Tomás García-Salgado

Distance to the Perspective Plane

Distance is an integral concept in perspective, both ancient and modern. Tomás García-Salgado provides a historical survey of the concept of distance, then goes on to draw some geometric conclusions that relate distance to theories of vision, representation, and techniques of observation in the field. This paper clarifies the principles behind methods of dealing with the perspective of space, in contrast to those dealing with the perspective of objects, and examines the perspective method of Pomponius Gauricus, contrasting it with the method of Alberti. Finally the symmetry of the perspective plane is discussed.

Introduction

The ability to measure without physical contact has been a constant pursuit throughout the history of science. We now know, for example, that the cosmic horizon—the limit to the observable universe—is at a distance of 1-2 x 10^{26} meters. Recently, in the search for planets beyond our solar system, the transit method of measurement has yielded astonishing results. Charbonneau and Henry asked themselves whether a planet passing before a star along our line of sight would cause the star's light to be diminished [Doyle, et al. 2000]. Doyle asserts that "[f]rom our perspective, the star would then dim in a distinctive way" [Doyle, et. al. 2000: 40]. The first logical question here concerns the probability of our view being aligned with the orbital plane of that "invisible" planet, no matter how distant it may be. His answer is that "[f]or planets that orbit very close to their stars, such as the one around HD 209458, the chance of the correct alignment is one in ten"[Doyle, et al. 2000: 40]. It may sound incredible, yet with the aid of the drawing below we shall see that the foundation of the argument is in fact quite simple (Fig. 1).

What I find interesting about all this is that perspective is part of this search both to appreciate the reduction in the light from the source, as well as to determine the inclination in the planet's orbit relative to our line of sight.

Distance is an integral concept in perspective, both ancient and modern. Indeed it is a central subject in the study of treatise writers from Euclid up to Pecham, Witelo, and Alhazen, through Alberti, Brunelleschi, Filarete, Piero della Francesca, Luca Pacioli, Leonardo da Vinci, Serlio, Vignola, Gauricus, Viator, Andrea Pozzo, and many others. Let us begin, therefore, with an historical overview of distance as our doorway into the particular methods of its geometric interpretation. The topic is undoubtedly extensive, which is why I recommend Kim Veltman's doctoral dissertation, which reviews the history of distance "… in an attempt to discern when and how the tenet of inverse size to distance began" [Veltman 1975: 3].

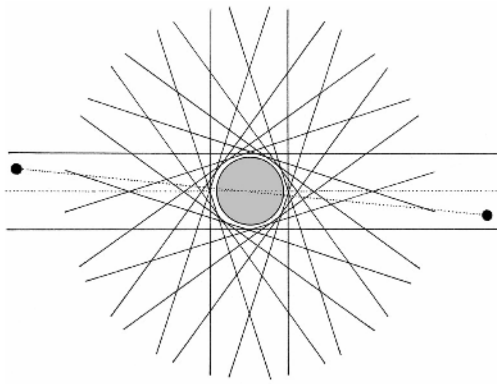


Fig. 1. At cosmic distances, the lines of sight passing through a star are considered parallel. The drawing therefore represents ten possibilities for the orbit of a planet to fall into the observer's line of sight of the star. Naturally, the planet's distance from, and size relative to the star are determining factors in the observation

Historical Survey

Aristarchus of Samos (c. 310-230 BC) set an interesting problem of measurement, declared in the title of his treatise *On the Sizes and Distances of the Sun and the Moon*. His solution was to measure the angle between the two bodies when the moon was half illuminated, thus determining the ratio of their distances. Based on the evidence from eclipses, he found the angle between the sun and the moon, yet only approximated its value. His reasoning was correct, but he erred greatly in calculating sizes and distances. In the tenth century, Al Farabi affirmed that the art of optics made it possible to measure distant objects such as the heights of trees, walls, and mountains, the breadths of valleys and rivers, and even the sizes of heavenly bodies.

Veltman identifies a dual origin of the concept of "distance". On the one hand are the theories of vision and representation, and on the other are the techniques of observation in the field. Optics is the first science to indirectly address the problem of distance by attempting to explain the appearance an object may have. Euclid (c. 300 BC) postulated that the magnitude of the visual angle determines the apparent size of the object viewed. Thus, in Euclidian geometry, distance represents an implicit, indeterminate value in angular measurement. Panofsky termed this principle the "angle axiom".[1]

After Euclid, Alhazen, Witelo, and Pecham, the subject of optics appeared to have been exhausted. Over time the techniques of field observation were systematized, giving way to a practical geometry, experimental to some extent, aimed at the design and construction of novel measurement tools. The commonest of these was the *rod*, a species of wooden surveying rod. Its use was as simple as it was illustrative. The rod was staked into the ground, then marked where the rod intersected the lines of sight from the viewer's eye to distant objects. This would be a useful method for recording the position of points on a reticulated ground relative to the eye. We could say that the rod is the predecessor of the Albertian perpendicular (*taglio*). Medieval engineers found this simple tool sophisticated enough for them to calculate the heights of walls and scale enemy fortifications.

When quattrocento artists sought a sensitive representation of the object seen—not its measurement as a visual or practical problem—they introduced the concept of distance as a measurable relationship between object, pictorial plane, and observer. This is the first geometric foundation upon which the theory of perspective is built. Its interpretation has diverse connotations in Medieval—particularly Alhazen's[2]—and Renaissance treatises due to the variety of methods and application procedures. Its geometric interpretation is still debated today, as no unified theory of perspective has yet come to fruition. We shall now turn to how some of these Renaissance authors, painters, and architects understood and applied the concept of distance.

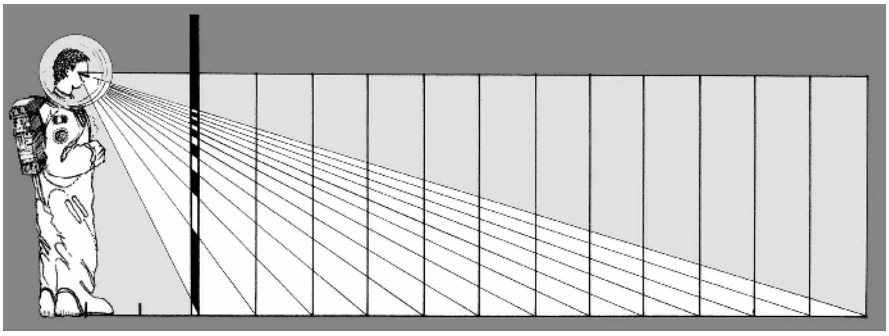


Fig. 2a. Measurement on a rod showing the intersection with the lines of sight, from Francesco di Giorgio Martini's La Pratica di Geometria, according to García-Salgado

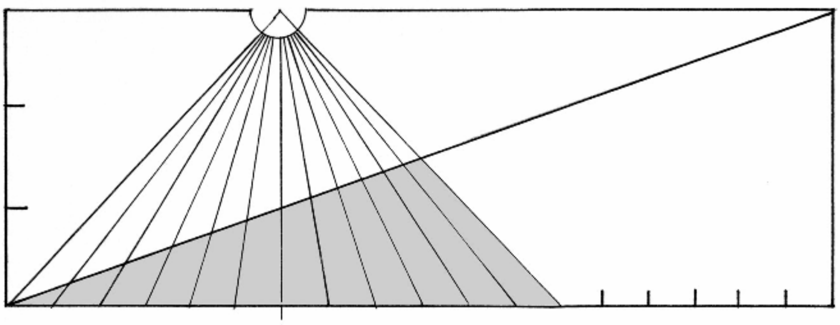


Fig. 2b. Center and countercenter of the model by Francesco di Giorgio Martini. These correspond to the vanishing point (vp) and the distance point (dv) in the Modular model. The intersection of dv with the sightlines directed toward vp determines depth

In his *Trattati* [1969], Francesco di Giorgio Martini described one of the first types of visual measurement in the field. He placed the rod vertically at a certain distance from the observer, then marked the visual rays directed toward each modulation point on the ground to obtain registry of the distances (Fig. 2a).

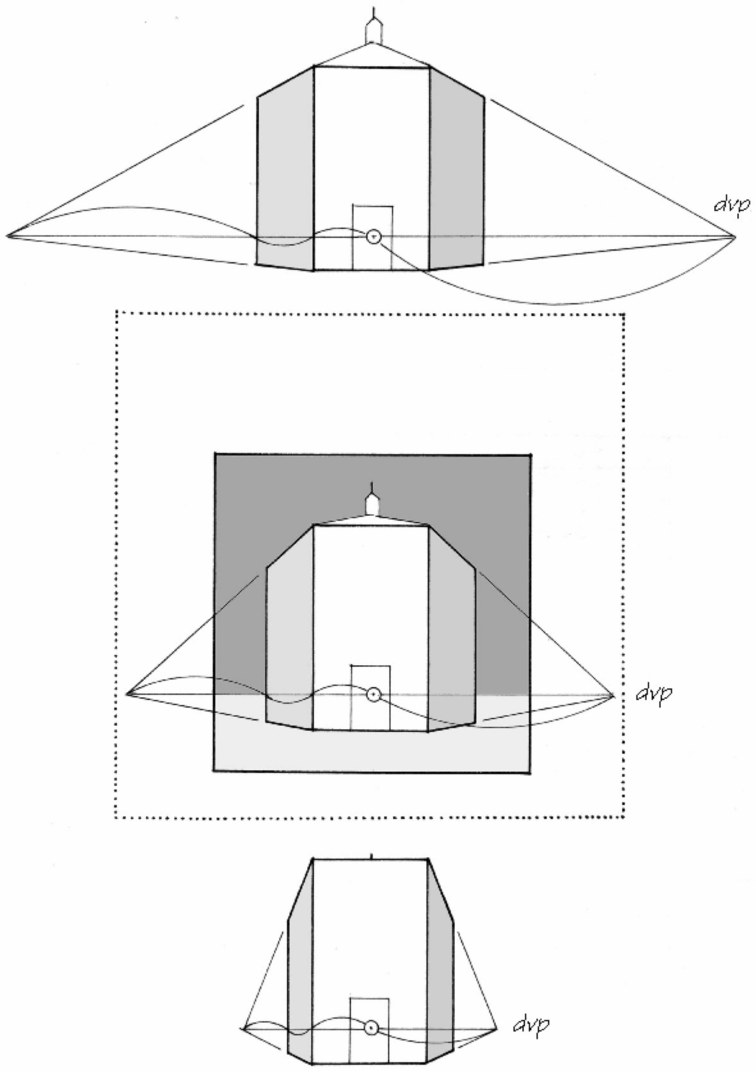


Fig. 3. a (top); b (middle); c (bottom). The Baptistery observed at three different distances. 3b (middle) approximates the observation distance from the Cathedral door. Since the axis of the Baptistery is off center to the left of the axis of the Cathedral (according to my measurements by half the width of the Baptistery's small lantern), the observer must also move so that his or her line of sight, directed toward the center of the doors of Heaven, aligns meticulously perpendicular to the plane

In another drawing, Francesco constructed a system of visual rays, measured in braccia, originating from a point he termed "the center". He used another point called "the countercenter" to associate the observer's lateral projection with the same frontal view (Fig. 2b).

The center, he explained, is the point and termination of all rays and of the eye. The countercenter is the eye that views the point generated by all oblique lines that cross the center rays, which cause the impression of reduction. He pointed out that the space between these two points, center and countercenter, may be as distant as one pleases, although recommending that they be neither too close nor too far. Francesco clearly understood that the geometric relationship of separation between these two points is the controlling factor in diminution of the ground checkerboard, because he considered the center and countercenter to be the fundamental principles of perspective. These principles remain valid in modern theory, even if Francesco only stated them in a general sense.

Manetti did not explain clearly how the panels (*tavole*) of San Giovanni and the Palazzo della Signoria were executed. This may be why some historians hold the hypothesis that Brunelleschi based his work on orthogonal geometrical drawings, but as we will see the most likely explanation is that the images were taken *in situ*.[3] Klein mentioned Brunelleschi was aware of the role played by the distance point, an allusion to Masaccio's *Trinity*.[4] Nonetheless, that principle need not be employed in the execution of the San Giovanni panel because the idea of the experiment was to show that the image had been taken from nature, not to deduce, or prove a perspective construction method.[5]

This hypothesis of orthogonal drawings presupposes a site plan and elevation of the Battistero and surrounding buildings, but even conceding that these could have been produced, the procedure would seem an unlikely one at that time.[6] Also, the octagonal nature of the Battistero floor meant that four of its eight faces generated 45° diagonals, such that when viewed from the Cathedral door its diagonal faces would necessarily run into the left and right distance points. This coincidence may have brought Klein to conjecture on the use of the distance point, or even what is known as the bifocal perspective mode.[7]

To explain this phenomenon, study Fig. 3 to appreciate how the separation—proportional separation—between the distance points and the object depends on the distance of observation.

Independent of the dimensions of the perspective plane, the distance points increasingly move away from the object as an observer recedes from the object; and vice versa, the points converge as an observer approaches. There is no indication whatsoever that Brunelleschi may have been aware of this geometric property. Had he been, he would possibly have prepared not one but several panels so as to fit the distance points onto them. Even supposing, to extrapolate from the logic of the drawing, that he wished to construct the drawing using the distance points, this approach would not have worked because they would have fallen outside the small panel (Fig. 3b). In my opinion, Brunelleschi was the first scientist of perspective who experimentally measured the natural distance of observation in real space. Yet unfortunately, as Vasari relates, his pragmatic personality and lack of *lettere*, lead him to give little importance to leaving written testimony of his findings.[8]

Alberti presented the first theoretical conceptualization of perspective in his treatise *De pictura* [1435]. His description was clear and orderly; each step in the procedure was introduced rigorously, defining the elements that comprised his geometric model. It is possible, however, to construct several differing interpretations of Alberti's description, partly because his work unfortunately lacked any illustrations. This lack that may have derived from his practical nature, the work being full of advice assisting his painter friends in their work setting, standing before walls to be painted and not before the draftsman's table replicating perspective lessons.

Alberti defined the concept of distance thus:

> *Dapoi ordino quanta distanza uoglio, che sia tra l'occhio di chi guarda, e la pittura: e quiui ordinato il loco del taglio, con una linea perpendiculare, come dicono i Mathematici, faccio il taglio di tutte le linee, ch'ella ha ritrouato* [Alberti 1435: 16].

He revisits the term *distanza* to explain how to deduce the successive separation between the checkerboard transversals,[9] so that the geometric interpretation of the term is inconsistent throughout the work, yet without contradicting its verbal function. Fig. 4 shows the step from the procedure that describes the construction of distance in accordance with the the Albertian model.

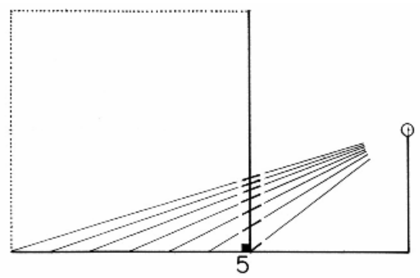
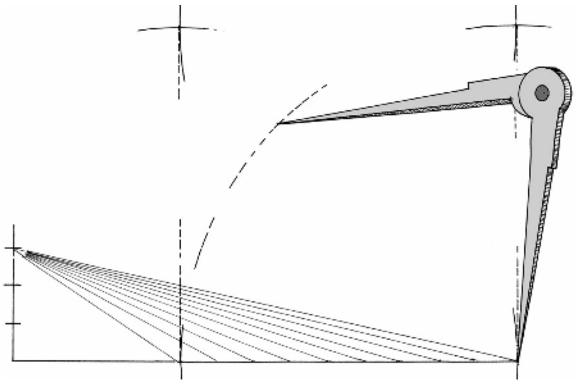


Fig. 4. The Albertian perpendicular is better understood as a plane than a as line, because this perpendicular represents the picture plane viewed laterally

In his *Trattato di Architettura* (1460-1465) , Filarete begins by constructing a square (the observer's visual field) with a compass.[10] In the same way as Francesco di Giorgio, he places the observer in a lateral view at an unspecified distance from the square, just as Alberti had done (Fig. 5a). Filarete draws rays, measured in braccia, from the lateral observer to the square's base modulation. The observer's height is measured as three braccia. Once the degradations on the square's edge are obtained—Alberti's perpendicular—, they are transferred to the opposite edge with the aid of a compass so that all the transversal lines may be drawn. This is how he theoretically demonstrated the construction of a grid in square braccia (*braccia quadratta*) in perspective, following the same idea as Alberti's checkerboard (*pavimento*), that is, creating a spatial reference system (Fig. 5b). Veltman thinks Filarete's procedure is clearly distinct from Alberti's because it does not use a second "panel" [Veltman 1975; Filarete 1972: 90], but in my view there is no evidence Alberti used any second "panel" (a sheet of paper, according to Panofsky and Klein). It all depends on how the expression *prendo uno piccolo spatio* is interpreted [García-Salgado 1998b: 121].



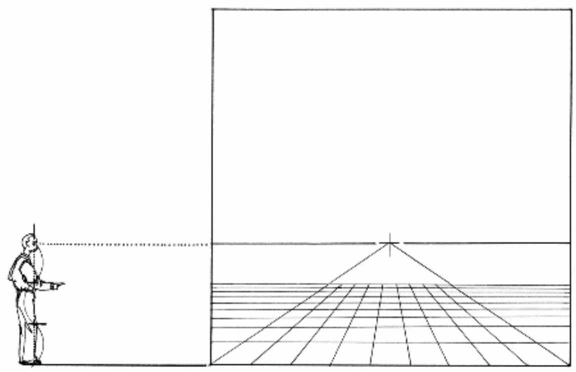


Fig. 5. a (top) Observer's visual field and distance, according to Filarete; b (bottom) Grid of braccia quadratta in perspective, according to Filarete

To execute the strokes precisely, Piero della Francesca recommended using nails, silk threads or horsetail hair. The Atlas of Drawings from *De prospectiva pingendi* (1480) gives testimony to his mastery of drawing. In my view this is the most complete work from the Italian quattrocento, not only through its sheer volume, but also for its rigorous demonstration of the theorems that shape the theoretical corpus. Piero developed the idea of the diminished square (*quadrato degradato*) identical to Filarete's, but encompassing the entire height of the pictorial plane. He placed the observer in the frontal and lateral views simultaneously, Alberti fashion, and defined distance as the geometrical interval between lateral observer and the near edge of the square (*quadrato*).

Piero said that the observer may be placed anywhere for a frontal view, but the most comfortable position is to place the eye at the center of the quadrato. We find the description of the first method in theorem XIII of his treatise [della Francesca 1984: 76], which we shall interpret step-by-step with the series of figures in Fig. 6.

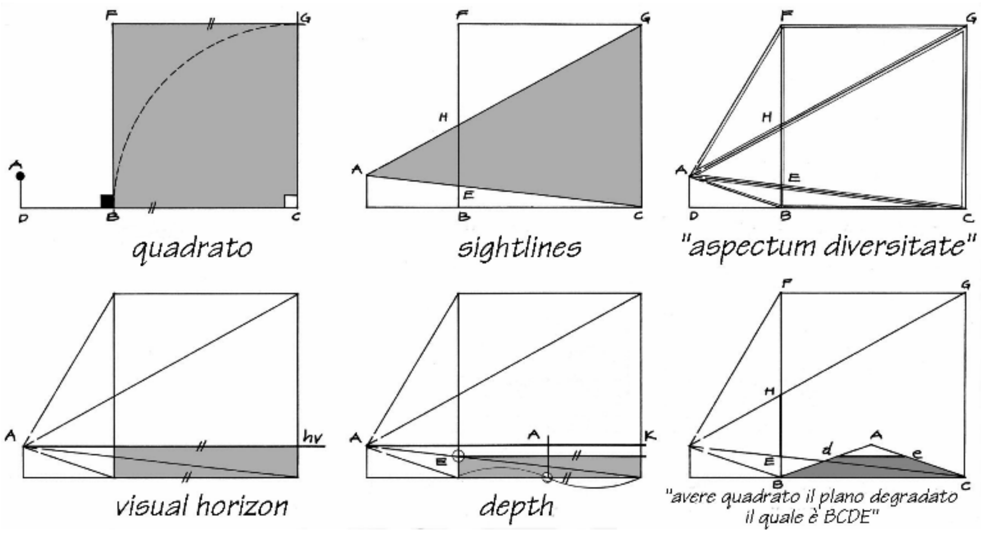


Fig. 6. Conceptualization of Piero della Francesca's Theorem XIII, according to García-Salgado: a) (top left) drawing of the quadrato; b) (top center) drawing of the visual rays or sightlines; c) (bottom left) visual horizon; d) (bottom center) depth; e) (top right) aspectum deversitate; f) (bottom right) dico avere quadrato il piano degradato il quale è BCDE

The function played by distance is clearly understood in Fig. 6b, from the observer's eye, indicated by point A, to the perpendicular BF, which also marks the border of the *quadrato*. Nonetheless, when Fig. 6f is compared with the series 6a to 6e, several constructive features arise meriting further scrutiny. These, however, I will set aside for a later study dedicated exclusively to Piero's treatise.

Perspectiva Pictorum et Architectorum (1693-1700) is a classic text on perspective, with truly advanced craftsmanship [Pozzo 1989]. The Jesuit friar Andrea Pozzo, the author of this masterpiece and other works, presented the monumental ceiling of Sant'Ignazio in Rome, possibly the most spectacular illusory fresco of all time. To execute this *trompe l'oeil*, Pozzo drafted 544 preparatory sketches, and an additional 1,088 sketches in order to transfer the projections from the virtual plane onto the hemicylindrical vault.

Pozzo presented the principles of perspective right from the first paragraph of his treatise. He dealt with an imaginary apse onto which an illusory fresco was to be painted. We have put aside the architecture of the church to simplify description of the procedure. In the series of Fig. 7, first the ground line (*linea terræ vel plani*) is drawn onto the diagonal section, then the eye point (*punctum oculi*) is placed on the visual horizon, and the distance points (*puncta distantiæ*) are fixed at its ends. The *puncta distantiæ* are the key to the process, as they allow for the determination of depth, height, and all other features of the illusory space to be represented. Increased distance between the *punctum distantiæ* and the *punctum oculi* naturally corresponds to increased observer distance in both the plan and the elevation.

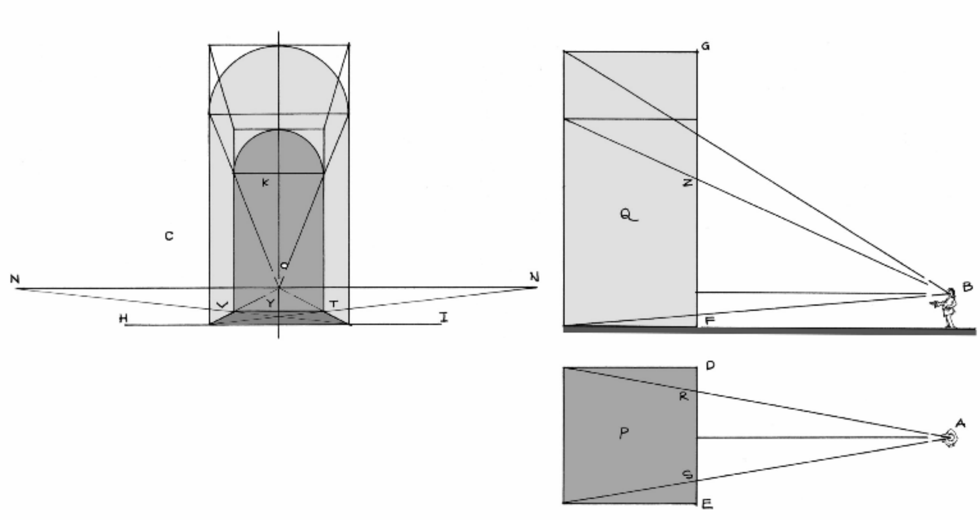


Fig. 7a (left) Explicatio Linearum Plani & Horizontis;
Fig. 7b (upper right) Ac Punctorum Oculi; c (lower right) Distantiæ.

Pozzo was the first commentator to systematize use of the 'vanishing distance' point (*punctum distantiæ*) in order to resolve a broad spectrum of perspective problems. He even anticipated the geometrical drawing technique, from descriptive geometry proper, by introducing the simultaneous use of plan and elevation to originate a detailed solution to architectural ornamentation of the classical orders.

Distance to the Perspective Plane vs. Distance to the Vanishing Point

The preceding historical review, which by no means exhausts or even treats the subject in much depth, is nonetheless sufficient for us to draw some geometric conclusions on the concept of distance that we will relate to theories of vision, representation, and techniques of observation in the field.

Obeying the Euclidian concept of angle size, Veltman made a drawing that exemplifies what happens when four objects of equal size are seen from point A. Fig. 8 establishes that BC<EF because angle BAC<angle EAF. It also establishes that BC<EF because BC is further away than EF. The proposition is correct but not entirely convincing because it raises the question of whether plane BF is the best measurement of the angle or whether there may a better means of measurement. Let us compare this with Veltman's idea of "equiangular" measurement.

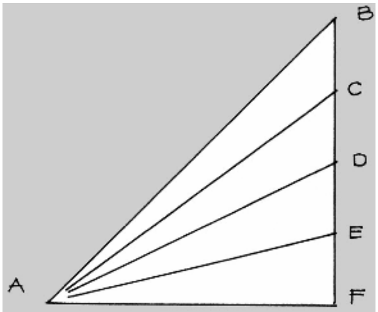


Fig. 8. Angular size of equal-sized objects, according to Veltman

Revisiting the concepts in Fig. 8, we now turn to Fig. 9a where we see that angular measurement definitely decreases even when the objects curve toward the observer.

In Fig. 9b, however, where the geometric condition of angular equality is met—for all measurement of objects of equal size—we deduce that the projection plane must necessarily be curved, not flat, replicating to a certain degree how the retina's curved surface receives visual rays. There is a definite difference between the two illustrations: while Fig. 9a is associated with geometric models of linear perspective, Fig. 9b represents an idealized construction of the retinal plane. We offer the following hypothesis as an option to the old dilemma of vision and representation:

The curvature of the human retina is the factor that controls and compensates the angular equidistance of vision, so that flat objects may be perceived as such rather than as curved.

Therefore, it is not a function of the perspective plane to interpret the retinal image, but rather to represent geometrically the perceivable image on its flat surface. So far so good. One could imagine there to be a multiplicity of geometric procedures in measuring distance, but fortunately they may be grouped into three types:

Type 1. The first type contains those cases of measurement aided by orthogonal drawings, plan, section, elevation, and axonometric, by any of the methods it may be applied. I will not comment here on this procedure, so common in the eighteenth century, because it belongs more to the discipline of descriptive geometry than to our discipline of perspective.

Type 2. The second type belongs to those cases where the distance to the picture plane is measured using any procedure, from the ingenious measurement by the rudimentary rod, to Brunelleschi's visual experiments, or the early formal methods of Alberti, Filarete, and Leonardo. This is commonly known as the distance point procedure. As we may appreciate in Fig. 10, its geometric approach is to superimpose the lateral and frontal views so that the distance is measured from the observer to the picture plane, the *finestra* viewed from the side, or the so-called *perpendiculare*.

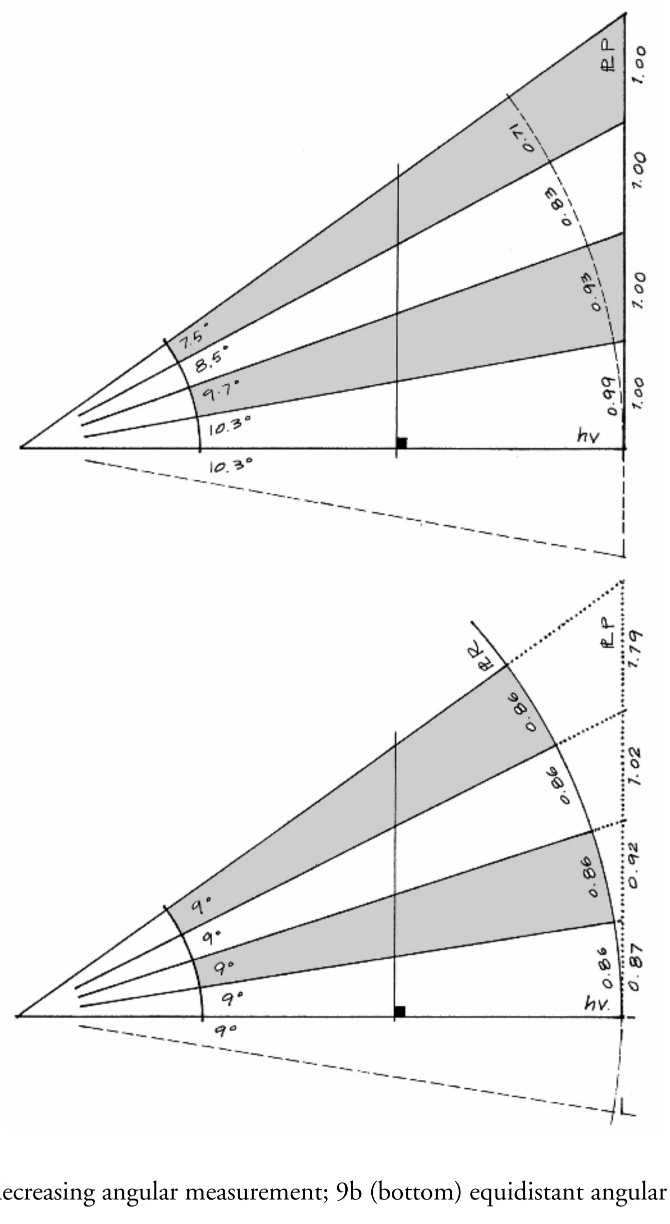


Fig. 9a (top) decreasing angular measurement; 9b (bottom) equidistant angular measurement

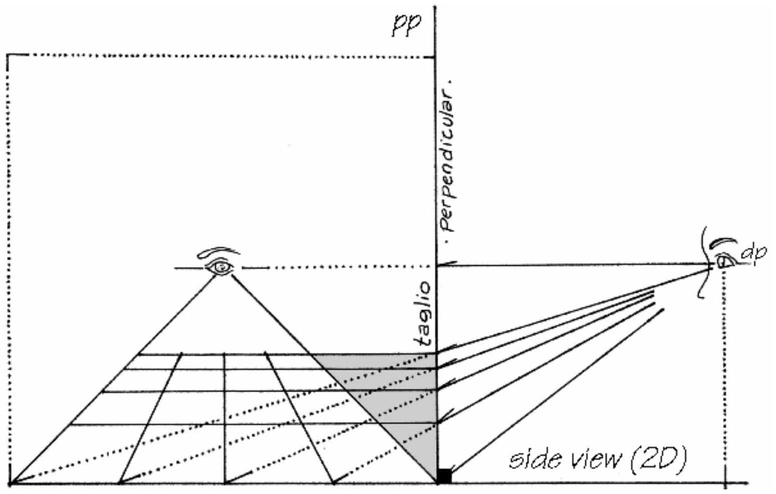


Fig. 10. Measurement of the distance to the perspective plane (PP) seen laterally

The depth of the ground checkerboard transversal lines is determined by where the visual rays from the baseline to the observer cut the picture plane. This procedure has three basic distinguishing elements: the observer, the picture plane (also called the perspective plane PP), and the object (the marks on the ground in this case). Some variations on this application have been ingenious, (Leonardo's geometric riddle in *L'Ultima Cena*, for instance [García-Salgado 1998c]), whilst others have not been as well appreciated as Serlio's [1982].

Type 3. The third type corresponds to measurements performed with rigorous perspective. As Fig. 11 shows, distance is measured directly at the picture plane PP, from the vanishing point (vp) to the distance vanishing point (dvp). Measurement is naturally made on the visual horizon understood as a plane, rather than a line, running into infinity at the height of the observer's eyes. Such measurement is feasible because dvp is by definition a perspective plane (PP) point that may be within or without the actual plane. Pozzo was able to comprehend the geometric subtleties of this procedure in its simplest expression, the author's interpretation of which is shown in Fig. 12.

If the reader analyzes this carefully, he will find not only a grand synthesis easily committed to memory, but also one of the best and briefest lessons in perspective. The whole dilemma in this discipline, of determining perspective depth in frontal modulation (*m*), is succinctly resolved through simultaneous application of vp and dvp, as demonstrated in Fig. 12. This is the golden rule of perspective. A visual ray is traced from *m*, the floor, to vp, and another is traced from one end of this measurement to dvp. Where the two rays intersect is the point that determines perspective depth (*m'*).[11]

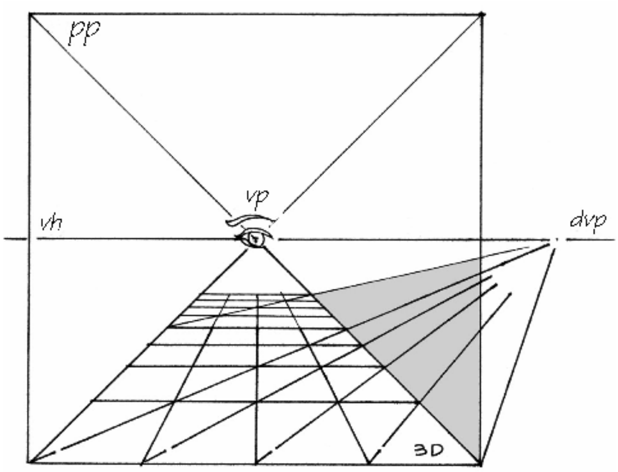


Fig. 11 (left). Direct measurement of depth in the perspective plane PP

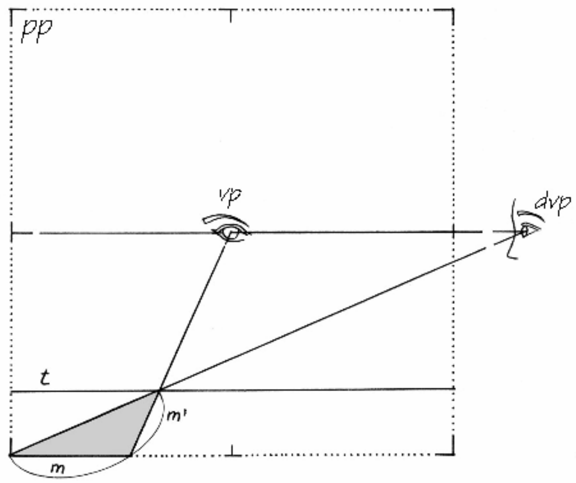


Fig. 12 (right). Square grid of module m

Concerning the distance vanishing point, I have previously established in my book, *Perspective Geometry*, the difference with the traditional concept of distance point:

> *[w]e adopt the term distance vanishing point because it suggests the idea of the observer oriented toward infinity in the perspective plane, even if some authors prefer the traditional denomination of distance point, which is adequate provided the observer's position is noted as being before the perspective plane. It is not appropriate when the position is represented as on the perspective plane* [García-Salgado 2000].

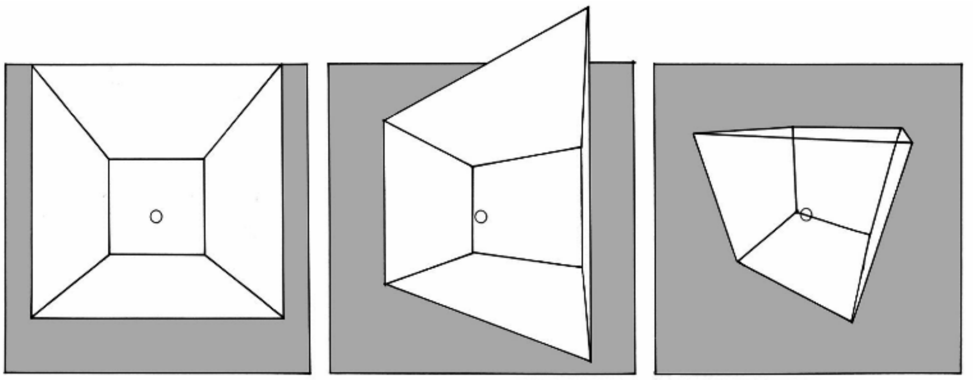


Fig. 13. To how many vanishing points does the cube in the perspective plane vanish?

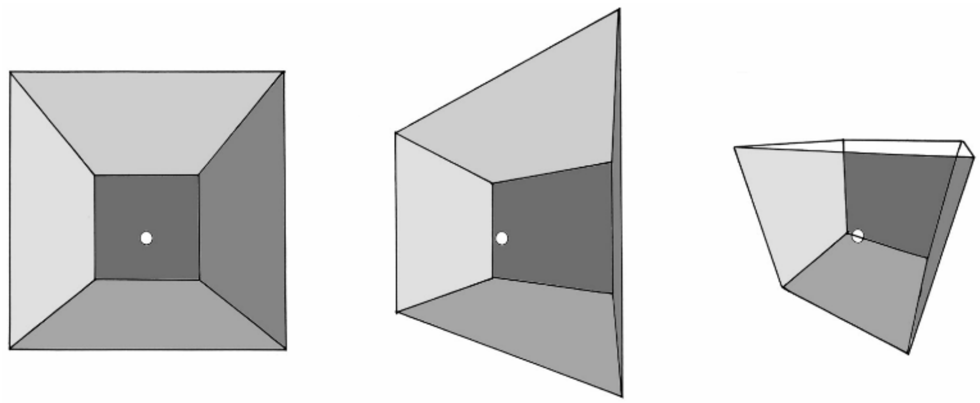


Fig. 14. To how many vanishing points does the cube in perspective vanish?

The concept of distance is universal, but variation in its application can lead to unreliable conclusions, as is the idea of bifocal perspective derived from Gauricus's method, an issue I will deal with below.

One Vanishing Point or Three?

A poor understanding of the concept of distance may be due not only to a lack of knowledge about its origins and geometric measurement procedures, but also to confusion concerning the principles behind methods of dealing with the perspective of space, in contrast to those dealing with the perspective of objects.

To overcome this ambiguity we shall delineate the difference between a space lattice reference system such as the Albertian checkerboard, and a grid to aid the drawing of objects in perspective. Both appear to possess the same properties, but the conception is different. A space lattice reference system represents the entirety of an observer's visual field in all its extent, modulated three dimensionally to its limits. On the other hand, the perspective life of an isolated object begins with its geometry, at times referred to a supporting grid, which cannot extend throughout an observer's visual field or is oriented in a different direction. I will now set two questions to demonstrate this constructive, but paradoxical difference.

Observe Fig. 13: to how many vanishing points does the cube in the perspective plane vanish? Now observe Fig. 14: to how many vanishing points does the cube in perspective vanish?

This pair of questions is my regular opening for the course on Modular Perspective, taught at the UNAM's faculty of architecture since 1992, normally eliciting the answers, "one, two, and three vanishing points" to each question. When the difference between the two questions is underlined, doubt sets in. The first question is recast without infringing on its content: "To how many vanishing points does the perspective plane vanish?" or "To how many vanishing points does the observer's field of vision vanish?" So the correct response is "ONE." Wherever sight is directed—center, up, side, or down—the observer's vanishing point will always be directly ahead along the line of sight: struck against something or lost in the horizon of the landscape or the cosmic horizon. Similarly, recasting of the second question as, "How many vanishing points does the cube have?" leaves the correct answer as "THREE," in all three cases.

Quite so. A cube or a rectangular prism has three parallel systems, necessarily having three asymmetric vanishing points, a*vp1*, a*vp2*, and a*va3*, regardless of its position in space relative to the observer. If any doubts still linger, return to Fig. 14 for proof. We see in the first cube a clear, visible vanishing point for lines orthogonal to the observer. A second vanishing point for horizontal lines runs into infinity. A third vanishing point for vertical lines also runs into infinity. Moving on to the second cube, we see two vanishing points that run along the horizon and a third vanishing point for the vertical lines that runs into infinity. The third cube demonstratively manifests the three asymmetric vanishing points, one for each system of parallels.

In conclusion, then, and strictly speaking, there is no correct taxonomy of perspective defined by whether it has one, two, and three vanishing points. One issue is the spatial structure of the observer's visual field, represented as the perspective plane, which has a single vanishing point. And a separate issue is the geometric structure of the object(s) in the visual field that will have as many asymmetrical vanishing points as there are parallel systems conforming to its shape.[12] I now put to the reader the standard question to close that first class in the course: "To how many vanishing points does an icosahedron in perspective vanish?"

Alberti vs. Gauricus

Although there are scholars who consider Gauricus's method to be a variation on Alberti's, his conception and procedure are distinct. We will examine three geometric differences between the two methods.

Alberti began by defining the observer's visual field as follows: … *faccio un quadrato grande, quanto mi piace d'anguli dritti: il quale mi serue per una finestra aperta, onde si possa uedere l'historia* [Alberti 1435: 15]. For Gauricus, however, the starting point is a vertical line on which lie the centers of two semi-circles, one the upper half of a circle, the other the lower half of a circle of equal size, and then tracing a horizontal line through their points of intersection.[13] Secondly, Alberti places the observer in a frontal view, while Gauricus gives a lateral placement -never frontal. Alberti's diagonal, which is drawn in step seven of his procedure, serves to legitimize the perspective construction of the ground's baseline; Gauricus on the other hand, deduces the construction of the grid by using a fan of diagonals starting from the ground's baseline. Confronted with these geometric differences, it is fairly meaningless to continue supposing, as Panofsky claimed, that these methods have much in common.

For Klein, the fundamental distinction between the methods of Alberti and Gauricus is that the former was unaware of the distance point,[14] while Gauricus was unaware both of the central vanishing point and the horizon line.[15] Neither of these claims is geometrically correct. Firstly, the term "distance point" is a way of constructively interpreting distance (*distanza*), which Alberti defines with perfect precision in the lateral view.[16] What is more, if we extend the ground's diagonal line (*diametro*) in the frontal perspective view onto the horizon or central line (*linea centrica*), the *distanza* would once again be defined by measuring from the center point (*punto centrico*), obviously. Distance is therefore implicit in the Albertian model, not overlooked. And secondly as we shall see, the central vanishing point and the horizon line are not completely overlooked in the perspective method by Gauricus.

The most controversial part of Alberti's method is the way of interpreting the correlation between frontal and lateral views. According to Klein, (and similarly, to Ivins and Panofsky):

> *Alberti divided the picture base into reduced braccia (…); he arbitrarily chose a central vanishing point and a height for his horizon; he then connected this point with the divisions of the base (…); then, on a smaller scale and a different piece of paper, he traced the profile of the pyramid, situating the taglio at a chosen distance (…); finally he copied onto the border of the picture the intervals he had obtained on the taglio of the auxiliary drawing, restored to the initial scale (…). The checkerboard was finished, and as a check, Alberti drew a diagonal which should cut through the corners of all the squares it passed through (…)* [Klein 1981: 112].

Let us go through this paragraph step by step, taking the author's interpretation of Alberti's method as the point of reference [García-Salgado 1998b: 123-128]. Firstly, Alberti did not begin by dividing the base line (linea bassa), but rather began with the observer's visual field. Secondly, he did not choose the central point arbitrarily, because its determination is a logical consequence of the first step: *Dopo questo faccio un punto solo nel quadrangulo in loco, doue sia ueduto; ilquale punto m'occupi quel loco istesso, alquale arriua il raggio centrico: e per questo lo chiamo punto centrico.*

Thirdly and most controversially, Klein asserted, "the profile line, on other paper and to scale." Let us take Alberti literally:

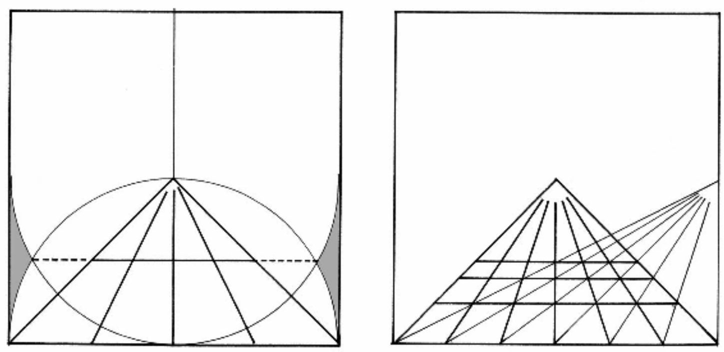
Io ho un picciolo campo, nelquale io descriuo una linea dritta. Questa io la parto per quelle parti, ne lequali la linea del quadrangulo, che sta a giacere, è diuisa. Dapoi metto su da questa linea a un punto solo tanto alto, quâto è lontano il punto centrico nel quadrangulo da la linea diuisa del quadrangulo, che sta a giacere: e poi tiro le linee d'una in una da questo punto a ciascuna diuisione di questa medesima linea.

This passage must surely refer to a complementary drawing, as at no time does it say, "on other paper, to scale, and that the outline must be restituted to the original scale." It would be risky to attribute to a text a particular geometrical meaning which it lacks. As we know, Alberti did not include a single illustration in his work; perhaps he considered a practical situation in aiding his painter friends, in front of the wall fresco to be painted (*pariete afrescata*) and not on a drafting table. A literal translation of the phrase, *Io ho un picciolo campo*, as "I take a little space" only denotes that he used a space smaller than the window (*finestra*) to facilitate geometric control of the cut (*taglio*) and guarantee correct deduction of the transversals, so long as it was a temporary, erasable drawing, or a contiguous drawing executed on cartoons (*cartoni*). Whatever the case, the *taglio* traces stayed on the border of the fresco itself without any need, contrary to what Klein claims, of restituting from a scale drawing for transfer onto the wall—the main practice during the quattrocento was to draw directly onto the fresco wall.[17]

Gauricus

One of the most polemical perspective methods is the one put forward in *De Sculptura, ubi agitura de symetria… et de perspectiva* (Florence, 1504) by Pomponius Gauricus (1481-1530) [Gauricus 1989]. The debate concerns how to interpret the geometrical meaning of the text, since Gauricus did not feel compelled, as Alberti had, to introduce the definitions of concepts in conformity with their descriptions. Before embarking on a new interpretation we will take a look at Gauricus's constructive principles,[18] centering on the concept of distance with regards to it being construed as a vanishing point on a perspective plane. This is the axis of our discussion in line with the principles of the author's Modular Perspective[19] and Veltman's ideas [Veltman 1975: 179-310, Chapter III, The Development of the Diagonals Method, the so-called "Distance Point Construction"] endorsed by comparison with the interpretations of Gauricus by Panofsky [1991], Gioseffi [1957],[20] Klein [1981], and Kitao [1962].[21]

Veltman began by explaining why Gauricus started his procedure with the two semi-circles, claiming that the horizontal line (parallel to the baseline) resulting from the intersection of the semi-circles determines the upper limit of the picture's diminished perspective. A couple of observations are in order at this point. One is that Gauricus's original description makes no mention of any central vanishing point, which leaves no geometric evidence for Veltman's deduction of a simplified outline (Fig. 15). On the contrary, Gauricus's procedure is mechanical and rather inexact in practice, because of the fact that the grid is not derived from the central vanishing point. As Klein pointed out, here the central vanishing point is no more than a test.[22]



Figs. 15a and 15b. Gauricus according to Veltman

The second observation concerns Veltman's claim that the countercenter must be situated at a precise height exactly vertically above the right edge of the square panel; but if it strays to left or right of this, the lower transversal will not coincide exactly with the line of intersection of the semi-circles.[23] Nonetheless, this lowest transversal must be calculated from baseline modulation (*Ducatur itaque quot uolueris pedum linea hec*). This is why the outline of the floor reticulation does not necessarily depend on fixing the lower transversal or in precision in placing the countercenter.[24]

To my way of thinking, Decio Gioseffi gave the clearest interpretation of Gauricus's text, because it follows most closely the text's geometrical description. Gioseffi conceived the procedure in twelve steps, of which steps 10 to 12 may be simplified (Fig. 16) if we assume—as Veltman suggested or interpreted and is shown here in Fig. 19e—that all the transversals run from the perpendicular, as opposed to only the limit line. This change does not constitute a significant departure, because the procedure is applied equally to all the transversals.

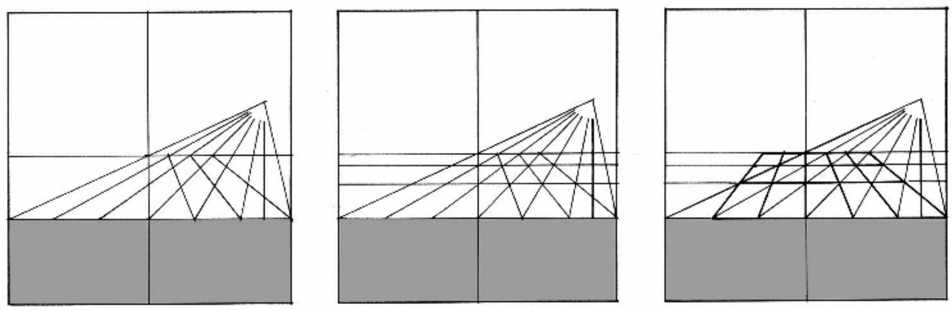


Fig. 16. Gauricus according to Gioseffi

Gioseffi's step 10 relies on the furthest transversal to generate the system of orthogonal lines, giving greater certainty in line orientation (Fig. 16a). Nonetheless, as is evident in Fig. 19f, the rest of the transversals serve as intermediate points in orienting the orthogonal lines -an advantage no geometrician or draughtsman would miss.

Panofsky introduced the central vanishing point and test diagonal in his interpretation, elements that may be clearly appreciated in his Fig. 23 which accompanied note 60 to his renowned essay "Die Perspective als 'Symbolische Form'" (Fig. 17).[25] The presence of these elements is inadmissible because they do not occur in Gauricus's description and because they invalidate the perpendicular's geometric function in Figs. 19e and 19f.

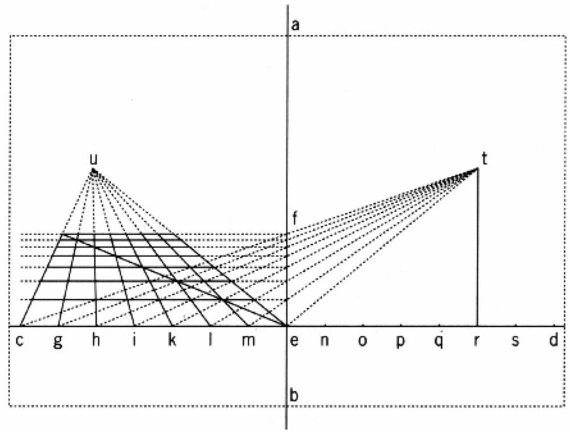


Fig. 17. Gauricus according to Panofsky

Klein's interpretation is practically a carbon copy of Gioseffi's step 12 (Fig. 18). What is notable is that the height of point O is less than the distance between the observer and the perpendicular, even though according to Gauricus's text, these two are meant to be equal.

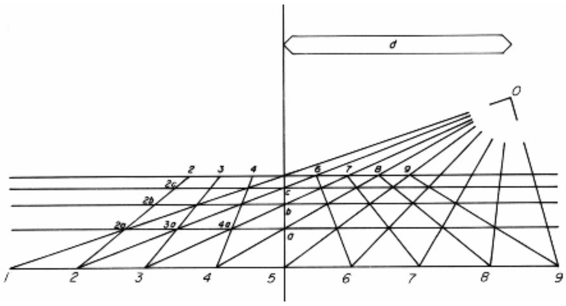


Fig. 18. Gauricus according to Klein

To recap on these comments I now present my interpretation of Gauricus's method, respecting the order and meaning of the description as it appears in the original text (which may be consulted in Veltman [1975] and Panofsky [1991]. The intention is not to produce a literal translation, but to achieve a comprehension of the passages in which he described the procedure. These I synthesize in six steps in Fig. 19.

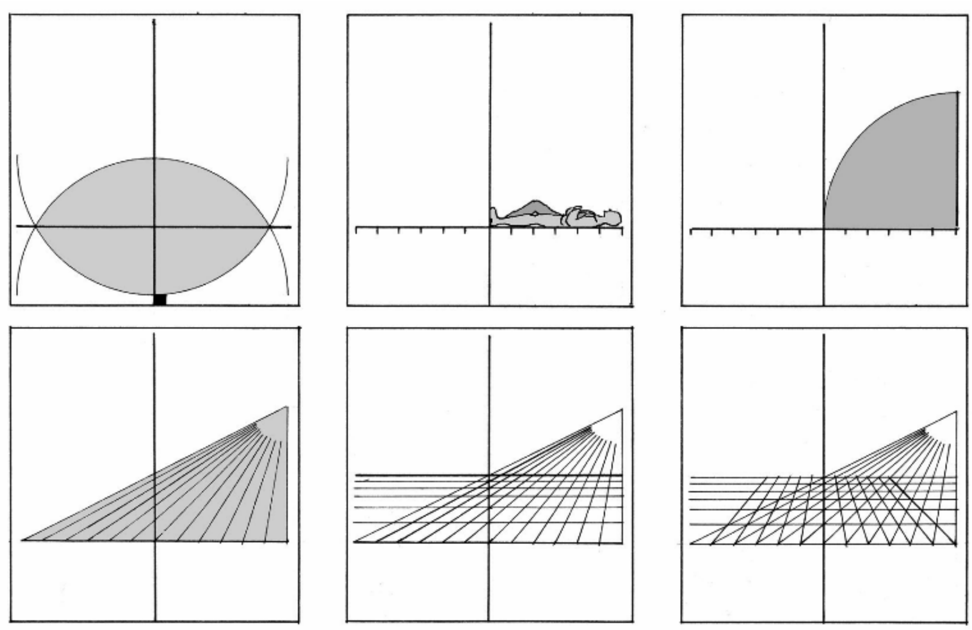


Fig. 19. Gauricus according to García-Salgado. a (upper left) On a square panel, a perpendicular is drawn to its base. Two semicircles are then drawn with their center points on this perpendicular, and the base line (aequoreae linea) parallel to the square base is drawn through their intersections. Gauricus then posed two questions, one concerning distance and one concerning the space to be represented: How far should the perpendicular (plani definitrix) be? Where should the persons be placed (corpora)? The procedure then goes on to answer these questions; b (upper middle) Observers, given their own height, will see at a distance from their feet at least equivalent to their height (ad minimum dimensione). Therefore, mark as many feet as needed on this line (parallel to the base line); c (upper right) Draw another vertical line a little away from the perpendicular, starting from the base line and up to a height equal to that of a person (6 feet); d (lower left) Draw a line from the top of this vertical line to one end of the base line, and likewise draw the lines to the markings (porcionum angulos); e (lower middle) The limit line (the furthest transversal) is situated where the perpendicular crosses the end line, which goes from the vertical line to the base line. The other transversals are drawn in (although Gauricus did not say so explicitly); f (lower right) The lines are drawn from the base line to the furthest transversal, from one side to the other, each taking its respective angle, to define the place (checkerboard) the (standing) people will occupy so that they are consistent among themselves regardless of the distance from the interval they occupy.

In sum, as may be seen in Fig. 19, Gauricus described a system for modular reticulation of the ground. Yet unlike Alberti, restricting his drawing to the furthest transversal and not making use of the vanishing point (Fig. 19f) left him without a geometrical solution to continue depth degradation to infinity. Yet curiously Gauricus did not discover the significance of the furthest transversal, that is, that it intersects all lines reaching the vanishing point at the same depth (Fig. 19e); and, moreover, that these lines all run to infinity at the distance vanishing point (the countercenter, in Veltman's parlance).

Gauricus was able to imagine the observer simultaneously before the lateral and frontal vistas, thanks to the geometric ambivalence of the perpendicular. But what he was unable to intuit or discover in the overlapped construction of these projections is that the perpendicular is not a line but a plane; viewed laterally it projects as a line but frontally it represents the perspective plane. The fact that these views are superimposed, makes it inevitable to interpret the furthest transversal as a flat drawing instead of as the projection of a three-dimensional construction, because there is no way to deduce its depth directly on the perspective plane.

Bifocal Perspective, or Symmetry of the Perspective Plane?

Klein pointed out that marginal distortions, are undeniably present in Albertian perspective, and pertain to a species of anamorphic projection.[26] Yet Alberti made no mention of them, not even hinting at them in his treatise. It was Leonardo da Vinci who studied the means of preventing them, without formulating a specific theory.[27] When dealing with the issue of anamorphic perspective, I have argued that its model may be articulated consistently with the Albertian model because the theoretical principle of deformations arises from linear perspective [García-Salgado 2000: Chapter 9: "Anamorphic Perspective", pp. 113-130].

What is known as bifocal perspective also arises from linear perspective. It consists of beginning the drawing of the grid from the distance points (or "distance vanishing points") which, due to symmetry with the picture plane, are situated at each side of the central vanishing point. Klein saw in Gauricus's system a moment of transition between the central pyramid and the oblique pyramid, attributing a wider angle to the bifocal construction than Alberti.[28] This idea sounds convincing in principle, but in geometric rigor the oblique (visual) pyramid may only be defined in anamorphic perspective, which is constructed precisely when the perspective plane is cut obliquely. This explains why marginal distortions are not anamorphic but rather the result of an excessively wide field of vision.

Another consideration is that the perspective called bifocal derives from the picture's diagonals, that is, from a two-dimensional geometric property. This property is retained in the perspective plane by virtue of symmetry with three-dimensional space. Therefore, the idea of an oblique pyramid, in Klein's terms, simply translates into reticulation of the ground based on the lines that generate the diagonals.

Viator (*De Artificiali Perspectiva*, 1505) was the first to sustain an argument on perspective construction with two distance points.[29] But scrutiny of some of his constructions raises the question of how legitimate it is to consider his procedure as a method distinct from Alberti's. Is it simply a different grid method? When I examined the problem of the cube's vanishing points, I mentioned that since a cube has three systems of parallel lines it must necessarily have three asymmetric vanishing points, a *vp1*, a*vp2*, and a*vp3*. We may now argue by extension that since a square has two systems of parallels it must necessarily have two asymmetric vanishing points, a*vp1* and a*vp2*.

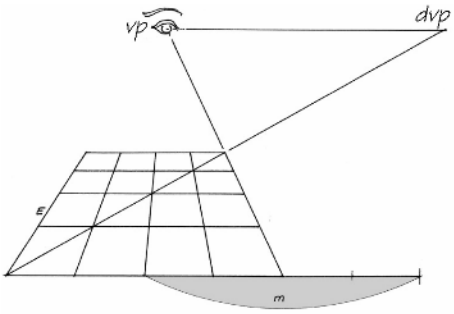
Now, whatever position the grid is transposed into, frontal or oblique, it must satisfy the properties of the square: the square's diagonals in both cases must generate their own asymmetric vanishing points. Note how the distance vanishing points or asymmetric vanishing points (a*vp*) coincide with the diagonal vanishing points (*dv*) when they are set at 45° relative the observer. But when the angle is any other than 45°, the diagonal vanishing points move along the visual horizon.

This geometric overlapping which at times occurs, has prevented the concepts of 'distance vanishing point' and 'diagonal vanishing point' from maturing, leading to the conclusion that Gauricus came to, that they are a single geometric element.

Finally, the following premise must be met to validate the procedure for drawing the grid when rotated on an axis:

Given a grid in frontal perspective and in the [*m:m*] format, the original format must be retained when rotating it to any horizontal or vertical direction.

I will use Fig. 20a to see the initial position of the grid in modular proportions *m:m*, and Fig. 20b to demonstrate the execution of an exact 45° rotation. In accordance with Modular Perspective, [*m:m*] modulation is established at the lower limit of the visual field. Modulation *m* is then transferred to any of the diagonals. Since this is a 45° rotation, the diagonal vanishing points will coincide with the distance vanishing points. Finally, we proceed to deduce complete construction of the grid.



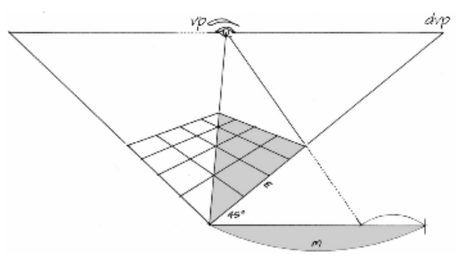


Fig. 20. a (left) Initial position of the grid of module m:m; b (right). Position of the grid rotated 45°. Note the modulation value m:m is conserved

In this fashion we have rotated the grid from position A to position B while conserving the [*m:m*] format. We can now compare how this geometric property was not respected in the Klein's example (Fig. 21) [Klein 1981: 116]. The oblique grid shown, despite a 45° rotation, does not preserve the format of the initial grid. A new and different grid was simply constructed from the initial grid's diagonals.

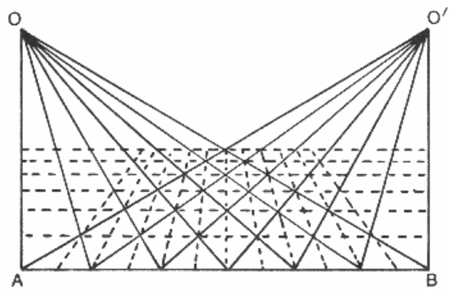


Fig. 21. Bifocal method according to Klein's Fig. 8: Ground-plane construction in the bifocal method.

When the rotation is other than 45°, the diagonal vanishing points will not coincide with the distance vanishing points, to which purpose one should follow the rotation procedures I set out in *Geometría Perspectiva* [García-Salgado 2000: Chapter 7: "Fuga de la Distancia (*fd*)", 81-96]. The interesting aspect of these procedures that rotate grids in space is that their geometric principle is grounded in a developing concept of distance, upon which the theory of perspective is built. In my opinion, the concept of distance vanishing point is second in importance only to the concept of vanishing point (or center vanishing point) discovered by Brunelleschi in Piazza San Giovanni. We will leave scrutiny of this issue for another occasion.

Acknowledgment

Special thanks to David A. Vila Domini for his valuable comments, observations and support during the review of this article.

Notes

1. "Evidently, the contradiction was felt between Euclid's *perspectiva naturalis* or *communis*, which sought simply to formulate mathematically the laws of natural vision (and so linked apparent size to the visual angle), and the *perspectiva artificialis* developed in the meantime, which on the contrary tried to provide a serviceable mehod for constructing images on two-dimensional surfaces. Clearly, this contradiction could be resolved only by abandoning the angle axiom…" [Panofsky 1991: 35-36]. Here the angle axiom concept means, in Panofsky's words, "… antique optics maintained, always and without exception, that apparent magnitudes (…) are determined not by the distances of the objects from the eye, but rather exclusively by the width of the angles of vision" [Panofsky 1991: 35]. This concept was valid until Alhazen introduced a new insight on the subject (see Note 2 on Alhazen below).
2. Alhazen (c. 965- c. 1039 A.D.) was a Muslim scientist and philosopher. He explained the concept of distance (*distantia*) by surveying—in some place—how a visible wall, at a certain distance from the observer, could match the size of his hand when it was placed between one eye and the wall while occluding the other eye. A remarkable experimental description is given in his book II.38:

   Et quantò remotius uisibile magis elongatibur, & uisus cetificauerit quantitatem remotionis eius, tantò comprehendetur maius. Verbi gratia: quando aliquis aspexerit parietem remotum à uisu remotione mediocri: & certificauerit uisus remotionem *illius parietis, & quantitatem eius: & certificauerit quantitatem latitudinis eius: deinde apposuerit manun uni uisui inter uisum & parietem: & clauserit alterum oculum: inueniet tunc, quòd manus eius cooperit portionem magnam illius parietis, & comprehendet quantitatem manus eius in ista dispositione, & comprehendet quòd quantitas cooperta à manu ex pariete, est multò maior quantitate manus eius: & uisus simul comprehendet uerticationes linearum radialium, & comprehendet angulum, quem continent lineæ radiales. Tunc ergo uisus comprehendet, quòd angulus, quem respiciunt manus & paries, est idem angulus: & tunc etiam comprehendet, quòd pars parietis cooperta manu eius, est multò maior manu* [Alhazen 1972: 52].

Thus, in Alhazen's optics, distance was understood as the relation between the size of the object and its subtended visual angle, and in its turn, the visual angle was understood as the quantity of the wall's remoteness.

3. To set up the San Giovanni experiment, Brunelleschi used a mirror both to capture the image of the Battistero and to observe it—through an orifice at the back of the *tavola*—making it improbable that he would have used geometrical drawings.
4. "[…] for Masaccio's *Trinity*, whose artificial architectural scaffolding was probably drawn, if not by Brunelleschi himself, at least according to his method, supposes an exact awareness of the role played by the point of distance, which is, moreover, indicated by a nail in the wall on which it is painted" [Klein 1981: 131].
5. The hypothesis of the execution is that Brunelleschi, with his back to the Battistero, placed —over the easel—a half braccio mirror and next to it a *tavola* (panel) of the same size onto which he painted what he saw and measured with the mirror. See [García-Salgado 1998a].
6. As we know, scale drawing is exactly what would facilitate application of perspective methods or procedures based on orthogonal drawings.
7. "We do not know the precise moment at which the two lateral points (…) received their theoretical explanation as the 'point of distance.' Brunelleschi probably extended their use from the plane to space; but did he know that their distance from the central vanishing point represented, according to the scale of the picture, the distance between the vantage point of an ideal spectator and the plane of the image?" [Klein 1981: 134].
8. *E se bene Filippo non aveva lettere, gli rendeva sí ragione delle cose, con il naturale della pratica e sperienza, che molte volte lo confondeva* [Vasari 1986: 280].
9. *Questa linea perpendiculare dunque mi darà ne i tagli suoi termini d'ogni distantia, lequali deuono essere fra le linee trauerse del pauimento egualmente lontane: nelqual modo io descritti tutti i paralelli dello spazzo* [Alberti 1435: 16].
10. *E con uno paio di seste farai quattro punti equidistanti, e con linee diritte le agiugni insieme, e fa' uno cuadro, o vuoi fare con la squadra; e fallo di quella grandezza che ti piaœ* [Filarete 1972, II: 651].
11. Note I intentionally avoid saying, "… where both visuals intersect determines the perspective depth of the transversal (m')", indicating that m' could correspond to another type of straight line or geometric shape (circle, ellipse, etc.).
12. The vanishing point of the perspective plane is defined by Modular Perspective as the vanishing point (*vp*), and the vanishing points belonging to the bodies are defined as asymmetrical vanishing points (a*vp1*, a*vp2*,… *avpn*).
13. Cennino Cennini describes a similar procedure in his work, *Il Libro d'Arte*, to secure drawing of a horizontal line on a wall (probably as a reference point to start drawing a fresco).

14. "Alberti, contrariwise, seems to have been ignorant of the real significance (almost of the existence) of the distance point; but he knew and well understood the central vanishing point and the horizon" [Klein 1981: 112].
15. "It is astonishing that this construction was completed without mentioning the central vanishing point or the horizon; Gauricus seems to describe a thoroughly mechanized procedure" [Klein 1981: 112].
16. *Dapoi ordino quanta distanza uoglio, che sia tra l'occhio di chi guarda, e la pittura: e quiui ordinato il loco del taglio, con una linea perpendiculare, come dicono i Mathematici, faccio il taglio di tutte le linee, ch'ella ha ritrouato* [Alberti 1435: 16].
17. "As assumed in Step One, the painter is before the wall he is about to paint, and therefore when he (Alberti) says, 'I take a small portion of space,' he is indicating that the lateral outline should occupy a small (*picciolo*) space (*spatio*), but that it should be at the side of the wall to be painted…" [García-Salgado 1998b: 121].
18. In Perspective Geometry, a constructive principle denotes—at least—two fundamental characteristics: order and structure.
19. In Modular Perspective, the perspective plane is considered as a true planar limit of 3D space. According to the geometrical model of Modular Perspective, the distance between the observer and the perspective plane is variable, and that between the observer and the object is constant (such concepts lack clarity in traditional methods, including that of Gauricus). Consequently, the distance between the observer and the perspective plane is ruled by the observer's visual angle and in its turn, interpreted as the distance vanishing point of the perspective plane. In traditional methods, on the other hand, the distance point will always remain in a two-dimensional plane either on a simple lateral projection or in a superimposed projection onto the picture plane.
20. With regard to the practice of Gauricus's method, Gioseffi suggests the employ of both the vanishing point and the distance point: "Gli artisti che practicarano il metodo del Gaurico avranno indubbiamente prolungato per controllo le linee di fuga fino al concorso nel punto principale: analogamente a chi per controllo tirasse le diagonali, secondo l'Alberti. Al quale la concorrenza in un punto delle diagonali stesse non è poi detto devesse necessariamente essere sfuggita, solo perchè non fu messa a profitto" [1957: 93].
21. Kitao closely analyzes the differences between Gioseffi's and Panofsky's interpretations of Gauricus's method; see [1962: 193].
22. "An elementary demonstration in geometry can prove that the orthogonals obtained by Gauricus's method should converge in a point situated on the median axis at the height of O, the classical principal vanishing point" [Klein 1981: 112].
23. "Another obvious problem with having the countercenter positioned beyond the frame itself is that it invites doubts whether it should be the central line or the boundary of the plane that serves to define the intersections" [Veltman 1975: 293, Chap. III, n. 55].
24. In any case, Gauricus's method remains valid even though it does not meet these conditions; as I postulated in my study on Perspective Geometry, the position—on the visual horizon—of the distance vanishing point (*dvp*) is variable.
25. Panofsky reproduced this passage:

    Ad perpendiculum mediam linea demittitio, Heinc inde semicirculos circunducito, Per eorum intersectiones lineam ipsam aequoream trahito, Nequis uero fiat in collocandis deinde personis error, fieri oportere demonstrant hoc modo, Esto iam in hac quadrata, nam eiusmodo potissimum utimur, tabula hec inquiunt linea, At quantum ab hac, plani definitrix distare debebit? Aut ubi corpora collocabimus? Qui prospicit, nisi iam in pedes despexerit, prospiciet a pedibus, unica sui ad minimum *dimensione, Ducatur itaque quot volueris pedum linea hec, Mox deinde heic longius atollatur alia in humanam staturam Sic, Ex huius autem ipsius uertice ducatur ad extremum aequoreae linea Sic, itidem ad omnium harum porcionum angulos Sic, ubi igitur a media aequorea perpendicularis hec, cum ea que ab uertice ad extremum ducta fuerat, se coniunxerit, plani finitricis Lineae terminus heic esto, quod si ab equorea ad hanc finitricem, ab laterali ad lateralem, absque ipsarum angulis ad angulos, plurimas hoc modo perduxeris lineas, descriptum etiam collocandis personis locum habebis, nam et cohaerare et distare uti oportuerit his ipsis debebunt intervallis* [Panofsky 1991:134-5].

26. "According to Gioseffi's justification of it (anamorphosis), Albertian perspective basically becomes a sort of anamorphosis, because its undeniable marginal 'errors' correct themselves for the 'right' spectator placed in the position designed for him" [Klein 1981: 138].

27. "By natural perspective I mean that the plane on which this perspective is represented is a flat surface, and this plane, although it is parallel both in length and height, is forced to diminish in its remoter parts more than in its nearer ones. … But artificial perspective, that is that which is devised by art, does the contrary; for objects equal in size increase on the plane where it is foreshortened in proportion as the eye is more natural and nearer to the plane, and as the part of the plane on which it is figured is farther from the eye" [Richter 1970, I: 63].

28. "practically and historically, it was a simplified bifocal construction; […] -more generally, of the oblique pyramid- […] in order to obtain a more supple and open perspective than Alberti's, thus infringing on one or the other of his postulates" [Klein 1981: 134].

29. "Le point principal en perspective doit estre constitue assis au nyueau de lueil: lequel point est appelle fix ou subiect. En apres/ une ligne produite et tiree des deux pars dudit point: et en icelle ligne doiuêt estre signez deux autres poits/ equedistans du subiect: plus prochains en presente/ et plus esloignez en distante beue: lesquelz sont appellez tiers poits" [Ivins 1973: Second Edition, A.ii]. Viator's *tiers poits* correspond to the so-called distance points.

References

ALBERTI, LEON BATTISTA. 1435. *La Pittura*. Tradotta per M. Lodovico Domenichi, Con Gratia e Privilegio, In Vinegia Apreßo Gabriel Giolito de Ferrari, MDXLVII (Milano, Biblioteca Ambrosiana SQLX 63).

ALHAZEN. 1972. *Opticae Thesaurus, Alhazeni Arabis Libri Septem, Nuncprimùm Editi. Eiusdem Liber De Crepusculis et Nubium Ascensionibus. Item Vitellonis Thuringopolini Libri X.* Basileæ, Per Episcopios, MDLXXII (With an Introduction to the Reprint Edition by David C. Lindberg). New York: Johnson Reprint Corporation.

DELLA FRANCESCA, PIERO. 1984. *De prospectiva pingendi.* G. Nicco-Fasola, ed. Florence: Le Lettere.

DOYLE, LAURENCE R., HANS-JÖRG DEEG and TIMOTHY M. BROWN. 2000. Searching for Shadows of Other Earths. *Scientific American*, Sept. 2000: 38-45.

FILARETE (Antonio Averlino). 1972. *Trattato di Architettura.* 2 vols. Anna Maria Finoli and Liliana Grassi, eds. Milan: Edizioni il Polifilo.

GARCÍA-SALGADO, TOMÁS. 1998a. Brunelleschi, Il Doumo y el Punto de Fuga. *Ciencias* **49**: 63-66.

———. 1998b. Geometric Interpretation of the Albertian Model. *Leonardo* **31**, 2: 119-123.

———. 1998c. La Última Cena de Leonardo da Vinci. *Ciencia y Desarrollo* **XXIV**, 143: 34-47.

———. 2000. *Geometría Perspectiva, Modular y Temas Clásicos*. México: Ed. TGS.

GAURICUS, POMPONIUS (Pomponio Gaurico). 1989. *Sobre la Escultura* (1504). André Chastel and Robert Klein, eds. Madrid: Ediciones Akal. (Italian ed: *De Sculptura*. Paolo Cutolo, trans. Naples: Edizioni Scientifiche Italiane, 1999.)

GIOSEFFI, DECIO. 1957. *Perspectiva Artificialis: Per la Storia della Prospettiva, Spigolature e Appunti*. Trieste: Università degli Studi.

IVINS, WILLIAM M., JR. 1973: *On the Rationalization of Sight*. Da Capo Press, Inc.

KITAO, TIMOTHY K. 1962. Prejudice in Perspective: A Study of Vignola's Perspective Treatise. *The Art Bulletin* 44: 173-194.

KLEIN, ROBERT. 1981. *Form and Meaning, Writings on the Renaissance and Modern Art*. New Haven: Princeton University Press.

MARTINI, FRANCESCO DI GIORGIO. 1969. *Trattati* (c. 1480), ed. C. Maltese. Milan.

PANOFSKY, ERWIN. 1991. *Perspective as Symbolic Form*. transl. Christopher S. Wood. New York: Zone Books.

POZZO, ANDREA. 1989. *Perspective in Architecture and Painting* (An Unabridged Reprint of the English-and-Latin Edition of the 1693 "Perspectiva, Pictorum et Architectorum"). New York: Dover.

RICHTER, JEAN PAUL. 1970. *The Notebooks of Leonardo da Vinci.* New York: Dover Publications, Inc.

SERLIO, SEBASTIANO. 1982. *The Five Books of Architecture*. New York: Dover.

VASARI, GIORGIO. 1986. *Le Vite de' piú eccellenti architetti, pittori, et scultori italiani, da Cimabue, insino a' tempi nostri* (Firenze 1550). Milan: Einaudi.

VELTMAN, KIM H. 1975. *Optics and Perspective: A Study in the Problems of Size and Distance*. PhD. Thesis, University of London, Warburg Institute.

About the Author

Tomás Garcia-Salgado is a researcher working on what he calls "perspective geometry". He has published several books and articles on the history, theory, and practice of perspective. The Modular Perspective method is one of his contributions in this field. A description of this method is available in English, in *Leonardo* (1988) or in his book *A Modular Perspective Handbook*. Some recent articles are: "Geometric Interpretation of the Albertian Model", *Leonardo* (1998); "The Geometry of the Pantheon's Vault" (http://www.generativeart.com/salgado/salgado.htm); "Anamorphic Perspective and Illusory Architecture" for the Generative Art 2001 Conference (Milano, Dec. 2001); and "Rational Design in Stained Glass Art vs. Artistic Intuition", to be published in *The Visual Mind II: Art and Mathematics*, Michele Emmer, ed. (MIT Press, in preparation).