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Distinguishability and Many-Particle Interference

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Quantum interference of two independent particles in pure quantum states is fully described by the particles' distinguishability: the closer the particles are to being identical, the higher the degree of quantum interference. When more than two particles are involved, the situation becomes more complex and interference capability extends beyond pairwise distinguishability, taking on a surprisingly rich character. Here, we study many-particle interference using three photons. We show that the distinguishability between pairs of photons is not sufficient to fully describe the photons' behavior in a scattering process, but that a collective phase, the *triad* phase, plays a role. We are able to explore the full parameter space of three-photon interference by generating heralded single photons and interfering them in a fiber tritter. Using multiple degrees of freedom—temporal delays and polarization—we isolate three-photon interference from two-photon interference. Our experiment disproves the view that pairwise two-photon distinguishability uniquely determines the degree of nonclassical many-particle interference.

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The famous Hong-Ou-Mandel (HOM) experiment in 1987 provided the first important example of nonclassical two-photon interference [1]. Two independent photons impinging on a beam splitter exhibit bunching behavior at the output ports that cannot be explained by a classical field model. The degree of bunching depends on how similar the two photons are in all degrees of freedom, for example, time, frequency, polarization, and spatial mode. Extending the study of interference to many particles is of interest from a fundamental as well as from a technological viewpoint [2–7]. The scattering of multiple photons in linear networks is related to solving problems in quantum information processing, metrology, and quantum state engineering [8–16]. Thus, understanding multiphoton interference is also of great relevance for practical applications.

Here, we demonstrate how many-particle interference is fundamentally richer than two-particle interference [17]. Two situations with the *same* pairwise distinguishability can lead to a *different* output distribution. This is due to a phase, the *triad* phase, that occurs only when more than two photons interfere.

We use independent photons and a tritter, a three-port symmetric beam splitter to investigate many-particle interference. We isolate the triad phase for the first time by interfering three photons in a tritter and exploiting multiple degrees of freedom, here time and polarization. We show that interfering three identical photons and varying time delays between them, as demonstrated in previous work [5,18,19], is not sufficient to study three-photon interference in full generality [20,21]. Our experiment allows us to isolate and tune the three-photon interference term as distinct from two-photon interference. In particular, manipulation of the triad phase goes beyond what is possible using temporal delays alone [5,6,19].

Theory.—The inner scalar product of two pure states $|\phi_i\rangle$ and $|\phi_j\rangle$ is

$$\langle\phi_i|\phi_j\rangle = r_{ij}e^{i\varphi_{ij}}, \tag{1}$$

where $r_{ij} \in (0,1)$ is the real modulus and $\varphi_{ij} \in (0,2\pi)$ is the argument. The modulus r_{ij} can be interpreted as a measure of the distinguishability of two photons in states $|\phi_i\rangle$ and $|\phi_j\rangle$, and equals zero (one) for two orthogonal (identical) states [22]. The argument φ_{ij} has, so far, received little attention due to its irrelevance in two-photon interference.

We consider two examples of devices that can be used to probe interference: a beam splitter and a tritter. The simplest device to probe interference is a balanced two-port beam splitter [see Figs. 1(a)and 1(b)]. When two photons $|\phi_1\rangle$ and $|\phi_2\rangle$ are injected into the beam splitter, the output statistics depend on the pairwise distinguishability of the incident photons

$$P_{11} = \frac{1}{2}(1 - r_{12}^2), \tag{2}$$

where P_{11} is the probability for detecting one photon in each of the output ports. If the photons are completely indistinguishable they always exit the same output port, in contrast to the classical behavior.

A tritter maps three spatial input modes onto three spatial output modes [see Fig. 1(c)]; the linear transformation for a balanced tritter is given by the unitary matrix

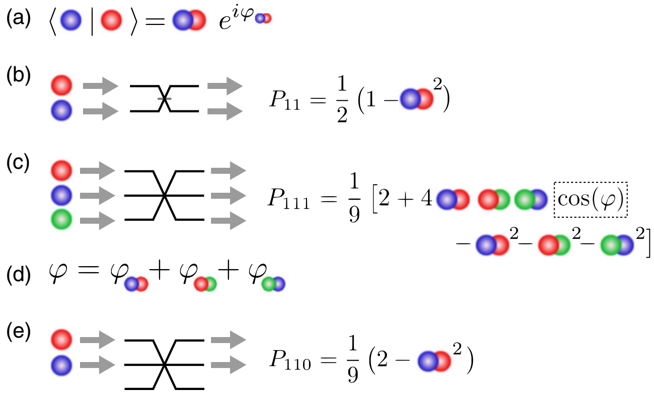

FIG. 1. Interference of photons in balanced beam splitters and tritters. (a),(b) The output statistics of two photons interfering in a beam splitter can be described via the pairwise distinguishability of the photons. (c) In the case of a tritter, the output statistics depend on an additional phase φ. (d) This *triad* phase φ is defined by the arguments of the pairwise complex scalar products. (e) φ only occurs in the interference of more than two photons.

$$U_{\text{tritter}} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \zeta^2 & \zeta \\ 1 & \zeta & \zeta^2 \end{pmatrix}, \tag{3}$$

where each output is equally likely and $\zeta = e^{i2\pi/3}$.

If we inject three photons into the tritter—a single photon in state $|\phi_i\rangle$ into each mode i for each $i = 1, 2, 3$—the probability P_{111} of having one photon in each of the output modes of the tritter is (see the Supplemental Material [23]) [24,25]

$$P_{111} = \frac{1}{9}[2 + 4r_{12}r_{23}r_{31}\cos(\varphi) - r_{12}^2 - r_{23}^2 - r_{31}^2], \tag{4}$$

where we define the collective *triad* phase $\varphi = \varphi_{12} + \varphi_{23} + \varphi_{31}$ as the sum of the three arguments. The dependence on φ appears only if the photons are partially distinguishable. If the states are orthogonal, the three moduli are zero; if they are identical, their scalar product will be equal to 1 and φ vanishes. Similar expressions can also be derived for the probabilities of having two or three photons in one of the output modes of the tritter (see Supplemental Material [23]).

Note that a global phase applied onto one of the input states does not lead to any change in the triad phase φ. Each phase φ_{ij} is only defined up to a global arbitrary phase. The sum of the phases, the triad phase, has physical meaning and is a measurable quantity. It remains unaffected by any global phase transformation and is crucial for the description of *partially distinguishable* photons [26,27].

However, dependence on the triad phase φ only occurs in measurements with more than two photons. The two-photon output coincidence probabilities P_{011} (one photon in outputs 2 and 3), P_{101}, P_{110} when sending two photons into different input ports of the tritter [as in Fig. 1(e)] are

$$P_{011} = P_{101} = P_{110} = \frac{1}{9}(2 - r_{ij}^2), \quad i,j = 1,2,3; \quad i \neq j, \tag{5}$$

and depend only on the mutual distinguishability of the incident photons $|\phi_i\rangle$ and $|\phi_j\rangle$.

Probing the triad phase and genuine three-photon interference.—We introduce a convenient implementation that allows us to control the moduli r_{ij} and the arguments φ_{ij} independently. We use 2 degrees of freedom for each spatial mode—time and polarization—to show that the addition of nonidentical polarization states can be used to create a nonzero φ. We consider the following input states to the tritter (see Fig. 2):

$$|\phi_i\rangle = |t_i\rangle \otimes (\cos\alpha_i|H\rangle + e^{i\eta_i}\sin\alpha_i|V\rangle), \tag{6}$$

where $|t_i\rangle$ is a temporal mode delayed by time t_i, $|H\rangle$ and $|V\rangle$ denote horizontal and vertical polarization, respectively, and $i = 1, 2, 3$ denotes the spatial mode. Using only temporal modes, $|t_1\rangle$, $|t_2\rangle$, $|t_3\rangle$, and otherwise identical photons with symmetric spectral intensities, the triad phase would always vanish, since $\langle t_1|t_2\rangle\langle t_2|t_3\rangle\langle t_3|t_1\rangle$ is real and non-negative (see Supplemental Material [23]).

In a first experiment, we aim to probe the triad phase directly. As a first step, we prepare the photons with the same polarization $|H\rangle$ in states (see Fig. 3(a)):

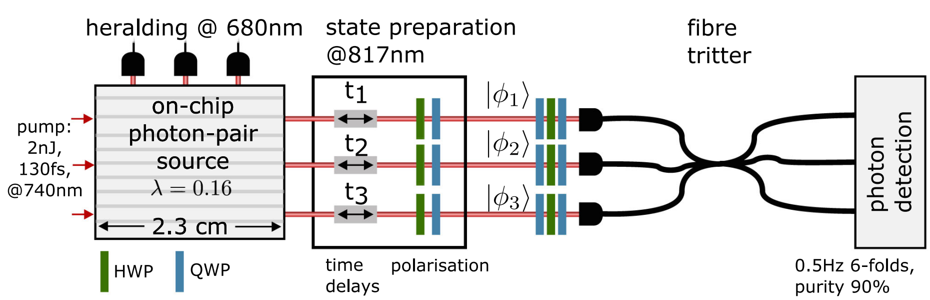

FIG. 2. Scheme of the experimental setup. The relative temporal delays of the three photons are adjusted using delay stages. We use sets of quarter-wave plates (QWPs) and half-wave plates (HWPs) to prepare the polarization state of each photon and to compensate for polarization rotations in the fibers. The outputs of the fiber tritter are monitored using multiplexed commercial avalanche photodiodes.

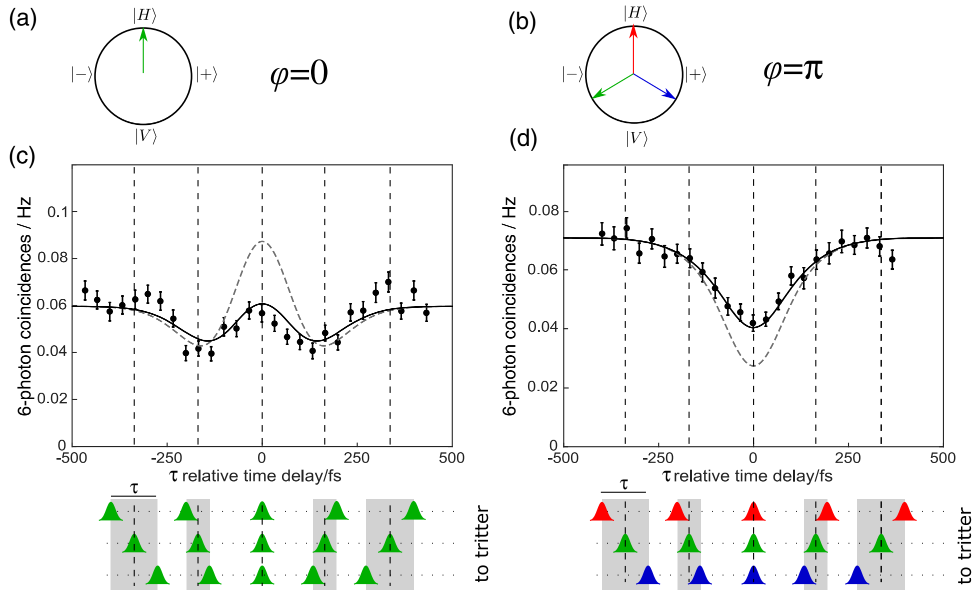

FIG. 3. Experimental heralded three-photon coincidences at the output of a fiber tritter for two values of the triad phase φ. (a),(b) We choose two polarization configurations so that $\varphi = 0$ (a) and $\varphi = \pi$ (b), see Eqs. (7) and (8). (c),(d) We measure heralded threefold coincidences ($\propto P_{111}$) between the different output ports of the tritter while varying the temporal delays of the photons. As shown pictorially beneath the plots, we start in a configuration where the photons are completely distinguishable in time; two of the photons are then scanned symmetrically across the third photon ($t_1 = t_2 - \tau/2$, $t_3 = t_2 + \tau/2$). The gray boxes show the region of temporal overlap of the photons. The nonmonotonic behavior in (c) arises because $\varphi = 0$ causes the three-photon interference term in Eq. (4) to have a contribution of opposite sign to those of the two-photon terms described by r_{ij}^2. In (d) $\varphi = \pi$ and so the contribution is of the same sign, resulting in monotonic behavior of the statistic. The dashed gray curve shows the theoretical prediction and the black curve is calculated using a model which includes experimental imperfections (see main text for details). The absolute number of counts per point were between 200 and 350 (250 and 450) for (a) [(b)]. Error bars are calculated from repeated measurements.

$$|\phi_i\rangle = |t_i\rangle \otimes |H\rangle \tag{7}$$

for $i = 1, 2, 3$, which sets $\varphi = 0$.

In the next step, we prepare photons in states [as depicted in the inset in Fig. 3(b)]

$$\begin{aligned} |\phi'_1\rangle &= |t_1\rangle \otimes |H\rangle, \\ |\phi'_2\rangle &= |t_2\rangle \otimes \frac{1}{2}(|H\rangle + \sqrt{3}|V\rangle), \\ |\phi'_3\rangle &= |t_3\rangle \otimes \frac{1}{2}(|H\rangle - \sqrt{3}|V\rangle). \end{aligned} \tag{8}$$

Here the scalar products $\langle\phi'_1|\phi'_2\rangle = 1/2\langle t_1|t_2\rangle$ and $\langle\phi'_3|\phi'_1\rangle = 1/2\langle t_3|t_1\rangle$, but $\langle\phi'_2|\phi'_3\rangle = -1/2\langle t_3|t_1\rangle$, setting $\varphi = \pi$. These two configurations demonstrate that using polarization as an additional degree of freedom allows us to vary the triad phase φ (see Supplemental Material [23] for more details).

In a second experiment, we *isolate* three-photon interference from two-photon interference. We explicitly show that control of φ allows manipulation of the three-photon term while leaving the two-photon interference terms constant. To do so, we prepare the following as input states to the tritter:

$$\begin{aligned} |\phi''_1\rangle &= |t_1\rangle \otimes [\cos(2\theta)|H\rangle + i\sin(2\theta)|V\rangle], \\ |\phi''_2\rangle &= |t_2\rangle \otimes \left[\frac{1}{2}(\sqrt{3}|H\rangle + |V\rangle)\right], \\ |\phi''_3\rangle &= |t_3\rangle \otimes \left[\frac{1}{2}(\sqrt{3}|H\rangle - |V\rangle)\right], \end{aligned} \tag{9}$$

where the state $|\phi''_1\rangle$ depends on a polarization rotation with angle θ and the polarizations of $|\phi''_2\rangle$ and $|\phi''_3\rangle$ are kept constant. With these states, we obtain the following moduli:

$$r_{12} = |\langle t_1|t_2\rangle| \times \frac{1}{2}\sqrt{2 + \cos(4\theta)}, \tag{10}$$

$$r_{31} = |\langle t_3|t_1\rangle| \times \frac{1}{2}\sqrt{2 + \cos(4\theta)}, \tag{11}$$

$$r_{23} = |\langle t_2|t_3\rangle| \times \frac{1}{2}, \tag{12}$$

and the triad phase

$$\varphi = 2\mathrm{Arg}(\sqrt{3}\cos(2\theta) + i\sin(2\theta)). \tag{13}$$

The angle θ affects both the triad phase φ and the moduli r_{12}, r_{31}; the temporal state $|t_1\rangle$ only affects r_{12} and r_{31}, but not φ. Combining control of both θ and $|t_1\rangle$ allows us to manipulate φ while r_{12} and r_{31} remain unchanged. For example, to keep $r_{12} = r_{23} = r_{31} = 1/2$, $|t_1\rangle$ must be chosen such that

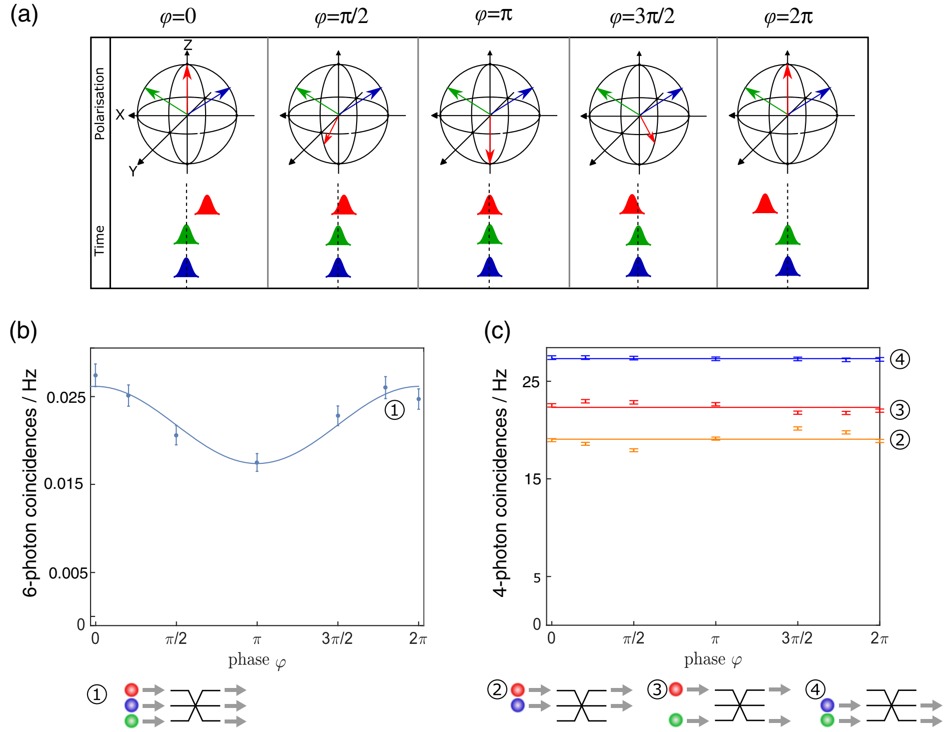

FIG. 4. Isolating two-photon from three-photon interference. (a) We vary the triad phase by rotating the polarization of photon $|\phi_1''\rangle$ as given in Eq. (9), leaving the polarization states of the two other photons fixed. To keep the moduli r_{12} and r_{31} constant, we adapt the temporal overlaps of the photons by tuning $|t_1\rangle$. (b) The three-photon signal P_{111} varies with the triad phase (absolute number of counts per data point is between 330 and 515). The plotted curve is a theory curve calculated based on our model of the experiment. (c) We plot a subset of two-photon distinguishability terms to demonstrate that these are kept constant.

$$|t_1 - t_2| = |t_1 - t_3| = \sigma\sqrt{2\ln[2+\cos(4\theta)]} \quad (14)$$

with $t_2 = t_3$ and σ^2 being the variance in time of the Gaussian wave packet (see Supplemental Material [23]).

Experiment and results.—To study the triad phase experimentally, we generate three heralded photons using spontaneous four-wave mixing (SFWM) in silica-on-silicon waveguides. The photon source is described in Ref. [19], along with 3-photon interference using only time delays ($\varphi = 0$).

We first probe the triad phase φ directly by choosing the input polarizations of the photons as given in Eqs. (7) and (8). By setting $t_1 = t_2 - \tau/2$, and $t_3 = t_2 + \tau/2$, and varying τ smoothly over the range shown in Figs. 3(c) and 3(d), we tune the degree of two- and three-photon interference. The results are shown in Fig. 3; we see a clear qualitative difference in behavior for the two cases of $\varphi = 0$ and $\varphi = \pi$.

We then demonstrate genuine three-photon interference by choosing the input states as given in Eq. (9), but now setting the time delay differences as in Eq. (14). We determined σ from a set of two-photon HOM dips with polarizations chosen as in Fig. 4(a) (first and third panel). The results are shown in Fig. 4; we observe good agreement of the measured curves with the theoretical prediction. The three-photon data follow a cosine shape as predicted by Eq. (4). The two-photon contributions P_{110}, P_{101}, P_{011} [see Eq. (5)] are nearly constant and show fluctuations of only on average 6% and the single photon detections at the tritter outputs vary only by a maximum of 3% due to polarization dependence. This verifies that these two-photon contributions are independent of the arguments. Detailed analysis suggests that polarization dependence of the tritter contributes to these fluctuations (see Supplemental Material [23]).

Our experimental data, both in Figs. 3 and 4, show the expected behavior, but there are some deviations from the probabilities given by Eqs. (4) and (5). To understand the influence of experimental imperfection, we performed a simulation of our experiment including the effects of higher-order photon emission, mixedness, and fluorescence noise. Based on our model, we calculated theory curves, including realistic experimental parameters (see lines in Figs. 3 and 4 and Supplemental Material [23] for more details) [28].

Conclusion.—In this work, we identify and describe a new phase that arises at the level of three photons: the triad phase. This new phase manifests itself in quantum interference and has implications for the scattering of many particles. In particular, the outcome of scattering events of more than two particles is determined not only by pairwise distinguishabilities of the particles' wave functions, but also on the collective properties of the particles. In this context, the triad phase emerges as a formal artifact [17,18,25,30–36]; we show here that it is of physical relevance.

There is a formal similarity between the triad phase and geometric phases that can be acquired by single photons, for instance, in the Pancharatnam-Berry phase [22,37–39]. Scaling up our study to more than three photons is ongoing work; for example, four interfering photons can be described by six two-photon measurements and three three-photon measurements.

Our work has implications for both linear-optical quantum computing and multiparticle scattering. There is an opportunity to use the triad phase to engineer the output state of a scattering process. For example, we are investigating properties of information encoded in a triad phase that result from the irreducibility of three-particle distinguishability. Another prospect is to efficiently characterize entanglement generation by linear optics using only partially mode-resolving detectors. Experimental approaches to boson sampling may benefit from partial distinguishability and using multiple degrees of freedom [36] as a means to increase the complexity of feasible tasks in an effort to surpass conventional computing power [40].

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