Distortion Analysis in Analog Integrated Circuits

Mohamed A. Y. Abdulla

I. Abstract

This paper reviews different techniques used in the literature to analyze distortion in analog integrated circuits. It concentrates on analytical techniques rather than on numerical techniques. Techniques such as Volterra series, harmonic injection method, and modeling of circuit non-linearities, are discussed with emphasis on the last two techniques. The theory behind each technique is explained, and they are compared together.

II. Introduction

In general, distortion is one of the most important undesired effects that appear in analog circuits due to non-linearities, which have many sources in integrated circuits [1]. In mixed-signal integrated circuits most of the functions are implemented in the digital domain, leaving the analog designer with tough specification to design analog circuits. Thus simply relying on the first-order analysis may not be enough. Thus exploring the analog circuits nonlinear behavior is a must. Also, in analog RF front-ends non-linearity becomes a very important issue, since the amplitude of the received signal may vary over a wide range depending on the distance between the transmitter and receiver. If the circuit exhibits non-linearities, this will generate higher order components called harmonics. On the other hand, if the desired signal is small such that it doesn't saturate the receiver circuits, a near-by out-of-band interferer may generate an undesired in-band component. These components are denoted as intermodulation products.

Distortion has been analyzed using different techniques in the literature [1-8], this paper will not discuss the numerical techniques used for distortion analysis. The Volterra series is the most popular symbolic method to analyze distortion. It combines the theory of convolution and Taylor series expansion to express non-linear systems with memory [1], [2]. It's a powerful technique, yet complicated, breaking the non-linear system down into an infinite parallel sub-systems: a linear sub-system, a second order sub-system, a third order sub-sytem,...to an infinite order subsystem, depending on the accuracy one needs. Most of the published work [3-8], till now, try to avoid using the Volterra series method to simplify the analysis, and merely approximating the system as a non-linear memoryless system. Thus, representing it by the Taylor expansion.

$$y(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots$$

For example in [3], the authors describe a simple way to model the non-linearities in a single-stage amplifier, and used it to derive expressions for the second and third harmonic distortion (HD_2), (HD_3), this will be discussed in section III. Furthermore, a new technique termed "Harmonic injection method" is presented in [4] which represents a non-linear system, excited by a certain frequency, by a linear system excited by the fundamental input and all its harmonics. To calculate the total harmonic distortion (THD), the authors propose a symbolic iterative method that will be discussed in section IV. Finally, in section V a comparison is made between the different techniques.

III. Distortion Analysis Using Non-Linearity Modeling

Single-stage amplifiers constitute the main building block for multi-stage amplifier, therefore, proper characterization of single-stage amplifiers is necessary. Any single-stage amplifier can be modeled, for small signal operation, by the circuit shown in Fig.1. Where, G_m is the transconductance of the input transistor, and R is the output resistance of the amplifier. In general the amplifier may have a cascode load to achieve a high gain from one stage. The compensation capacitor C is usually connected to the output node, which already has a high output impedance. In this model the authors in [3] argued that the sources of non-linearity are: the non-linear voltage-to-current conversion, and the non-linearity associated with the output resistance. Since high gain stages are usually operated in negative feedback loops, the input signal will always be a very small signal, and the output will be multiplied by the gain of the amplifier, thus the dominant source of non-linearity will be the output resistance. On the other hand, at very high frequencies the output capacitor shunts the output resistance and the open-loop gain of the amplifier is reduce, thus the error signal applied to the amplifier begins to increase causing the non-linearities due to the input transconductance to increase and dominate at high frequencies. From this argument the authors suggested that the non-linearity effects can be studied separately.

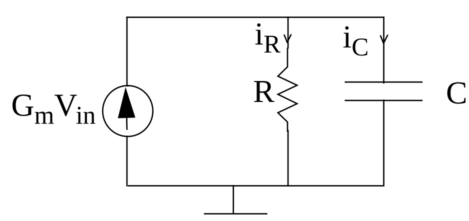


Figure (1): Small signal model of a single-stage amplifier [3]

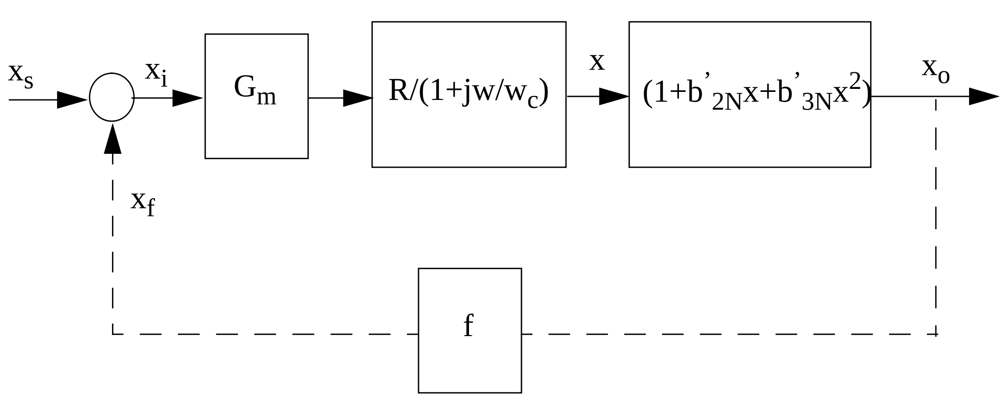
First, assume that the transconductance is constant and resistance non-linearity can be expressed as:

$$i_R = \frac{1}{R}(1 + g_{2N}v_{out} + g_{3N}v_{out}^2)v_{out}$$

Where g_{2N} and g_{3N} are the non-linear coefficients. So if the input can be represented by a single tone, $v_{in} = V_M \exp(j\omega t)$, then the non-linear representation of the resistance causes harmonics to appear at the output, which can be represented as:

$$v_{out} = b_1(j\omega)V_M \exp(j\omega t) + b_2(j\omega)V_M^2 \exp(j2\omega t) + b_3(j\omega)V_M^3 \exp(j3\omega t)$$

To find the harmonic distortion we have to solve for $b_1(j\omega)$, $b_2(j\omega)$, $b_3(j\omega)$. This can be done by writing a node equation at the output node and expressing the capacitor current as $i_C = C\frac{d}{dt}v_{out}$. To visualize the dynamic behavior of the amplifier, the author represented these results in a block diagram which is reproduced in Fig.2. Where b'_{2N} and b'_{3N} are the normalized coefficients, and w_c=1/RC.



Figure(2): Block diagram representing the single-stage amplifier with non-linearities [3]

Now it is desired to calculate the harmonic distortion when this amplifier is operated in a closed loop system, assuming a linear frequency independent feedback network f. In [5] the same authors derived expressions for the harmonic distortion, for a closed loop non-linear amplifier represented by the block diagram of Fig.2. Combining the two results we can finally reach expressions for the HD due to the non-linear resistance R.

$$HD^{R}{}_{2f}(j\omega) = \frac{1}{2}\frac{G_m R}{(1+T_o)^2} \times \frac{g_{2N}}{\left|1 + j\frac{\omega}{\omega_{GBW}}\right|\left|1 + j\frac{2\omega}{\omega_{GBW}}\right|} X_s$$

$$HD^{R}{}_{3f}(j\omega) = \frac{1}{4}\frac{(G_m R)^2}{(1+T_o)^3} \times \frac{\left|g_{3N} - \frac{2g_{2N}^2}{1 + \frac{j2\omega}{\omega_c}}\right|}{\left|1 + j\frac{\omega}{\omega_{GBW}}\right|^2\left|1 + j\frac{3\omega}{\omega_{GBW}}\right|} X_s^2$$

Where $T_o = fG_m R$, $\omega_c = 1/(RC)$, and $\omega_{GBW} = (1+T_o)\omega_c$.

The same can be repeated to take the effect of the non-linear transconductance assuming that the resistance is linear, and similar equations are derived. In this case the output current of the transconductor can be expressed as:

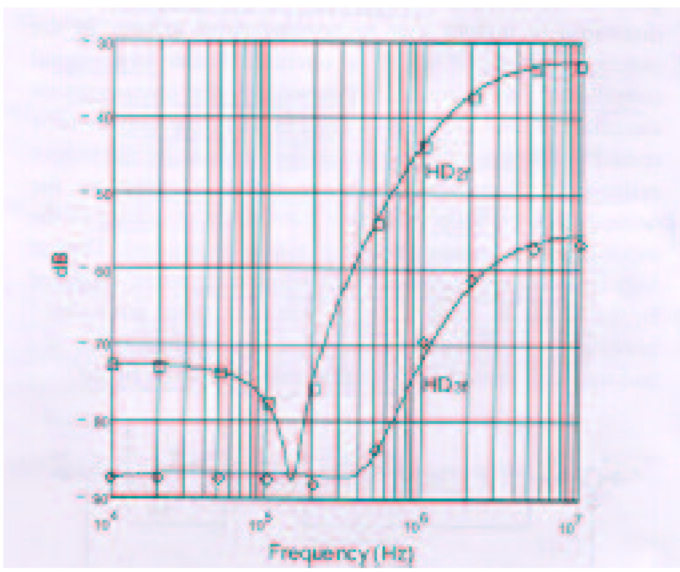
$$i = G_m(1 + a_{2N}x_i + a_{3N}x_i^2)x_i$$

Following the authors argument about different distortion sources, and that each source is dominant in a certain frequency range, they add them algebraically to arrive to the following expressions for the HD.

$$HD_{2f}(j\omega) \approx \frac{1}{2}\frac{G_m R}{(1+T_o)^2} \times \frac{\left|g_{2N} + \frac{a_{2N}}{G_m R}\left(1 + \frac{j\omega}{\omega_c}\right)^2\right|}{\left|1 + j\frac{\omega}{\omega_{GBW}}\right|\left|1 + j\frac{2\omega}{\omega_{GBW}}\right|} X_s$$

$$HD_{3f}(j\omega) \approx \frac{1}{4}\frac{(G_mR)^2}{(1+T_o)^3} \times \frac{\left| g_{3N} - \frac{2g_{2N}^2}{1+\frac{j2\omega}{\omega_c}} + \frac{a_{3N}-2a_{2N}^2}{(G_mR)^2}\left(1+\frac{j\omega}{\omega_c}\right)^3 \right|}{\left|1+j\frac{\omega}{\omega_{GBW}}\right|^2 \left|1+j\frac{3\omega}{\omega_{GBW}}\right|} X_s^2$$

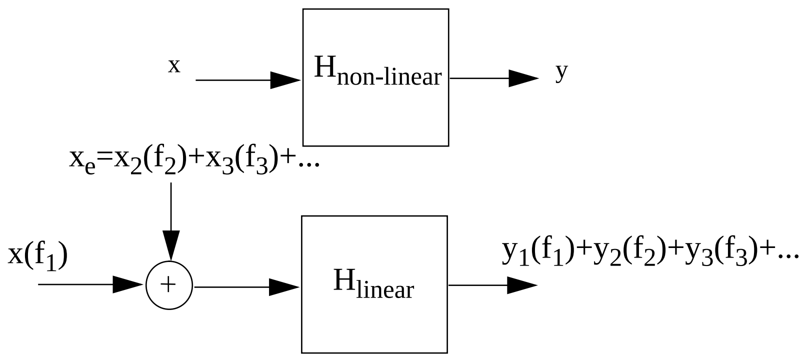
To validate these results the authors designed a cascode amplifier in a $0.35\mu m$ process. A low frequency signal was applied to the open-loop amplifier and from the Furrier analysis of the output voltage the coefficients g_{2N} and g_{3N} were evaluated. Then to evaluate a_{2N} and a_{3N} the short circuit output current was measured. The simulation results of the HD_2 and the HD_3 are reproduced in Fig.3 showing good matching between the analytical and simulated results.



Figure(3): Simulation results (HD_2 & HD_3) plotted with theoretical values vs. frequency [3]

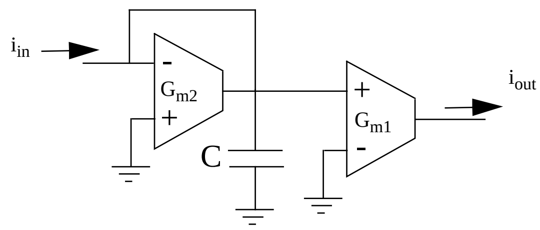
IV. Distortion Analysis Using Harmonic Injection Method

In [4] the harmonic injection method was proposed. Its basic idea is that, the output of a non-linear system excited by a single tone can be calculated by a linear system excited by the fundamental frequency and all its harmonics with different amplitudes, Fig.4. To estimate the harmonic distortion (THD), the authors proposed a symbolic iterative method to calculate these this additional input x_e.

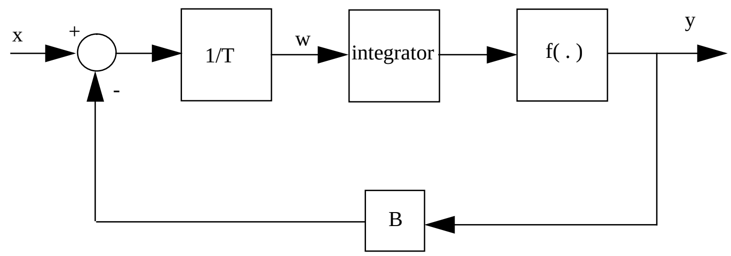


Figure(4): Block diagram of a non-linear system, and it's equivalent linear system [4]

The authors relay on a current-mode first order low pass G_m-C filter, Fig.5, to validate their method. In this circuit the source of non-linearity is the transconductors G_m. The transconductances can be written as: $G_1(v) = I_a f\left(\frac{v}{V_a}\right), G_1(v) = I_b f\left(\frac{v}{V_b}\right)$. Where I_a, I_b and V_a, V_b are constants. The value of the small signal transconductances were shown to be: $g_{m1} = \frac{I_a}{V_a}, g_{m2} = \frac{I_b}{V_b}$. This system was then represented with the block diagram reproduced in Fig.6, assuming that V_a=V_b.



Figure(5): A first order current-mode G_m-C low pass filter [4].



Figure(6): The block diagram representation of the circuit in Figure (5) [4]

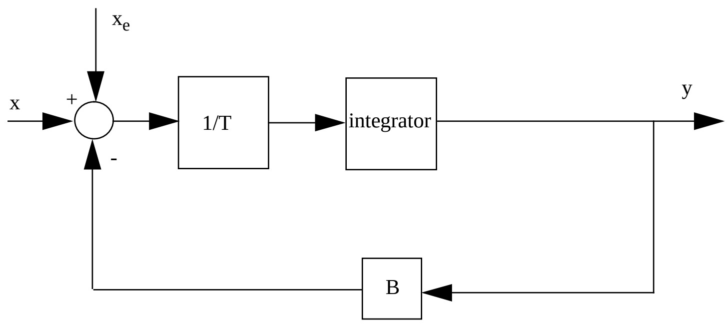
In this block diagram $B=I_b/I_a$, x and y are the normalized input and output currents with respect to I_a, and $T=CV_a/I_b$, and the function f is the source of non-linearity in the system. It can be easily shown that the first order transfer function can be given as:

$$H(s) = \frac{H_o}{\left(1 + \frac{s}{\omega_o}\right)}$$

Where $H_o = \frac{1}{B}$, $\omega_o = \frac{B}{T}$. Using some mathematical formulation is was shown that the same input-output relation can be represented by the linear block diagram shown in Fig.7, such that the input x_e is given by:

$$x_e = z \times Q(y)$$

Where $z = x - By$, and $Q(y) = \frac{1}{p(y)}$, and p(y) is given by: $p(y) = \frac{d}{dy}(f^{-1}(y))$. But to calculate the additional input x_e we need to know the output y of the system, which is already unknown. So the authors used iteration by assuming an initial value for y and calculating x_e, then using it to predict the new output and so one.



Figure(7): Linearized block diagram with the fundamental excitation and the harmonics.

The authors applied this method to different types of transconductors: BJT with exponential characteristics, MOS with quadratic characteristics, emitter-coupled-pairs with hyperbolic tanget characteristics, and mixed-translinear-loops with hyperbolic sine characteristics. In all the mentioned cases the number of iterations needed for the output to converge never exceeds three iterations for weakly non-linear circuits. That's why this method although iterative gives symbolic expressions for the THD. The expressions derived for BJT and MOS G_m-C filters are reproduced here:

$$THD_{BJT} = \frac{A\omega\omega_o}{2\sqrt{\omega^2 + \omega_o^2} \times \sqrt{4\omega^2 + \omega_o^2}}$$

$$THD_{MOS} = \frac{A\omega\omega_o}{4(2)\sqrt{\omega^2 + \omega_o^2} \times \sqrt{4\omega^2 + \omega_o^2}}$$

The simulation results provided in [4] demonstrate a good matching with the analytical results. But the iteration formulas derived are specific for G_m-C first order low-pass filters. To apply this circuit to any different class of analog circuits some steps should be done to obtain its block diagram representation, and then obtain the formulas used for iteration, as well as it is not yet tested to converge quickly, as in the demonstrated cases.

V. Conclusion

Throughout this paper, we tried to focus on symbolic techniques for distortion analysis in analog circuit, to try to provide the analog designers with insight in the dynamic behavior of the non-linear circuits. Numerical techniques are used, by simulators, to validate a certain distortion performance, but they don't guide throughout the design process. Analytical techniques vary from circuit-specific, to general techniques.

Circuit-specific techniques are based on good modeling of the devices including all the necessary effects, such as in most low noise amplifier (LNA) designs [6-8]. In [7] the square law of the MOS devices wasn't used but more complicated expressions were derived, and used to calculate the intermodulation products. While, for example in [8] short-channel MOSFET's were used to design a LNA, so current expressions taking into account velocity saturation and mobility degradation were necessary. But all these example are specific to a certain circuit or group of circuits.

General techniques, are more flexible to adapt to any analog circuit. The Volterra series method provides the most powerful and accurate tool to analyze distortion in non-linear analog circuits, yet it is, till now, complicated in its analysis. So most analog designers tend to develop other techniques, assuming a memoryless non-linear system, from which we discussed: modeling the circuit non-linearities, and harmonic injection method. Modeling the non-linearities seems a direct and simple way to analyze distortion, but it is restricted to single-stage amplifiers. The harmonic injection technique, provides an easy way to find the distortion, it is a general concept for any analog circuit. But it needs formulation for each class of different circuits, to obtain the block diagram representation and the equations used for iteration. Also, it is not guaranteed to converge fast in all cases. In conclusion, trying to re-approach the Volterra series method to make it simpler [2] will give analog designers a very powerful tool, and accurate tool in analyzing distortion.

VI. References

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