Research Article

Distance-Based Congestion Pricing with Day-to-Day Dynamic Traffic Flow Evolution Process

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This paper studies the distance-based congestion pricing in a network considering the day-to-day dynamic traffic flow evolution process. It is well known that, after an implementation or adjustment of a new congestion toll scheme, the network environment will change and traffic flows will be nonequilibrium in the following days; thus it is not suitable to take the equilibrium-based indexes as the objective of the congestion toll. In the context of nonequilibrium state, prior research proposed a mini–max regret model to solve the distance-based congestion pricing problem in a network considering day-to-day dynamics. However, it is computationally demanding due to the calculation of minimal total travel cost for each day among the whole planning horizon. Therefore, in order to overcome the expensive computational burden problem and make the robust toll scheme more practical, we propose a new robust optimization model in this paper. The essence of this model, which is an extension of our prior work, is to optimize the worst condition among the whole planning period and ameliorate severe traffic congestions in some bad days. Firstly, a piecewise linear function is adopted to formulate the nonlinear distance toll, which can be encapsulated to a day-to-day dynamics context. A very clear and concise model named logit-type Markov adaptive learning model is then proposed to depict commuters' day-to-day route choice behaviors. Finally, a robust optimization model which minimizes the maximum total travel cost among the whole planning horizon is formulated and a modified artificial bee colony algorithm is developed for the robust optimization model.

1. Introduction

Congestion toll is generally regarded as a potent economic instrument for transportation demand management (TDM) to alleviate the traffic congestion and improve the system performance in urban areas and also has received more and more attention both academically and practically. Since the successful implementation of congestion pricing in Singapore from 1975, many countries and cities (such as Norway, London, Stockholm and Milan) have implemented a road congestion pricing policy, which has achieved remarkable success in terms of easing urban traffic congestion [1]. However, all of the existing and implemented congestion toll schemes adopt a unique toll method, which makes the inequitable and ineffective problem. Therefore, in order to give full play to

congestion pricing in alleviating urban traffic congestion and improve the fairness and effectiveness of congestion pricing, it is necessary to consider the travel distance inside the charging cordon and establish the distance-based congestion toll scheme.

The distance-based toll scheme receives more and more attention recently in both the academic community and industrial circles. References [2–4] assumed that the distance-based toll is linearly proportional to the distance traveled in the charging area. However, it may be more efficient and effective in using the nonlinear toll function in the practical congestion toll scheme according to some recent studies. Lawphongpanich and Yin [5] assumed that the distance-based toll is a nonlinear form and used a piecewise linear function with two intervals to represent it. References [1, 6–9]

studied the distance-based toll using the piecewise linear toll function with multiple intervals to approximate the nonlinear toll function. It is worth noting that the next generation of road pricing systems in Singapore will adopt the distance-based toll scheme based on global navigation satellite system (GNSS) technology [10].

Generally, the total travel cost is regarded as the optimization objective of the congestion toll design problem. Most of the literatures studying the congestion pricing problem calculate the total travel cost (TTC) based on the equilibrium flows and make an evaluation based on the calculated TTC. However, when a toll pattern is implemented, the route flows will be totally different from day to day, because the implemented toll policy is an important component which will influence travelers' route choice behaviors. The system cannot reach an equilibrium state overnight. Therefore, for the optimal toll design problem during the whole planning period, the day-to-day dynamics models can better describe the network flow conditions, rather than the final equilibrium state. Besides, to avoid the complicated implementation of governments and confusions of travelers on the toll in practice, it is necessary to levy an unchanged toll in the whole period D; for instance, Singapore's electronic road pricing (ERP) toll is adjusted every three months [4, 5] and kept unchanged in-between; thus D can be set as three months in this study.

During the whole period of D, the TTC will change from day to day because the traffic flows will change from day to day. Therefore, no toll pattern can give rise to a minimal TTC in all days of D. Liu et al. [7] proposed a mini–max regret model to solve the day-to-day dynamic congestion pricing (DCP) problem. However, this model is computationally demanding due to the calculation of minimal total travel cost for each day among the whole planning horizon. Therefore, in order to overcome the expensive computational burden problem and make the robust toll scheme more practical, we can use the mini–max total travel cost model to replace the mini–max regret model. The essence of this model, which is an extension of our previous work, is to optimize the worst condition among the whole planning period and ameliorate severe traffic congestions in some bad days.

The day-to-day dynamic flow evolution process is the foundation for the day-to-day DCP problem; lots of research work focus on the day-to-day dynamic flow evolution process [6–14]. As for the DCP, Wie and Tobin [15] considered the day-to-day DCP problem and used a convex control model to solve it. Friesz et al. [16] studied the day-to-day DCP aiming at maximizing the net present value of social welfare. More recently, Guo et al. [17] studied the dynamic tolls on each day and they are merely determined by the flows and tolls on the previous day. Ye et al. [18] studied the marginal-cost pricing scheme considering the day-to-day dynamics based on the trial-and-error method. Tan et al. Reference [19] studied the day-to-day DCP problem aiming at minimizing the total system cost and time. However, all of the aforementioned studies focus on deterministic day-to-day DCP problem. Recently, Rambha and Boyles [20] studied the stochastic day-to-day DCP problem. However, the objective

function in Rambha and Boyles [20] does not consider the network performances of each day. Cheng et al. [21] made a comprehensive review of urban dynamic congestion pricing and highlighted that there was an emerging research need to investigate the DCP problem.

The contributions of this paper are twofold. On the one hand, a finite learning process model named logit-type Markov adaptive learning model is proposed to depict commuters' day-to-day route choice behaviors. On the other hand, a mini–max total travel cost model, which can overcome the expensive computational burden problem in previous work and make the congestion toll scheme more practical, is proposed to solve the congestion toll problem considering nonequilibrium flow evolution processes. This paper is structured as follows. The next section first introduces the nonlinear distance toll which can be approximated with a piecewise linear toll function. A logit-type Markov adaptive learning model is then proposed in Section 3. Afterwards, a mini–max model for the optimal toll pattern that minimizes the maximum total travel cost among the whole planning horizon is introduced in Section 4, and a modified artificial bee colony (ABC) algorithm is developed for the robust optimization model in Section 5. Finally, conclusions are drawn in Section 6.

2. Problem Statement

A strongly connected network, denoted by $G = (N, A)$, is considered. N denotes the set of nodes and A denotes the set of directed links. W denotes the set of origin-destination (OD) pairs, and R^w denotes the set of routes connecting an OD pair $w \in W$. q^w is the traffic demand connecting the OD pair $w \in W$, and $\mathbf{q} = (q^w, w \in W)^T$. f_{wr} is the traffic flow on route $r \in R^w$ connecting OD pair $w \in W$, and $\mathbf{f} = (f_{wr}, r \in R^w, w \in W)^T$. v_a is the traffic flow on link $a \in A$, and $t_a(v_a)$ is the travel time (or link performance) function of link $a \in A$, which is assumed to be increasing, convex and continuously differentiable. The notations in this paper mostly follow that in Liu et al. [1, 7], which are summarized in Table 1.

A nonlinear-type distance-toll function is preferred according to Liu et al. [1, 7] and Meng et al. [8]; the general idea is to formulate the nonlinear distance-toll function $\phi(\boldsymbol{\eta})$ as a piecewise linear function in terms of the travel distance $\boldsymbol{\eta}$ in a cordon. It is clear to define the toll function $\phi(\boldsymbol{\eta})$ on the range $[\eta_0, \eta_K]$ with K equal intervals as shown in Figure 1. Note that η_0 and η_K are the minimal and maximal route length in the charging cordon, respectively. Obviously, the piecewise linear toll function composes K straight line sections, and each line section is uniquely defined by the two ends points of each interval $\mathbf{y} = (y_0, y_1, y_2, \ldots y_k, \ldots y_K)^T$ whose corresponding value of distance is $\boldsymbol{\eta} = (\eta_0, \eta_1, \eta_2, \ldots \eta_k, \ldots, \eta_K)^T$. With this piecewise linear toll function discussed above, the continuous curve of the nonlinear distance-toll function can be characterized as a number of straight lines, which can be determined by $K + 1$

| Notation	Explanation
D	The total planning period for one toll pattern.
d	The number of days after the toll implementation, $d = 1, 2, \cdots, D$.
W	The set of OD pairs.
R^w	The set of routes connecting an OD pair $w \in W$.
c_{wr}	The travel cost on route $r \in R^w$ connecting OD pair $w \in W$.
$\mathbf{f}$	The route flows over the entire network, $\mathbf{f} = (f_{wr}, r \in R^w, w \in W)^T$.
f_{wr}	The traffic flow on route $r \in R^w$ connecting OD pair $w \in W$.
$\mathbf{q}$	The travel demands, $\mathbf{q} = (q^w, w \in W)^T$.
q^w	The travel demand connecting OD pair $w \in W$.
$\mathbf{t(v)}$	The link travel time functions, $\mathbf{t(v)} = (t_a(v_a), a \in A)^T$.
$\mathbf{v}$	The link flows, $\mathbf{v} = (v_a, a \in A)^T$.
$t_a(v_a)$	The travel time function of link $a \in A$.
v_a	The traffic flow on link $a \in A$.
δ_{ar}^w	$\delta_{ar}^w = 1$ if route $r \in R^w$ contains link a, and $\delta_{ar}^w = 0$ otherwise.
$\mathbf{y}$	The vertex values, $\mathbf{y} = (y_0, y_1, y_2, \ldots y_k, \ldots y_K)^T$ of the piecewise linear toll function.
$y_{\min}, y_{\max}$	The lower and upper bound of the distance-based toll.
$\phi(\boldsymbol{\eta})$	The toll charge function.
K	The total number of the intervals in the toll function $\phi(\boldsymbol{\eta})$.
$\boldsymbol{\eta}$	Column vector of the travel distance in a cordon, $\boldsymbol{\eta} = (\eta_{wr}, r \in R^w, w \in W)^T$.
$\boldsymbol{\tau}$	Column vector of the distance-based toll $\boldsymbol{\tau} = (\tau_{wr}, r \in R^w, w \in W)^T$.
N_c, N_e, N_o	Number of the colony size, the employed bees, and the onlookers.
$\gamma_0, \gamma_1, \gamma_2$	Parameters used in the day-to-day dynamics model.
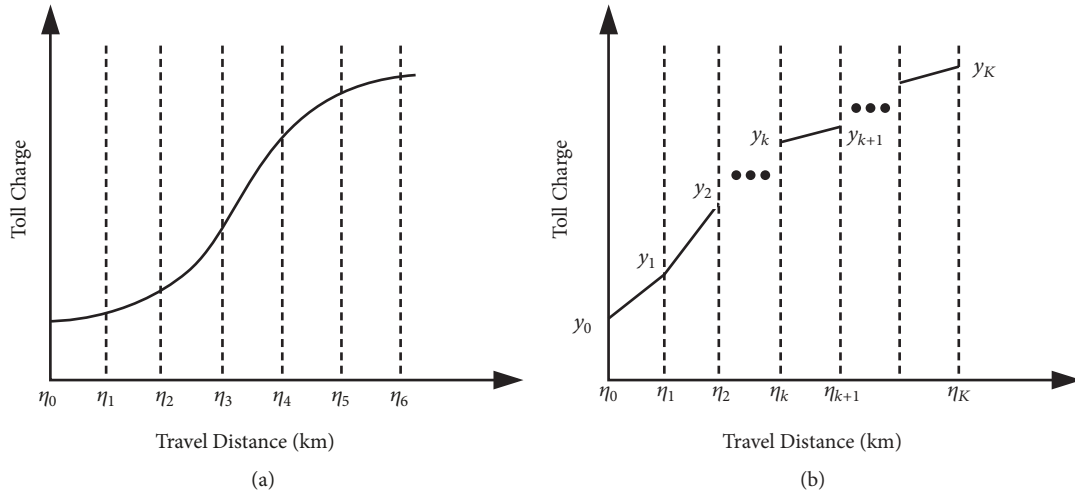
FIGURE 1: Piecewise linear toll.

points. For example, assume that $y_{\min} = 2.0$, $y_{\max} = 5.0$, and $\Delta_y = 0.5$, and then $K = (5.0 - 2.0)/0.5 = 6$, which means the piecewise linear toll function can be determined with 6 intervals, and these 6 intervals can be uniquely defined by 7 vertexes. By the way, the proposed methodology here can be easily generalized for the case of unequal intervals between the range $[\eta_0, \eta_K]$.

It is practical to define a nondecreasing distance-toll function in real life; thus we should have

$$y_{\min} = y_0 \leq y_1 \leq y_2 \leq \ldots \leq y_k \leq \ldots \leq y_K \leq y_{\max} \quad (1)$$

In a real application, we only need to know a particular route length, then we can calculate the corresponding toll value in terms of the piecewise linear toll function. Let η_{wr} denote the

length portion of route $r \in R^w$ in the cordon. Suppose η_{wr} locates in the kth distance interval of $\phi(\boldsymbol{\eta})$ shown in Figure 1, then the distance toll of route $r \in R^w$ is

$$\tau_{wr} = \phi\left(\eta_{wr}\right) = y_{k-1} + \frac{\eta_{wr} - \eta_{k-1}}{\eta_k - \eta_{k-1}}\left(y_k - y_{k-1}\right) \qquad (2)$$

The total/generalized travel cost on route $r \in R^w$ connecting OD pair $w \in W$

$$c_{wr} = \sum_a t_a \delta_{ar}^w + \frac{\tau_{wr}}{\kappa} \qquad (3)$$

where κ is the travelers' value-of-time.

From (2) we can see that each toll pattern τ is uniquely determined by the vertex value $\mathbf{y} = (y_0, y_1, y_2, \ldots y_k, \ldots y_K)^T$. Let Ω_y be the set of all the feasible $\mathbf{y}$. Then, the optimal congestion pricing becomes a problem to obtain the optimal $\mathbf{y}^* \in \Omega_y$. Before introducing the model for the optimal $\mathbf{y}^*$, the day-to-day dynamics model is first discussed in the next section.

3. Day-To-Day Dynamics Model

A reasonable day-to-day dynamics model should well reflect the realistic route adjustment process and learning behavior of commuters [15, 16]. As for the travel behavior of an individual commuter, his/her route choice on current day is dependent on his/her route choice decision as well as what others did in the previous day. Essentially, this reconsideration process of the route choice from day to day is an experienced weighted learning model [22]. Compared to the reinforcement learning model in market entry games [23], the modification of this model is that commuter's route choice in that past may influence his/her own current route choice, and this route choice may also influence other commuters' route choice behaviors [24]. As for an ordinary commuter, he/she would concern about the utilization for every route (no matter whether it was his/her currently chosen route) from day to day to make his/her route choice in the next day more rational.

Erev et al. [25] showed that subjects in the market entry games were mainly influenced by what happened in the most recent times, and this effect would be more obvious in the day-to-day dynamics process [24]. For instance, commuters' route choice behaviors are highly affected by the most recently unexpected incidents, such as unexpected network disruptions and adverse weather conditions. Thus, in the proposed day-to-day dynamics model of this paper, we assume that a commuter's route choice on the current day is only dependent on his/her and other commuters' route choice decisions in the previous day, and this is a finite learning process in essence. The most obvious characteristic is that the route flow on day $d + 1$ decreases as its actual travel cost of that route on day d increases and vice versa. It is obvious that the route choice decision of the current day is the baseline of the route choice decision of the next day. For the sake of presentation but without loss of generality, all of the commuters are assumed at the initial state of day 1, and then traffic flows of the whole network evolve from day to day.

A very clear and concise finite learning process named logit-type Markov adaptive learning model is proposed to depict commuters' day-to-day route choice behaviors. Specifically, the baseline probability of choosing route r on day $d+1$, which is the same route as day d, can be expressed through the following multinomial logit-type function:

$$\mathrm{Pr}^{baseline}\left(R_{d+1} = r\right) = \frac{\exp\left(A_{wr}^{baseline}\left(\mathbf{y}, d\right)\right)}{\sum_k \exp\left(A_{wk}^{baseline}\left(\mathbf{y}, d\right)\right)} \qquad (4)$$

where R_{d+1} is the route choice decision on day $d + 1$, $A_{wr}^{baseline}(\mathbf{y}, d) = -\gamma_1 C_{wr}(\mathbf{y}, d)$ is the *attraction of* or the *propensity towards* choosing route $r \in R^w$ with the toll pattern $\mathbf{y}$ on day d, and γ_1 is a positive response sensitivity parameter.

Then, the flow of route r can be calculated by

$$f_{wr}^{baseline}\left(\mathbf{y}, d + 1\right) = q^w \cdot \mathrm{Pr}^{baseline}\left(R_{d+1} = r\right) \qquad (5)$$

From this baseline model, we can see clearly that the routes with high travel costs on day d hold weaker attraction to commuters compared to those routes with low travel costs on the same day, and this is coincided with the nature of day-to-day dynamics process.

According to the baseline model, the following model is proposed to estimate the actual attraction of route r on day d:

$$A_{wr}\left(\mathbf{y}, d\right) = \begin{cases} \gamma_1\left(C_{wi}\left(\mathbf{y}, d\right) - C_{wr}\left(\mathbf{y}, d\right)\right) & \text{if } r \neq i \text{ and } C_{wi}\left(\mathbf{y}, d\right) < C_{wr}\left(\mathbf{y}, d\right) \\ \gamma_0 & \text{if } r = i \\ \gamma_2\left(C_{wi}\left(\mathbf{y}, d\right) - C_{wr}\left(\mathbf{y}, d\right)\right) & \text{if } r \neq i \text{ and } C_{wi}\left(\mathbf{y}, d\right) \geq C_{wr}\left(\mathbf{y}, d\right) \end{cases} \qquad (6)$$

where γ_0 and γ_2 are positive parameters. The probability of choosing route r on day $d + 1$ and the route flow can

be expressed through the following multinomial logit-type functions:

$$\Pr(R_{d+1} = r) = \frac{\exp(A_{wr}(\mathbf{y}, d))}{\sum_k \exp(A_{wk}(\mathbf{y}, d))} \qquad (7)$$

$$f_{wr}(\mathbf{y}, d+1) = q^w \cdot \Pr(R_{d+1} = r) \qquad (8)$$

It is worth noting that γ_0 and γ_2 are inertia and regret effect in this model, respectively. Compared with other day-to-day dynamics models, e.g., Guo et al. [26], He et al. [27], and Cantarella and Watling [28], the proposed model in this paper is more concise, and there is no explicit route adjustment parameter, which is actually reflected by the inertia and regret effect of γ_0 and γ_2, respectively.

4. Robust Optimization Model

As discussed in Introduction, the whole network environment will be changed after a period of days, and the days $D = 90$ in this paper. Therefore, after 90 days, a new toll scheme will be performed to optimize the whole network with the already changed network environment, making a new process of day-to-day dynamics. At same time, d should be reset to 1 when the new toll scheme is implemented. Hence, the study period is from $d = 1$ to $d = D$.

When a toll pattern $\mathbf{y}$ is implemented, the route flows will be totally different from day to day, because the implemented toll scheme is an important component which will influence travelers' route choice behaviors. Let $\mathbf{f}(\mathbf{y}, d)$ be the column vector of route flows on day d in terms of a toll pattern $\mathbf{y}$, and $\mathbf{f}(\mathbf{y}, d)$ is clearly determined by (4)-(8). The authorities' objective is to minimize the total travel costs for each day rather than merely for the final equilibrium state. Since the commuters' route choice behavior follows the logit-type Markov adaptive learning model introduced in Section 3, the system's optimal performance is reflected by the one with minimal total travel cost. On day d, the total travel cost can be calculated by

$$TTC(\mathbf{y}, d) = \mathbf{f}(\mathbf{y}, d)^T \cdot \mathbf{c}(\mathbf{y}, \mathbf{f}, d) \qquad (9)$$

It may be impossible for just one particular toll pattern to optimize the system's performance for all the days/scenarios (from day 1 to day D). If a particular toll scheme $\mathbf{y}$ is implemented on $d = 1$, then there would be a new day-to-day route flow evolution process. In the context of day-to-day dynamics, network flows are varying each day, so any toll pattern that can give rise to minimum total travel cost in a particular day may lead to bad traffic conditions in some other days. From the viewpoints of policy-maker, the deterioration of some worst days is more harmful than the loss of efficiency on the good cases, both temporally and spatially. Thus, it is a better strategy in practice that: compromising the efficiency on the optimal end of some good days so that the severe traffic congestions in some bad days could be ameliorated. Following this logic, this paper then takes the mini–max total travel cost as the objective; thus

$$\mathbf{y}(d) \in \arg\min_{\mathbf{y} \in \Omega_y} \max TTC \qquad (10)$$

subject to the day-to-day route flows introduced in Section 3.

It is clear that model (10) is a robust optimization model, which can also be deemed as a bilevel model, where the upper level is a mini–max total travel cost model, and the lower level reflects the day-to-day dynamic flows, which is discussed in Section 3. The optimal solution of model (10) is a robust toll scheme that considers the system's performance on each day of the study horizon.

5. Solution Algorithms

All of the existing solution methods (such as sensitivity analysis method, system optimal relaxation method, and gap function method) are not suitable to solve the proposed bilevel robust model due to the complexity of the flow evolution process $\mathbf{f}(\mathbf{y}, d)$. More specifically, the day-to-day dynamics model of $\mathbf{f}(\mathbf{y}, d)$ has no closed form when $d > 2$, in spite of an initial flow pattern $\mathbf{f}(\mathbf{y}, 1)$ is given. Therefore, a heuristic algorithm named artificial bee colony algorithm is adopted in this paper.

The ABC algorithm was originally proposed by Karaboga [29] for solving unimodal and multimodal numerical optimization problems. Recently, this algorithm is used to solve transportation problems [3, 22–25, 27]. We also use the ABC algorithm to figure out the mini–max TTC problem here due to its advantages of good local search mechanism which can enhance the solution quality compared to other evolutionary algorithms such as genetic algorithm [30]. In this paper, we proposed a modified ABC algorithm to solve the robust programming model for the optimal toll design. According to an initial food source, we can obtain the TTC for each day and the maximum TTC among the whole planning period, then the mini–max TTC can be calculated through the modified ABC algorithm. Finally, the robust optimal toll pattern can be output. The procedures of this algorithm are summarized in Figure 2. For a detailed description of the ABC algorithm, the readers are referred to some other references, such as [3, 25, 27].

6. Conclusions

This paper studies the nonlinear distance-based toll with day-to-day dynamic traffic flow evolution. After an implementation or adjustment of a new congestion toll scheme, the network environment will change and traffic flows will be nonequilibrium in the following days, which makes it not suitable to take the equilibrium-based indexes as the objective of the congestion toll. A mini–max total travel cost model, which can overcome the expensive computational burden problem in previous work and make the congestion toll scheme more practical, is then proposed to solve the congestion toll problem considering nonequilibrium flow evolution processes. The essence of the mini–max total travel cost model is to optimize the worst condition among the whole planning period and ameliorate severe traffic congestions in some bad days, and this takes into consideration the network performance on each day of the study horizon rather than the final equilibrium state.
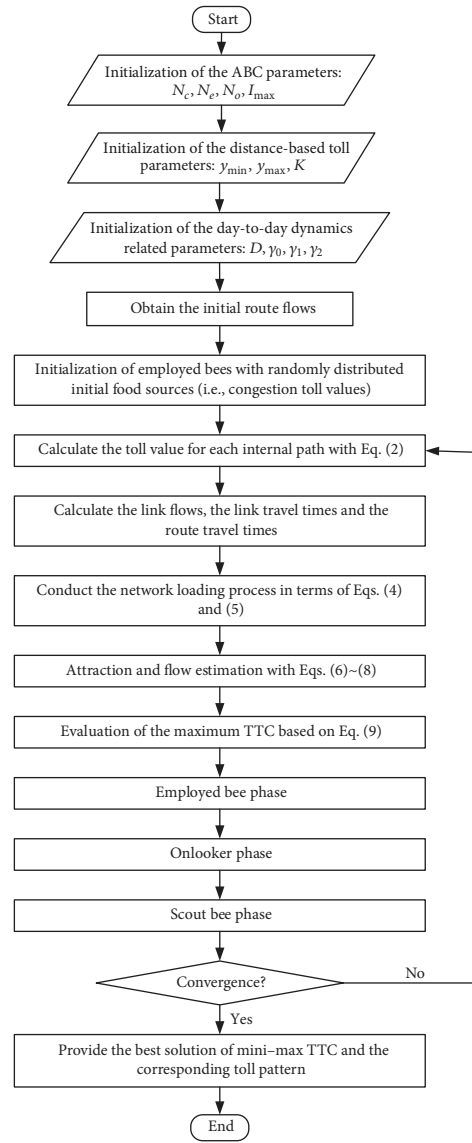
FIGURE 2: Flowchart of the solution algorithm.

Data Availability

No external data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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