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Distributed Global Output-Feedback Control for a Class of Euler–Lagrange Systems

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Abstract—This paper investigates the distributed tracking control problem for a class of Euler–Lagrange multiagent systems when the agents can only measure the positions. In this case, the lack of the separation principle and the strong nonlinearity in unmeasurable states pose severe technical challenges to global output-feedback control design. To overcome these difficulties, a global nonsingular coordinate transformation matrix in the upper triangular form is first proposed such that the nonlinear dynamic model can be partially linearized with respect to the unmeasurable states. And, a new type of velocity observers is designed to estimate the unmeasurable velocities for each system. Then, based on the outputs of the velocity observers, we propose distributed control laws that enable the coordinated tracking control system to achieve uniform global exponential stability. Both theoretical analysis and numerical simulations are presented to validate the effectiveness of the proposed control scheme.

Index Terms—Coordinate transformation, distributed control, Euler–Lagrange systems, global output feedback.

I. INTRODUCTION

Recently, intensive attention has been paid to distributed control for Euler–Lagrange systems due to its broad applications. Several approaches have been proposed to deal with the distributed tracking problem, see, for example, sliding-mode method [1], [2], disturbance observer [3], and extended proportional-integral control scheme [4]. Extension to handle unknown parameter uncertainties can be found in [5], where an adaptive controller is proposed to synchronize nonidentical Euler–Lagrange systems with communication time delays. Later, [6] solves the synchronization problem of networked robotic systems with both the kinematic and dynamic uncertainties using passivity theory. It has also been shown that, under a jointly connected switching network topology, leader-following consensus can be achieved for multiple Euler–Lagrange systems by employing adaptive control [7], in which various reference signals, such as sinusoidal and ramp signals, generated by an exosystem are considered.

In order to relax the restrictive requirement for full state measurements in designing the controllers in the existing results, some efforts on partial state feedback control have been made. To estimate the unmeasurable velocity, observers are constructed by invoking the Immersion and Invariance (I&I) techniques [8]–[10]. Through introducing two extra states, some lower dimensional observer is proposed in [9] in comparison to that in [8], and moreover, the explicit expressions of the observer have been given. In [10], dynamic scaling and high-gain terms have been adopted to perform the Lyapunov stability analysis. Note that the dynamics of the observers relying on I&I techniques are generally high dimensional and complex. In addition, it is required to find a certain attractive and invariant manifold in the extended state space of the plant and the observer, which will likely increase the computational burden.

For multiagent systems, a consensus algorithm using linear observers is first proposed in [11], and, in [12], a distributed control law with time-varying control gains is designed to compensate for the lack of neighbors' velocity measurements. For the distributed tracking problem, [13] presents a sliding mode observer-based controller to track the leader with constant velocity in finite time. The more challenging problems of tracking a leader with varying velocity have also been investigated in [14]–[16]. When only nominal parameters of Euler–Lagrange systems are available, global asymptotic stability can be ensured using continuous control algorithms with adaptive coupling gains [14]. More generally, in cases when we do not have access to any velocity measurement, it is desirable to coordinate the agents using output-feedback strategies. However, some drawbacks of the available results still exist. For example, tracking errors can only be guaranteed to be uniformly ultimately bounded but not converging to zero [15] and the resulted closed-loop system is only locally but not globally stable under the designed control laws [16]. For some specific class of nonlinear systems, the global output feedback control problem has been investigated recently in [17] and [18], where the cyclic-small-gain approach and distributed internal model have been introduced, respectively, to achieve global convergence. To deal with the leader's unavailable velocity measurements, distributed observers are designed for second-order agents in [19].

The goal of this paper is to address the problem of distributed global output-feedback tracking for multiple Euler–Lagrange systems modeling a class of two-link revolute robot manipulators. Up to now, there is no known result for such distributed global tracking algorithms due to several technical challenges. The main difficulties in achieving global stability lie in the quadratic nonlinearities and the cross terms of the unmeasurable velocity states derived from the Coriolis and centrifugal torques. To eliminate such quadratic terms, different state transformation methods have been utilized in [20]–[23]. In [20] and [21], the coordinate transformation strategies are first applied to simplify the nonlinear models, and then controllers are proposed for 1 DOF Euler–Lagrange systems and underactuated mechanical systems in their Hamiltonian forms, respectively. It should also be noted that both of the techniques in [22] and [23] pose constraints on the system model, i.e., a class of nonlinear systems that are linear in unmeasured states, globally stabilizable using output feedback [22], and with skew-symmetric Coriolis terms [23]. However, the models of the two-link revolute robot manipulators considered in this paper do not possess any of the properties just mentioned that contribute to the simplification of the system model. So, all of these approaches cannot be directly applied to the robot manipulators discussed in this paper. Inspired by the works [22] and [23], we shall focus on how to partially linearize the dynamics of the robot manipulators through coordinate transformation and state reconstruction. With the help of the model transformation, a distributed velocity observer is proposed, which enables us to implement the output-feedback control for multiple robot manipulators such that the tracking errors uniformly globally exponentially converge to zero.

The rest of the paper is organized as follows. Section II reviews the system dynamics and presents the method on how to partially linearize the nonlinear system through coordinate transformation. In Section III, an observer-based control strategy is proposed based on the partially linearized system. Section IV gives the main result of this paper, followed by the numerical simulations in Section V. Finally, conclusions are provided in Section VI.

Notations: $|X|$ denotes the determinant of a real square matrix X. $\|x\|$ is used to denote the two norm of a vector x. I_n represents the identity matrix with dimension n, and $\mathbf{1}_n$ denotes the column vector whose components are all one. We use $^iX_{jk}$ to denote the (j,k)th element of matrix X_i, and $\overline{\lambda}_X$ and $\underline{\lambda}_X$ are the largest and smallest eigenvalues of a real symmetric matrix X, respectively.

II. PARTIAL LINEARIZATION

In this section, we first briefly introduce the general expression of Euler–Lagrange systems, followed by the specific dynamics of two-link revolute robot manipulators. Then, we present the process removing the cross terms of the velocity states via coordinate transformation.

A. Dynamics of Robot Manipulator

We consider here a group of n mechanical robots, each of which is described by a Euler–Lagrange equation as follows:

$$M_i(q_i)\ddot{q}_i + C_i(q_i,\dot{q}_i)\dot{q}_i + G_i(q_i) = \tau_i, \quad i = 1,\ldots,n \quad (1)$$

where q_i is the vector of the generalized coordinates, $M_i(q_i)$ is the symmetric positive-definite inertia matrix, $C_i(q_i,\dot{q}_i)\dot{q}_i$ is the Coriolis and centrifugal torque, $G_i(q_i)$ is the vector of the gravitational torques, and τ_i is the control torque on robot i.

The neighbor relationships between the robots are described by a directed graph $\mathbb{G}$ with the vertex set $\mathcal{V} = \{1,2,\ldots,n\}$ and the edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. We use $\mathcal{A} = [a_{ij}]_{n \times n}$ to denote the adjacency matrix, where $a_{ij} = 1$ means there is an edge (j,i) between robots i and j,

and robot i can obtain information from robot j, but not vice versa, and $a_{ij} = 0$, otherwise. There is one leader robot and the rest are followers. The interaction relationships among the followers and the leader is denoted by the matrix $B = \text{diag}\{b_1,\ldots,b_n\}$, where $b_i = 1$ if the leader is a neighbor of robot i, and $b_i = 0$, otherwise. The Laplacian matrix $L = [l_{ij}]_{n \times n}$ is defined by $l_{ii} = \sum_{j \in \mathcal{N}_i} a_{ij}$ and $l_{ij} = -a_{ij}$, $i \neq j$, where $\mathcal{N}_i$ denotes the set of neighbors of robot i.

It is well known that a wide range of mechanical systems can be represented by Euler–Lagrange equations, such as robot manipulators, mobile robots, and rigid bodies. Here, we focus on a class of two-link revolute robot manipulators, whose dynamics are given by (see [24])

$$M_i(q_i) = \begin{bmatrix} O_{i(1)} + 2O_{i(2)}\cos(q_{i(2)}), & O_{i(3)} + O_{i(2)}\cos(q_{i(2)}) \\ O_{i(3)} + O_{i(2)}\cos(q_{i(2)}), & O_{i(3)} \end{bmatrix}$$

$$C_i(q_i,\dot{q}_i) = \begin{bmatrix} -O_{i(2)}\sin(q_{i(2)})\dot{q}_{i(2)}, & -O_{i(2)}\sin(q_{i(2)})(\dot{q}_{i(1)}+\dot{q}_{i(2)}) \\ O_{i(2)}\sin(q_{i(2)})\dot{q}_{i(1)}, & 0 \end{bmatrix}$$

$$G_i(q_i) = \begin{bmatrix} O_{i(4)}g\cos(q_{i(1)}) + O_{i(5)}g\cos(q_{i(1)} + q_{i(2)}) \\ O_{i(5)}g\cos(q_{i(1)} + q_{i(2)}) \end{bmatrix}$$

where g is the acceleration of gravity, $q_i = [q_{i(1)}, q_{i(2)}]^T$ represents the joint angles of the two links and $O_i = [O_{i(1)}, O_{i(2)}, O_{i(3)}, O_{i(4)}, O_{i(5)}] = [m_1 l_{c1}^2 + m_2(l_1^2 + l_{c2}^2) + J_1 + J_2, m_2 l_1 l_{c2}, m_2 l_{c2}^2 + J_2, m_1 l_{c1} + m_2 l_1, m_2 l_{c2}]$, in which the variables m_i, l_i, and J_i are, respectively, used to denote the masses, the lengths, and the moments of inertia of link i, and l_{ci} represents the distance from the previous joint to the center of mass of link i, $i = 1, 2$. The inertia matrix $M_i(q_i)$ satisfies the following property: for all $q_i \in \mathbb{R}^2$, there exist positive constants k_m and k_M, such that $k_m I_2 \leq M_i(q_i) \leq k_M I_2$.

B. Coordinate Transformation

In order to linearize the quadratic velocity terms in $C_i(q_i,\dot{q}_i)\dot{q}_i$ and to simplify the dynamics model, motivated by [23], we introduce the following coordinate transformation

$$z_i = T_i(q_i)\dot{q}_i \quad (2)$$

where $T_i(q_i) \in \mathbb{R}^{2 \times 2}$, a nonsingular matrix with bounded elements to be determined, is constructed as follows:

$$T_i(q_i) = \begin{bmatrix} ^iT_{11} & ^iT_{12} \\ ^iT_{21} & ^iT_{22} \end{bmatrix} = \begin{bmatrix} ^iM_{11} & ^iM_{12} \\ 0 & ^iT_{22} \end{bmatrix} \quad (3)$$

where $^iT_{22}$ needs to be determined. Here, instead of fully linearizing system (1), we aim at partially linearizing the nonlinear mechanical system. Hence, the transformation matrix is chosen to be in its upper triangular form (3), which not only simplifies the system model, but also reduces the computational complexity greatly when solving a set of partial differential equations (PDEs). Considering the system model (1), the dynamics of the new state z_i can be described by

$$\dot{z}_i = \left(\dot{T}_i(q_i)\dot{q}_i - T_i(q_i)M_i(q_i)^{-1}C_i(q_i,\dot{q}_i)\dot{q}_i\right)$$
$$+ T_i(q_i)M_i(q_i)^{-1}(\tau_i - G_i(q_i)). \quad (4)$$

Note that the matrix $T_i(q_i)$ is globally nonsingular as long as $^iT_{22}$ is not equal to zero. In order to determine $^iT_{22}$, substituting (3) into (4)

yields

$$\dot{z}_{i(2)} = \frac{\partial {}^i T_{22}}{\partial q_{i(1)}} \dot{q}_{i(1)} \dot{q}_{i(2)} + \frac{\partial {}^i T_{22}}{\partial q_{i(2)}} \dot{q}_{i(2)}^2$$

$$- \frac{{}^i T_{22} \, O_{i(2)} \sin(q_{i(2)})}{|M_i(q_i)| {}^i M_{11}} \left({}^i M_{11} \dot{q}_{i(1)} + {}^i M_{12} \dot{q}_{i(2)} \right)^2$$

$$+ \frac{{}^i T_{22}}{|M_i(q_i)|} O_{i(2)} \sin(q_{i(2)}) \frac{{}^i M_{12} \left({}^i M_{12} - {}^i M_{11} \right)}{{}^i M_{11}} \dot{q}_{i(2)}^2$$

$$+ {}^i T_{22} (M_i^{-1})_{21} u_{i(1)} + {}^i T_{22} (M_i^{-1})_{22} u_{i(2)}. \quad (5)$$

Here, for the purpose of removing the cross coupling term $\dot{q}_{i(1)} \dot{q}_{i(2)}$ in $\dot{z}_{i(2)}$, we let

$$\frac{\partial {}^i T_{22}}{\partial q_{i(1)}} \dot{q}_{i(1)} \dot{q}_{i(2)} + \frac{\partial {}^i T_{22}}{\partial q_{i(2)}} \dot{q}_{i(2)}^2$$

$$= \frac{{}^i T_{22}}{|M_i(q_i)|} O_{i(2)} \sin(q_{i(2)}) \frac{{}^i M_{12} \left({}^i M_{11} - {}^i M_{12} \right)}{{}^i M_{11}} \dot{q}_{i(2)}^2. \quad (6)$$

With (6), the dynamics of $\dot{z}_{i(2)}$ reduce to

$$\dot{z}_{i(2)} = - \frac{{}^i T_{22} \, O_{i(2)} \sin(q_{i(2)})}{|M_i(q_i)| {}^i M_{11}} \left({}^i M_{11} \dot{q}_{i(1)} + {}^i M_{12} \dot{q}_{i(2)} \right)^2$$

$$+ {}^i T_{22} (M_i^{-1})_{21} u_{i(1)} + {}^i T_{22} (M_i^{-1})_{22} u_{i(2)}. \quad (7)$$

One can check that one solution to (6) is

$$^i T_{22} = \sqrt{\frac{|M_i(q_i)|}{{}^i M_{11}}}. \quad (8)$$

So, the globally nonsingular transformation matrix $T_i(q_i)$ is obtained as follows:

$$T_i(q_i) = \begin{bmatrix} {}^i M_{11} & {}^i M_{12} \\ 0 & \sqrt{\frac{|M_i(q_i)|}{{}^i M_{11}}} \end{bmatrix}. \quad (9)$$

Consequently, the coordinate transformation (2) results in the partially linearized system with the state $[q_i^T, z_i^T]^T$, output y_i and input $u_i = \tau_i - G_i(q_i)$

$$\begin{cases} \dot{q}_i = A_i(q_i) z_i \\ \dot{z}_i = f_i(q_i, z_i) + D_i(q_i) u_i \\ y_i = q_i \end{cases} \quad (10)$$

where

$$A_i(q_i) = \begin{bmatrix} \frac{1}{{}^i M_{11}} & -\frac{{}^i M_{12}}{\sqrt{{}^i M_{11} |M_i(q_i)|}} \\ 0 & \sqrt{\frac{|M_i(q_i)|}{{}^i M_{11}}} \end{bmatrix}$$

$$D_i(q_i) = \begin{bmatrix} 1 & 0 \\ -\frac{{}^i M_{12}}{\sqrt{{}^i M_{11} |M_i(q_i)|}} & \sqrt{\frac{{}^i M_{11}}{|M_i(q_i)|}} \end{bmatrix}$$

and

$$f_i(q_i, z_i) = \left[0, -\frac{O_{i(2)} \sin(q_{i(2)})}{\sqrt{{}^i M_{11} |M_i(q_i)|} {}^i M_{11}} z_{i(1)}^2 \right]^T.$$

It can be seen that the quadratic cross terms of the unmeasurable velocities have been removed from the system dynamics (10). Moreover, the matrices A_i and D_i are both independent of the velocity states and bounded. Both of the above properties will facilitate the design of globally stable observers and controllers.

Remark 1: For future reference, denote

$$\delta_i(q_{i(2)}) \triangleq -\frac{O_{i(2)} \sin(q_{i(2)})}{\sqrt{{}^i M_{11} |M_i(q_i)|} {}^i M_{11}}. \quad (11)$$

It follows from the positive definiteness of the inertia matrix $M_i(q_i)$ that ${}^i M_{11} > 0$ and $\inf_t {}^i M_{11}(q_i(t)) = O_{i(1)} - 2O_{i(2)}$. Since $M_i(q_i)$ is bounded, we have

$$\sup_t \delta_i[q_{i(2)}(t)] = \frac{O_{i(2)}}{\sqrt{k_m}} (O_{i(1)} - 2O_{i(2)})^{2/3} \triangleq \bar{\delta}_i > 0. \quad (12)$$

Remark 2: The simplification of Euler–Lagrange systems was previously studied in [25] and [26], where the conditions for the existence of the transformation matrix $T_i(q_i)$ were presented based on the equation $\dot{T}_i(q_i) = T_i(q_i) M_i(q_i)^{-1} C_i(q_i, \dot{q}_i)$. However, for a class of Euler–Lagrange systems, such as the robot manipulators we discussed here and unicycle-type mobile robots [23], such a nonsingular matrix solution $T_i(q_i)$ does not exist. So, in this paper, a wide class of transformation matrices is derived from the relaxed equation, i.e., $\dot{T}_i(q_i)\dot{q}_i = T_i(q_i) M_i(q_i)^{-1} C_i(q_i, \dot{q}_i)\dot{q}_i$ resulted from (4).

Remark 3: It can be seen that the computation of the nonsingular coordinate transformation matrix (9) relies on the exact knowledge of the inertia parameters. When the parameter uncertainties are taken into account, the construction of robust adaptive controllers needs to be considered based on the parameter linearizability property of Euler–Lagrange systems.

C. Problem Formulation

Consider a group of n followers modeled by (1), and the leader labeled by 0 with the same dynamics as the followers. Hence, by employing (2), the leader's dynamics can also be transformed to (10) with the states (q_0, z_0). The distributed global output-feedback tracking problem is to design local control protocols u_i using only output information for all the followers, such that all the followers' states synchronize to the leader's state globally, i.e., $\lim_{t \to \infty} q_i(t) - q_0(t) = \mathbf{0}, i = 1, \ldots, n$.

III. OUTPUT-FEEDBACK TRACKING CONTROL

The purpose of this section is to present an observer-based control law to solve the distributed output-feedback tracking problem. Toward this end, we first design the observers to estimate the unmeasurable velocities.

A. Observer Design

Note that system (10) can be rewritten into the following two subsystems

$$\begin{cases} \dot{q}_{i(1)} = {}^i A_{11} z_{i(1)} + {}^i A_{12} z_{i(2)} \\ \dot{z}_{i(1)} = u_{i(1)} \end{cases} \quad (13)$$

$$\begin{cases} \dot{q}_{i(2)} = {}^i A_{22} z_{i(2)} \\ \dot{z}_{i(2)} = \delta_i z_{i(1)}^2 + {}^i D_{21} u_{i(1)} + {}^i D_{22} u_{i(2)}. \end{cases} \quad (14)$$

It can be observed from (13) that the dynamics of $q_{i(1)}$ incorporate the state $z_{i(2)}$ of the second subsystem. Similarly, the dynamics of $z_{i(2)}$ also depend on the state $z_{i(1)}$ of the first subsystem in (14). This implies that when we design the velocity observers for both of the subsystems, the convergence analysis of the observation errors for each subsystem is still related to each other, which makes it challenging to design globally stable observers. To handle this problem, motivated by [26], we aim at fully decoupling the subsystems by constructing the new

sates $x_i = [x_{i(1)}, x_{i(2)}]^T$ as follows:

$$x_{i(1)} = \int_0^{q_{i(1)}} {}^i M_{11}(q_{i(2)}) e^{-(q_{i(1)} - s)} ds + \int_0^{q_{i(1)}} {}^i M_{12}(s) ds$$

$$x_{i(2)} = \int_0^{q_{i(2)}} {}^i M_{12}(s) ds. \tag{15}$$

Combining (2), (9), (13), and (14) and taking derivative of (15), the dynamics of (x, z) are given by

$$\begin{cases} \dot{x}_{i(1)} = z_{i(1)} \\ \dot{z}_{i(1)} = u_{i(1)} \\ \dot{x}_{i(2)} = z_{i(2)} \\ \dot{z}_{i(2)} = \delta_i(q_{i(2)}) z_{i(1)}^2 + {}^i D_{21} u_{i(1)} + {}^i D_{22} u_{i(2)} \end{cases}, \tag{16}$$

in which the dynamics of the first subsystem $(x_{i(1)}, z_{i(1)})$ are independent of the second one $(x_{i(2)}, z_{i(2)})$. Consequently, it is relatively straightforward to design the observers for the two subsystems in (16). For the first subsystem, the observer is designed as

$$\begin{cases} \dot{\hat{x}}_{i(1)} = -k_{o,1}(\hat{x}_{i(1)} - x_{i(1)}) + \hat{z}_{i(1)} \\ \dot{\hat{z}}_{i(1)} = -k_{o,2}(\hat{x}_{i(1)} - x_{i(1)}) + u_{i(1)} \end{cases} \tag{17}$$

where $\hat{x}_i$ and $\hat{z}_i$ are the observations of x_i and z_i, respectively. Here, $k_{o,1}$ and $k_{o,2}$ are positive observer gains. Correspondingly, the observation errors are defined as $\tilde{x}_i = \hat{x}_i - x_i$ and $\tilde{z}_i = \hat{z}_i - z_i$, whose dynamics are of the form

$$\begin{bmatrix} \dot{\tilde{x}}_{i(1)} \\ \dot{\tilde{z}}_{i(1)} \end{bmatrix} = \begin{bmatrix} -k_{o,1} & 1 \\ -k_{o,2} & 0 \end{bmatrix} \begin{bmatrix} \tilde{x}_{i(1)} \\ \tilde{z}_{i(1)} \end{bmatrix} \triangleq \tilde{A} \begin{bmatrix} \tilde{x}_{i(1)} \\ \tilde{z}_{i(1)} \end{bmatrix}. \tag{18}$$

It can be easily checked that matrix $\tilde{A}$ is Hurwitz and, therefore, system (18) is exponentially stable at the origin. So

$$\lim_{t \to \infty} \begin{bmatrix} \tilde{x}_{i(1)}(t) \\ \tilde{z}_{i(1)}(t) \end{bmatrix} = 0. \tag{19}$$

For the second subsystem $(x_{i(2)}, z_{i(2)})$, the observer is constructed as

$$\begin{cases} \dot{\hat{x}}_{i(2)} = -k_{o,1}(\hat{x}_{i(2)} - x_{i(2)}) + \hat{z}_{i(2)} \\ \dot{\hat{z}}_{i(2)} = -k_{o,2}(\hat{x}_{i(2)} - x_{i(2)}) + \delta_i \hat{z}_{i(1)}^2 + {}^i D_{2j} u_{i(j)} \end{cases} \tag{20}$$

where $j = 1, 2$. In view of (16) and (20), we have

$$\begin{bmatrix} \dot{\tilde{x}}_{i(2)} \\ \dot{\tilde{z}}_{i(2)} \end{bmatrix} = \tilde{A} \begin{bmatrix} \tilde{x}_{i(2)} \\ \tilde{z}_{i(2)} \end{bmatrix} + \begin{bmatrix} 0 \\ h_i(t) \tilde{z}_{i(1)} \end{bmatrix} \tag{21}$$

where $h_i(t) = \delta_i(q_{i(2)}(t))(\tilde{z}_{i(1)} + 2z_{i(1)}(t))$ is continuous in t and $\tilde{z}_{i(1)}$, and locally Lipschitz in $\tilde{z}_{i(1)}$. Note that both (18) and the nominal part of (21) are uniformly globally exponentially stable (UGES). Then, the origin of the cascaded system (18) and (21) is UGES [27], namely, $\tilde{x}$ and $\tilde{z}$ uniformly globally exponentially converge to zero.

B. Observer-Based Control Law Design

The following assumptions are made throughout this paper.

Assumption 1: The leader's state information $(x_0(t), z_0(t))$ satisfies $\sup_t \|\dot{z}_0(t)\| \le \bar{z}_0$.

Assumption 2: The communication relationships among the $n + 1$ robots form a directed graph $\mathbb{G}$ that contains a spanning tree rooted at the leader.

In order to keep this paper self-contained, two lemmas are presented.

Lemma 1 (see [28]): Let $A \in \mathbb{R}^{m \times n}, B \in \mathbb{R}^{r \times s}, C \in \mathbb{R}^{n \times p}, D \in \mathbb{R}^{s \times t}$. Then

$$(A \otimes B)(C \otimes D) = AC \otimes BD \quad (\in \mathbb{R}^{mr \times pt}) \tag{22}$$

and, for all A and B

$$(A \otimes B)^T = A^T \otimes B^T. \tag{23}$$

Lemma 2 (see [29], [30]): Under Assumption 2, $(L + B)$ is a nonsingular M-matrix. Define

$$H = [h_1, \dots, h_n]^T = (L + B)^{-1} \mathbf{1}_n$$

$$P = \text{diag}\{p_i\} = \text{diag}\{1/h_i\}. \tag{24}$$

Then, P is positive definite and the matrix Q defined as

$$Q = P(L + B) + (L + B)^T P \tag{25}$$

is also positive definite.

To come up with the observer-based distributed control laws, an auxiliary variable is introduced as follows:

$$\xi_i = \hat{z}_i - z_0 + \kappa(x_i - x_0) \tag{26}$$

where $\kappa > 1$ is a constant. The local differences are defined as

$$\begin{cases} x_{ir} = \sum_{j \in \mathcal{N}_i} a_{ij}(x_i - x_j) + b_i(x_i - x_0) \\ z_{ir} = \sum_{j \in \mathcal{N}_i} a_{ij}(\hat{z}_i - \hat{z}_j) + b_i(\hat{z}_i - z_0) \end{cases} \tag{27}$$

and

$$s_i = z_{ir} + \kappa x_{ir}. \tag{28}$$

The auxiliary variable s_i can be written into a compact form

$$s = ((L + B) \otimes I_2) \xi. \tag{29}$$

The distributed control law for robot i is proposed as follows:

$$u_i = D_i^{-1}(q_i)(k_{c,1} \tilde{x}_i - k_{c,2} s_i - k_{c,3} \text{sign}(s_i) - f_i(q_i, \hat{z}_i)), \tag{30}$$

where $\text{sign}(s_i) = [\text{sign}(s_{i(1)}), \text{sign}(s_{i(2)})]^T \in \mathbb{R}^2$. Here, $k_{c,1}$ can be any positive number, and $k_{c,2}$ and $k_{c,3}$ are positive numbers satisfying

$$\begin{cases} k_{c,2} > \dfrac{1}{\underline{\lambda}_Q}\left(3\kappa\bar{\lambda}_P + \kappa^2\bar{\lambda}_P + \dfrac{\kappa^2\bar{\lambda}_P}{2(\kappa - 1)} + |k_{c,1} - k_{o,2}|\bar{\lambda}_P\bar{\sigma}_{(L+B)}\right) \\ k_{c,3} > \bar{z}_0 \end{cases} \tag{31}$$

where the real symmetric matrices P and Q are defined in (24) and (25), respectively.

IV. MAIN RESULTS

The main result of this paper is given below.

Theorem 1: Under Assumptions 1 and 2, consider the system (16) transformed from the mechanical system (1) in closed loop with the observer-based controllers given by (30). Then, the origin of the closed-loop system is UGES for the control gains satisfying (31) and any positive observer gains $k_{o,1}$ and $k_{o,2}$.

Proof of Theorem 1: The Lyapunov function candidate is chosen as

$$V_c = \frac{1}{2} s^T (P \otimes I_2) s + \frac{\varpi}{2} x_r^T x_r$$

$$= \frac{1}{2} \begin{bmatrix} s \\ x_r \end{bmatrix}^T \begin{bmatrix} P \otimes I_2 & 0 \\ 0 & \varpi I_{2n} \end{bmatrix} \begin{bmatrix} s \\ x_r \end{bmatrix}$$

$$\triangleq \frac{1}{2} y^T \bar{P} y \tag{32}$$

where $y \triangleq [s^T, x_r^T]^T$, and ϖ is a positive scalar satisfying $\varpi > \kappa^2 \bar{\lambda}_P / 2(\kappa - 1)$. It is straightforward to check that matrix $\bar{P}$ is positive definite and

$$\frac{\underline{\lambda}_{\bar{P}}}{2} \|y\|^2 \le V_c(y) \le \frac{\bar{\lambda}_{\bar{P}}}{2} \|y\|^2. \tag{33}$$

The generalized derivative of V_c (see [31, Remark 3.7]) is given by

$$\dot{V}_c = s^T (P \otimes I_2)((L + B) \otimes I_2)[\dot{\tilde{z}} - (\mathbf{1}_n \otimes \dot{z}_0)$$
$$+ \kappa z - \kappa(\mathbf{1}_n \otimes z_0)] + \varpi x_r^T (s - \kappa x_r - \tilde{z}_r) \quad (34)$$

where $\tilde{z}_r = ((L + B) \otimes I_2)\tilde{z}$. Note that from (17) and (20), we know

$$\dot{\tilde{z}} = -k_{o,2}\tilde{x} + f(q, \hat{z}) + D(q)u. \quad (35)$$

Also, the control input (30) can be written in its stacked form as

$$u = D(q)^{-1} (k_{c,1}\tilde{x} - k_{c,2}s - k_{c,3}\text{sign}(s) - f(q, \hat{z})) \quad (36)$$

where $D(q)^{-1} = \text{blockdiag}\{D_1^{-1}, \ldots, D_n^{-1}\} \in \mathbb{R}^{2n \times 2n}$ and $\text{sign}(s) = [\text{sign}(s_1)^T, \ldots, \text{sign}(s_n)^T]^T \in \mathbb{R}^{2n}$. Substituting (35) and (36) into (34), we get

$$\dot{V}_c = - k_{c,2}s^T (P(L + B) \otimes I_2)s - \varpi \kappa x_r^T x_r$$
$$+ (k_{c,1} - k_{o,2})s^T (P(L + B) \otimes I_2)\tilde{x}$$
$$+ \kappa s^T (P(L + B) \otimes I_2)(z - \hat{z} + \hat{z} - \mathbf{1}_n \otimes z_0)$$
$$+ \varpi x_r^T s - \varpi x_r^T \tilde{z}_r - k_{c,3}s^T (PL \otimes I_2)\text{sign}(s)$$
$$- k_{c,3}s^T (PB \otimes I_2)\text{sign}(s) + s^T (PB \otimes I_2)(\mathbf{1}_n \otimes \dot{z}_0) \quad (37)$$

where Lemma 1 and the equality that $(L \otimes I_2)(\mathbf{1}_n \otimes \dot{z}_0) = \mathbf{0}$ have been used.

Note that

$$z - \hat{z} = -\tilde{z} = -((L + B) \otimes I_2)^{-1}\tilde{z}_r \quad (38)$$

and

$$\hat{z} - \mathbf{1}_n \otimes z_0 = ((L + B) \otimes I_2)^{-1}z_r = ((L + B) \otimes I_2)^{-1}(s - \kappa x_r). \quad (39)$$

Then, substituting (38) and (39) into (37) yields

$$\dot{V}_c = - k_{c,2}s^T (P(L + B) \otimes I_2)s - \varpi \kappa x_r^T x_r$$
$$- \kappa s^T (P(L + B) \otimes I_2)((L + B) \otimes I_2)^{-1}\tilde{z}_r$$
$$+ \kappa s^T (P(L + B) \otimes I_2)((L + B) \otimes I_2)^{-1}(s - \kappa x_r)$$
$$+ (k_{c,1} - k_{o,2})s^T (P(L + B) \otimes I_2)\tilde{x} + \varpi x_r^T s - \varpi x_r^T \tilde{z}_r$$
$$- k_{c,3}s^T (PL \otimes I_2)\text{sign}(s) - k_{c,3}s^T (PB \otimes I_2)\text{sign}(s)$$
$$+ s^T (PB \otimes I_2)(\mathbf{1}_n \otimes \dot{z}_0). \quad (40)$$

Therefore

$$\dot{V}_c = - k_{c,2}s^T (P(L + B) \otimes I_2)s - \varpi \kappa x_r^T x_r - \kappa s^T (P \otimes I_2)\tilde{z}_r$$
$$+ \kappa s^T (P \otimes I_2)(s - \kappa x_r) + \varpi x_r^T s - \varpi x_r^T \tilde{z}_r$$
$$+ (k_{c,1} - k_{o,2})s^T (P(L + B) \otimes I_2)\tilde{x}$$
$$- k_{c,3}s^T (PL \otimes I_2)\text{sign}(s) - k_{c,3}s^T (PB \otimes I_2)\text{sign}(s)$$
$$+ s^T (PB \otimes I_2)(\mathbf{1}_n \otimes \dot{z}_0)$$
$$= - \frac{k_{c,2}}{2}s^T (Q \otimes I_2)s - \varpi \kappa x_r^T x_r + \kappa s^T (P \otimes I_2)s$$
$$- \kappa s^T (P \otimes I_2)\tilde{z}_r - \kappa^2 s^T (P \otimes I_2)x_r + \varpi x_r^T s - \varpi x_r^T \tilde{z}_r$$
$$+ (k_{c,1} - k_{o,2})s^T (P(L + B) \otimes I_2)\tilde{x}$$
$$- k_{c,3}s^T (PL \otimes I_2)\text{sign}(s) - k_{c,3}s^T (PB \otimes I_2)\text{sign}(s)$$
$$+ s^T (PB \otimes I_2)(\mathbf{1}_n \otimes \dot{z}_0). \quad (41)$$

From Lemma 2, we know matrix Q and P are positive definite. Then, it follows

$$\dot{V}_c \leq - \frac{k_{c,2}}{2}\underline{\lambda}_Q \|s\|^2 - \varpi \kappa \|x_r\|^2 + \kappa \overline{\lambda}_P \|s\|^2$$
$$+ \frac{\kappa}{2}\overline{\lambda}_P (\|s\|^2 + \|\tilde{z}_r\|^2) + \frac{\kappa^2}{2}\overline{\lambda}_P (\|s\|^2 + \|x_r\|^2)$$
$$+ \frac{\varpi}{2}(\|s\|^2 + \|x_r\|^2) + \frac{\varpi}{2}(\|\tilde{z}_r\|^2 + \|x_r\|^2)$$
$$+ \frac{|k_{c,1} - k_{o,2}|}{2}\overline{\lambda}_P \overline{\sigma}_{(L+B)}(\|s\|^2 + \|\tilde{x}\|^2)$$
$$- k_{c,3}\sum_{i=1}^{n} p_i b_i \|s_i\|_1 + \sum_{i=1}^{n} p_i b_i \bar{z}_0 \|s_i\|$$
$$\leq - \alpha_1 \|s\|^2 - \alpha_2 \|x_r\|^2 + \alpha_3 \|\tilde{z}_r\|^2 + \alpha_4 \|\tilde{x}\|^2 \quad (42)$$

where $\alpha_i, i = 1, \ldots, 4$, are given by

$$\alpha_1 = \frac{1}{2}\left(k_{c,2}\underline{\lambda}_Q - 3\kappa\overline{\lambda}_P - \kappa^2\overline{\lambda}_P - \varpi - |k_{c,1} - k_{o,2}|\overline{\lambda}_P \overline{\sigma}_{(L+B)}\right)$$

$$\alpha_2 = \varpi \kappa - \varpi - \frac{\kappa^2}{2}\overline{\lambda}_P$$

$$\alpha_3 = \frac{\kappa}{2}\overline{\lambda}_P + \frac{\varpi}{2}$$

$$\alpha_4 = \frac{|k_{c,1} - k_{o,2}|}{2}\overline{\lambda}_P \overline{\sigma}_{(L+B)}. \quad (43)$$

$\overline{\sigma}_{(X)}$ represents the largest singular value of matrix X, and here we have used the facts that $s^T (PL \otimes I_2)\text{sign}(s) \geq 0$ and $\|s_i\|_1 \geq \|s_i\|$. Under the condition (31) and the constraint for ϖ in (32), the parameters $\alpha_i > 0, i = 1, \ldots, 4$. Hence, $\dot{V}_c$ satisfies

$$\dot{V}_c \leq - \min\{\alpha_1, \alpha_2\}\|y\|^2 + \alpha_3 \|\tilde{z}_r\|^2 + \alpha_4 \|\tilde{x}\|^2. \quad (44)$$

Combining (33) and (44), we get

$$\dot{V}_c(y) \leq -\frac{2\min\{\alpha_1, \alpha_2\}}{\overline{\lambda}_{\bar{P}}}V_c(y) + \alpha_3 \|\tilde{z}_r\|^2 + \alpha_4 \|\tilde{x}\|^2. \quad (45)$$

Recall that $\|\tilde{z}\|$ and $\|\tilde{x}\|$ converge to zero exponentially from Section III-A, so does $\|\tilde{z}_r\|$ due to the fact that $\tilde{z}_r = ((L + B) \otimes I_2)\tilde{z}$. Then from the Converse Theorem [32], there exist a function $V_o(t, \tilde{z}_r, \tilde{x})$ that satisfies the inequalities

$$\dot{V}_o \leq -\beta(\|\tilde{z}_r\|^2 + \|\tilde{x}\|^2) \quad (46)$$

where β is a positive constant. Then, we choose the overall Lyapunov function candidate as

$$V = V_c + \varrho V_o \quad (47)$$

where ϱ is a positive constant satisfying $\varrho > \max\{\frac{\alpha_3}{\beta}, \frac{\alpha_4}{\beta}\}$. For simplicity, we use $\bar{\alpha} \triangleq 2\min\{\alpha_1, \alpha_2\}/\overline{\lambda}_{\bar{P}}$. Taking the time derivative of both sides of (47), and combining (45) and (46), we have

$$\dot{V} \leq -\bar{\alpha}V_c - \max\{(\beta\varrho - \alpha_3), (\beta\varrho - \alpha_4)\}(\|\tilde{z}_r\|^2 + \|\tilde{x}\|^2). \quad (48)$$

Denoting $\bar{\beta} \triangleq \max\{(\beta\varrho - \alpha_3), (\beta\varrho - \alpha_4)\}$, one has

$$\dot{V} \leq \begin{cases} -\bar{\alpha}V - (\bar{\beta} - \bar{\alpha}\varrho)V_o, & \text{if } \bar{\beta} > \bar{\alpha}\varrho \\ -\frac{\bar{\beta}}{\varrho}V - (\bar{\alpha} - \frac{\bar{\beta}}{\varrho})V_c, & \text{if } \bar{\beta} \leq \bar{\alpha}\varrho. \end{cases} \quad (49)$$

Consequently, we get

$$\dot{V} \leq - \min\{\bar{\alpha}, \bar{\beta}/\varrho\}V. \quad (50)$$

It then follows from [32, Th 4.10] that $[s^T, x_r^T]^T = \mathbf{0}$ is UGES, which implies x_{ir} and z_{ir} converge to $\mathbf{0}$ according to the definitions (27) and
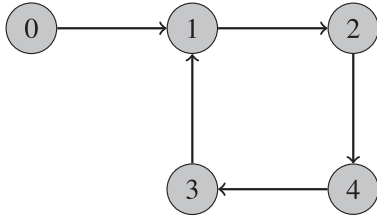
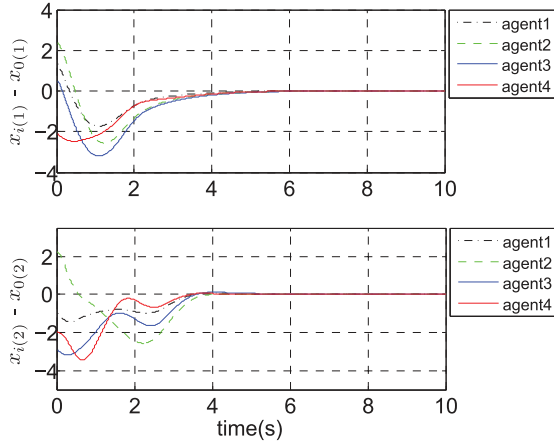Fig. 1.   Interaction topology, where agent 0 is the leader.

Fig. 2.   Position tracking errors for each agent.

Fig. 3.   Velocity tracking errors for each agent.

Fig. 4.   Velocity observation errors for each agent.

(28). By invoking [1, Th. 4] and $\lim_{t \to \infty} \hat{z} = z$, we know z_i and x_i, respectively, converge to z_0 and x_0 in the sense of uniform global exponential stability. This implies $q_i(t)$ exponentially globally converges to $q_0(t)$ due to the same integrand in definition (15).

Till now, it has been proved that the observation errors uniformly globally exponentially converge to zero for any positive observer gains $k_{o,1}$ and $k_{o,2}$, and the origin of the closed-loop system is UGES for the control gains satisfying (31). In addition, the global convergence of the tracking errors can still be guaranteed if the observer states $(\hat{x}, \hat{z})$ are replaced by the real states (x, z) in controller (30). These imply that the observer and the controller can be designed separately, namely the separation principle holds. ∎

Remark 4: In comparison with the $I\&I$ technique reported in [8]–[10], our proposed approach can reduce the complexity of the observer and the distributed controller design. Moreover, the theoretical analysis can also be more easily carried out based on the resulted partially linearized cascaded system. The method to partially linearize the class of Euler–Lagrange system can be applied to deal with the systems of n DOFs, $n > 2$, by setting the transformation matrix in a particular upper triangular form. However, to solve a set of PDEs with high order is a challenging, yet interesting issue for future research.

Remark 5: To eliminate the chattering behavior caused by the signum function, in practice, various continuous functions such as the hyperbolic tangent function and the saturation function [33] have been employed to approximate the discontinuous signum function. Although the damage exerted on the actuator caused by discontinuity could be avoided, the states of the closed-loop system might only be stabilized within a bounded neighborhood of the equilibrium, instead of converging to the equilibrium.

V.   SIMULATIONS

To validate the theoretical results derived in the preceding sections, we shall consider four robot manipulators modeled by (1),
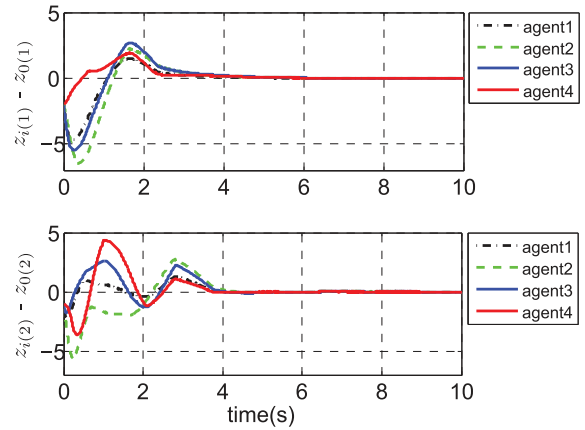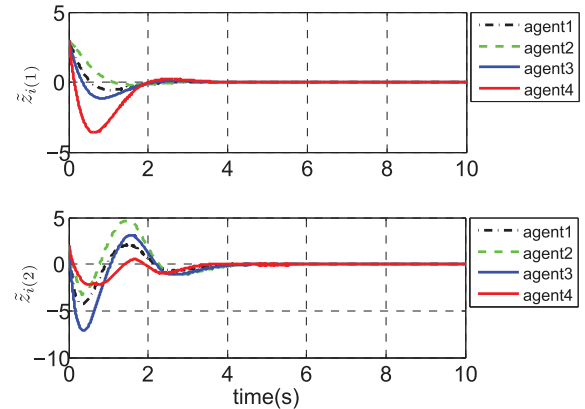
with the physical parameters taken from [11] as $[m_1, m_2, l_1, l_2, l_{c1}, l_{c2}, J_1, J_2, g] = [0.5, 0.4, 0.4, 0.3, 0.2, 0.15, 0.0067, 0.003, 9.8]$. The communication topology among the four followers and the leader is represented by Fig. 1.

The initial values of $x_i(t)$ and $\hat{z}_i(t), i = 1, \ldots, 4$, are set as $3 * \text{rands}(2, 1)$ and $[0, 0]^T$, respectively, and the leader's trajectory is $[2 * t, \sin(t)]^T$, satisfying Assumption 1. The following parameters are used in the simulation. Observer parameters: $k_{o,1} = 3, k_{o,2} = 5$, control gains: $k_{c,1} = 5, k_{c,2} = 6, k_{c,3} = 3$. By employing the distributed control laws (30), the numerical simulation results are shown in Figs. 2 and 4. It can be seen from Figs. 2 and 3 that the tracking errors for each event converge to zero. Fig. 4 indicates that each agent can precisely observe the unmeasurable velocities using the proposed observer (17) and (20).

VI.   CONCLUSION

In this paper, for a class of Euler–Lagrange systems that cannot be fully linearized by output-feedback, we have constructed a nonsingular coordinate transformation matrix to partially linearize the nonlinear systems. Then, observers have been designed to overcome the unavailability of the velocity measurements. We have also proposed observer-based control laws by output-feedback such that the followers uniformly globally exponentially track the leader. It should be noted that the system discussed here is fully actuated. Future research directions may include distributed global output-feedback control for a class of underactuated Euler–Lagrange systems, such as

nonholonomic wheeled mobile robots studied in [23]. We are also interested in distributed observerless global output-feedback control for Euler–Lagrange systems with parameter uncertainties reported recently in [34].

REFERENCES

[1] S. Khoo, L. Xie, and Z. Man, "Robust finite-time consensus tracking algorithm for multirobot systems," *IEEE Trans. Mechatronics*, vol. 14, no. 2, pp. 219–228, Apr. 2009.

[2] J. Mei, W. Ren, and G. Ma, "Distributed coordinated tracking with a dynamic leader for multiple Euler–Lagrange systems," *IEEE Trans. Autom. Control*, vol. 56, no. 6, pp. 1415–1421, Jun. 2011.

[3] Z. Yang, Y. Shibuya, and P. Qin, "Distributed robust control for synchronised tracking of networked Euler–Lagrange systems," *Int. J. Syst. Sci.*, vol. 46, no. 4, pp. 720–732, 2015.

[4] F. Chen, G. Feng, L. Liu, and W. Ren, "Distributed average tracking of networked Euler–Lagrange systems," *IEEE Trans. Autom. Control*, vol. 60, no. 2, pp. 547–552, Feb. 2015.

[5] E. Nuno, R. Ortega, L. Basanez, and D. Hill, "Synchronization of networks of nonidentical Euler–Lagrange systems with uncertain parameters and communication delays," *IEEE Trans. Autom. Control*, vol. 56, no. 4, pp. 935–941, Apr. 2011.

[6] H. Wang, "Passivity based synchronization for networked robotic systems with uncertain kinematics and dynamics," *Automatica*, vol. 49, no. 3, pp. 755–761, 2013.

[7] H. Cai and J. Huang, "Leader-following consensus of multiple uncertain Euler-Lagrange systems under switching network topology," *Int. J. General Syst.*, vol. 43, no. 3/4, pp. 294–304, 2014.

[8] A. Astolfi, R. Ortega, and A. Venkatraman, "A globally exponentially convergent immersion and invariance speed observer for mechanical systems with non-holonomic constraints," *Automatica*, vol. 46, no. 1, pp. 182–189, 2010.

[9] Ø. N. Stamnes, O. M. Aamo, and G. O. Kaasa, "A constructive speed observer design for general Euler–Lagrange systems," *Automatica*, vol. 47, no. 10, pp. 2233–2238, 2011.

[10] I. Sarras, E. Nuño, M. Kinnaert, and L. Basañez, "Output-feedback control of nonlinear bilateral teleoperators," in *Proc. Amer. Control Conf.*, pp. 3490–3495, 2012.

[11] W. Ren, "Distributed leaderless consensus algorithms for networked Euler–Lagrange systems," *Int. J. Control*, vol. 82, no. 11, pp. 2137–2149, 2009.

[12] J. Mei, W. Ren, J. Chen, and G. Ma, "Distributed adaptive coordination for multiple Lagrangian systems under a directed graph without using neighbors' velocity information," *Automatica*, vol. 49, no. 6, pp. 1723–1731, 2013.

[13] Y. Zhao, Z. Duan, and G. Wen, "Distributed finite-time tracking of multiple Euler–Lagrange systems without velocity measurements," *Int. J. Robust Nonlinear Control*, vol. 25, pp. 1688–1703, 2015.

[14] Z. Meng, D. V. Dimarogonas, and K. H. Johansson, "Leader-follower coordinated tracking of multiple heterogeneous Lagrange systems using continuous control," *IEEE Trans. Robot.*, vol. 30, no. 3, pp. 739–745, Jun. 2014.

[15] G. Chen, "Cooperative controller design for synchronization of networked uncertain Euler–Lagrange systems," *Int. J. Robust Nonlinear Control*, vol. 25, no. 11, pp. 1721–1738, 2014.

[16] Q. Yang, H. Fang, J. Chen, and X. Wang, "Distributed observer-based coordination for multiple Lagrangian systems using only position measurements," *IET Control Theory Appl.*, vol. 8, no. 17, pp. 2102–2114, Nov. 2014.

[17] T. Liu and Z. Jiang, "Distributed output-feedback control of nonliear multi-agent systems," *IEEE Trans. Autom. Control*, vol. 58, no. 11, pp. 2912–2917, Nov. 2013.

[18] Y. D. Ang and J. Huang, "Cooperative global output regulation for a class of nonlinear multi-agent systems," *IEEE Trans. Autom. Control*, vol. 59, no. 5, pp. 1348–1354, May 2014.

[19] Y. Hong, G. Chen, and L. Bushnell, "Distributed observers design for leader-following control of multi-agent networks," *Automatica*, vol. 44, no. 3, pp. 846–850, 2008.

[20] G. Besançon, "Global output feedback tracking control for a class of lagrangian systems," *Automatica*, vol. 36, no. 12, pp. 1915–1921, 2000.

[21] I. Sarras, R. Ortega, and E. Panteley, "Asymptotic stabilization of nonlinear systems via sign-indefinite damping injection," in *Proc. 51st IEEE Conf. Decision. Control*, pp. 2964–2969, 2012.

[22] Z. Jiang and I. Kanellakopoulos, "Global output-feedback tracking for a benchmark nonlinear system," *IEEE Trans. Autom. Control*, vol. 45, no. 5, pp. 1023–1027, May 2000.

[23] K. Do, Z. Jiang, and J. Pan, "A global output-feedback controller for simultaneous tracking and stabilization of unicycle-type mobile robots," *IEEE Trans. Robot. Autom.*, vol. 20, no. 3, pp. 589–594, Jun. 2004.

[24] M. W. Spong, S. Hutchinson, and M. Vidyasagar, *Robot Modeling and Control*, vol. 3. New York, NY, USA: Wiley, 2006.

[25] M. Mabrouk, S. Ammar, and J. C. Vivalda, "Transformation synthesis for Euler–Lagrange systems," *Nonlinear Dyn. Syst. Theory*, vol. 7, no. 2, pp. 197–216, 2007.

[26] M. Mabrouk, "Triangular form for Euler–Lagrange systems with application to the global output tracking control," *Nonlinear Dyn.*, vol. 60, no. 1/2, pp. 87–98, 2010.

[27] E. Panteley, E. Lefeber, A. Loria, and H. Nijmeijer, "Exponential tracking control of a mobile car using a cascaded approach," in *Proc. IFAC Workshop Motion Control*. Grenoble, France, pp. 221–226, 1998.

[28] A. J. Laub, *Matrix Analysis for Scientists and Engineers*. Philadelphia, PA, USA: SIAM, 2005.

[29] Z. Qu, *Cooperative Control of Dynamical Systems: Applications to Autonomous Vehicles*. Berlin, Germany: Springer Science & Business Media, 2009.

[30] H. Zhang and F. L. Lewis, "Adaptive cooperative tracking control of higher-order nonlinear systems with unknown dynamics," *Automatica*, vol. 48, no. 7, pp. 1432–1439, 2012.

[31] S. Ghapani, J. Mei, W. Ren, and Y. Song, "Fully distributed flocking with amoving leader for Lagrange networks with parametric uncertainties," *Automatica*, vol. 67, no. 5, pp. 67–76, 2016.

[32] H. K. Khalil, *Nonlinear Systems*, 3rd ed. Upper Saddle River, NJ, USA: Prentice-Hall, 2002.

[33] J. J. E. Slotine and W. Li, *Applied Nonlinear Control*, vol. 199. Englewood Cliffs, NJ, USA: Prentice-Hall, 1991.

[34] A. Loria, "Observers are unnecessary for output-feedback control of lagrangian systems," *IEEE Trans. Automat. Control*, vol. 61, no. 4, pp. 905–920, Apr. 2016.