Distributed Control of Multi–Robot Systems with Global Connectivity Maintenance

Lorenzo Sabattini, Cristian Secchi, Nikhil Chopra and Andrea Gasparri

***Abstract*—This work introduces a control algorithm that, exploiting a completely decentralized estimation strategy for the algebraic connectivity of the graph, ensures the connectivity maintenance property for multi–robot systems, in the presence of a generic (bounded) additional control term. This result is obtained driving the robots along the negative gradient of an appropriately defined function of the algebraic connectivity. The proposed strategy is then enhanced, with the introduction of the concept of critical robots, that is robots for which the loss of a single communication link might cause the disconnection of the communication graph. Limiting the control action to critical robots will be shown to reduce the control effort introduced by the proposed connectivity maintenance control law and to mitigate its effect on the additional (desired) control term.**

***Index Terms*—Distributed Robot Systems, Networked Robots, Global Connectivity Maintenance.**

I. Introduction

In this paper a decentralized control strategy is proposed for the maintenance of the global connectivity in multi–robot systems. The connectivity maintenance problem turns out to be a relevant problem for achieving a wide range of collaborative tasks, for instance collaborative exploration, coverage or formation control [1]–[3]. As a matter of fact, disconnection among the members of a multi–robot system might appear in several cases, such as for the presence of collision avoidance control actions. Nevertheless, the majority of techniques proposed for achieving these collaborative tasks do not take the connectivity maintenance into account.

In the literature, several approaches have been proposed to preserve the connectivity of a multi–robot system [4]–[10]. Roughly speaking, these approaches can be divided into two categories: approaches to maintain the local connectivity, and approaches to maintain the global connectivity.

The local connectivity maintenance problem aims at preserving over time the original set of links that define the connectivity graph. Clearly, the preservation of each link of the communication graph is a very restrictive requirement which significantly limits the capability of the multi–robot system itself. As a matter of fact, in order to ensure the information exchange among all the robots, it is necessary to guarantee only the *global* connectivity of the communication graph. Generally speaking, it is acceptable that a some links could be removed while others could be added, as long as the overall connectivity of the graph is preserved. Thus, imposing the global connectivity maintenance ensures that none of the robots loses connectivity from the rest of the group, without unnecessarily constraining the motion of the multi–robot system.

Global connectivity maintenance may be addressed implementing a gradient based control strategy to increase the algebraic connectivity [10]. On these lines, a bounded error estimation procedure was proposed in [11] to estimate the second–smallest eigenvalue of the Laplacian matrix, as well as its gradient. These estimates were exploited to formally guarantee connectivity maintenance. Alternative strategies for the estimation of the algebraic connectivity of an undirected graph and its related eigenvector can be found in [12]–[14].

In this work the connectivity maintenance problem for distributed control applications is addressed. Specifically, the contribution of the paper is the following:

1) The scope of the control strategy proposed in [11] is extended by taking into account the presence of an additional (bounded) control law.
2) The proposed control law is then enhanced by limiting its effect to appropriately defined *critical* robots, namely robot for which a disconnection might cause the split of the communication graph.

This paper extends the preliminary results on these topics that have been presented in [15], [16].

II. Preliminaries

A. Definition of the Multi–Robot System

Consider a group of N mobile robots, each one with single integrator dynamics defined as follows:

$$\dot{p}_i = u_i \tag{1}$$

where $p_i \in \mathbb{R}^m$ is the position of the i–th robot, and u_i is the control input. Let $p = \left[p_1^T \dots p_N^T\right]^T \in \mathbb{R}^{Nm}$ be the state vector of the multi–robot system.

Assume that each robot has the capability to localize itself with respect to a common global reference frame. Note that, if only relative distance and angular measurements are available, this could be achieved using the algorithm described in [17].

Let $\mathcal{N}_i$ be the neighborhood of the i–th robot, that is the set of robots that can exchange information with the i–th one. The following simplified communication model is assumed: a bidirectional communication link is established between two robots if their distance is smaller than the maximum communication range. Let then the communication architecture among the robots be described by means of an undirected graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$. Each robot i corresponds to a node $i \in \mathcal{V}$ of the graph, and each link between two robots i and j corresponds to an edge $e_{ij} \in \mathcal{E}$ of the graph. Then, $\mathcal{N}_i$ is defined as follows:

$$\mathcal{N}_i = \{j \in \mathcal{V} \text{ such that } j \neq i \text{ and } \|p_i - p_j\| \leq R\} \tag{2}$$

where R is the maximum communication range. It is worth noting that all the proposed concepts can be extended to more realistic communication models, considering for instance *line–of–sight* constraints, exploiting the concept of generalized connectivity introduced in [18].

Let then $A \in \mathbb{R}^{N \times N}$ be the adjacency matrix of the communication graph, and let each element a_{ij} be defined as the weight of the edge e_{ij}. Namely, a_{ij} is a positive number if $j \in \mathcal{N}_i$, zero otherwise. Since undirected graphs are considered, it follows that $a_{ij} = a_{ji}$. Let $L \in \mathbb{R}^{N \times N}$ be the (weighted) Laplacian matrix of the graph, and let λ_2 be the second smallest eigenvalue of L. As is well known, $\lambda_2 > 0$ if and only if the graph is connected: then, λ_2 is defined as the algebraic connectivity of the graph. Further details on graph theory can be found in [19].

B. Connectivity maintenance and estimation procedure

In [11], the following control law is introduced:

$$\dot{p}_i = u_i^c = \operatorname{csch}^2(\lambda_2 - \epsilon) \frac{\partial \lambda_2}{\partial p_i} \tag{3}$$

with ϵ the desired lower–bound for the value of λ_2.

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According to the communication model described in Section II-A, the edge–weights a_{ij} are defined as follows:

$$a_{ij} = \begin{cases} e^{-\left(\|p_i - p_j\|^2\right)/\left(2\sigma^2\right)} & \text{if } \|p_i - p_j\| \le R \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

The scalar parameter σ is chosen to satisfy the threshold condition $e^{-\left(R^2\right)/\left(2\sigma^2\right)} = \Delta$, where Δ is a small predefined threshold. This definition of the edge–weights introduces a discontinuity in the control action, that can be avoided introducing a smooth bump function, as in [20].

Let v_2 be the eigenvector corresponding to the eigenvalue λ_2. Given the definition of the edge–weights in Eq. (4), the value of $\frac{\partial \lambda_2}{\partial p_i}$ can be computed as shown in [10]:

$$\frac{\partial \lambda_2}{\partial p_i} = \sum_{j \in \mathcal{N}_i} -a_{ij} \left(v_2^i - v_2^j\right)^2 \frac{p_i - p_j}{\sigma^2} \tag{5}$$

where v_2^k is the k–th component of v_2.

The computation of the eigenvectors of the Laplacian matrix is a centralized operation. Hence, the actual values of λ_2, v_2 and $\frac{\partial \lambda_2}{\partial p_i}$ are not available to the robots. Nevertheless, it is possible to exploit a decentralized procedure that allows each robot to obtain an estimate of these values. Specifically, the power iteration procedure described in [21] may be exploited to design the following update law:

$$\dot{\tilde{v}}_2 = -k_1 \text{Ave}\left(\left\{\tilde{v}_2^i\right\}\right) \mathbf{1} - k_2 L \tilde{v}_2 - k_3 \left(\text{Ave}\left(\left\{\left(\tilde{v}_2^i\right)^2\right\}\right) - 1\right) \tilde{v}_2 \tag{6}$$

where $k_1, k_2, k_3 > 0$ are the control gains, and $\text{Ave}\left(\cdot\right)$ is the averaging operation. Furthermore, $\tilde{v}_2^i$ is defined as the i–th robot's estimate of v_2^i, that is the i–th component of the eigenvector v_2, and $\tilde{v}_2 = \left[\tilde{v}_2^1, \ldots, \tilde{v}_2^N\right]^T$.

To implement the update law in Eq. (6) in a decentralized way, the averaging operation is implemented by means of the PI average consensus estimator described in [22].

Since there are two averaging operations in the update law in Eq. (6), two PI consensus estimators must be run:

- the first one, whose input is $\tilde{v}_2^i$, provides z_1^i as the i–th robot's estimate of $\text{Ave}\left(\left\{\tilde{v}_2^i\right\}\right)$;
- the second one, whose input is $\left(\tilde{v}_2^i\right)^2$, provides z_2^i as the i–th robot's estimate of $\text{Ave}\left(\left\{\left(\tilde{v}_2^i\right)^2\right\}\right)$.

Thus, the following decentralized version of the update law in Eq. (6) is obtained according to [11]:

$$\dot{\tilde{v}}_2^i = -k_1 z_1^i - k_2 \sum_{j \in \mathcal{N}_i} a_{ij} \left(\tilde{v}_2^i - \tilde{v}_2^j\right) - k_3 \left(z_2^i - 1\right) v_2^i - k_4 \left|\tilde{v}_2^i\right| \tilde{v}_2^i \tag{7}$$

for some constant $k_4 > 0$. As shown in [11], the presence of this term is fundamental to guarantee the boundedness of the estimation error. As will be shown later on, the boundedness of the estimation error is necessary to ensure connectivity maintenance.

Let $\tilde{\lambda}_2$ be the value that the second smallest eigenvalue of the Laplacian matrix would take if $\tilde{v}_2$ were the corresponding eigenvector. As shown in [10], [11], $\tilde{\lambda}_2$ can be computed as follows:

$$\tilde{\lambda}_2 = \frac{k_3}{k_2} \left[1 - \text{Ave}\left(\left\{\left(\tilde{v}_2^i\right)^2\right\}\right)\right]. \tag{8}$$

Moreover, $\frac{\partial \tilde{\lambda}_2}{\partial p_i}$ can be computed as:

$$\frac{\partial \tilde{\lambda}_2}{\partial p_i} = \tilde{v}_2^T \frac{\partial L}{\partial p_i} \tilde{v}_2 = \sum_{j \in \mathcal{N}_i} \frac{\partial a_{ij}}{\partial p_i} \left(\tilde{v}_2^i - \tilde{v}_2^j\right)^2 \tag{9}$$

Then, from the definition of the edge–weights a_{ij} given in Eq. (4):

$$\frac{\partial \tilde{\lambda}_2}{\partial p_i} = \sum_{j \in \mathcal{N}_i} -a_{ij} \left(\tilde{v}_2^i - \tilde{v}_2^j\right)^2 \frac{p_i - p_j}{\sigma^2} \tag{10}$$

The actual value of $\tilde{\lambda}_2$ can not be computed by each robot. In fact, the real value of $\text{Ave}\left(\left\{\left(\tilde{v}_2^i\right)^2\right\}\right)$ is not available. Nevertheless, an estimate of this average, namely z_2^i, is available to each robot. Hence, the i–th robot can compute its own estimate of λ_2, namely λ_2^i, as follows:

$$\lambda_2^i = \frac{k_3}{k_2} \left(1 - z_2^i\right) \tag{11}$$

As shown in [11], λ_2^i is a good estimate of both λ_2 and $\tilde{\lambda}_2$. More specifically, it has been proven that $\exists\, \Xi,\ \Xi' > 0$ such that

$$\begin{aligned} \left|\lambda_2 - \lambda_2^i\right| &\le \Xi \quad \forall i = 1, \ldots, N \\ \left|\tilde{\lambda}_2 - \lambda_2^i\right| &\le \Xi' \quad \forall i = 1, \ldots, N \end{aligned} \tag{12}$$

From Eq. (12), it follows that

$$\left|\lambda_2 - \tilde{\lambda}_2\right| \le \Xi + \Xi' \tag{13}$$

The control law introduced in Eq. (3) will then be implemented introducing each robot's estimate, that is:

$$u_i^c = \text{csch}^2\left(\lambda_2^i - \tilde{\epsilon}\right) \frac{\partial \tilde{\lambda}_2}{\partial p_i} \tag{14}$$

with $\tilde{\epsilon} \ge \epsilon + \Xi + \Xi'$.

III. Connectivity maintenance in the presence of an external control term

In this section the control law given in Eq. (3) is extended by considering an additional desired control term as follows:

$$\dot{p}_i = u_i^c + u_i^d \tag{15}$$

where u_i^c is the control term introduced in Eq. (14), while u_i^d is a control term used to obtain some desired behavior. Namely, the control term u_i^d is an unknown bounded function, that is $\left\|u_i^d\right\| \le u_M$.

The effectiveness of the control strategy defined so far will now be proven. More specifically, connectivity maintenance will be analytically demonstrated hereafter. For this purpose, as in [11], consider the following energy function:

$$V\left(p\right) = \text{coth}\left(\tilde{\lambda}_2 - \tilde{\epsilon}\right) \tag{16}$$

The energy function $V(p)$ given in Eq. (16) is non–increasing with respect to $\tilde{\lambda}_2$ and non–negative for any $\tilde{\lambda}_2 > \tilde{\epsilon}$.

Lemma 1 *Assume the value of the algebraic connectivity to be $\lambda_2 > \tilde{\epsilon} + \Xi + \Xi'$. Then the following condition holds:*

$$csch^2\left(\lambda_2^i - \tilde{\epsilon}\right) \ge csch^2\left(\tilde{\lambda}_2 + \Xi' - \tilde{\epsilon}\right),\ \forall\, i = 1, \ldots, n. \tag{17}$$

Proof: According to Eq. (12), if the value of λ_2 is greater than $\tilde{\epsilon} + \Xi + \Xi'$, then the value of λ_2^i is greater than $\tilde{\epsilon}$, $\forall i = 1, \ldots, N$ as well. Therefore, since the function $\text{csch}^2\left(\lambda_2^i - \tilde{\epsilon}\right)$ is monotonically decreasing with respect to λ_2^i, the following condition holds:

$$\text{csch}^2\left(\lambda_2^i - \tilde{\epsilon}\right) \ge \text{csch}^2\left(\lambda_2^{MAX} - \tilde{\epsilon}\right) \tag{18}$$

where, according to Eq. (12), λ_2^{MAX} is defined as follows:

$$\lambda_2^{MAX} = \max_{i=1,\ldots,N} \left\{\lambda_2^i\right\} \le \tilde{\lambda}_2 + \Xi' \tag{19}$$

■

Proposition 1 *Consider the dynamical system described by Eq.* (15). *Let* Ξ, Ξ' *be defined according to Eq.* (12). *Then,* $\exists\, \epsilon$, $\tilde{\epsilon}$ *such that, if the initial value of* $\lambda_2 > \tilde{\epsilon} + \Xi + \Xi'$, *the control law defined in Eq.* (15) *ensures that the value of* λ_2 *never goes below* ϵ.

Proof: In order to prove the statement, the partial derivative of the energy function introduced in Eq. (16) is computed, with respect to robot i, as:

$$\frac{\partial V}{\partial p_i} = \frac{\partial V}{\partial \tilde{\lambda}_2}\frac{\partial \tilde{\lambda}_2}{\partial p_i} = -\text{csch}^2\left(\tilde{\lambda}_2 - \tilde{\epsilon}\right)\frac{\partial \tilde{\lambda}_2}{\partial p_i} \tag{20}$$

From Eqs. (14), (15), (20), it follows that the time derivative of $V(p)$ can be computed as follows:

$$\begin{gathered}\dot{V}(p) = \nabla_p V(p)^T \dot{p} = \sum_{i=1}^{N} \frac{\partial V}{\partial p_i}^T \dot{p}_i = \\ \sum_{i=1}^{N}\left[-\text{csch}^2\left(\tilde{\lambda}_2 - \tilde{\epsilon}\right)\frac{\partial \tilde{\lambda}_2}{\partial p_i}\right]^T\left[\text{csch}^2\left(\lambda_2^i - \tilde{\epsilon}\right)\frac{\partial \tilde{\lambda}_2}{\partial p_i} + u_i^d\right]\end{gathered} \tag{21}$$

Given the boundedness of the additional control term u_i^d the time derivative $\dot{V}(p)$ can be restated as:

$$\dot{V}(p) \leq \text{csch}^2\left(\tilde{\lambda}_2 - \tilde{\epsilon}\right)\sum_{i=1}^{N}\left[-\text{csch}^2\left(\lambda_2^i - \tilde{\epsilon}\right)\left\|\frac{\partial \tilde{\lambda}_2}{\partial p_i}\right\|^2 + \left\|\frac{\partial \tilde{\lambda}_2}{\partial p_i}\right\| u_M\right] \tag{22}$$

As a result, the time derivative $\dot{V}(p) \leq 0$ if the following condition holds:

$$\sum_{i=1}^{N}\left[\text{csch}^2\left(\lambda_2^i - \tilde{\epsilon}\right)\left\|\frac{\partial \tilde{\lambda}_2}{\partial p_i}\right\|^2\right] \geq u_M \sum_{i=1}^{N}\left\|\frac{\partial \tilde{\lambda}_2}{\partial p_i}\right\| \tag{23}$$

Furthermore, according to Lemma 1, a sufficient condition to satisfy the inequality in Eq. (23) may be written as:

$$\text{csch}^2\left(\tilde{\lambda}_2 + \Xi' - \tilde{\epsilon}\right)\sum_{i=1}^{N}\left\|\frac{\partial \tilde{\lambda}_2}{\partial p_i}\right\|^2 \geq u_M \sum_{i=1}^{N}\left\|\frac{\partial \tilde{\lambda}_2}{\partial p_i}\right\| \tag{24}$$

Assume now that the following condition holds:

$$\sum_{i=1}^{N}\left\|\frac{\partial \tilde{\lambda}_2}{\partial p_i}\right\|^2 \neq 0 \tag{25}$$

Then, the inequality in Eq. (24) can be rewritten as follows:

$$\text{csch}^2\left(\tilde{\lambda}_2 + \Xi' - \tilde{\epsilon}\right) \geq u_M \frac{\sum_{i=1}^{N}\left\|\frac{\partial \tilde{\lambda}_2}{\partial p_i}\right\|}{\sum_{i=1}^{N}\left\|\frac{\partial \tilde{\lambda}_2}{\partial p_i}\right\|^2} = H(p) > 0 \tag{26}$$

which implies

$$\tilde{\lambda}_2 \leq \bar{\lambda}_2(p) = \text{settcsch}\left(\sqrt{H(p)}\right) + \epsilon' \tag{27}$$

where $\epsilon' = \tilde{\epsilon} - \Xi'$, and settcsch$(\cdot)$ is the inverse function of csch$(\cdot)$. At this point, it should be noticed that $\bar{\lambda}_2(p) > \epsilon'$ always exists such that the condition in Eq. (27) is satisfied. This implies that:

$$\dot{V}(p) \leq 0, \qquad \forall \tilde{\lambda}_2 \leq \bar{\lambda}_2(p) \tag{28}$$

Therefore, $\forall \tilde{\lambda}_2 \leq \bar{\lambda}_2(p)$, the energy function $V(p)$ does not increase over time.

With a slight abuse of notation, let $\lambda_2(t)$ and $\tilde{\lambda}_2(t)$ be the values of λ_2 and $\tilde{\lambda}_2$ at time t.

It is worth remarking that $\lambda_2(0) > \tilde{\epsilon} + \Xi + \Xi'$. Subsequently, it follows that, according to Eq. (13), $\tilde{\lambda}_2(0) > \tilde{\epsilon}$.

It is possible to show that the same condition holds even if Eq. (25) is not verified. In fact, in this case, the value of $\tilde{\lambda}_2(t)$ does not change, and it is then trivially lower–bounded by its initial value.

Therefore, from Eq. (27), it follows that, if $\tilde{\lambda}_2(0) > \epsilon'$, then $\tilde{\lambda}_2(t)$ will never go below ϵ' for any $t > 0$.

Then, the control law in Eq. (15) ensures that the value of λ_2 never goes below $\epsilon = \epsilon' - \Xi = \tilde{\epsilon} - \Xi' - \Xi$. ∎

IV. Enhanced control action

As will be clearly shown in the simulations described in Section V-A, the control strategy described in Section III influences the behavior of the system, interfering with the primary goal of the multi–robot system. Hence, in this section, a selective action is introduced to enhance this control strategy. The objective is twofold:

1) to reduce the overall control effort introduced by the connectivity maintenance control action.
2) to reduce the interference between the connectivity maintenance control action and the main task of the system.

In order to achieve these goals, the control law given in Eq. (14) is modified as follows:

$$u_i^c = \gamma_i \,\text{csch}^2\left(\lambda_2^i - \tilde{\epsilon}\right)\frac{\partial \tilde{\lambda}_2}{\partial p_i} \tag{29}$$

where the coefficient $\gamma_i \in \mathbb{R}$ is used to modulate the control action as will be explained hereafter.

The neighborhood of the i–th robot, that is $\mathcal{N}_i$ defined in Eq. (2), can be decomposed as follows:

$$\mathcal{N}_i = \mathcal{N}_i^c + \mathcal{N}_i^f \tag{30}$$

where:

- $\mathcal{N}_i^c$ is the set of the *close* neighbors of the i–th robot,
- $\mathcal{N}_i^f$ is the set of the *far* neighbors of the i–th robot.

These two sets are defined as follows:

$$\begin{aligned}\mathcal{N}_i^c &= \{j \in \mathcal{N}_i \text{ such that } \|p_i - p_j\| \leq \delta R\} \\ \mathcal{N}_i^f &= \{j \in \mathcal{N}_i \text{ such that } \|p_i - p_j\| > \delta R\}\end{aligned} \tag{31}$$

where $\delta \in (0,1)$ is a predefined threshold. Note that, according to this definition, $\mathcal{N}_i^f \cap \mathcal{N}_i^c = \emptyset$.

Moreover, the definition of *isolated robot* is now introduced.

Definition 1 (Isolated robot) *A robot* j *is considered* isolated*, from the robot* i*'s perspective, if it belongs to* $\mathcal{N}_i^f$ *and it does not belong to the* $\mathcal{N}_k^c$ *for any of the* $k \in \mathcal{N}_i^c$*, that is:*

$$j \in \mathcal{N}_i^f, \text{ and } \nexists\, k \in \mathcal{N}_i^c \text{ such that } j \in \mathcal{N}_k^c \tag{32}$$

Hence, the following definition of *critical robot* is introduced.

Definition 2 (Critical robot) *The* i*–th robot identifies itself as* critical *if at least one of its neighbors is* isolated*.*

The definition of critical robot exhibits a symmetry property, that is: *If the* i*–th robot considers itself as critical by identifying the* j*–th one as isolated, then the* j*–th robot considers itself as critical by identifying the* i*–th one as isolated, as well.*

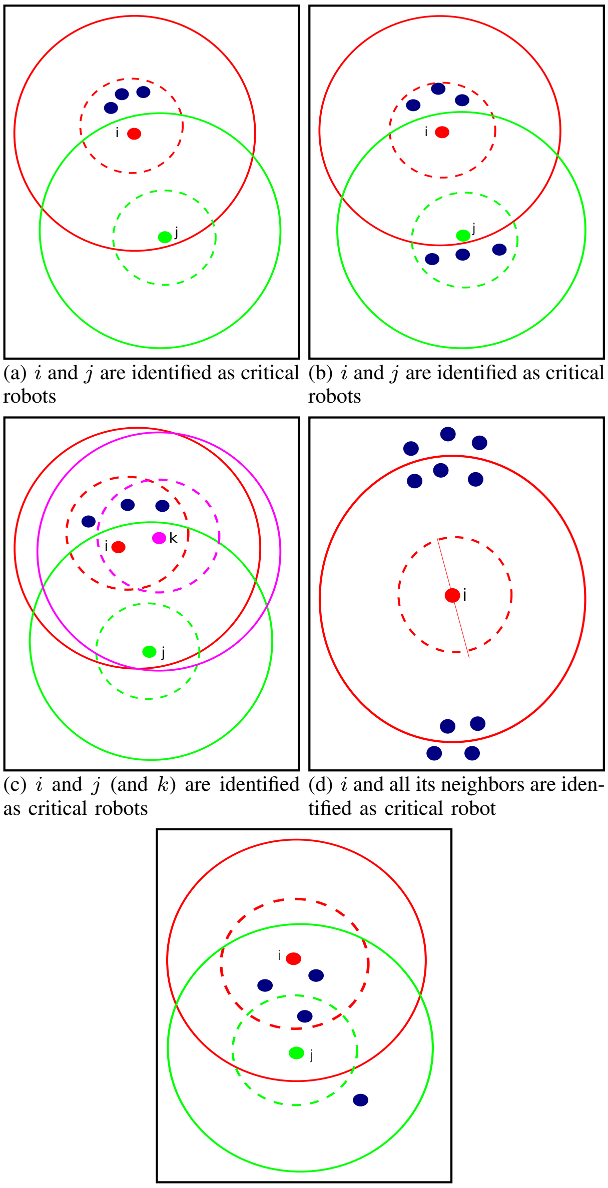
This is a simple consequence of some geometrical facts, under the assumption of common communication range R.

As a result, the connectivity maintenance control action is limited to those robots whose disconnection may lead to the loss of connectivity. Thus, the coefficient γ_i in Eq. (29) can be defined as follows:

$$\gamma_i = \begin{cases} 1 & \text{if the } i\text{–th robot is critical} \\ \rho & \text{otherwise} \end{cases} \tag{33}$$

with $\rho \in (0,1)$ arbitrarily small.

Proposition 2 *Consider the dynamical system described by Eq.* (15). *Let* Ξ, Ξ' *be defined according to Eq.* (12). *Then,* $\exists\, \epsilon, \tilde{\epsilon}$ *such that, if*



(a) i and j are identified as critical robots
(b) i and j are identified as critical robots
(c) i and j (and k) are identified as critical robots
(d) i and all its neighbors are identified as critical robot
(e) i and j are clearly identified as non–critical robots

Fig. 1. Decision algorithm to define the critical robots: some examples

the initial value of $\lambda_2 > \tilde{\epsilon} + \Xi + \Xi'$, *the control law given in Eq.* (15) *with the connectivity control term* u_i^c *defined in Eq.* (29) *ensures that the value of* λ_2 *never goes below* ϵ.

Proof: The proof is analogous to that of Proposition 1, and is then omitted. ∎

The five configurations shown in Fig. 1 for the communication graph are representative of all the possible scenarios:

- in Fig. 1(a), the j–th robot is isolated: only the i–th one is in its neighborhood. Then, they are both considered critical, and $\gamma_i = \gamma_j = 1$.
- in Fig. 1(b), the link between the i–th and the j–th robots links two different components of the graph. Both robots are considered critical, and $\gamma_i = \gamma_j = 1$.
- in Fig. 1(c), the j–th robot is identified as critical. Consequently, i–th robot is considered critical as well.
- in Fig. 1(d), losing the i–th robot would create two separate sub–graphs. Hence, the i–th robot is critical.
- Fig. 1(e) represents a situation where the connectivity maintenance action is not needed.

Notably, critical robots are identified by a local policy that only requires locally available quantities.

Referring to Definition 2, the local policy may be described with Algorithm 1.

Algorithm 1 Local policy to identify the critical robots

$\gamma_i = 1$
if $\left\{\mathcal{N}_i^f = \emptyset\right\}$ **then**
  $\gamma_i = \rho$
end if
if $\left\{\forall j \in \mathcal{N}_i^f \;\; \exists k \in \mathcal{N}_i^c \text{ s.t. } j \in \mathcal{N}_k^c\right\}$ **then**
  $\gamma_i = \rho$
end if

V. Simulations and Experimental Results

Several Matlab simulations and experiments on e–puck robots have been carried out to evaluate the performance of the proposed control strategy within a formation control application, exploiting the control law introduced in [23].[1]

A. Simulations

Simulations have been carried out with a varying number of agents ranging from $N = 3$ to $N = 30$, with randomized initial conditions, and considering the following set of parameters: $\{\rho = 10^{-5}, \delta = 0.8\}$.

1) Comparison between standard and enhanced control action: In order to carry out a quantitative analysis of the advantage introduced by the enhanced action, a measurement of the required *control effort* may be defined as the integral of the control action over time. As shown in Fig. 2, data have been acquired for a number of simulated robots ranging from $N = 3$ to $N = 30$. For each setup, 15 simulations have been run, randomly changing the initial positions of the agents, as well as the obstacles' positions. From the statistical analysis of the acquired data, it turns out that the introduction of the enhanced action drastically reduces the required control effort. Even though the actual value of the reduction of the required control action heavily depends on the particular application and environment, it is worth noting that, in the simulation scenario described above, the effort is always reduced, on average, by more than 40%. Moreover, it is important to highlight the fact that the control effort reduction appears increasing with the number of robots involved. This can be explained considering that for a randomly generated graph (that is without imposing any particular structure on the topology, such as a ring or a chain) the percentage of critical robots decreases as the overall number of robots increases. Hence, the introduction of the enhanced control action (described in Section IV) increases the scalability of the connectivity maintenance strategy.

2) Comparison with a local connectivity maintenance control strategy: The performance of the proposed control strategy has been evaluated by comparing it with a typical local connectivity maintenance control strategy [6]. To perform the comparison, the following simulation setup has been implemented: a formation of 6 agents is controlled to reach a target position, while moving though randomly placed point obstacles. Robots initial positions are randomly chosen, guaranteeing that the initial distance of the barycenter of the robots' positions and the target is equal to $15m$.

The steady state distance of the barycenter of the formation from the target has then been evaluated. Table I summarizes the results of

[1] A video clip containing different runs of simulations and experiments is freely available online at http://www.arscontrol.unimore.it/troconn13

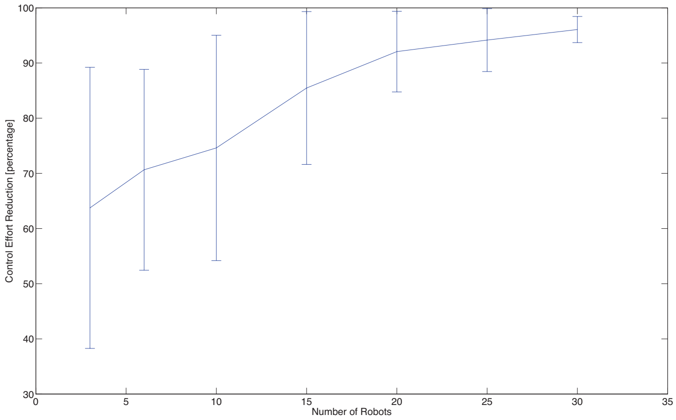


Fig. 2. Reduction of the control effort provided by the enhanced action

	Distance of the barycenter from the target	
	Mean	Standard deviation
Local action [6]	6.344	3.124
Standard action	0.0	0.0
Enhanced action	0.0	0.0

TABLE I
COMPARISON BETWEEN LOCAL AND GLOBAL CONNECTIVITY MAINTENANCE CONTROL ACTIONS

the simulations. Specifically, 30 simulation runs have been performed with three different connectivity maintenance control actions: the local action [6], the standard action proposed in Section III, and the enhanced action proposed in Section IV. It turns out that the local connectivity maintenance control action often makes the group stop in undesired positions: this is due to the fact that keeping each link among the robots causes a rigid behavior for the formation, that is then not allowed to overcome the obstacles. Conversely, as expected, the proposed global connectivity maintenance control strategy always allows the formation to reach the target. Notably, the same result is obtained both with the standard and with the enhanced action.

3) Perturbations on the desired behavior: Simulations have been performed to evaluate the perturbation on the desired behavior of the system introduced by the connectivity maintenance control action. For this purpose, the following analysis has been carried out over repeated simulations: a group of six point agents, starting from random initial positions, has been controlled to perform formation control in a free environment (i.e. without obstacles), respectively with and without connectivity maintenance. The results of 50 simulation runs are represented in Fig. 3. Specifically, the figure represents the absolute value of the difference (mean and standard deviation) between the control law applied to the agents with and without connectivity maintenance: blue solid line represents the effect of the standard action, while red dashed line represent the effect of the enhanced action. Both for the standard and the enhanced actions, it is possible to notice that the difference is greater than zero only during the initial transient, when the estimation process has not converged yet. It is worth noting that this effect can be avoided letting the estimation system evolve for a small amount of time before applying control laws to the system.

B. Experiments

Experiments have been carried out on a group of four e–puck robots moving in a bounded arena. To implement the proposed strategy on nonholonomic systems, the well known feedback linearization technique was applied [24]. The control algorithm was implemented on a centralized PC, but emulating a decentralized

	Average control effort: $\bar{u}^c = \frac{1}{N}\sum_{i=1}^{N} \lvert u_i^c \rvert$	
	Mean	Standard deviation
Standard action	61.9517	155.8315
Enhanced action	4.9586	6.4957

TABLE II
COMPARISON BETWEEN STANDARD AND ENHANCED CONTROL ACTIONS

computation procedure [25]. In order to carry out a statistical analysis, 30 experimental runs have been performed, with four e–puck robots performing a rendezvous task moving in a bounded arena, where ten point obstacles were placed. The obstacles' positions, as well as the robots' initial positions, were randomly changed within a $100cm$ radius circle. Results (mean value and standard deviation) are summarized in Table II. The obtained results clearly highlight the advantages introduced by the use of the enhanced control law.

VI. CONCLUSION

In this work the connectivity maintenance problem for a multi–robot system has been investigated. First, a control strategy that ensures connectivity maintenance in the presence of an external bounded control term has been proposed. Successively, an enhanced decentralized control law has been proposed based on the concept of critical robots, that is robots for which a disconnection might cause a split of the communication graph. By exploiting this concept, a more selective control action has been designed. This allows to both reduce the control effort and avoid unnecessary action of the original connectivity maintenance control law, thereby reducing its effect on the overall performance of the system. A theoretical analysis of the effectiveness of the proposed control law has been carried out along with simulations and experiments, to corroborate the obtained theoretical results. Future work will be focused on the analysis of the proposed connectivity control law for different connectivity modeling as well as for multi–robot systems characterized by a directed communication graph.

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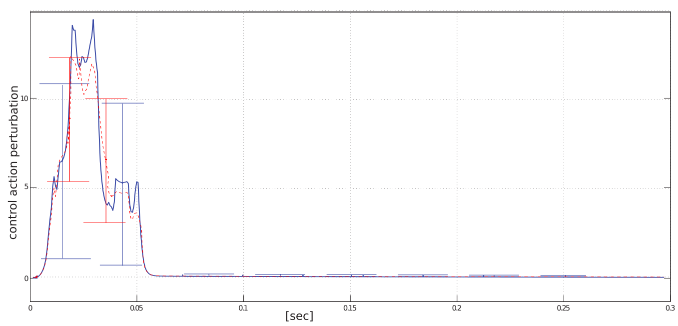

Fig. 3. Perturbation introduced by the connectivity maintenance control action: blue solid line represents the effect of the standard action, while red dashed line represent the effect of the enhanced action

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