \tag{38}$$

Remark 5. To reduce the computational complexity, the provided sufficient criteria in Theorem 3 can guarantee that the error fuzzy DMJNNs (35) are stochastically dissipative, which can not verify that the fuzzy neural networks (1) are stochastically dissipative. However, the obtained feasible criteria in Theorem 2 can ensure stochastic dissipativity of the fuzzy DMJNNs (1), and we can see from the following numerical examples that the estimated states can better approximate the real states. Moreover, delay-divisioning technique and slack matrix approach have been also adapted to get the dissipative conditions with less conservatism at the expense of heavier computing burden in [34].

Assuming $m = q$, and choosing $\mathcal{R}_l = \delta\gamma^2 I, \gamma > 0$, $S_l = (1-\delta)I, \delta \in [0,1]$ and $\mathcal{X}_l = -\delta I$, we give the mixed H_∞ and passive performance of the error fuzzy DMJNNs (35) as follows.

Corollary 2. Given scalar $\delta \in [0,1]$, under Assumption 1, the error fuzzy DMJNNs (35) are with a strictly mixed H_∞ and passive performance, if there exist scalars $\alpha > 0, \gamma > 0$, sets of scalars $\{\theta_{1,l} > 0, l \in \mathcal{L}\}$ and $\{\theta_{2,l} > 0, l \in \mathcal{L}\}$, symmetric positive-definite matrices $\hat{Q}_1, \hat{Q}_2, \hat{Q}_3$, sets of symmetric positive-definite matrices $\{P_l, l \in \mathcal{L}\}$, a set of matrices $\{W_{i,l}, i \in \Lambda, l \in \mathcal{L}\}$, for all $l \in \mathcal{L}$ and $i, j \in \Lambda$, such that

$$\tilde{\Lambda}_{ij,l} + \tilde{\Lambda}_{ji,l} < 0, \quad i < j, \tag{39a}$$
$$\tilde{\Lambda}_{ii,l} < 0, \tag{39b}$$

where

$$\begin{aligned}
&\tilde{\Lambda}_{ij,l} = \begin{bmatrix} \tilde{\Lambda}_{11i,l} & * & * \\ \Lambda_{21ij,l} & -\Lambda_{22,l} & * \\ \tilde{\Lambda}_{31i,l} & 0 & -\delta I_m \end{bmatrix},\\
&\tilde{\Lambda}_{11i,l} = \begin{bmatrix} \Lambda_{11,l}^{11} & * \\ -(1-\delta)E_{i,l}e_1 & -(\delta\gamma^2 - \alpha)I_m \end{bmatrix},\\
&\tilde{\Lambda}_{31i,l} = \begin{bmatrix} -\delta E_{i,l} e_1 & 0_{m\times m} \end{bmatrix},
\end{aligned}$$

and other matrix parameters are defined in Theorem 3. Moreover, the desired state estimator is in the form of (38).

Remark 6. In our main results, the criteria of dissipativity and mixed H_∞ and passivity are derived for fuzzy DMJNNs in terms of LMIs. The conditions can be readily checked by the solvability of feasible problems based on LMIs. Note that, given specific parameters of $\mathcal{X}_l, \mathcal{S}_l$ and $\mathcal{R}_l$ in (7), related results on stability analysis, H_∞ performance and positive real performance can be also derived for the class of fuzzy DMJNNs by applying the similar techniques.

IV. Numerical examples

In this section, we give two simulation examples to illustrate the effectiveness of the developed methods.

Example 1. Consider the fuzzy DMJNNs involving two modes in model (1) described as follows:

Plant Rule 1: IF $x_1(k)$ is μ_{11}, THEN

$$\begin{aligned}
x(k+1) &= A_{1,l}x(k) + B_{1,l}f(x(k)) \\
&\quad + C_{1,l}g(x(k-d(k))) + B_{\omega 1,l}\omega(k),\\
y(k) &= C_{11,l}x(k) + C_{21,l}x(k-d(k)),\\
z(k) &= E_{1,l}x(k),
\end{aligned}$$

Plant Rule 2: IF $x_1(k)$ is μ_{21}, THEN

$$\begin{aligned}
x(k+1) &= A_{2,l}x(k) + B_{2,l}f(x(k)) \\
&\quad + C_{2,l}g(x(k-d(k))) + B_{\omega 2,l}\omega(k),\\
y(k) &= C_{12,l}x(k) + C_{22,l}x(k-d(k)),\\
z(k) &= E_{2,l}x(k),
\end{aligned}$$

where the membership functions $h_1(k)$ and $h_2(k)$ are defined, respectively, as $h_1(k) = h(x_1(k))$ and $h_2(k) = 1 - h(x_1(k))$ with

$$h(x_1(k)) = \begin{cases} 0.5(1 - x_1(k)), & |x_1(k)| < 1 \\ 1, & |x_1(k)| \geq 1. \end{cases}$$

and

$$\begin{aligned}
&A_{1,1} = \begin{bmatrix} 0.05 & 0 \\ 0 & 0.04 \end{bmatrix}, B_{1,1} = \begin{bmatrix} 0.03 & 0.02 \\ -0.02 & 0.01 \end{bmatrix},\\
&C_{1,1} = \begin{bmatrix} 0.05 & 0.03 \\ -0.01 & 0.03 \end{bmatrix}, A_{2,1} = \begin{bmatrix} 0.06 & 0 \\ 0 & 0.03 \end{bmatrix},\\
&B_{2,1} = \begin{bmatrix} 0.05 & 0.02 \\ -0.04 & 0.02 \end{bmatrix}, C_{2,1} = \begin{bmatrix} 0.02 & 0 \\ -0.02 & 0.01 \end{bmatrix},\\
&A_{1,2} = \begin{bmatrix} 0.06 & 0 \\ 0 & 0.04 \end{bmatrix}, B_{1,2} = \begin{bmatrix} 0.04 & 0.02 \\ -0.04 & 0.01 \end{bmatrix},\\
&C_{1,2} = \begin{bmatrix} -0.05 & 0.03 \\ -0.01 & 0.03 \end{bmatrix}, A_{2,2} = \begin{bmatrix} 0.06 & 0 \\ 0 & 0.05 \end{bmatrix},
\end{aligned}$$

$$B_{2,2} = \begin{bmatrix} 0.04 & 0.02 \\ -0.01 & 0.02 \end{bmatrix}, C_{2,2} = \begin{bmatrix} 0.06 & 0 \\ 0.05 & 0.01 \end{bmatrix},$$
$$B_{w1,1} = B_{w2,1} = B_{w1,2} = B_{w2,2} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$
$$C_{11,1} = C_{12,1} = C_{11,2} = C_{12,2} = \begin{bmatrix} 1 & 1 \end{bmatrix},$$
$$C_{21,1} = C_{22,1} = C_{21,2} = C_{22,2} = \begin{bmatrix} -0.01 & 0.02 \end{bmatrix},$$
$$E_{1,1} = E_{2,1} = E_{1,2} = E_{2,2} = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

The nonlinear activation functions $f(x(k))$ and $g(x(k))$ are taken as

$$f(x(k)) = g(x(k)) = \begin{bmatrix} 0.01x_1 + \tanh(0.01x_2) \\ 0.01x_1 - 0.01x_2 + \tanh(0.02x_1) \end{bmatrix}.$$

Then, it follows that the conditions (5a) and (5b) can be satisfied with

$$U_1 = V_1 = \begin{bmatrix} 0.01 & 0 \\ 0.01 & -0.01 \end{bmatrix}, U_2 = V_2 = \begin{bmatrix} 0.01 & 0.01 \\ 0.03 & -0.01 \end{bmatrix}.$$

In addition, assume that the transition rate matrix is given by $\Pi = \begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{bmatrix}$.

Furthermore, we choose $\mathcal{R}_1 = \mathcal{R}_2 = \mathcal{S}_1 = \mathcal{S}_2 = I_2$, $\mathcal{X}_1 = \mathcal{X}_2 = -I_2$ and $\alpha = 0.01$. The time-varying delay is chosen by $d(k) = 0.5(3 - \cos(k))$, which satisfies $1 \le d(k) \le 2$. Applying Theorem 2, the feasible solutions of dissipative performance are given by

$$P_{1,1} = \begin{bmatrix} 14.8489 & 2.4280 \\ 2.4280 & 13.9756 \end{bmatrix},$$
$$P_{2,1} = \begin{bmatrix} 17.5554 & 2.0248 \\ 2.0248 & 16.2925 \end{bmatrix},$$
$$P_{1,2} = \begin{bmatrix} 13.1281 & 3.2245 \\ 3.2245 & 12.2657 \end{bmatrix},$$
$$P_{2,2} = \begin{bmatrix} 14.3863 & 3.0052 \\ 3.0052 & 12.9964 \end{bmatrix},$$
$$Q_1 = \begin{bmatrix} 1.2387 & 0.2445 & -0.0279 & 0.0085 \\ 0.2445 & 2.1862 & 0.0775 & -0.0117 \\ -0.0279 & 0.0775 & 3.4007 & 0.3812 \\ 0.0085 & -0.0117 & 0.3812 & 3.5489 \end{bmatrix},$$
$$Q_2 = \begin{bmatrix} 2.9509 & 0.5073 & -0.0327 & 0.0094 \\ 0.5073 & 2.9336 & 0.0771 & -0.0117 \\ -0.0327 & 0.0771 & 3.4006 & 0.3810 \\ 0.0094 & -0.0117 & 0.3810 & 3.5451 \end{bmatrix},$$
$$Q_3 = \begin{bmatrix} 3.1566 & 0.6289 & -0.0276 & 0.0057 \\ 0.6289 & 3.1847 & 0.0824 & -0.0083 \\ -0.0276 & 0.0824 & 2.4433 & 0.3893 \\ 0.0057 & -0.0083 & 0.3893 & 2.8312 \end{bmatrix},$$
$$Z_1 = \begin{bmatrix} 3.0822 & -0.2040 \\ -0.2040 & 3.0928 \end{bmatrix},$$
$$Z_2 = \begin{bmatrix} 3.0817 & -0.2616 \\ -0.2616 & 2.4913 \end{bmatrix},$$
$$W_{1,1} = \begin{bmatrix} -1.8019 \\ -1.1148 \end{bmatrix}, W_{2,1} = \begin{bmatrix} -0.8192 \\ -0.2271 \end{bmatrix},$$
$$W_{1,2} = \begin{bmatrix} -0.6393 \\ 0 \end{bmatrix}, W_{2,2} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$
$$\theta_{1,1} = 0.8611, \theta_{2,1} = 0.8361, \theta_{1,2} = 1.1060, \theta_{2,2} = 0.7866.$$

Therefore, the augmented fuzzy DMJNNs (4) are strictly stochastically (I_2, I_2, I_2)-0.01-dissipative, and the designed state estimator is derived by

$$H_{1,1} = \begin{bmatrix} -0.0961 \\ -0.0565 \end{bmatrix}, H_{2,1} = \begin{bmatrix} -0.0457 \\ -0.0083 \end{bmatrix},$$
$$H_{1,2} = \begin{bmatrix} -0.0467 \\ 0.0108 \end{bmatrix}, H_{2,2} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

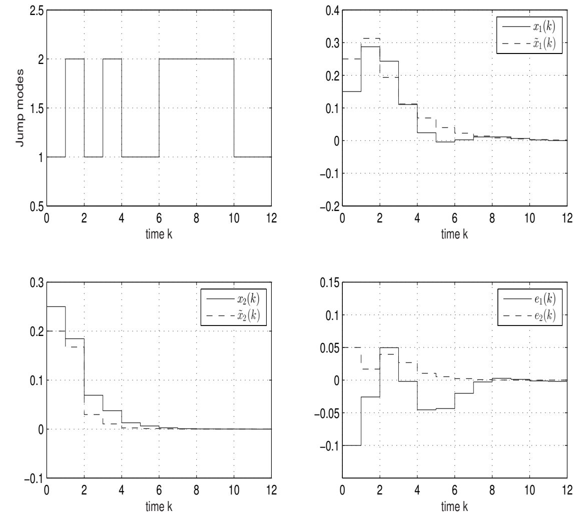

Fig. 1. The simulation of the system (4) in Example 1.

Now, we assume that the external disturbance input $\omega(k) = \begin{bmatrix} \omega_1(k) & \omega_2(k) \end{bmatrix}^T$ is a two-dimensional vector, which both $\omega_1(k)$ and $\omega_2(k)$ are Gaussian white noises with mean 0 and variance 1. With the initial mode $r_0 = 1$ and the initial states $\phi^T(j) = \begin{bmatrix} 0.15 & 0.25 \end{bmatrix}$ and $\tilde{\phi}^T(j) = \begin{bmatrix} 0.25 & 0.2 \end{bmatrix}$ for all $j \in \{-2, -1, 0\}$, the jump modes and state response of the system (4) are given in Figure 1.

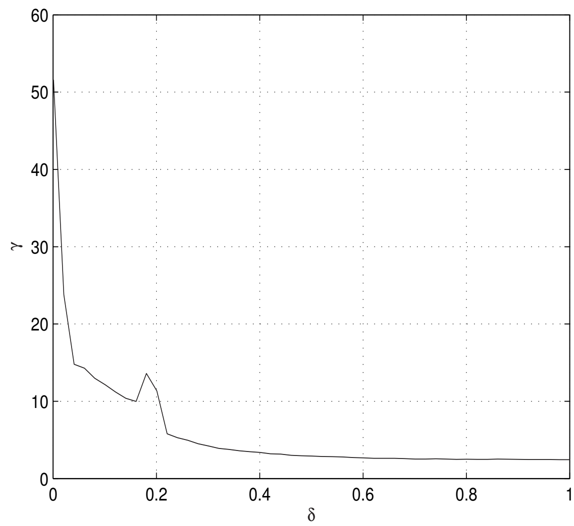

Fig. 2. The optimal value of γ.

Remark 7. Assume that $\alpha = 0$ and the other parameters are defined in Example 1, LMIs (34a) and (34b) are feasible and the optimal value with different values of $\delta \in [0,1]$ is shown in Figure 2. We can obtain from Corollary 1 that the H_∞ norm condition satisfies $\gamma \le 2.4479$ when $\delta = 1$.

Example 2. Consider the fuzzy DMJNNs (1) involving two modes with the following parameters

$$B_{w1,1} = B_{w2,1} = B_{w1,2} = B_{w2,2} = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}.$$

Moreover, the other matrix parameters, the nonlinear activation functions, the time-varying delay and the transition rate matrix are defined in Example 1.

Let $\mathcal{R}_1 = \mathcal{R}_2 = \mathcal{S}_1 = \mathcal{S}_2 = 1$, $\mathcal{X}_1 = \mathcal{X}_2 = -1$ and $\alpha = 0.01$. Then, LMIs (37a) and (37b) are feasible, and the optimal solutions of dissipative performance can be obtained from Theorem 3 as follows

$$\begin{aligned}
P_1 &= \begin{bmatrix} 25.8457 & 0.0098 \\ 0.0098 & 25.6312 \end{bmatrix}, \\
P_2 &= \begin{bmatrix} 26.2510 & 0.0074 \\ 0.0074 & 26.0203 \end{bmatrix}, \\
\hat{Q}_1 &= \begin{bmatrix} 6.1179 & 0.0045 \\ 0.0045 & 6.3091 \end{bmatrix}, \\
\hat{Q}_2 &= \begin{bmatrix} 6.1179 & 0.0045 \\ 0.0045 & 6.3091 \end{bmatrix}, \\
\hat{Q}_3 &= \begin{bmatrix} 3.6147 & 0.0042 \\ 0.0042 & 3.7877 \end{bmatrix}, \\
W_{1,1} &= \begin{bmatrix} 0.2169 \\ 0.5031 \end{bmatrix}, W_{2,1} = \begin{bmatrix} 0.3486 \\ 0.3717 \end{bmatrix}, \\
W_{1,2} &= \begin{bmatrix} 0.3241 \\ 0.4649 \end{bmatrix}, W_{2,2} = \begin{bmatrix} 0.4242 \\ 0.6865 \end{bmatrix}, \\
&\theta_{1,1} = 14.0885, \theta_{2,1} = 14.0827, \\
&\theta_{1,2} = 14.0861, \theta_{2,2} = 14.1041.
\end{aligned}$$

Therefore, the designed state estimator is derived by

$$\begin{aligned}
H_{1,1} &= \begin{bmatrix} 0.0084 \\ 0.0196 \end{bmatrix}, H_{2,1} = \begin{bmatrix} 0.0135 \\ 0.0145 \end{bmatrix}, \\
H_{1,2} &= \begin{bmatrix} 0.0123 \\ 0.0179 \end{bmatrix}, H_{2,2} = \begin{bmatrix} 0.0162 \\ 0.0264 \end{bmatrix}.
\end{aligned}$$

Given that the same initial conditions as Example 1, and supposed that the external disturbance $\omega(k)$ is a zero mean Gaussian white noise with variance 1, then the simulation of the model (35) is depicted in Figure 3, which reveals that the model in (35) is exponentially stable in mean square sense.

Remark 8. Assume that $\alpha = 0$ and the other parameters are defined in Example 2. We can choose γ^2 as the optimal value and optimize over value γ^2 in LMIs (39a) and (39b). Figure 4 shows the optimal value with different values of $\delta \in [0,1]$. When $\delta = 1$, we can obtain from Corollary 2 that the H_∞ norm condition satisfies $\gamma \le 8.0573$.

From the above simulation results, we can see that all the state variables including real states and estimated states converge to their equilibrium points. Moreover, the estimated states of Example 1 can better approximate real states.

V. Conclusions

This paper studied the dissipativity analysis and design of DMJNNs with time-varying delays and involving sector-bounded activation functions represented by Takagi-Sugeno fuzzy model. Applying piecewise Lyapunov-Krasovskii functional approach and stochastic analysis technique, sufficient criteria were presented to guarantee that the augmented or error fuzzy jump neural networks are stochastically dissipative. Then, feasible problems of dissipativity were established to solve the dissipative state estimation problems by using matrix decomposition approaches. The mixed H_∞ and passive analysis and design are also derived for fuzzy DMJNNs. These criteria can be developed in terms of LMIs. Numerical examples are also given to illustrate the effectiveness of the proposed approaches. An important future research direction is to extend our dissipative conditions to the case when there exist multiple discrete delays and distributed delays.

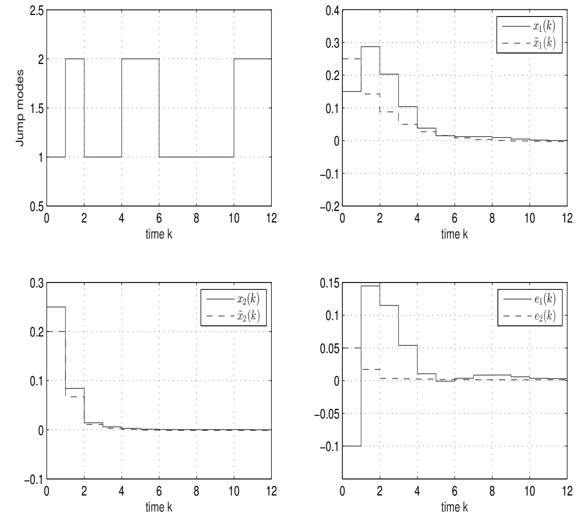

Fig. 3. The simulation of the system (35) in Example 2.

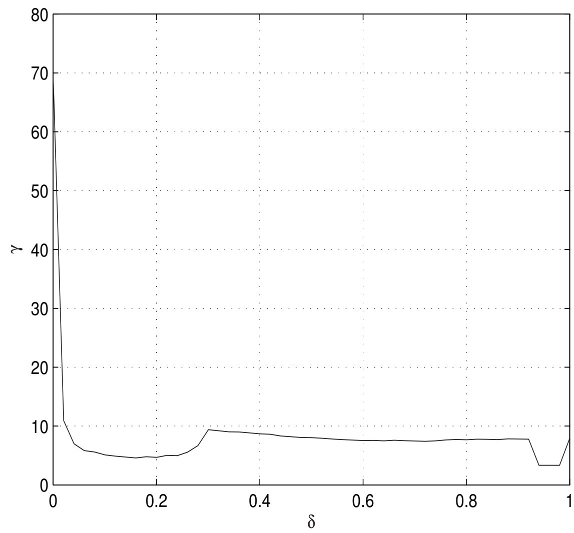

Fig. 4. The optimal value of γ.

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