

# Distributed Antenna Systems and Linear Relaying for Gigabit MIMO Wireless

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**Abstract**— Spatial multiplexing is mandatory to achieve the extreme bandwidth efficiency of future Gigabit/sec WLANs. Both distributed Antenna Systems (DAS) at the access point and cooperative relaying (the infrastructureless counterpart) have been recognized as means to meet coverage/range requirements and to enable spatial multiplexing in a low scattering environment. In this paper we evaluate three candidate schemes under a two-hop (relay) traffic pattern: (i) DAS with decode&forward in the access point (DDAS), (ii) DAS with linear processing in the access point (LDAS) and (iii) linear relaying without any information exchange between the relay nodes. We give lower bounds on the capacity of LDAS and DDAS. A main contribution of this paper is a systematic derivation of local gain allocation strategies for linear relaying with multi-antenna source and destination nodes, which are based on large system analysis and do not require global channel knowledge at the relays. We derive approximate expression for the ergodic capacity. We show for a source and destination with  $M$  antennas, that asymptotically (large number of relays) linear relaying with  $MN$  support nodes performs similar to LDAS with  $M$  distributed antenna elements. Finally we propose a zero forcing gain allocation, which enables spatial multiplexing of *multiple single antenna source/destination pairs* based on a small number of autonomous relays. The theory is supported by comprehensive performance results.

## I. INTRODUCTION

Pervasive wireless access networks (PWAN) provide ubiquitous short range wireless connectivity for a variety of heterogeneous nodes. They encompass applications ranging from RF identification, Wireless Sensor Networks, Wireless Personal Area Networks to Wireless Local Area Networks. Pervasive wireless networks introduce fundamental new characteristic and requirements: (i) seamless integration of heterogeneous nodes, (ii) heterogeneous quality of service (QoS) requirements, (iii) extreme scalability and adaptivity, (iv) high node density, (v) extremely low cost per node required for ubiquity (vi) low EM exposure required for user acceptance and (vii) extremely nonuniform data traffic with Gbps local hot spots around access points. For a variety of technical and political reasons it is unlikely that the community will agree on a generic pervasive wireless access air interface. To meet the PWAN challenges however we have to design the constituent systems for *cooperation* rather than for *coexistence* only. In this paper we consider cooperative approaches to utilize the high density of single and multi-antenna nodes in a PWAN to optimize the spectral efficiency. In order to facilitate

link adaptation we have a primary focus on linear cooperation schemes, which are transparent to the modulation and coding schemes used by the source. Specifically we will consider both distributed antenna systems (DAS) with decode&forward (DDAS) and with linear processing (LDAS) as well as linear relaying (amplify&forward relaying) for PWAN.

Spatial multiplexing is mandatory to achieve the extreme bandwidth efficiency of future Gigabit/sec WLANs [1]. These Multiple Input/Multiple Output (MIMO) systems achieve an unprecedented spectral efficiency in a rich scattering environment. As opposed to conventional MIMO systems, distributed antenna systems (DAS) employ multiple antennas, which are not co-located at one site [4,5]. Recently cooperative relaying schemes have been proposed to improve wireless communication in multi-node networks. They are based on the idea to have multiple idle nodes assist in the communication of active nodes. To date cooperative relaying schemes have primarily been proposed to achieve diversity [2,3]. In [6,7] we propose distributed antenna systems and linear relaying to relax the rich scattering requirement of conventional MIMO signaling. Upper and lower bounds on the capacity of wireless networks with a relay traffic pattern have been determined in [8]. The system model consists of one source/destination pair, while all other nodes operate as relays in order to assist this transmission. In [9] the analysis of [8] is extended and upper and lower bounds on the capacity of MIMO wireless networks are given.

The remainder of the paper is organized as follows: in Section II we describe the signal model and summarize assumptions and notation. In Section III we consider lower bounds on the capacity of distributed antenna systems. In Section IV we treat linear relaying and give an approximate system model, which is valid, if the number of support nodes is large. In Section V we introduce the notions of zero forcing and asymptotic zero forcing relay gain allocation, propose suitable quadratic gain allocation schemes and give approximate expressions for the ergodic capacity. In Section VI we give comprehensive performance results for DDAS, LDAS and linear relaying with multi-antenna source and destination. In a network with single antenna sources and destinations it is obvious that both DDAS and LDAS may utilize up/downlink beamforming to achieve a space division multiple access gain. We show, that the proposed null space projection of the gain vector provides an

efficient way to utilize space division multiple access in *linear relaying* even if the number of relays is as small as four.

## II. SIGNAL MODEL

In Fig. 1 we show a typical application scenario. At the perimeter of a room a number of support nodes are placed. Communication between source and destination follows a two-hop pattern: source - support nodes - destination. In DAS the support nodes (remote radio front-ends) are connected to a central processor by a wired backbone (dashed lines). In relaying schemes the relays operate autonomously without backbone. In this paper we compare three different schemes. In the order of decreasing complexity: (i) DAS with decode&forward in the central processor, (ii) DAS with linear processing in the central processor and (iii) linear relaying without any information exchange between the relay nodes. The beauty of the linear approaches is simplicity and their transparency to adaptive modulation techniques used by the source. Source and destination may have multiple antennas. For linear relaying we also consider a scenario with multiple co-active source/destination pairs, where each node has only one antenna. Here we use the relaying to orthogonalize the individual links. We refer to this scenario as multiple source/multiple destination (MSMD) scenario.

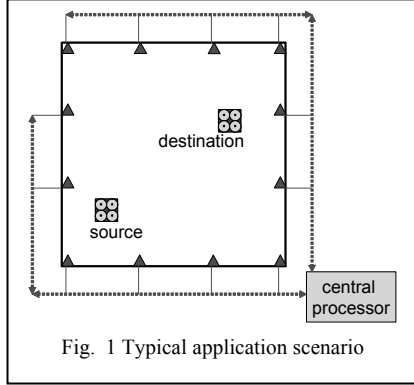


Fig. 1 Typical application scenario

Fig. 2 shows the corresponding signal model. The source transmits the vector  $\vec{s}$  through the uplink channel matrix  $H_{sr}$  to the support nodes (either DAS radio front ends or autonomous relay nodes).  $H_{sr}[k, q]$  denotes the channel gain between source antenna  $q$  and the antenna of relay  $k$ .  $\vec{m}$  comprises the AWGN contributions at the support nodes. It has i.i.d. elements with variance  $\sigma_m^2$ . The received signal at the support nodes is multiplied with the gain matrix  $G_r$ . The support node transmit signal  $\vec{r}$  is passed through the downlink channel matrix  $H_{rd}$  to the destination.  $\vec{w}$  is the destination AWGN contribution. It has i.i.d. elements with variance  $\sigma_w^2$ . For the linear distributed antenna system (LDAS) no constraints other than a total transmit power constraint are imposed on the gain matrix  $G_r$ . In linear relaying (LinRel) the gain matrix  $G_r$  is *diagonal*.

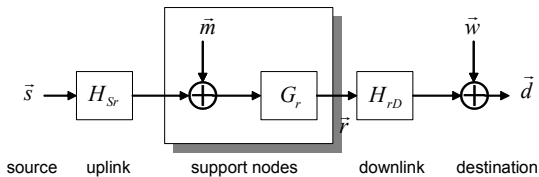


Fig. 2 System model

In decode distributed antennas systems (DDAS) a central processor decodes the uplink signal and re-encodes it for transmission on the downlink. Throughout the paper we assume a two-hop traffic pattern, i.e. the direct link from source to destination is not utilized.

### A. Notation and Assumptions

$N_r$  is the number of support nodes,  $N_{as}$  the number of source antenna elements and  $N_{ad}$  the number of destination antenna elements.  $P_s = E[\vec{s}^H \cdot \vec{s}]$  is the source transmit power. Column vectors are denoted as  $\vec{h}$ , matrices as  $H$ , the  $p$ -th column of a matrix is  $H[:, p]$ , the element  $(p, q)$  is denoted as  $H[p, q]$  and  $H^H$  indicates hermitian transpose. The source has no channel state information (CSI), the destination perfect CSI. The support nodes have only one antenna element each and operate half-duplex (either TX or RX). The source symbol vector  $\vec{s}$  has i.i.d. complex normal elements with variance  $\sigma_s^2$  (no power loading).

## III. DISTRIBUTED ANTENNA SYSTEM

Let  $H_{sr} = U_{sr} \cdot \Sigma_{sr} \cdot V_{sr}^H$  and  $H_{rd} = U_{rd} \cdot \Sigma_{rd} \cdot V_{rd}^H$  be the singular value decompositions of the uplink and downlink channel matrix, respectively. Furthermore, let  $\sigma_{sr}^{(q)} = \Sigma_{sr}[q, q]$  and  $\vec{u}_{sr}^{(q)} = U_{sr}[:, q]$  be the  $q$ -th singular value and the  $q$ -th left-hand eigenvector of the uplink channel matrix; equivalently  $\sigma_{rd}^{(p)}$  and  $\vec{v}_{rd}^{(p)}$  are the  $p$ -th singular value and the  $p$ -th right-hand eigenvector of the downlink channel matrix. As we assume no channel state information at the source, the capacity achieving source symbol vector  $\vec{s}$  has i.i.d. complex normal elements. The unitary matrix  $V_{sr}$  has no impact on correlation matrix of the source symbols in this case. Thus for capacity considerations we may drop  $V_{sr}$  without loss of generality. As the additive noise  $\vec{w}$  at the destination is white, we may drop  $U_{rd}$  for the same reasons (perfect CSI at the destination) and obtain the parallel system model in Fig. 3.

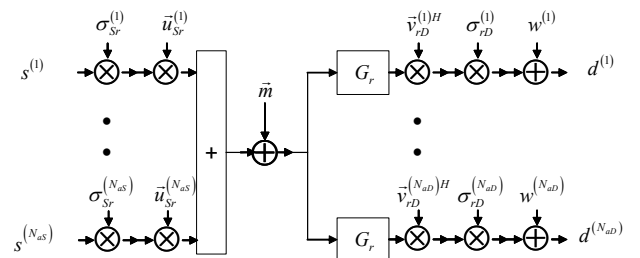


Fig. 3: Parallel system model

The  $q$ -th element  $\vec{s}[q]$  of the (equivalent) transmit symbol vector is weighed with the  $q$ -th singular value of the uplink channel matrix. Multiplication with the vector  $\vec{u}_{sr}^{(q)}$  yields the contribution of the  $q$ -th (equivalent) transmit antenna to the receive signal vector at the support nodes. We refer to  $\vec{u}_{sr}^{(q)}$  as

the  $q$ -th uplink eigenmode. Conversely  $\bar{v}_{rD}^{(p)}$  is the  $p$ -th downlink eigenmode. It determines the mapping of the support node transmit vector to the received signal at the  $p$ -th (equivalent) destination antenna.

In the DAS case a central processor knows the signals at all support nodes. Thus without loss of generality we may expand the gain matrix as  $G_r = U_{Sr}^H \cdot G_{r,2} \cdot V_{rD}$  and obtain a diagonal system model with orthogonal uplink/downlink subchannels (Fig. 4).

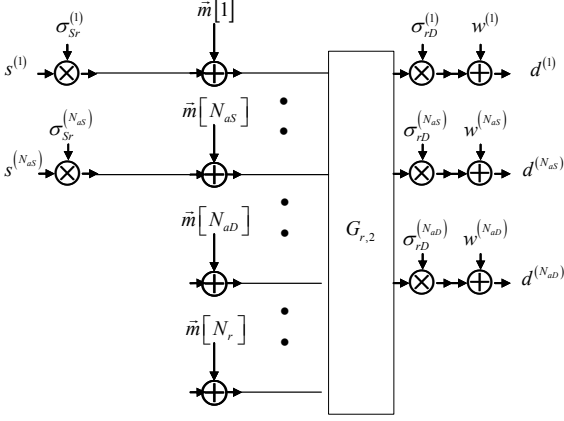


Fig. 4: Diagonal system model for DAS

#### A. Linear Distributed Antenna System

To derive a lower bound on the capacity in the LDAS case we impose the following constraints:

- the equivalent gain matrix is diagonal:  $G_{r,2} \equiv D_r$ . We refer to this constraint as *stream-LDAS*.
- the singular values are ordered in decreasing order
- uniform allocation of the total support node transmit power  $P_{r,g}$  across the first  $N = \min(N_{as}, N_{ad}, N_r)$  downlink eigenmodes
- no transmit power allocated to the remaining downlink eigenmodes.

We define the uplink and downlink SNR on the  $n$ -th eigenmodes:

$$SNR_U^{(n)} = \frac{\sigma_s^{2(n)} \cdot \sigma_{Sr}^{(n)2}}{\sigma_m^2}; \quad SNR_D^{(n)} = \frac{P_{r,g} \cdot \sigma_{rD}^{(n)2}}{N \cdot \sigma_w^2} \quad (1)$$

The concatenation of the  $n$ -th uplink and downlink eigenmodes yields the destination SNR in the  $n$ -th subchannel

$$SNR^{(n)} = \frac{SNR_U^{(n)} \cdot SNR_D^{(n)}}{SNR_U^{(n)} + SNR_D^{(n)} + 1} \quad (2)$$

The aggregate mutual information of all subchannels follows readily. In [bit/complex dimension] we obtain

$$I_{LDAS} = 0.5 \cdot \sum_{n=1}^N \log_2(1 + SNR^{(n)}) \leq C_{LDAS} \quad (3)$$

Note that the factor 0.5 accounts for the 2 channel uses, which are required for our two-hop traffic pattern. The mutual information of stream-LDAS is a lower bound on the capacity  $C_{LDAS}$  of a general LDAS system. Let the uplink and downlink

channel matrices have i.i.d. complex normal elements with variance  $\sigma_h^2$ . Then for large  $N_r$  their singular values  $\sigma_{Sr}^{(n)}; \sigma_{rD}^{(n)}$  are well approximated by  $\sigma_{Sr}^{(n)}; \sigma_{rD}^{(n)} \approx \sqrt{\sigma_h^2 \cdot N_r}$ . With (1) and (2) we obtain the approximate asymptotic subchannel SNR for LDAS

$$\widehat{SNR}_{LDAS} = N_r \cdot \frac{\sigma_s^2 \cdot \sigma_h^2}{\frac{\sigma_w^2}{P_{r,0}} \cdot \left( \sigma_s^2 + \frac{\sigma_m^2}{N_r \cdot \sigma_h^2} \right) + \sigma_m^2} \quad (4)$$

#### B. DDAS

In the DDAS case the support nodes jointly decode the  $N_U = \min(N_{as}, N_r)$  received signal streams. The decoded data is jointly re-encoded for the  $N_D = \min(N_{ad}, N_r)$  downlink spatial subchannels. The capacity  $C_{DDAS}$  is the minimum of the aggregate uplink capacity and the aggregate downlink capacity. Without support node power loading, the total support node transmit power  $P_{r,g}$  is uniformly allocated across all nonzero downlink eigenmodes. In the stream-LDAS case the power is distributed over  $N = \min(N_r, N_{ad}, N_{as})$  eigenmodes. Thus the effective downlink SNR for DDAS is a scaled version of the SNR defined in (1) and we obtain

$$C_{DDAS} = \frac{1}{2} \cdot \min(C_U, C_D)$$

$$C_U = \sum_{m=1}^{N_U} \log_2(1 + SNR_U^{(m)}) \quad (5)$$

$$C_D = \sum_{n=1}^{N_D} \log_2\left(1 + \frac{N}{N_D} SNR_D^{(n)}\right)$$

Note the different ranges of the summation indices. It is not necessarily optimum to use all subchannels on the downlink. The available transmit power per subchannels increases as the number of active subchannels decreases (this is of course not true with power loading). In the remainder of this section we will constrain our attention to  $N_{as} = N_{ad} \leq N_r$ , so as to facilitate the comparison with stream-LDAS.

A lower bound on the capacity of the DDAS system is obtained by decoding/encoding each stream independently (Fig. 5). We refer to this variant as *stream-DDAS*. The capacity follows readily from (5)

$$C_{1,DDAS} = \frac{1}{2} \cdot \sum_{m=1}^N \log_2\left(1 + \min(SNR_U^{(m)}, SNR_D^{(m)})\right) \quad (6)$$

In the following we consider one subchannel. For convenience we drop the subchannel index of the SNR. Without loss of generality we let  $SNR_D / SNR_U = a \geq 1$ . The relevant SNR in (6) thus is  $SNR_U$ . With (2) we obtain for the LDAS

$$SNR = \frac{SNR_U \cdot a \cdot SNR_U}{SNR_U + a \cdot SNR_U + 1} \approx \frac{a}{1+a} \cdot SNR_U \quad \text{for } SNR_U \gg 1 \quad (7)$$

Thus the maximum loss of LDAS w.r.t. stream-DDAS (Fig. 5) is 3dB in the large SNR regime.

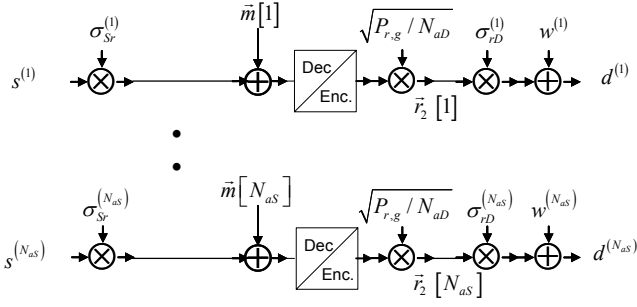


Fig. 5: System model for a decode&forward DAS with independent encoding/decoding of the data streams (stream-DDAS).

#### IV. LINEAR RELAYING

Fig. 6 shows the system model, which we will use as starting point of the analysis of linear relaying. The scaling factor  $c_n$  of the destination noise vector  $\bar{w}$  incorporates the total power constraint at the relays.

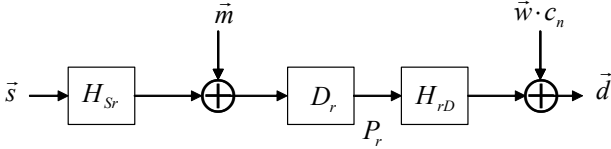


Fig. 6: Linear relaying system model.

The total relay transmit power  $P_r$  is given by

$$P_r = \sum_{p=1}^{N_{as}} \left( \sum_{k=1}^{N_r} |H_{Sr}[k, p]|^2 \cdot |\bar{d}_r[k]|^2 \cdot \sigma_w^2 \right) + \sigma_m^2 \cdot \sum_{k=1}^{N_r} |\bar{d}_r[k]|^2 \quad (8)$$

The noise scaling factor follows with the desired total relay transmit power  $P_{r,g}$  as

$$c_n^2 = P_r / P_{r,g} \quad (9)$$

An equivalent ( $N_{as} \times N_{ad}$ ) system model is depicted in Fig. 7.

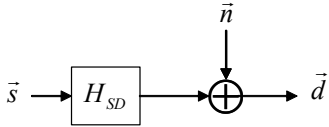


Fig. 7: Equivalent system model

The equivalent channel matrix  $H_{SD}$  has the elements

$$H_{SD}[p, q] = \sum_{k=1}^{N_r} H_{rD}[p, k] \cdot \bar{d}_r^*[k] \cdot H_{Sr}[k, q] \equiv \sum_{k=1}^{N_r} a^{(k)}(p, q) \quad (10)$$

The equivalent noise vector  $\bar{n}$  comprises the relay and the destination noise. It has the correlation matrix

$$\Lambda_{nn}[p_1, p_2] = P_r / P_{r,g} \cdot \sigma_w^2 \cdot \delta(p_1, p_2) + \sigma_m^2 \cdot \sum_{k=1}^{N_r} |H_{rD}[p_1, k]|^2 \cdot |\bar{d}_r[k]|^2 \cdot H_{rD}^*[p_2, k] \quad (11)$$

The  $\sigma_w^2$ -term in (11) is due to the destination noise and the  $\sigma_m^2$ -term describes the relay noise contribution.  $\delta(p_1, p_2)$  is the Kronecker delta.

#### A. Large System Analysis

In the sequel we assume, that the set of up- and downlink channel coefficients to relays  $k_1$  and  $k_2$  are independent realizations of the same random process. Furthermore we assume a *local gain allocation* at the relays, i.e. the gain allocation depends only on the up- and downlink channel coefficients of the considered relay. As a result the relay can determine the gain locally without information exchange with other nodes. With these assumptions the variables  $a^{(k)}(p, q)$  in (10) are i.i.d. realizations of a random variable

$$a(p, q) \equiv h_{rD}(p) \cdot d_r^* \cdot h_{Sr}(q) \quad (12)$$

The random variable  $h_{Sr}(q)$  characterizes the uplink channel coefficients from the  $q$ -th source antenna and  $h_{rD}(p)$  the downlink channel coefficients to the  $p$ -th destination antenna. Let  $\bar{a}(p, q) = E[a(p, q)]$  be the mean of  $a(p, q)$ . Then with (10) we may express the normalized equivalent channel matrix as

$$1/N_r \cdot H_{SD}[p, q] = \bar{a}(p, q) + 1/\sqrt{N_r} \cdot \alpha(p, q) \quad (13)$$

where

$$\alpha(p, q) = 1/\sqrt{N_r} \cdot \sum_{k=1}^{N_r} (a^{(k)}(p, q) - \bar{a}(p, q)) \quad (14)$$

is a zero mean random variable with the same variance as  $a(p, q)$ . The noise correlation matrix  $\Lambda_{nn}$  in (8) and (11) involves sums of independent random variables. Asymptotically for a large number of relays  $N_r$  we obtain

$$\begin{aligned} \bar{\Lambda}_{nn}[p_1, p_2] &= \lim_{N_r \rightarrow \infty} 1/N_r \cdot \Lambda_{nn}[p_1, p_2] \\ &= \bar{P}_r / P_{r,g} \cdot \sigma_w^2 \cdot \delta(p_1, p_2) + \sigma_m^2 \cdot E[|d_r|^2 \cdot h_{rD}(p_1) \cdot h_{rD}^*[p_2]] \end{aligned} \quad (15)$$

where

$$\bar{P}_r = \sigma_s^2 \cdot \sum_{p=1}^{N_{as}} E[|d_r|^2 \cdot |h_{Sr}(p)|^2] + \sigma_m^2 \cdot E[|d_r|^2] \quad (16)$$

Noting that both the noise correlation matrix in (15) and the channel matrix in (13) have been normalized by the number of relays, we obtain the asymptotic system model in Fig. 8. The matrix  $\bar{A}$  may be regarded as "Line-of-Sight" component whereas the lower branch resembles a "diffuse" component of the equivalent channel matrix. We will use the asymptotic system model in the sequel to optimize the relay gain allocation.

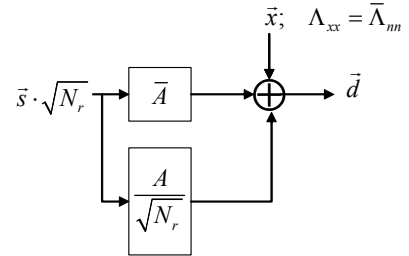


Fig. 8: Asymptotic system model for  $N_r \gg 1$ .

## V. GAIN ALLOCATION STRATEGIES

For single antenna source and destination, i.e.  $N_a = 1$ , the optimum relay gain allocation problem has an analytical solution [10]

$$d_r = c \cdot \frac{h_{sr} \cdot h_{rd}}{\left( (\sigma_m^2 + \sigma_s^2 \cdot |h_{sr}|^2) \cdot \sigma_w^2 + P_{r,g} \cdot \sigma_m^2 \cdot |h_{rd}|^2 \right)} \quad (17)$$

The constant  $c$  is chosen such, that the total transmit power constraint in (8) is met.  $h_{sr}, h_{rd}$  is the up-/downlink channel coefficient of the considered relay. For convenience we have dropped the relay index (k). For  $N_a > 1$  antenna elements at source and destination we are not aware of an analytical solution to the gain allocation problem. In the sequel we will study different suboptimal approaches.

### A. Quadratic Gain Allocation

In this paragraph we consider gain allocations  $d_r$ , which may be expressed as

$$d_r = \bar{h}_{rD}^T \cdot G \cdot \bar{h}_{Sr} \quad (18)$$

The elements of the random vector  $\bar{h}_{Sr}$  are the uplink channel coefficients  $h_{sr}(q)$  to a specific relay and  $\bar{h}_{rD}$  downlink channel coefficients  $h_{rd}(p)$  from that relay. For the remainder of this section we assume that all elements of the channel vectors are i.i.d. complex normal with zero mean and variance  $\sigma_h^2$ . We consider the asymptotic case  $N_r \rightarrow \infty$ . For this reason the system performance is determined by the matrix  $\bar{A}$  in Fig. 8, as the variance of the elements of the diffuse contribution tends towards zero. With (18) and (12) we obtain immediately

$$\bar{A}[p, q] = E \left[ h_{rd}(p) \cdot \bar{h}_{rD}^H \cdot G^* \cdot \bar{h}_{Sr}^* \cdot h_{sr}(q) \right] = \sigma_h^4 \cdot G^* [p, q] \quad (19)$$

and after some calculations

$$\begin{aligned} \bar{\Lambda}_m[p_1, p_2] = & \\ & \sigma_h^6 \cdot \sigma_w^2 / P_{r,g} \cdot \delta(p_1, p_2) \cdot b_i \cdot \left( (N_{as} + 1) \cdot \sigma_s^2 + \sigma_m^2 / \sigma_h^2 \right) + \\ & + \sigma_m^2 \cdot \sigma_h^6 \cdot \left( c_i \cdot \delta(p_1, p_2) + C(p_2, p_1) \right) \end{aligned} \quad (20)$$

where

$$\begin{aligned} C &= G \cdot G^H \quad \text{and} \quad c_i = \text{trace}(C) \\ b_i &= \text{trace}(G^H \cdot G) \end{aligned} \quad (21)$$

### B. Asymptotic Zero Forcing

We refer to the class of gain allocations, which lead to a unitary matrix  $\bar{A}$  in Fig. 8 as *asymptotic zero forcing* gain allocation. For quadratic gain allocation schemes and  $N_{as} = N_{ad} = N_a$  this implies with (10):  $G^H \cdot G = G \cdot G^H = I_{N_a}$ . A particularly simple choice is

$$d_r = \bar{h}_{rD}^T \cdot \bar{h}_{Sr} \quad (22)$$

With (20) the equivalent noise is white and has the correlation matrix

$$\bar{\Lambda}_m = \sigma_h^6 \cdot \left[ \sigma_w^2 / P_{r,0} \cdot \left\{ \sigma_s^2 + \frac{\sigma_m^2}{\sigma_h^2 \cdot (N_a + 1)} \right\} + \sigma_m^2 \right] \cdot (N_a + 1) \cdot I \quad (23)$$

with  $P_{r,0} = P_{r,g} / N_a$ . As a result, the destination may decode the different data stream independently (i.e. the destination antennas need not cooperate and thus may belong to different users). The approximate asymptotic SNR per spatial subchannel follows readily

$$\widehat{SNR}_B = \frac{N_r}{(N_a + 1)} \cdot \frac{\sigma_s^2 \cdot \sigma_h^2}{\sigma_w^2 / P_{r,0} \cdot \left\{ \sigma_s^2 + \frac{\sigma_m^2}{\sigma_h^2 \cdot (N_a + 1)} \right\} + \sigma_m^2} \quad (24)$$

It is interesting to compare this SNR to the approximate SNR of LDAS. A comparison with (4) reveals that linear relaying in the large system limit essentially requires  $N_a$  times as many support nodes as does LDAS.

In the single antenna case and a noiseless destination ( $\sigma_w^2 = 0$ ) we obtain with (17) the relay gain  $d_r \sim h_{sr} / h_{rd}^*$ . The downlink channel coefficient is inverted and the destination receive signal realizes a maximum ratio combining of the relay receive signals. Obviously in this case there is no performance difference to LDAS. From (12) and (15) we obtain the approximate asymptotic SNR for a noiseless destination

$$\widehat{SNR}_A = N_r \cdot \sigma_s^2 \cdot \frac{\sigma_h^2}{\sigma_m^2} \quad (25)$$

A comparison with (24) yields a 3dB improvement over the simple gain allocation  $d_r = h_{sr} \cdot h_{rd}$ . Our experience has shown that the asymptotic zero forcing gain allocation in (22) is a viable suboptimal solution for  $N_a \geq 4$ . In order to improve the performance for a small number of antennas, we propose to normalize this gain by an extension of the denominator in (17)

$$d_r = c_1 \cdot \frac{\bar{h}_{Sr}^T \cdot \bar{h}_{rD}}{\left( N_a \cdot \sigma_m^2 + \sigma_s^2 \cdot \bar{h}_{Sr}^H \bar{h}_{Sr} \right) \cdot \sigma_w^2 + P_{r,g} \cdot \sigma_m^2 \cdot \bar{h}_{rD}^H \bar{h}_{rD}} \quad (26)$$

Note that this gain allocation is still asymptotic zero forcing, because the denominator involves only the magnitude and not the phase of the channel coefficients.

### C. Zero Forcing Projection

Let  $\bar{h}_{rD}^{(p)} = H_{rD}[p, :]^T$  be the p-th row of the downlink channel matrix and  $\bar{h}_{Sr}^{(q)} = H_{Sr}[:, q]$  the q-th column of the uplink channel matrix. Then the element  $H_{SD}[p, q]$  of the equivalent channel matrix in (10) is the projection of the gain vector on the Hadamard product (element-wise multiplication) of the respective channel vectors

$$H_{SD}[p, q] = \bar{d}_r^H \cdot \left( \bar{h}_{rD}^{(p)} \odot \bar{h}_{Sr}^{(q)} \right) \quad (27)$$

For clarity of exposition we assume, that source and destination have the same number  $N_a$  of antenna elements. If the matrix  $H_{SD}$  is diagonal, there is no intersymbol interference between the  $N_a$  spatial subchannels. For any gain vector  $\bar{d}$ , a corresponding zero forcing gain vector  $\bar{d}_{r,zf}$  is

obtained by a projection of  $\vec{d}_r$  onto the nullspace of the set of the  $N_a \cdot (N_a - 1)$  vectors defined by

$$\left\{ \left( \vec{h}_{r,D}^{(p)} \odot \vec{h}_{s,r}^{(q)} \right) \mid p, q \in [1, N_a] \text{ and } p \neq q \right\} \quad (28)$$

Note that a sufficient condition for a nonempty set is  $N_r > (N_a - 1) \cdot N_a$ , because otherwise the set of vectors may have full rank.

## VI. PERFORMANCE RESULTS

In this section we compare stream-DDAS, stream-LDAS and linear relaying with different gain allocation strategies. Both the uplink channel matrix  $H_{sr}$  and downlink channel matrix  $H_{rd}$  have i.i.d. complex normal elements with unit variance and zero mean. We use the notation  $(N_{as} \times N_r \times N_{ad})$  to denote a system with  $N_{as}$  source antennas,  $N_r$  support nodes (relays) and  $N_{ad}$  destination antennas. In this section we will constrain our attention to  $(N_a \times N_r \times N_a)$  systems with the same number of source and destination antennas. All nodes are perfectly synchronized, i.e. there is a global phase reference. The source has no channel state information (CSI) and the transmit symbols at each source antenna element are i.i.d. complex normal random variables with variance  $\sigma_s^2$  (independent streams). We impose a total power constraint on the source, i.e.  $\sigma_s^2 = 1/N_a$ . For the DAS the central processor has perfect CSI, it does not utilize power loading across the downlink spatial eigenmodes however. For linear relaying each relay has perfect local CSI (i.e. it knows its own channel coefficients). Only in the case of linear relaying with Zero Forcing perfect global CSI (uplink and downlink channel matrix) is required at each relays in order to calculate the zero forcing gain coefficient. The support nodes are subject to a total power constraint  $P_{r,G} = 1$ . The average uplink/downlink SNR is defined as the average support node/destination SNR in a  $(1 \times 1 \times 1)$  system. All systems in this section have the same average uplink and downlink SNR (i.e. the local noise contribution at any one relay and at the destination has the same variance:  $\sigma_m^2 = \sigma_w^2$ ). When not otherwise stated we assume  $\overline{SNR} = 20$  dB. For linear relaying we consider the local gain allocations

- *LinRel A*; Eq. (17) with the uplink channel coefficient from the source antenna 1 and the downlink channel coefficients to the destination antenna 1
- *LinRel B*; Eq. (22)
- *LinRel C*; Eq. (26).

In these cases the destination jointly processes all receive signals (co-located antenna elements). For gain allocation *LinRel B<sub>ZF</sub>* the nullspace projection (28) is applied to the gain vector from *LinRel B* and the decoder at the destination operates independently on each stream (antenna element). Thus *LinRel B<sub>ZF</sub>* does not require co-located antenna elements. As the source uses independent streams either, the *LinRel B<sub>ZF</sub>* results also apply directly to a system with multiple single antenna sources and destinations (MSMD). In all figures we consider the ergodic capacity per channel use and complex

dimension. Due to the two-hop traffic pattern one transmission from source to destination requires two channel uses. For this reason a spatial multiplexing gain of 1 corresponds to a  $0.5 \cdot \log_2 10 = 1.65$  bit increase of the ergodic capacity for every 10 dB of SNR improvement in the large SNR regime. In all figures we show the performance of linear relaying system with single antenna source and destination as reference.

In Fig. 9 we compare the approximate capacity (dashed) as obtained from the approximate SNR expressions for *LinRel B* (24) and *LDAS* (4) with simulation results. Parameter of the curves is the number  $N_a$  of source/destination antennas. For *LinRel B* the approximation is close in all cases. As one would expect the approximation for *LDAS* is close in the large support node regime.

In Fig. 10 we compare *DDAS*, *LDAS* and linear relaying in the large support node regime. Due to the relay array gain the reference  $(1 \times N_r \times 1)$  system essentially experiences a SNR, which is proportional to the number of relays and the capacity is proportional to  $0.5 \log_2 N_r$ . Despite the  $N_a = 4$  source/destination antennas *LinRel A* achieves only the same slope as the reference system. This is essentially due to the fact, that *LinRel A* realizes coherent combining on the downlink for the first source/destination antenna pair only. In contrast *LinRel B* and *LinRel C* achieve the same slope as *LDAS* and *DDAS*. For a small number of support nodes *LinRel A/B/C* have similar performance, as the diffuse component of the equivalent channel matrix dominates (Fig. 8). Note the good asymptotic performance of *LinRel B<sub>ZF</sub>*. This is a consequence of *LinRel A/B/C* being asymptotically zero forcing (Section V).

In Fig. 11 we compare *DDAS*, *LDAS* and *LinRel B* for 4 and 8 source/destination antenna elements. The slope of the curves show, that all settings achieve the full spatial multiplexing gain. As derived in Section III the loss of *LDAS* w.r.t. *DDAS* is 3dB. The figure confirms the observation from (24) and (4), that the loss of *LinRel B* w.r.t. *LDAS* is proportional to  $(N_a + 1)$ ; here this amounts to 7dB and 9.5 dB respectively.

In Fig. 12 we compare the MSMD scenario (multiple co-active single antenna nodes) with the MIMO source/destination scenario (with multiple co-located antennas). For *LinRel B* the destination decodes all data stream jointly (i.e. the antenna elements cooperate). For *LinRel B<sub>ZF</sub>* neither sources nor destinations cooperate and the relay gain vector is chosen such that no multiuser interference arises at the destinations. Parameter of the curves is the number of support nodes. As outlined in Section V, zero forcing requires  $N_r > (N_a - 1) \cdot N_a$  relay nodes. For this reason e.g. the capacity of the  $N_r = 50$  *LinRel B<sub>ZF</sub>* case drops to zero for  $N_a \geq 8$ . For each number of relays there is an optimum number of co-active source/destination links, which maximizes the aggregate capacity of all links. Note the good performance of *LinRel B<sub>ZF</sub>* in comparison to the  $(1 \times N_r \times 1)$  reference system.

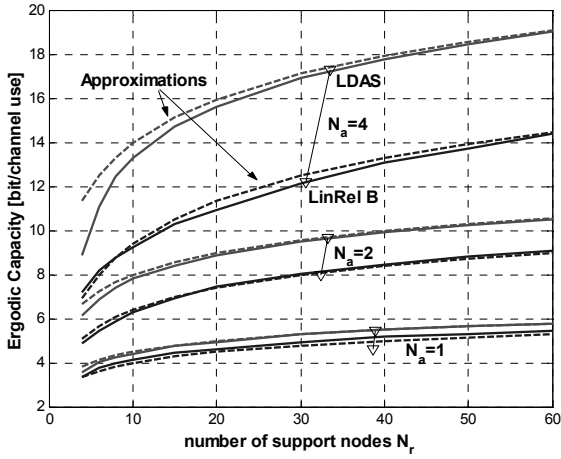


Fig. 9: Comparison of approximation (dashed) and simulation (solid) of the ergodic capacity of LDAS and linear relaying (B).

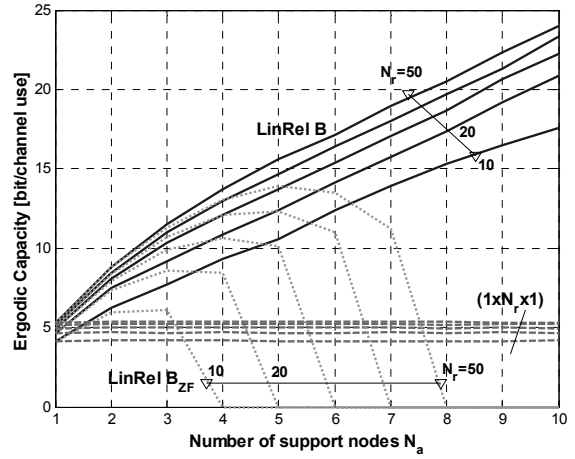


Fig. 12: Comparison of the ergodic capacity of linear relaying (B) with and without Zero Forcing.

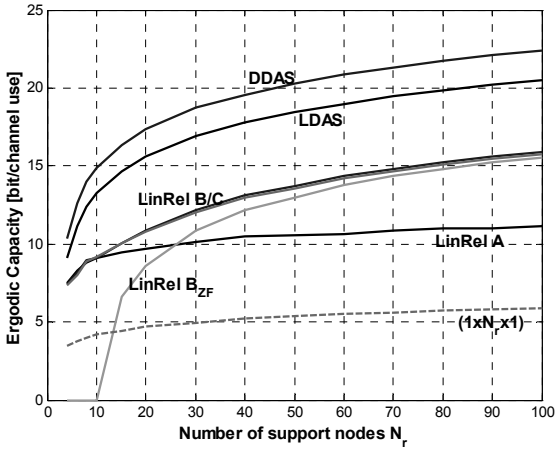


Fig. 10: Ergodic capacity of DDAS, LDAS and linear relaying with different gain allocation schemes in a  $(4 \times N_r \times 4)$  configuration.

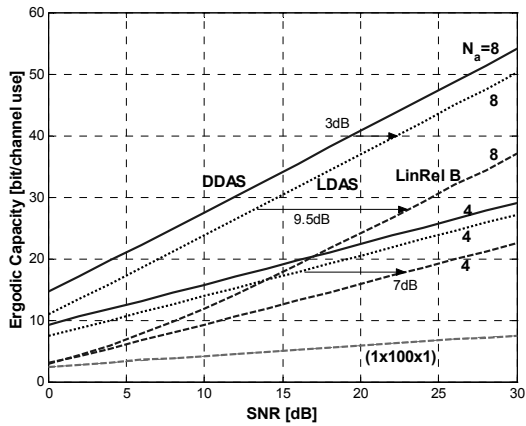


Fig. 11: Ergodic capacity of DDAS, LDAS and linear relaying (B) versus the SNR.

## REFERENCES

- [1] *Proceedings of the IEEE*, Special Issue on Gigabit Wireless, Vol. 92, No. 2, Feb. 2004.
- [2] N. Laneman, D. Tse, and G. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behaviour," *IEEE Trans. Inf. Theory*, to appear.
- [3] A. Sendonaris, E. Erkip, B. Aazhang, "User Cooperation Diversity - Part I and II," *IEEE Trans. on Communications*, Nov. 2003.
- [4] M. V. Clark et al, "Distributed Versus Centralized Antenna Arrays in Broadband Wireless Networks," *IEEE Vehicular Technology Conference VTC 2001*.
- [5] W. Roh, A. Paulraj, "Outage performance of the distributed antenna systems in a composite fading channel," *IEEE Vehicular Technology Conference, VTC Fall 2002*.
- [6] A. Wittneben, B. Rankov, "Impact of Cooperative Relays on the Capacity of Rank-Deficient MIMO Channels," *Proceedings of the 12th IST Summit on Mobile and Wireless Communications*, Aveiro, Portugal, pp. 421-425, June 2003.
- [7] A. Wittneben, B. Rankov, "Distributed Antenna Arrays versus Cooperative Linear Relaying for Broadband Indoor MIMO Wireless," *International Conference on Electromagnetics in Advanced Applications, ICEAA'03*, Torino, Italy, Sept. 2003, (invited paper).
- [8] M. Gastpar and M. Vetterli, "On the capacity of wireless networks: The relay case", in *Proc. IEEE INFOCOM*, (New York), pp. 1577-1586, Jun. 2002.
- [9] R. U. Nabar, O. Oyman, H. Bölcskei, and A. Paulraj, "Capacity scaling laws in MIMO wireless networks", in *Proc. Allerton Conf. Comm., Contr. and Comp.*, pp. 378-389, Oct. 2003.
- [10] I. Hammerstroem, M. Kuhn and A. Wittneben, "Impact of Relay Gain Allocation on the Performance of Cooperative Diversity Networks," *IEEE Vehicular Technology Conference VTC 2004 Fall, Los Angeles, USA*.