A DISTRIBUTED FINITE-ELEMENT
MODELING AND CONTROL APPROACH FOR
LARGE FLEXIBLE STRUCTURES

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INTRODUCTION

This paper describes an unconventional framework for the design of decentralized controllers for large flexible structures. In contrast to conventional control system design practice which begins with a model of the open loop plant, the controlled plant is assembled from *controlled components* in which the modeling phase and the control design phase are integrated at the component level.

The developed framework is called *Controlled Component Synthesis* (CCS) to reflect that it is motivated by the well developed *Component Mode Synthesis* (CMS) methods which have been demonstrated to be effective for solving large complex structural analysis problems for almost three decades.

The design philosophy behind CCS is also closely related to that of the *Subsystem Decomposition Approach* in decentralized control.

CONTROLLED COMPONENT SYNTHESIS

CCS is a framework for an integrated, component oriented, finite-element modeling and structural control design. Similar to CMS methods, CCS is developed on the premise that a large complex controlled structure is to be built from *controlled components*: The finite element modeling and control design are carried out for the individual components; the model of the large complex structure is assembled from the controlled components only for the purpose of performance evaluation.

The CCS method developed herein adopts the following modeling and control design considerations at the component level: Instead of using either the boundary loading, or the constraint modes approach as in CMS, we introduce a new approach called *Isolated Boundary Loading* for the development of component models. For the design of controllers for the component, an *InterLocking Control* concept is developed to minimize the motion of the nodes that are adjacent to the boundary, thereby suppressing the transmission of mechanical disturbance from component to component in the coupled structure.

The major ideas behind CCS are:

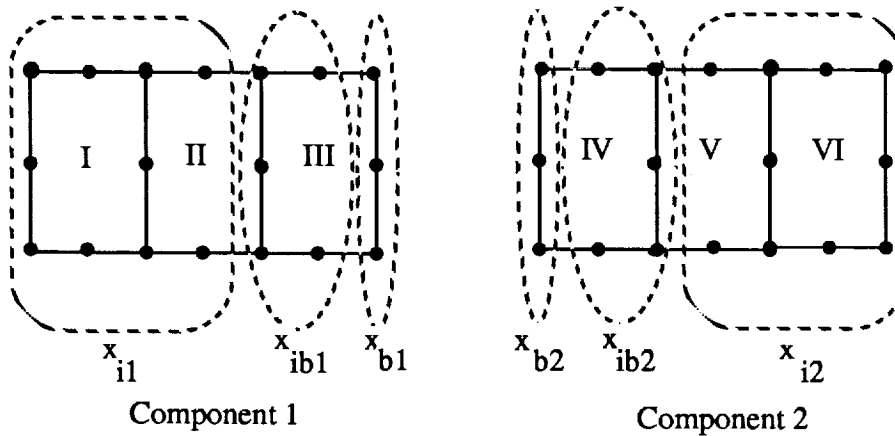Component modeling using Isolated Boundary loading

Connections to Overlapping Decomposition

Intelocking Control Concept

COMPONENT MODELING

A two component structure, as shown below, will be used to outline the modeling and design procedure of the CCS method. Each of the structure component is composed of three finite elements. Identified in the figure by Roman numerals are the finite elements, and by solid circles are the element node points.

In the **CCS** method, the nodal coordinates of a component are partitioned into three groups: The internal coordinates are subdivided into a group of internal boundary coordinates x_{ibs} and a group of internal coordinates x_{is}. The boundary coordinates x_{bs} remain in a single group. The boundary coordinates are coordinates of the boundary element, such as element III of component 1, which are on the boundary. The remaining coordinates of the boundary element are designated the internal boundary coordinates. The remaining coordinates of the component are the internal coordinates.

Component 1   Component 2

A two component structure modeled with finite elements

ISOLATED BOUNDARY LOADING

The component mass and stiffness matrices are obtained from the finite-element modeling of an expanded component, *i.e.*, the original boundary of the component is extended one finite element into the adjacent component. The nodes of the expanded component consist of the original nodes of the component, and the internal boundary coordinates of the adjacent component. The mass and stiffness matrices are obtained from the mass and stiffness matrices of the expanded component by deleting the rows and columns corresponding to the nodes in the expanded portion.


$\blacksquare$: nodes to be truncated
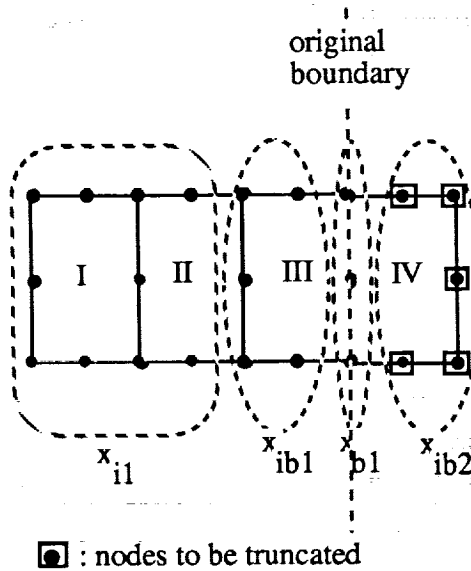
Expanded component 1

CONNECTION TO OVERLAPPING DECOMPOSITION

The component models developed using isolated boundary loading have direct connection with the Subsystem Decomposition Approach. These models are identical to the decoupled subsystem models if an overlapping decomposition is applied to the finite-element model of the coupled structure. This is a key connection which allows the use of tools developed by Siljak[1] and his co-workers for evaluating the performance of the controlled coupled structure, after the controlled component designs have been completed.

The mass and stiffness matrix connectivity is illustrated in the following diagram showing how the component models can be "contracted" to form the coupled structure finite-element model.

b1 and b2 blocks are identical due to boundary loading

b2 and b3 blocks are identical due to boundary loading

CONTRACTION
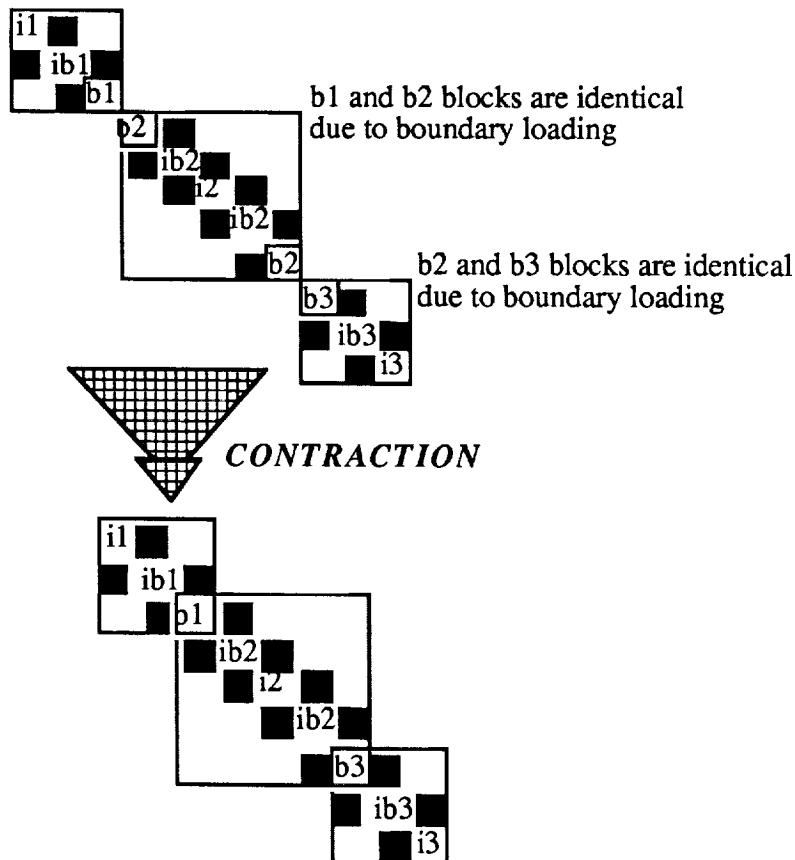
Overlapping Decomposition Connection

155

INTERLOCKING CONTROL CONCEPT

The new insights gained from the developed component modeling approach in turn motivate a new component level control design concept which we call *InterLocking Control* (**ILC**) in which collocated actuator and sensors are placed at the internal boundary degrees of freedom, and the control law is designed, using the developed component model for **CCS**, to minimize the internal boundary coordinate motion.

Such minimization would localize the dynamic interactions of the coupled structure in the components. The component control action is designed to lock up its own internal boundary to realize a boundary condition which better approximates the one assumed in the component modeling of its adjacent components.

A convenient control design technique for this concept is the linear quadratic optimal regulator approach in which the internal boundary coordinates are considered as regulated outputs of the component to be weighted together with the component control inputs in the quadratic performance index. The resulting component control law minimizes this index.

The **ILC** concept translates into a two step component control design process summarized below:

1. For the **sth** component, use the component model

$$
\begin{bmatrix} M_{ii}^s & M_{iib}^s & 0 \\ M_{bii}^s & M_{ib}^s & M_{ibb}^s \\ 0 & M_{bib}^s & M_{bb,iL}^s \end{bmatrix} \begin{bmatrix} \ddot{x}_{is} \\ \ddot{x}_{ibs} \\ \ddot{x}_{bs} \end{bmatrix} + \begin{bmatrix} K_{ii}^s & K_{iib}^s & 0 \\ K_{bii}^s & K_{ib}^s & K_{ibb}^s \\ 0 & K_{bib}^s & K_{bb,iL}^s \end{bmatrix} \begin{bmatrix} x_{is} \\ x_{ibs} \\ x_{bs} \end{bmatrix} = \begin{bmatrix} 0 \\ u^s \\ 0 \end{bmatrix}
$$

$$
y^s = x_{ibs},
$$

for control system design, where u^s and y^s denotes respectively the control force exerted by the actuators, and the sensor outputs, at the internal boundary coordinates.

2. Derive the component control law by minimizing the performance index,

$$
J_c^s = \frac{1}{2} \int_0^\infty (y^{sT} y^s + u^{sT} R^s u^s) dt .
$$

156

APPLICATION TO TRUSS STRUCTURE CONTROL

The developed CCS method is applied to the design of structural control laws for a planar truss structure for a preliminary assessment of its feasibility toward solving more complex structural control design problems. This truss structure which is depicted below has six bays, and the nodal coordinates are defined as the vertical and horizontal displacements at the joints.
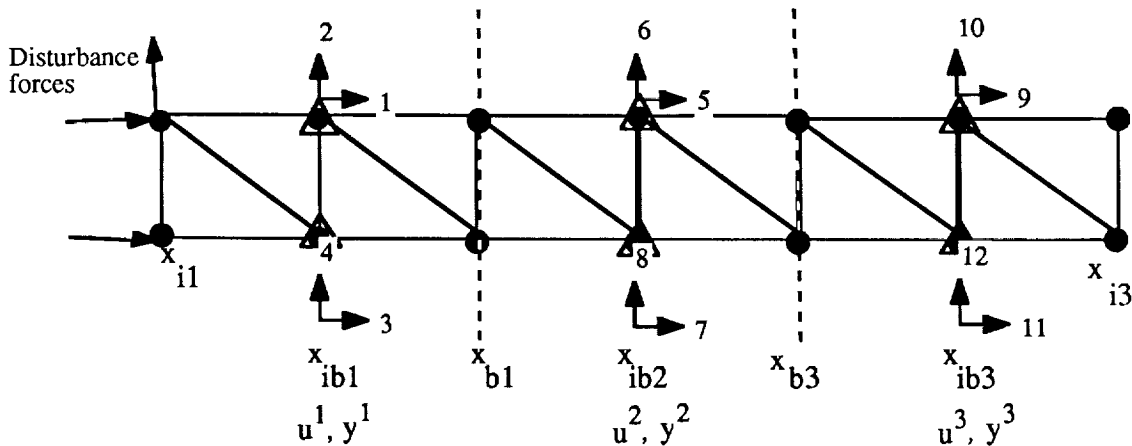
Planar truss for CCS evaluation

External forces applied at the nodes are decomposed into orthogonal components. The assumptions made are that the truss members are subjected to axial forces alone, and not bending moments; and the members are uniform rods of identical lengths L, mass per unit length m, cross-section area per unit length A and modulus of elasticity E.

The six bay truss can be viewed as a structure that consists of three identical components, namely the left component, the center component, and the right component, which are composed of the left-most, the middle, and the right-most two bays respectively. The six bay/three component truss structure is chosen to capture the essential characteristics of a truss consists of an arbitrary number of identical components, i.e., a truss structure with an arbitarily large number of two bay components is composed of the three same types of components identified in the six bay truss, with the center component duplicated as necessary. Thus, conclusions from the six bay/three component design apply equally well to the design of structural controls for a multiple bay truss.

For CCS, the component models are developed using the expanded component introduced in Isolated Boundary Loading. The mass and stiffness matrices of the expanded component are derived using a finite-element method with the Ritz-Rayleigh approximation. The truss member mass and stiffness matrices used in the assembly process are:

$$K_{member} = \frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad , \quad M_{member} = \frac{mL}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

The component model is further scaled to remove the effects of the material properties: a new time variable
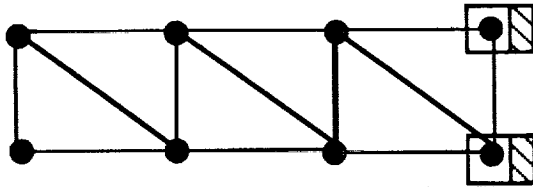
$$\tau = \sqrt{\frac{m}{6EA}}\, Lt$$

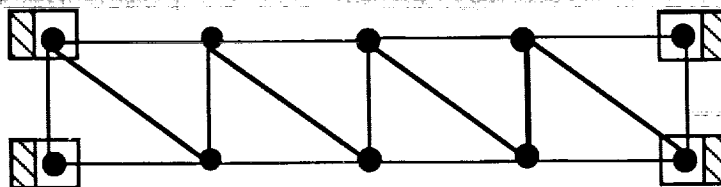is introduced, and the nodal forces are scaled by $L\,(EA)^{-1}$.

The three expanded components from which the component models are derived are shown below. The internal boundary degrees of freedom at which collocated force actuators and displacement sensors are placed are marked by Δ.
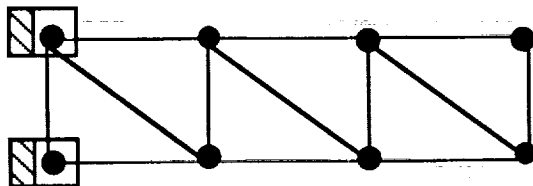
Right-most Component

Left-most Component

Center Component

◼●◻ : Nodes to be truncated/Degrees of Freedom to be constrained

The three expanded component

TRUSS STRUCTURE CONTROL – INTERLOCKING CONTROL DESIGN

The component level control design using the Interlocking Control concept is carried out with a 4 × 4 identical control weighting matrix $R=.001$ I for all three components. The control designs for the left-most and right-most components are identical due to symmetry. Therefore, we only need to carry out a center component design and an end component control design.

The controlled components' poles, as well as the poles of the controlled truss structure, are plotted in the figure below. Since the left-most and right-most components are identical, we plot only the poles of one of them which are denoted by End Component Poles in these figures. All the poles of the controlled structure have negative real parts, indicating that the closed-loop system is asymptotically stable.

That the pole locations of the controlled components are close to that of the controlled coupled structure indicates that the component models developed for CCS are effective for this structural control design.
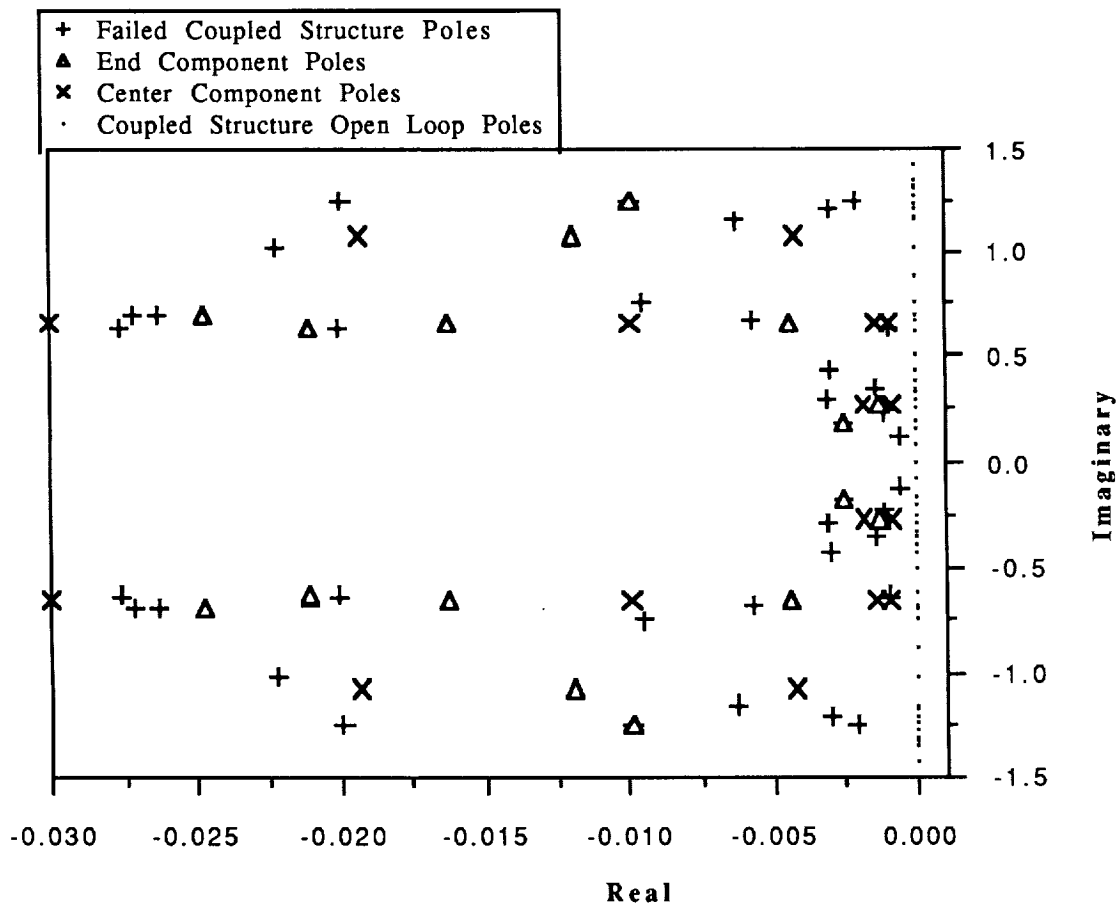

Closed up view of pole location near the imaginary axis of the controlled components and the controlled structure, open loop poles of the coupled structure are inserted for reference

159

TRUSS STRUCTURE CONTROL – SIMULATION RESULTS

For transient response studies, the response of the controlled structure to three disturbance force pulses of 0.5 seconds is examined, simultaneously applied to the left-most nodes of the truss, as shown in the three expanded component figure. The coupled structure is assumed to be in static equilibrium initially in the simulation, of which samples of the sensor output time responses are shown in the top two figures below. Two of the twelve sensor channels, one horizontal (Channel 1 in the three expanded component figure) and one vertical nodal displacements (Channel 10), are selected. The magnitudes of the displacement response drop by an order of magnitude per component for nodes that are farther away from the disturbances. The delay effect of the force pulses on the displacements shown in the Channel 10 displacement figure below is typical for the right-most component.

The developed CCS method inherits the capability to withstand system failures from decentralized control developed using the Subsystem Decomposition Approach. The controlled structure, in which the center component controller failed, is simulated for the same disturbances and initial conditions as before. The two bottom figures below show an order-of-magnitude performance degradations for one of the displacements (Channel 5) at the center component. However, despite the center component controller failure, the neighboring components stabilize the vibrations in the center component with interlocking controls.
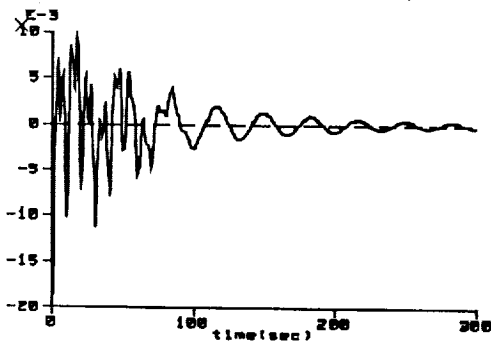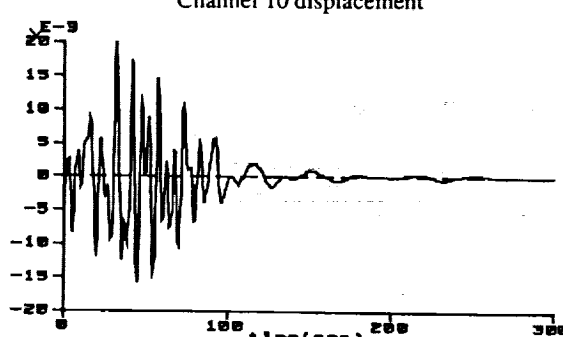
Channel 1 displacement
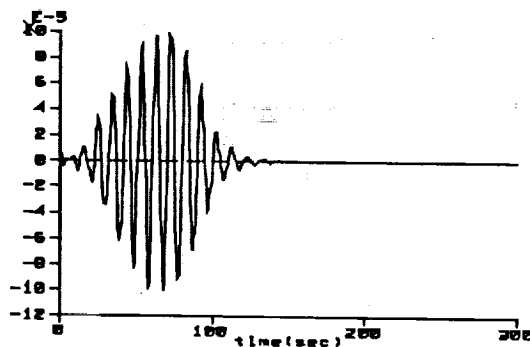
Channel 10 displacement
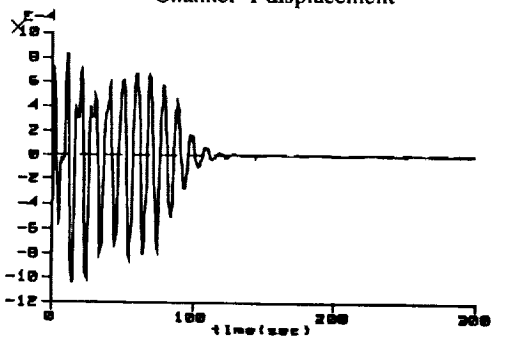
Channel 5 displacement, without failure

Channel 5 displacement,
with center component controller failure

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2. Ikeda M. and D. D. Siljak(1980). Overlapping Decompositions, Expansions, and Contractions of Dynamic Systems. *Large Scale Systems*, Vol. 1, pp. 29-38.
3. Ikeda M., D. D. Siljak, and D. E. White(1981). Decentralized Control with Overlapping Information Set. *J. Opt. Theory Appl.*, Vol. 34, pp. 279-310.