

# Distributed Cooperative Coverage Control of Sensor Networks

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**Abstract**— We present a distributed coverage control scheme for cooperating mobile sensor networks. The mission space is modeled using a density function representing the frequency of random events taking place, with mobile sensors operating over a limited range defined by a probabilistic model. A gradient-based algorithm is designed requiring local information at each sensor and maximizing the joint detection probabilities of random events. We also incorporate communication costs into the coverage control problem, viewing the sensor network as a multi-source, single-basestation data collection network. Communication cost is modeled as the power consumption needed to deliver collected data from sensor nodes, thus trading off sensing coverage and communication cost. The control scheme is tested in a simulation environment to illustrate its adaptive, distributed, and asynchronous properties.

## I. INTRODUCTION

A sensor network consists of a collection of (possibly mobile) sensing devices that can coordinate their actions through wireless communication and aim at performing tasks such as reconnaissance, surveillance, target tracking or environmental monitoring over a specific region, often referred to as the “mission space”. Collected field data are further processed and often support higher-level decision making processes. Nodes in such networks are generally inhomogeneous, they have limited on-board resources (e.g., power and computational capacity), and they may be subject to communication constraints. The performance of a sensor network is sensitive to the location of its nodes in the mission space. This leads to the basic problem of deploying sensors in order to meet the overall system objectives, which is referred to as the *coverage control* or *active sensing* problem [1],[2],[3]. In particular, sensors must be deployed so as to maximize the information extracted from the mission space while maintaining acceptable levels of communication and energy consumption. The *static* version of this problem involves positioning sensors without any further mobility; optimal locations can be determined by an off-line scheme which is akin to the widely studied facility location optimization problem. The *dynamic* version allows the coordinated movement of sensors, which may adapt to changing conditions in the mission space, typically deploying them into geographical areas with the highest information density.

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Because of the similarity of coverage control with facility location optimization, the problem is often viewed in that framework. In [2], the authors develop a decentralized coverage control algorithm which is based on Voronoi partitions and the Lloyd algorithm. In [1] a coverage control scheme is proposed which aims at the maximization of target exposure in some surveillance applications, while in [4] a heuristic algorithm based on “virtual forces” is applied to enhance the coverage of a sensor network. Much of the active sensing literature [3] also concentrates on the problem of tracking specific targets using mobile sensors and the Kalman filter is extensively used to process observations and generate estimates.

Some of the methods that have been proposed for coverage control assume uniform sensing quality and an unlimited sensing range. Partition-based deployment methods, on the other hand, tend to overlook the fact that the overall sensing performance may be improved by sharing the observations made by multiple sensors. There are also efforts which rely on a centralized controller to solve the coverage control problem. A centralized approach, however, does not suit the distributed communication and computation structure of sensor networks. In addition, the combinatorial complexity of the problem constrains the application of such schemes to limited-size sensor networks. Finally, another issue that appears to be neglected is the cost of relocating sensors. The movement of sensors not only impacts sensing performance, but it also influences other quality-of-service aspects in a sensor network, especially those related to wireless communication: because of the limited on-board power and computational capacity, a sensor network is not only required to sense but also to collect and transmit data as well. For this reason, both sensing quality and communication performance need to be jointly considered when controlling the deployment of sensors.

In this paper, we develop a distributed coverage control approach for cooperative sensing. The mission space is modeled using a density function representing the frequency that specific events take place (e.g., data are generated at a certain point). We assume that a mobile sensor has a limited range which is defined by a probabilistic model. A deployment algorithm is applied at each mobile node and it maximizes the joint detection probabilities of random events. We assume that the event density function is fixed and given; however, in the case that the mission space (or

our perception of the mission space) changes over time, the adaptive relocation behavior naturally follows from the optimal coverage formulation.

Another contribution of this paper is the incorporation of communication cost into the coverage control problem, viewing the sensor network as a multi-source, single-basestation data collection network. Communication cost is modeled as the power consumption needed to deliver collected data from sensor nodes (data sources) to the basestation using wireless multi-hop links. Thus, the coverage problem we formulate trades off sensing coverage and communication cost.

The remainder of the paper is organized as follows. In Section II, we formulate the coverage control problem and develop the distributed deployment algorithm. In Section III, the communication cost is defined and added into the original problem. Section IV presents some simulation results which illustrate the effectiveness of the proposed schemes and compare sensor deployments obtained with and without communication considerations. Section V concludes the paper and describes directions for future work.

## II. PROBLEM FORMULATION AND DISTRIBUTED COVERAGE CONTROL

### A. Mission Space and Sensor Model

We model the mission space as a polyhedron  $\Omega \subset \mathbb{R}^2$ , over which there exists an event density function  $R(x)$ ,  $x \in \Omega$ , that captures the frequency or density that a specific random event takes place (in  $H\text{z}/\text{m}^2$ ).  $R(x)$  satisfies  $R(x) \geq 0$  for all  $x \in \Omega$  and  $\int_{\Omega} R(x) < \infty$ . Depending on the application,  $R(x)$  may be the frequency that a certain type of vehicle appears at  $x$ , or it could be the probability that the temperature at  $x$  exceeds a specific threshold. In the mission space  $\Omega$ , there are  $N$  mobile sensors located at  $\mathbf{s} = (s_1, \dots, s_N)$ ,  $s_i \in \mathbb{R}^2$ ,  $i = 1, \dots, N$ . When an event occurs at point  $x$ , it emits a signal and this signal is observed by a sensor at location  $s_i$ . The received signal strength generally decays with  $\|x - s_i\|$ , the distance between the source and the sensor. We represent this degradation by a monotonically decreasing differentiable function  $p_i(x)$ , which expresses the probability that sensor  $i$  detects the event occurring at  $x$ .

As an example, if we assume signal strength declines polynomially with distance and taking into consideration environmental noise, the signal strength received at  $s_i$  is expressed by  $E_i(x) = \frac{E}{\|x - s_i\|^n} + \eta_i$ , where  $E$  is the total energy emitted when an event takes place,  $\eta_i$  is the noise, and  $n$  is a decay coefficient (typically selected between 2 to 5). If a sensor detects an event when  $E_i$  is beyond some threshold, then  $p_i(x)$  can be expressed as

$$p_i(x) = \text{Prob} \left[ \frac{E}{\|x - s_i\|^n} + \eta_i > c \right]$$

With a given probability distribution of noise (e.g., additive white Gaussian noise), this may be used as the sensor model. Alternatively, a sensor model with a simpler form may be:

$$p_i(x) = p_{0i} e^{-\lambda_i \|x - s_i\|} \quad (1)$$

where the detection probability declines exponentially with distance, and  $p_{0i}$ ,  $\lambda_i$  are determined by physical characteristics of the sensor.

### B. Optimal Coverage Formulation and Distributed Solution

When deploying mobile sensors into the mission space, we want to maximize the probability that events are detected. This motivates the formulation of an optimal coverage problem. Throughout this paper, we assume that sensors make observations independently. Then, given the mission space and sensor model, when an event takes place at  $x$  and it is observed by the sensors, the joint probability that this event is detected can be expressed by

$$P(x, \mathbf{s}) = 1 - \prod_{i=1}^N [1 - p_i(x)] \quad (2)$$

The optimal coverage problem can be formulated as an optimization problem to maximize the expected event detection frequency by the sensors over the mission space  $\Omega$ :

$$\max_{\mathbf{s}} \int_{\Omega} R(x) P(x, \mathbf{s}) dx \quad (3)$$

In this optimization problem, the controllable variables are the locations of mobile sensors contained in  $\mathbf{s}$ . This problem may be solved by applying a non-linear optimizer with an algorithm which can evaluate integrals numerically. In this case, a centralized controller with intensive computational capacity is required.

Thus, instead of using a centralized scheme, we will develop a distributed method to solve the optimal coverage problem. We denote the objective function in (3) by

$$F(\mathbf{s}) = \int_{\Omega} R(x) P(x, \mathbf{s}) dx \quad (4)$$

When taking partial derivatives with respect to  $s_i$ ,  $i = 1, \dots, N$ , we have

$$\frac{\partial F}{\partial s_i} = \int_{\Omega} R(x) \frac{\partial P(x, \mathbf{s})}{\partial s_i} dx \quad (5)$$

If this partial derivative can be evaluated locally by each mobile sensor  $i$ , then a gradient method can be applied which directs mobile sensors towards locations that maximize  $F(\mathbf{s})$ . In view of (2), the partial derivative (5) can be rewritten as

$$\frac{\partial F}{\partial s_i} = \int_{\Omega} R(x) \prod_{k=1, k \neq i}^N [1 - p_k(x)] \frac{dp_i(x)}{ds_i} \frac{s_i - x}{d_i(x)} dx \quad (6)$$

where  $d_i(x) \equiv \|x - s_i\|$ . It is hard for a mobile sensor to directly compute (6), since it requires global information such as the value of  $R(x)$  over the whole mission space and the exact locations of all other sensors. In addition, the evaluation of integrals remains a significant task for a mobile sensor to carry out. To address these difficulties, we first truncate the sensor model and constrain its sensing capability by applying a *sensing radius*. This approximation is based on the physical observation that when  $d_i(x) \gg 1$ ,  $p_i(x) = 0$  for most sensing devices. Let

$$p_i(x) = 0, \frac{dp_i(x)}{ds_i} = 0 \text{ for all } x \text{ s.t. } d_i(x) \geq D \quad (7)$$

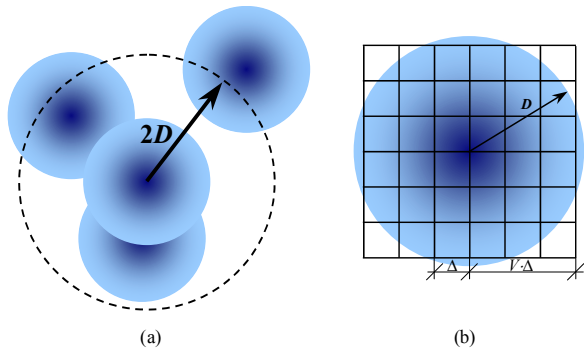


Fig. 1. Neighbor set and grid layout

where  $D$  denotes the sensing radius. Thus, (7) defines sensor  $i$ 's region of coverage, which is represented by  $\Omega_i = \{x : d_i(x) \leq D\}$ . Since  $p_i(x) = 0$ ,  $dp_i(x)/dd_i(x) = 0$  for all  $x \notin \Omega_i$ , we can use  $\Omega_i$  to replace  $\Omega$  in (6). Another byproduct of using (7) is the emergence of the concept of *neighbors*. In (6), for a point  $x \in \Omega_i$  and a sensor  $k \neq i$ , a necessary condition for the detection probability  $p_k(x)$  to be greater than 0 is  $d_k(x) \leq D$ . As shown in Fig. 1-(a), when the distance between sensors  $i$  and  $k$  is greater than  $2D$ , every point  $x$  in  $\Omega_i$  satisfies  $d_k(x) > D$ , thus  $p_k(x) = 0$  and  $[1 - p_k(x)] = 1$  for all  $x \in \Omega_i$ . If we define a set  $\mathcal{B}_i = \{k : \|s_i - s_k\| < 2D, k = 1, \dots, N, k \neq i\}$ , then, any sensor node  $k \notin \mathcal{B}_i$  ( $k \neq i$ ) will not contribute to the integral in (6).

After applying (7) and using  $\mathcal{B}_i$ , (6) reduces to

$$\frac{\partial F}{\partial s_i} = \int_{\Omega_i} R(x) \prod_{k \in \mathcal{B}_i} [1 - p_k(x)] \frac{dp_i(x)}{dd_i(x)} \frac{s_i - x}{d_i(x)} dx \quad (8)$$

The final step in making (8) computable is to discretize the integral evaluation. As shown in Fig. 1-(b), a grid is applied over the coverage region  $\Omega_i$ . The grid has a resolution  $\Delta \ll D$ , and  $\Omega_i$  is represented by a  $(2V + 1) \times (2V + 1)$  grid with  $V = \lfloor D/\Delta \rfloor$ . On the grid of each sensor  $i$ , a Cartesian coordinate system is defined, with its origin located at  $s_i$ , its axes parallel to the grid's setting, and the unit length being  $\Delta$ . In this local coordinate system, let  $(u, v)$  denote the location of a point  $x$ . Then, the transformation that maps  $(u, v)$  onto the global coordinate system is  $x = s_i + [u\Delta \quad v\Delta]^T$ . Upon switching to this local coordinate system, the terms in (8) become:

$$R(x) = \tilde{R}_i(u, v), \quad p_i(x) = \tilde{p}_i(u, v), \quad \frac{dp_i(x)}{dd_i(x)} = \tilde{p}'_i(u, v)$$

where  $\tilde{R}_i(u, v)$  indicates sensor  $i$ 's local perception (map) on the event density of the mission space. In a typical dynamic deployment application, all sensors start with the same copy of an estimated event density function at the beginning of the deployment. As sensors are deployed and data are collected, an individual sensor may update its local map through merging new observations into its perception, and by exchanging information with nearby neighbors.

We also rewrite the product term in (8) as

$$\begin{aligned} \tilde{B}_i(u, v) &\equiv \prod_{k \in \mathcal{B}_i} [1 - p_k(x)] \\ &= \prod_{k \in \mathcal{B}_i} [1 - \tilde{p}_k(u - \frac{s_{k1} - s_{i1}}{\Delta}, v - \frac{s_{k2} - s_{i2}}{\Delta})] \end{aligned}$$

where  $(u - \frac{s_{k1} - s_{i1}}{\Delta}, v - \frac{s_{k2} - s_{i2}}{\Delta})$  are the coordinates of  $x$  in the  $k$ th sensor's local coordinate system. By applying the grid and the coordinate transformation, (8) can be rewritten as

$$\begin{aligned} \frac{\partial F}{\partial s_{i1}} &\approx \Delta^2 \sum_{u=-V}^V \sum_{v=-V}^V \frac{\tilde{R}_i(u, v) \tilde{B}_i(u, v) \tilde{p}'_i(u, v) u}{\sqrt{u^2 + v^2}} \quad (9) \\ \frac{\partial F}{\partial s_{i2}} &\approx \Delta^2 \sum_{u=-V}^V \sum_{v=-V}^V \frac{\tilde{R}_i(u, v) \tilde{B}_i(u, v) \tilde{p}'_i(u, v) v}{\sqrt{u^2 + v^2}} \end{aligned}$$

These derivatives can be easily computed by mobile sensors using only the local information available.

The gradient information above provides direction for a mobile sensor's movement. The precise way in which this information is used depends on the choice of motion scheme. The most common approach in applying a gradient method is to determine the next waypoint on the  $i$ th mobile sensor's motion trajectory through

$$s_i^{k+1} = s_i^k + \alpha_k \frac{\partial F}{\partial s_i^k} \quad (10)$$

where  $k$  is an iteration index, and the step size  $\alpha_k$  is selected according to standard rules (e.g., see [5]) in order to guarantee the convergence of motion trajectories.

The computational complexity in evaluating the gradient shown in (9) depends on the scale of the grid and the size of neighbor set  $\mathcal{B}_i$ . In the worst case, node  $i$  has  $N - 1$  neighbors and the number of operations needed to compute  $\frac{\partial F}{\partial s_i}$  is  $O(NV^2)$ . The best case occurs when there is no neighbor for node  $i$ , and the corresponding complexity is  $O(V^2)$ . In both cases, the complexity is quadratic in  $V$ .

### III. ADDING COMMUNICATION COSTS

#### A. System Structure and Communication Energy Model

Besides sensing and collecting data from the mission space  $\Omega$ , another important task of a sensor network is to forward field data to a *basestation*, denoted by  $b$ . Most current sensor networks assume a two-layer structure [6], in which all sensor nodes form the first layer. The second layer consists of a unique basestation, which is the common destination for all data.

In a two-layer data collection network, the cost of communication mainly comes from the power consumption for wireless transmissions. In order to ensure reliable data forwarding, a wireless link must preserve some basic channel quality which is measured by its Signal to Interference and noise Ratio (SIR). To preserve a given SIR, the power of the transmitter is a monotonically increasing function of the length of the current link [7]. For a single-hop link, the key energy parameters are the energy needed to transmit a bit ( $E_{tx}$ ) and to receive a bit ( $E_{rx}$ ) over a distance  $d$ . Assuming

a  $1/d^n$  path loss, these parameters take the form (see also [6]):

$$E_{tx} = \alpha_{11} + \alpha_2 d^n, \quad E_{rx} = \alpha_{12} \quad (11)$$

where  $\alpha_{11}$  is the energy/bit consumed by the transmitter electronics,  $\alpha_2$  accounts for energy dissipated in the transmit op-amp, and  $\alpha_{12}$  is the energy/bit consumed by the receiver electronics. Hence, the energy consumed by a node acting as a relay that receives a bit and then transmits it a distance  $d$  onward is

$$e(d) = \alpha_{11} + \alpha_2 d^n + \alpha_{12} \equiv \alpha_1 + \alpha_2 d^n \quad (12)$$

Typical numbers for current radios are  $\alpha_1 = 180nJ/bit$  and  $\alpha_2 = 10pJ/bit/m^2$  ( $n = 2$ ) or  $0.001pJ/bit/m^4$  ( $n = 4$ ) [8].

### B. Optimal Coverage Problem with Communication Costs

The previous discussion provides some background to incorporate communication costs into the optimal coverage problem. Consider a mobile sensor network with  $N$  sensors, each located at  $s_i$ ,  $i = 1, \dots, N$ , and a single basestation  $b$  that resides at  $s_0 \in \mathbb{R}^2$ . The data rate originating from the  $i$ th sensor is denoted by  $r_i(s_i)$ ,  $i = 1, \dots, N$ . Note that  $r_i$  is defined as a function of  $s_i$  because the amount of data forwarded at  $i$  is determined by the number of events detected, and the latter depends on the sensor's location. Here we assume that  $r_i(s_i)$  is proportional to the frequency that events are detected, i.e.,

$$r_i(s_i) = \alpha_3 \int_{\Omega} R(x) p_i(x) dx \quad (13)$$

where  $\alpha_3$  (bit/detection) is the amount of data forwarded when the sensor detects an event.

Data originating at each sensor are finally delivered to basestation  $b$ . Let  $c_i(\mathbf{s})$  be the *total* power consumed by the network in order to deliver a bit of data from sensor  $i$  to  $b$ . Then, the optimal coverage problem can be revised by combining sensing coverage and communication cost as follows:

$$\max_{\mathbf{s}} \left\{ w_1 \int_{\Omega} R(x) P(x, \mathbf{s}) dx - w_2 \sum_{i=1}^N r_i(s_i) c_i(\mathbf{s}) \right\} \quad (14)$$

where  $w_1, w_2$  are weighting factors.

Let us denote the communication cost by  $G(\mathbf{s}) = \sum_{i=1}^N r_i(s_i) c_i(\mathbf{s})$  and, recalling (4), the overall objective function is written as

$$J(\mathbf{s}) = w_1 F(\mathbf{s}) - w_2 G(\mathbf{s}) \quad (15)$$

In order to derive partial derivatives  $\frac{\partial J}{\partial s_i}$  similar to Section II, we shall focus on the evaluation of  $\frac{\partial G}{\partial s_i}$ , which can be expressed as

$$\frac{\partial G}{\partial s_i} = c_i(\mathbf{s}) \frac{dr_i(s_i)}{ds_i} + \sum_{k=1}^N r_k(s_k) \frac{\partial c_k(\mathbf{s})}{\partial s_i} \quad (16)$$

In this expression, both  $r_i$  and  $\frac{\partial r_i}{\partial s_i}$  can be obtained by applying the same method as the one described in Section

II. That is, recalling that  $x = s_i + [u\Delta \quad v\Delta]^T$ ,

$$r_i \approx \alpha_3 \Delta^2 \sum_{u=-V}^V \sum_{v=-V}^V \tilde{R}(u, v) \tilde{p}_i(u, v)$$

$$\frac{dr_i}{ds_{i1}} \approx \alpha_3 \Delta^2 \sum_{u=-V}^V \sum_{v=-V}^V \frac{\tilde{R}(u, v) \tilde{p}'_i(u, v) u}{\sqrt{u^2 + v^2}} \quad (17)$$

$$\frac{dr_i}{ds_{i2}} \approx \alpha_3 \Delta^2 \sum_{u=-V}^V \sum_{v=-V}^V \frac{\tilde{R}(u, v) \tilde{p}'_i(u, v) v}{\sqrt{u^2 + v^2}}$$

The only term remaining to derive in  $\frac{\partial G}{\partial s_i}$  is  $c_i(\mathbf{s})$  and its gradient. The cost of delivering a bit of data from  $i$  to  $b$ ,  $c_i(\mathbf{s})$ , is determined by the way in which data forwarding paths are constructed, i.e., the precise routing protocol used. Many wireless routing protocols are available to build reliable and efficient multi-hop paths between a data source and its destination (e.g., see [9]). Among them, we are interested in those that can generate minimal power consumption paths (e.g. [10]).

Let us represent the sensor network by a graph  $\mathcal{G} = (\mathcal{N}, \mathcal{E})$ , where  $\mathcal{N} = \{0, 1, \dots, N\}$  denotes the set for all nodes ( $b$  is indexed by 0) and  $\mathcal{E} = \{(i, j) | i, j \in \mathcal{N}\}$  is the set for all links. A cost  $e_{ij}$  is defined for link  $(i, j) \in \mathcal{E}$  to be

$$e_{ij} = e(\|s_i - s_j\|) \quad (i, j) \in \mathcal{E} \quad (18)$$

where  $e(\cdot)$  is the communication power consumption on edge  $(i, j)$  as in (12). Over  $\mathcal{G}$ , a routing protocol is executed which generates a set of "shortest" paths  $\mathcal{L} = \{l_1, \dots, l_N\}$  between each sensor and the basestation  $b$ . Here, a path  $l_i = \{(i, j), \dots, (k, 0)\}$  is said to be a "shortest" path between node  $i$  and 0 in the sense that  $c_i = \sum_{(j,k) \in l_i} e_{jk}$  is minimized over all possible paths between  $i$  and 0. It is well-known that the set of "shortest" paths  $\mathcal{L}$  forms a *tree* structure [11], and it can be expressed by a *forward index vector*  $H = (h_1, \dots, h_N)$ , where  $h_i \in \{0, 1, \dots, N\}$  denotes the index of the next-hop node when forwarding data from  $i$ .

At each sensor  $i$ , the forward index  $h_i$  and forward cost  $c_i$  are given by the routing protocol. The routing protocol also provides sensor  $i$  an *upstream vector*  $U_i = (u_1^i, \dots, u_N^i)$  and a *cumulative flow factor*  $z_i$  defined as

$$u_j^i = \mathbf{1}[h_j = i], \quad z_i = r_i + \sum_{j=1}^N u_j^i z_j$$

where  $u_j^i$  indicates whether  $j$  is  $i$ 's upstream node and  $z_i$  records the total data rate originated from  $i$ :  $r_i$  accounts for data collected at  $i$  and  $\sum_{j=1}^N u_j^i z_j$  is the total traffic from upstream nodes.

Given  $h_i$ ,  $c_i$ ,  $U_i$  and  $z_i$ , a node  $i$  can evaluate  $\frac{\partial G}{\partial s_i}$  locally. To accomplish this, let us rewrite  $G(\mathbf{s})$  in (15) as

$$\begin{aligned} G(\mathbf{s}) &= \sum_{i=1}^N r_i(s_i) c_i(\mathbf{s}) \\ &= \sum_{(j,k) \in \mathcal{E}} e_{jk} \left\{ \sum_{i=1}^N \mathbf{1}[(j, k) \in l_i] r_i \right\} \end{aligned} \quad (19)$$

where  $\sum_{i=1}^N \mathbf{1}[(j, k) \in l_i] r_i$  is actually equivalent to the total flow on link  $(j, k)$ . Because of the tree structure of  $\mathcal{L}$ , we have

$$\sum_{i=1}^N \mathbf{1}[(j, k) \in l_i] r_i = \begin{cases} z_j & \text{if } k = h_j \\ 0 & \text{otherwise} \end{cases}$$

By removing all the terms with value zero in (19), we get  $G(s) = \sum_{i=1}^N e_{ih_i} z_i$  and the term  $\sum_{k=1}^N r_k \frac{\partial c_k}{\partial s_i}$  in (16) is equivalent to

$$\sum_{k=1}^N r_k \frac{\partial c_k}{\partial s_i} = \sum_{k=1}^N z_k \frac{\partial e_{kh_k}}{\partial s_i}$$

where we assume that network routing ( $\mathcal{L}$ ) remains fixed when sensor locations ( $s$ ) change slightly. Using (12) and (18),

$$\frac{\partial e_{kh_k}}{\partial s_i} = \begin{cases} n\alpha_2 \|s_i - s_j\|^{(n-2)} (s_i - s_j) & \text{if } k = i \text{ or } h_k = i \\ 0 & \text{otherwise} \end{cases}$$

Thus,

$$\sum_{k=1}^N r_k \frac{\partial c_k}{\partial s_i} = \left[ z_i \frac{\partial e_{ih_i}}{\partial s_i} + \sum_{\{j|u_j^i=1\}} z_j \frac{\partial e_{jh_j}}{\partial s_i} \right] \quad (20)$$

By combining (9), (16), (17) and (20), sensor node  $i$  can derive  $\frac{\partial J}{\partial s_i}$  locally. Then, each sensor uses gradient information to direct motion control as in (10) with  $\frac{\partial J}{\partial s_i^k}$  replacing  $\frac{\partial F}{\partial s_i^k}$ . With properly selected step sizes, mobile sensors will finally converge to a maximum point of  $J(s)$ .

#### IV. SIMULATION RESULTS

The previously presented distributed deployment algorithm has been implemented in a Java-based simulation environment (see [frontera.bu.edu/Applets/CoverageContr/](http://frontera.bu.edu/Applets/CoverageContr/)). As shown in Fig. 2, a team of 6 mobile sensors is waiting to be deployed into a  $40 \times 40$  (meter) mission space. The event density function  $R(x)$  is given by,

$$R(x) = R_0 - \beta \|x - x_0\| \quad (21)$$

where  $R_0 = 3.0$ ,  $\beta = 0.1$ ,  $x_0 = [0, 20]$ . According to (21), the event density of a point  $x$  ( $x \in \Omega$ ) declines linearly with the distance between  $x$  and the center point  $x_0$  of the mission space.

At time  $t = 0$ , mobile sensors reside near the origin of the mission space. Each mobile node is equipped with a sensor whose detection probability is modeled as in (1) by  $p_i(x) = p_{0i} e^{-\lambda_i \|x - s_i\|}$  where  $p_{0i} = 1.0$ ,  $\lambda_i = 1.0$  for all  $i = 1, \dots, N$ . The sensing radius is  $D = 5.0$ , as illustrated by black circles in Fig. 2. A mobile sensor also has a wireless transceiver whose power consumption is modeled by (12) with  $\alpha_1 = 0.01nJ/bit$ ,  $\alpha_2 = 0.001nJ/bit/m^4$  and  $n = 4$ . In the mission space, there is a radio basestation residing at  $s_0 = [0, 0]$ , (marked by a red square in Fig. 2). Upon a sensor detecting an event, it collects 32 bits of data and forwards them back to the basestation (so that  $\alpha_3 = 32$  in (13)).

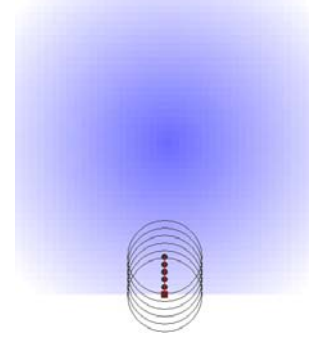


Fig. 2. Cooperative coverage control problem with 6 mobile sensors

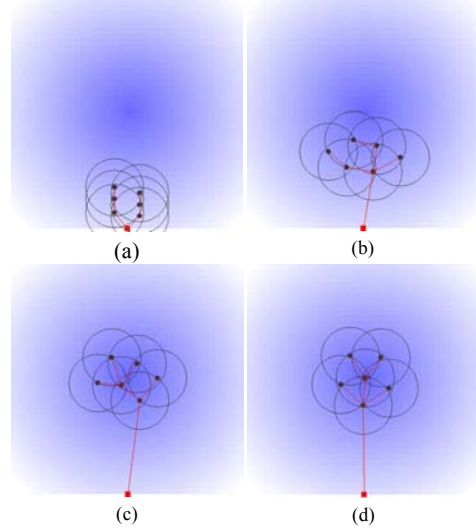


Fig. 3. Sensors deployment without communication cost consideration

We will present simulation results for this coverage control problem by looking at two distinct cases. In the first case, no communication cost is considered, which corresponds to  $w_1 > 0$ ,  $w_2 = 0$  in the optimal coverage formulation (14). In the second case, both sensing coverage and communication cost are included ( $w_1, w_2 > 0$ ).

Figure 3 presents several snapshots taken during the deployment process of the first case. Starting with Fig. 3-(a), 6 sensors establish a formation and move towards the center of the mission space. During its movement, the formation keeps evolving, so that sensors expand the overall area of sensing and at the same time jointly cover the points with high event density. In addition, sensors also maintain wireless communication with the basestation. This is shown in Fig. 3 as links between sensor nodes and the basestation. The team of sensors finally converges to a stationary formation as shown in Fig. 3-(d). It can be seen in this symmetric formation that all 6 sensors are jointly sensing the area with the highest event density.

We incorporate communication cost into the optimal coverage formulation by setting  $w_2 = 0.0008$  and  $w_1 = 1 - w_2$  in (14). The corresponding deployment simulation results

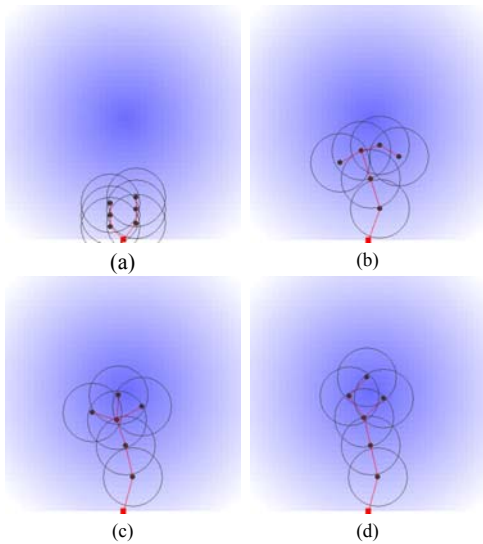


Fig. 4. Sensors deployment with communication cost consideration

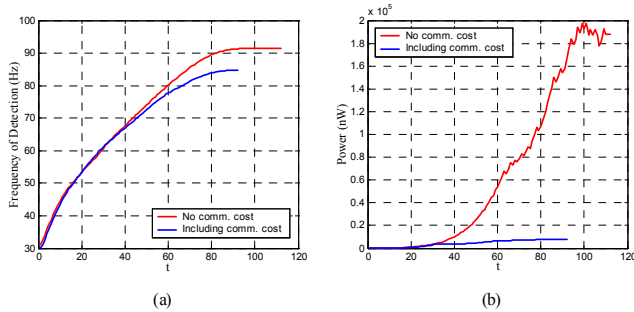


Fig. 5. Comparison on sensing coverage and communication costs

are shown in Fig. 4. Comparing with the first case, a critical difference can be observed in the formation of mobile sensors: sensors not only move towards the area with high event density, but they also maintain an economical multi-hop path to the basestation. The team of sensors reaches a stationary deployment as illustrated in Fig. 4-(d). In contrast to the final formation of the first case (Fig. 3-(d)), only 4 sensors gather around the center of the mission space. The other 2 sensors are aligned as relays to support the communication with the basestation.

Figure 5 demonstrates the sensing coverage and communication cost associated with the previously shown two cases. Fig. 5-(a) depicts the change in sensing coverage (measured by the expected frequency of event detection) when sensors move towards the optimal deployment. A direct observation is that in both cases, sensing coverage increases monotonically with the evolution of formations. If no communication cost is considered during sensor deployment, sensing coverage reaches a maximum at  $91.47 Hz$ . However, in the case that communication cost is considered, when sensors reach optimal deployment, only  $84.74$  events can be detected per second, which corresponds to a  $7.36\%$  coverage loss. This coverage loss is natural, since the optimal coverage

formulation (14) actually trades off sensing coverage for a lower communication cost. This tradeoff can be further examined by looking at Fig. 5-(b). If communication cost is considered, the final power consumption is  $8.01 \times 10^3 nW$ . Compared to the communication cost of the first case ( $1.877 \times 10^5 nW$ ), there is a  $95.73\%$  power saving.

## V. CONCLUSIONS

We have developed a distributed coverage control scheme for cooperating mobile sensor networks. The mission space is modeled using a density function representing the frequency of random events taking place. We assume that a mobile sensor has a limited range which is defined by a probabilistic model. A deployment algorithm is applied at each mobile node so that it maximizes the joint detection probabilities of random events. We also incorporate communication costs into the coverage control problem, viewing the sensor network as a multi-source, single-basestation data collection network. Communication cost is modeled as the power consumption needed to deliver collected data from sensor nodes to a basestation using wireless multi-hop paths. Thus, the coverage problem we formulate trades off sensing coverage and communication cost.

This distributed deployment algorithm has been extensively tested in a simulation environment. Experimental results indicate that this scheme is efficient and it can generate a quality deployment scheme. In addition, by applying gradient methods and geographic routing techniques, the algorithm avoids solving global optimization problems, which in turn guarantees real-time performance.

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