# Distributed Interference Alignment and Power Control for Wireless MIMO Interference Networks with Noisy Channel State Information

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***Abstract*—This paper considers a multi-input multi-output (MIMO) interference network in which each transmitter intends to communicate with its dedicated receiver at a certain fixed rate. It is known that when perfect CSI is available at each terminal, the interference alignment technique can be applied, to align the interference signals at each receivers in a subspace independent of the desired signal subspace. The impact of interference can hence be eliminated. In practice, however, terminals in general can acquire only noisy CSI. Interference alignment cannot be perfectly performed to avoid interference leakage in the signal subspace. Thus, the quality of each communication link depends on the transmission power of the unintended transmitters. To solve this problem, we propose an iterative algorithm to perform stochastic power control and transceiver design based on only noisy local CSI. The transceiver design is conducted based on the interference alignment concept, and the power control seeks solutions of efficiently assigning transmit powers to provide successful communications for all transmitter-receiver pairs.**

## I. Introduction

A wireless *interference network* refers to a communication network in which multiple source-destination pairs share the radio spectrum. It is a model for a large class of wireless communication systems including cellular communication networks. The design of transmission schemes for such networks, hence, has a broad range of possible applications. In an interference network, the reception at each destination can be interfered by the transmitted signals of unintended sources which potentially degrades network's performance. Therefore, a proper interference management solution is required. Conventional interference management techniques (e.g. time-division-multiple-access, TDMA, or frequency-division-multiple-access, FDMA) tend to orthogonalize the transmissions of different source-destination pairs. This leads to the fact that at each destination the subspaces of different interference signals are orthogonal to that of the desired signal and also orthogonal to each other. Interference is avoided at the cost of low spectral efficiency.

The *interference alignment* concept [1], [2], however, reveals that with proper transmission design, different interference signals at each destination can be aligned together, such that more radio resources can be assigned to the desired transmission. For instance, consider a multiple-input multiple-output (MIMO) interference network with more than two source-destination pairs. In certain cases, the sources can perform linear beamforming to send their signals simultaneously in such a way that at each destination interference signals are aligned together to span only half of the available signal space. Thus, the interference can be completely eliminated with simple linear zero-forcing filters [2]. At high-SNR regime, each source-destination pair can potentially attain half of its interference-free achievable transmission rate.

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The solution for interference alignment proposed in [2] requires the global channel state information (CSI) to be perfectly known at all terminals. This is a challenging problem in practice. In most cases, it is more convenient for each terminal to obtain local CSI (i.e. the CSI of the channels directly connected to the terminal). An iterative algorithm for distributed interference alignment in such a situation has been proposed in [3], and its implementation on a hardware test-bed has been reported in [4]. To deploy the transceiver design algorithm proposed in [3], an adaptive coding and modulation is required to adapt the transmission system to channel variations. This increases the system complexity. Also, in certain delay-sensitive applications such as network control systems, and voice and video communication systems, it is desired to ensure data transmission at certain fixed rates [5]. Therefore, power control (see e.g. [6]–[10]) is required to efficiently use available resources and provide the demanded communication quality. An iterative algorithm for power control and interference alignment based on perfect local CSI has been proposed in [11], and its convergence has been shown.

One may ask whether it is possible to design transceiver and perform power control when only noisy local CSI is available at each terminal. We address this issue in this paper. Specifically, a stochastic power control and interference alignment is applied in MIMO interference networks. We propose an iterative algorithm which jointly updates transceiver filters and power control solutions, to provide successful communications for all transmitter-receiver pairs. The convergence of the algorithm is shown via both theoretic proof and computer simulations.

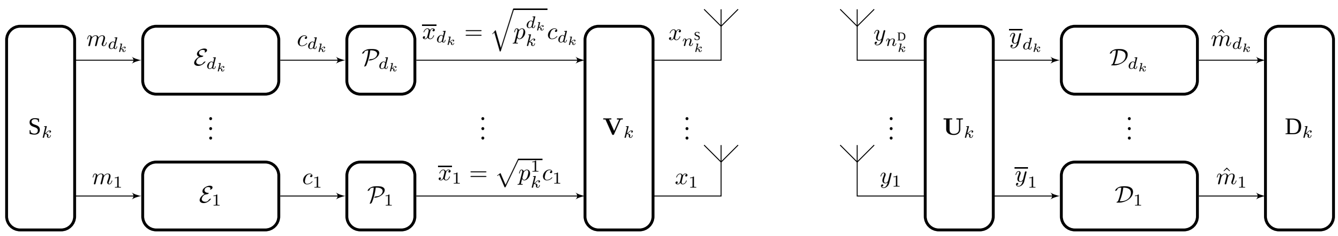


Fig. 1: The structure of a transmitter-receiver pair.

## II. System Model

Consider a MIMO interference network with $K$ sources and $K$ destinations in which each source intends to communicate to the corresponding destination. We denote the sources as $\mathrm{S}_k$ and the destinations as $\mathrm{D}_k$ $(k \in \{1, 2, ..., K\})$. $\mathrm{S}_k$ and $\mathrm{D}_k$ are equipped with $n_k^{\mathrm{S}}$ and $n_k^{\mathrm{D}}$ antennas, respectively. The architecture of one transmitter and receiver pair is shown in Fig. 1. The source $\mathrm{S}_k$ sends $d_k$ independent messages $m_d$ $(d \in \{1, ..., d_k\})$. The encoder $\mathcal{E}_k$ encodes $m_d$ to a unit-power codeword $c_d$ chosen from a Gaussian codebook. The power controller $\mathcal{P}_d$ scales this codeword to $\overline{x}_d = \sqrt{p_k^d} c_d$ where $p_k^d$ is the power of the transmitted signal. The $d_k \times 1$ vector $\overline{\mathbf{x}}_k$ denotes these scaled codewords. Let $\mathbf{V}_k$ be an $n_k^{\mathrm{S}} \times d_k$ beamforming matrix with orthogonal column vectors $\mathbf{v}_k^d$ $(d \in \{1, ..., d_k\})$. The transmitted signal of $\mathrm{S}_k$ is

$$\mathbf{x}_k = \mathbf{V}_k \overline{\mathbf{x}}_k. \tag{1}$$

Let $\mathbf{U}_k$ denote an $n_k^{\mathrm{D}} \times d_k$ receiver filtering matrix with orthogonal column vectors $\mathbf{u}_k^d$ $(d \in \{1, ..., d_k\})$. The filter output of $\mathrm{D}_k$ is

$$\overline{\mathbf{y}}_k = \mathbf{U}_k^* \mathbf{H}_{kk} \mathbf{V}_k \overline{\mathbf{x}}_k + \sum_{l=1, l \neq k}^{K} \mathbf{U}_k^* \mathbf{H}_{kl} \mathbf{V}_l \overline{\mathbf{x}}_l + \mathbf{U}_k^* \mathbf{z}_k, \tag{2}$$

where $\mathbf{U}_k^*$ denotes the conjugate transpose of matrix $\mathbf{U}_k$; $\mathbf{H}_{kl}$ is the channel matrix from $\mathrm{S}_l$ to $\mathrm{D}_k$; $\mathbf{z}_k$ is zero mean complex Gaussian noise, i.e. $\mathbf{z}_k \sim \mathcal{CN}(\mathbf{0}, N_0 \mathbf{I}_{n_k^{\mathrm{D}}})$ in which $N_0$ is the noise power and $\mathbf{I}_{n_k^{\mathrm{D}}}$ is the $n_k^{\mathrm{D}} \times n_k^{\mathrm{D}}$ identity matrix. The decoder $\mathcal{D}_l$ $(l \in \{1, ..., d_k\})$ decodes received signal $\overline{y}_l$ to a message $\hat{m}_l$. In the considered network, it is desired to design $\mathbf{V}_k$, $\mathbf{U}_k$, and $p_k^d$ such that, for all realizations of channel matrices, each source $\mathrm{S}_k$ be able to communicate to the intended destination $\mathrm{D}_k$ at a specific rate $R_k$.

### A. Channel State Information

We assume that $\mathrm{D}_k$ $(k \in \{1, 2, ..., K\})$ perfectly knows direct channel $\mathbf{H}_{kk}$, however, it knows only noisy version of the interference channels $\widehat{\mathbf{H}}_{kl}$ $(l \in \{1, 2, ..., K\}, l \neq k)$ which follows the following model

$$\widehat{\mathbf{H}}_{kl} = \mathbf{H}_{kl} + \mathbf{E}_{kl}, \ k \neq l, \tag{3}$$

where $\mathbf{E}_{kj} \sim \mathcal{CN}(\mathbf{0}, \sigma_e^2 \mathbf{I}_{n_D})$ is the channel estimation error matrix. This model is motivated by the fact that a linear estimation of Gaussian variables induces a Gaussian distributed estimation error. The parameter $\sigma_e^2$ indicates the accuracy of the channel estimation. For instance, $\sigma_e^2 = 0$ is corresponding to the case that perfect CSI is available. In general, the channel matrices corresponding to different links may have different accuracy, however, in this work for the sake of simplicity we assume that their accuracy are the same.

We assume that reciprocity holds, i.e. $\mathbf{H}_{kl}^* = \mathbf{H}_{lk}^r$, where $\mathbf{H}_{lk}^r$ is the channel matrix from $\mathrm{D}_k$ to $\mathrm{S}_l$. Therefore, each transmitter can estimate its corresponding channels from the training sequences transmitted by destinations.

## III. Distributed interference alignment

The beamforming mentioned in Section II can be performed such that, at each destination, interference signals are aligned in the same subspace which is distinct from the desired signal subspace. Consequently, the desired signals can be recovered by eliminating the interference with proper filtering [2]. For general MIMO interference networks, interference alignment may not be always feasible. In the considered network, if $(d_1, ..., d_K)$, are carefully chosen such that interference alignment is feasible, then there exist transmitter beamforming and receiver filtering matrices that satisfy the following conditions:

$$\begin{aligned} \mathbf{U}_k^* \mathbf{H}_{kj} \mathbf{V}_j &= \mathbf{0}, \ \forall j \neq k: \ j, k \in \{1, ..., K\}, \\ \operatorname{rank}\left(\mathbf{U}_k^* \mathbf{H}_{kk} \mathbf{V}_k\right) &= d_k, \ \forall k \in \{1, ..., K\}. \end{aligned} \tag{4}$$

In general, perfect global CSI is required at all terminals to find a solution of this problem. An iterative optimization of the transmitter beamforming and the receiver filtering matrices is proposed in [3] which demands only local CSI at each terminal. Applying this method incurs some interference to be leaked to the desired signal subspace at each destination. The receiver filter can be designed such that the power of the leakage interference be minimized. At time index $n \in \mathbb{N}$, using $\mathbf{U}_k(n)$ to denote the receiver filtering matrix, the total power of the leaked interference at $\mathrm{D}_k$ is

$$IF_k(n) = \operatorname{Tr}\left[\big(\mathbf{U}_k(n)\big)^* \mathbf{Q}_k(n-1) \mathbf{U}_k(n)\right], \tag{5}$$

where $\operatorname{Tr}[\mathbf{A}]$ denotes the trace of a matrix $\mathbf{A}$ and $\mathbf{Q}_k(n-1)$ is the interference covariance matrix:

$$\mathbf{Q}_k(n-1) = \sum_{\substack{j=1 \\ j \neq k}}^{K} \sum_{d=1}^{d_j} p_j^d(n-1) \mathbf{H}_{kj} \mathbf{v}_j^d(n-1) \big(\mathbf{v}_j^d(n-1)\big)^* \mathbf{H}_{kj}^*. \tag{6}$$

The following solution minimizes $IF_k(n)$ [3]:

$$\mathbf{u}_k^d(n) = \nu^d[\mathbf{Q}_k(n-1)], \tag{7}$$

where $\nu^d[\mathbf{A}]$ is the eigenvector corresponding to the $d$th smallest eigenvalue of matrix $\mathbf{A}$ and $\mathbf{u}_k^d(n)$ is the $d$th column of matrix $\mathbf{U}_k(n)$. Since the exact value of $\mathbf{Q}_k(n-1)$ is unknown at $\mathrm{D}_k$, it chooses the receiver filters as follows

$$\mathbf{u}_k^d = \nu^d\left[\widehat{\mathbf{Q}}_k(n-1)\right],\ d=1,...,d_k. \tag{8}$$

where $\widehat{\mathbf{Q}}_k$ is the unbiased estimation of the interference covariance matrix:

$$\widehat{\mathbf{Q}}_k(n-1) = \sum_{\substack{j=1\\j\neq k}}^{K}\sum_{d=1}^{d_j} p_j^d(n-1)\widehat{\mathbf{H}}_{kj}\mathbf{v}_j^d(n-1)\big(\mathbf{v}_j^d(n-1)\big)^*\widehat{\mathbf{H}}_{kj}^* - \sigma_e^2\sum_{\substack{j=1\\j\neq k}}^{K}\sum_{d=1}^{d_j} p_j^d(n-1)\mathbf{I}_{n_D}. \tag{9}$$

Next, to design the beamforming matrices, the destinations broadcast training sequences and the sources update their beamforming matrices based on an estimation of the channel. Specifically, $\mathrm{D}_k$ beamforms its training sequences with a fixed power $p_F$ uniformly allocated to different sequences, using an $n_k^{\mathrm{D}} \times d_k$ matrix $\mathbf{V}_k^r$. At the same time, $\mathrm{S}_k$ applies an $n_k^{\mathrm{S}} \times d_k$ filtering matrix $\mathbf{U}_k^r$ to its received signal. If $\mathbf{V}_k^r$ and $\mathbf{U}_k^r$ satisfy the following conditions

$$\begin{aligned}(\mathbf{U}_k^r)^*\mathbf{H}_{kj}^r\mathbf{V}_j^r &= \mathbf{0},\ \forall j\neq k:\ j,k\in\{1,...,K\},\\ \mathrm{rank}\left((\mathbf{U}_k^r)^*\mathbf{H}_{kk}^r\mathbf{V}_k^r\right) &= d_k,\ \forall k\in\{1,...,K\},\end{aligned} \tag{10}$$

then matrices $\mathbf{V}_k = \mathbf{U}_k^r$ and $\mathbf{U}_k = \mathbf{V}_k^r$ also satisfy the conditions in (4). This property along with the channel reciprocity can be exploited to optimize the beamforming matrices. Specifically, $\mathrm{D}_k$ sets its beamforming matrix as $\mathbf{V}_k^r(n) = \mathbf{U}_k(n)$ in which $\mathbf{U}_k(n)$ is obtained by (8). Similarly, the sources choose the following filter to minimize the received interference

$$\left(\mathbf{u}_l^r\right)^d(n) = \nu^d\left[\widehat{\mathbf{Q}}_l^r(n-1)\right], \tag{11}$$

where

$$\widehat{\mathbf{Q}}_l^r(n-1) = \sum_{j=1,j\neq l}^{K}\frac{p_F}{d_j}\widehat{\mathbf{H}}_{lj}^r\mathbf{V}_j^r(n-1)\left(\mathbf{V}_j^r(n-1)\right)^*\left(\widehat{\mathbf{H}}_{lj}^r\right)^* - (K-1)\sigma_e^2 \tag{12}$$

is an unbiased estimate of the reverse covariance matrix. Next, $\mathrm{S}_l$ sets $\mathbf{V}_l(n) = \mathbf{U}_l^r(n)$ as the updated beamforming matrix. Due to the channel reciprocity, such choice would minimize the interference to the unintended destinations in the forward direction.

## IV. Distributed Power Control

To update the powers in the $n$th iteration, the updated beamforming and filtering matrices at $\mathrm{S}_k$ and $\mathrm{D}_k$ are $\mathbf{V}_k(n-1)$ and $\mathbf{U}_k(n)$, respectively. For the simplicity of presentation, let $\mathbf{U}_k$, $\mathbf{V}_k$, and $p_k^l$ denote $\mathbf{U}_k(n)$, $\mathbf{V}_k(n-1)$, and $p_k^l(n)$, respectively. The SINR of the signal corresponding to the $l$th message at $\mathrm{D}_k$ is

$$\mathrm{SINR}_k^l = \frac{\left|\left(\mathbf{u}_k^l\right)^*\mathbf{H}_{kk}\mathbf{v}_k^l\right|^2 p_k^l}{\varphi_k^l(\mathbf{p}) + N_0}, \tag{13}$$

where

$$\varphi_k^l(\mathbf{p}) = \sum_{j=1}^{K}\sum_{d=1}^{d_j}\left|\left(\mathbf{u}_k^l\right)^*\mathbf{H}_{kj}\mathbf{v}_j^d\right|^2 p_j^d - \left|\left(\mathbf{u}_k^l\right)^*\mathbf{H}_{kk}\mathbf{v}_k^l\right|^2 p_k^l, \tag{14}$$

and $\mathbf{p} = \left[p_1^1,...,p_1^{d_1},...,p_K^1,...,p_K^{d_K}\right]^T$ is a $\left(\sum_{k=1}^K d_k\right)\times 1$ power vector. The mutual information of the source-destination pair $\mathrm{S}_k-\mathrm{D}_k$ is $\sum_{l=1}^{d_k}\log_2\left(1+\mathrm{SINR}_k^l\right)$. For the successful transmission, the following condition should be satisfied:

$$\sum_{l=1}^{d_k}\log_2\left(1+\mathrm{SINR}_k^l\right) \geq R_k. \tag{15}$$

The following requirements will guarantee the above condition to be met:

$$\log_2\left(1+\mathrm{SINR}_k^l\right) \geq \frac{R_k}{d_k}\ \forall l\in\{1,...,d_k\}. \tag{16}$$

Using (13) we can rewrite (16) as a power constraint

$$p_k^l \geq I_k^l(\mathbf{p}), \tag{17}$$

where

$$I_k^l(\mathbf{p}) = \frac{\left(2^{R_k/d_k}-1\right)\left(\varphi_k^l(\mathbf{p})+N_0\right)}{\left|\left(\mathbf{u}_k^l\right)^*\mathbf{H}_{kk}\mathbf{v}_k^l\right|^2}. \tag{18}$$

Therefore, all the power constraints can be represented as

$$\mathbf{p} \succeq \mathbf{I}(\mathbf{p}), \tag{19}$$

where $\mathbf{I}(\mathbf{p}) = \left[I_1^1(\mathbf{p}),...,I_1^{d_1}(\mathbf{p}),...,I_K^1(\mathbf{p}),...,I_K^{d_K}(\mathbf{p})\right]^T$ is called *interference function*, and the operator $\succeq$ denotes an *element-wise* inequality. For a given set of transmitter beamforming and receiver filtering matrices, the power control problem is to find the minimum powers which satisfy the inequality in (19). A deterministic power control algorithm for the case in which CSI is perfectly known at terminals, i.e. $\sigma_e^2 = 0$, has been proposed in [11]. In this paper, we present a stochastic power control algorithm for the case in which only noisy CSI is known at terminals.

### A. Stochastic Power Control Algorithm

The stochastic power control algorithm initializes power vector and iteratively updates the power vector as follows

$$\mathbf{p}(i+1) = (1-\alpha(i))\,\mathbf{p}(i) + \alpha(i)\widehat{\mathbf{I}}\left(\mathbf{p}(i),\theta\right), \tag{20}$$

where $\alpha(i)$ is the step size at the $i$th iteration and $\widehat{\mathbf{I}}(\mathbf{p}(i),\theta)$ is an estimation of the interference function for a given $p(i)$ in which $\theta$ is a random variable. To show the convergence of this algorithm, we first provide the definition of the standard stochastic interference function which is consistent with the one in [12].

*Definition 1:* $\widehat{\mathbf{I}}(\mathbf{p},\theta)$ is called *standard stochastic interference function* if for all vectors $\mathbf{p},\mathbf{p}'\succeq 0$, it satisfies

1) *Mean condition*:

$$\mathbb{E}_\theta\left[\widehat{\mathbf{I}}(\mathbf{p},\theta)|\mathbf{p}\right]=\mathbf{I}(\mathbf{p}), \tag{21}$$

where $\mathbf{I}(\mathbf{p})$ is a standard interference function defined in [8].

2) *Lipschitz condition*: There exists $K_1>0$ such that $\forall\mathbf{p}_1,\mathbf{p}_2\succeq\mathbf{0}$,

$$\|\mathbf{I}(\mathbf{p}_1)-\mathbf{I}(\mathbf{p}_2)\|^2\le K_1\|\mathbf{p}_1-\mathbf{p}_2\|^2. \tag{22}$$

3) *Growing condition*: There exists $K_2>0$ such that

$$\mathbb{E}_\theta\left[\left\|\widehat{\mathbf{I}}(\mathbf{p},\theta)-\mathbf{I}(\mathbf{p})\right\|^2\Big|\mathbf{p}\right]\le K_2\left(1+\|\mathbf{p}\|^2\right). \tag{23}$$

Next, we propose an estimation of the interference function based on noisy CSI which can be used in the stochastic power control algorithm in (20).

*Theorem 1:* For the interference function

$$\hat{I}_k^l(\mathbf{p})=\frac{\left(2^{R_k/d_k}-1\right)\left(\hat{\varphi}_k^l(\mathbf{p})+N_0\right)}{\left|\left(\mathbf{u}_k^l\right)^*\mathbf{H}_{kk}\mathbf{v}_k^l\right|^2}, \tag{24}$$

with

$$\begin{aligned}\hat{\varphi}_k^l(\mathbf{p}) &= \sum_{j=1}^{K}\sum_{d=1}^{d_j}\left|\left(\mathbf{u}_k^l\right)^*\widehat{\mathbf{H}}_{kj}\mathbf{v}_j^d\right|^2p_j^d-\left|\left(\mathbf{u}_k^l\right)^*\widehat{\mathbf{H}}_{kk}\mathbf{v}_k^l\right|^2p_k^l\\ &\quad-\sigma_e^2\left(\sum_{j=1}^{K}\sum_{d=1}^{d_j}p_j^d-p_k^{d_k}\right),\end{aligned} \tag{25}$$

the stochastic power control algorithm in (20) converges to a vector denoted as $\mathbf{p}^*$ if the step-size $\alpha(i)$ satisfies

$$\sum_{n=0}^{\infty}\alpha(i)=\infty,\ \ \sum_{n=0}^{\infty}\alpha(i)^2<\infty. \tag{26}$$

*Proof:* According to [12, *Theorem* 1], the stochastic power control algorithm in (20) converges to $\mathbf{p}^*$, if the function $\widehat{\mathbf{I}}(\mathbf{p}(n),\theta)$ is standard stochastic interference function and the step-size sequence $\alpha(n)$ satisfies the conditions in (26). In the following we prove that the estimated interference function in (24) satisfies the mean condition, the Lipschitz condition, and the growing condition. For the mean condition we have:

$$\begin{aligned}\mathbb{E}\left[\hat{I}_k^l(\mathbf{p})\right]&=\mathbb{E}\left[\frac{\left(2^{R_k/d_k}-1\right)\left(\hat{\varphi}_k^l(\mathbf{p})+N_0\right)}{\left|\left(\mathbf{u}_k^l\right)^*\mathbf{H}_{kk}\mathbf{v}_k^l\right|^2}\right]\overset{(a)}{=}k_1\mathbb{E}\left[\hat{\varphi}_k^l(\mathbf{p})+N_0\right]\\ &=k_1\mathbb{E}\left[\hat{\varphi}_k^l(\mathbf{p})\right]+k_1N_0\overset{(b)}{=}k_1\varphi_k^l(\mathbf{p})+k_1N_0\overset{(c)}{=}I_k^l(\mathbf{p}),\end{aligned}$$

where $(a)$ follows the fact that $\mathbf{H}_{kk}$ is perfectly known at $\mathrm{D}_k$ and defining $k_1\triangleq\left(2^{R_k/d_k}-1\right)/\left|\left(\mathbf{u}_k^l\right)^*\mathbf{H}_{kk}\mathbf{v}_k^l\right|^2$; $(b)$ follows the computation of $\mathbb{E}\left[\hat{\varphi}_k^l\right]$ in Appendix A; and $(c)$ follows the definition in (18). As shown in [11], the function $I_k^l(\mathbf{p})$ is a standard interference function, thus, the function in (24) satisfy the mean condition.

To verify Lipschitz condition, consider two power vectors $\mathbf{p}$ and $\tilde{\mathbf{p}}$, $\forall k\in\{1,2,\cdots,K\}$ and $\forall l\in\{1,2,\cdots,d_k\}$ we have

$$\begin{aligned}&I_k^l(\mathbf{p})-I_k^l(\tilde{\mathbf{p}})=k_1\left(\varphi_k^l(\mathbf{p})-\varphi_k^l(\tilde{\mathbf{p}})\right)=\\ &k_1\sum_{j=1}^{K}\sum_{d=1}^{d_j}\left|\left(\mathbf{u}_k^l\right)^*\mathbf{H}_{kj}\mathbf{v}_j^d\right|^2\left(p_j^d-\tilde{p}_j^d\right)-k_1\left|\left(\mathbf{u}_k^l\right)^*\mathbf{H}_{kk}\mathbf{v}_k^l\right|^2\left(p_k^l-\tilde{p}_k^l\right).\end{aligned} \tag{27}$$

This can be written in matrix form

$$\mathbf{I}(\mathbf{p})-\mathbf{I}(\tilde{\mathbf{p}})=\mathbf{A}(\mathbf{p}-\tilde{\mathbf{p}}), \tag{28}$$

where $\mathbf{A}$ is a $\left(\sum_{k=1}^{K}d_k\right)\times\left(\sum_{k=1}^{K}d_k\right)$ matrix. Now, let define norm of a matrix as $|||\mathbf{A}|||=\max_{\|\mathbf{x}\|=1}\|\mathbf{A}\mathbf{x}\|$, where $\|.\|$ is a vector norm. According to (28) we have,

$$\|\mathbf{I}(\mathbf{p})-\mathbf{I}(\tilde{\mathbf{p}})\|=\|\mathbf{A}(\mathbf{p}-\tilde{\mathbf{p}})\|\le|||\mathbf{A}|||\times\|\mathbf{p}-\tilde{\mathbf{p}}\|, \tag{29}$$

where the last inequality follows [13, *Theorem* 5.6.2]. We can choose $K_1=|||\mathbf{A}|||$ to satisfy Lipschitz condition.

To verify the growing condition, consider a power vector $\mathbf{p}$, $\forall k\in\{1,2,\cdots,K\}$ and $\forall l\in\{1,...,d_k\}$ we have

$$\begin{aligned}\hat{I}_k^l(\mathbf{p})-I_k^l(\mathbf{p})&=k_1(\hat{\varphi}_k^l-\varphi_k^l)-k_1\sigma_e^2\left(\sum_{j=1}^{K}\sum_{d=1}^{d_j}p_j^d-p_k^{d_k}\right)\\ &=k_1\Bigg(\sum_{j=1}^{K}\sum_{d=1}^{d_j}\Big(\left(\mathbf{u}_k^l\right)^*\mathbf{H}_{kj}\mathbf{v}_j^d\left(\mathbf{v}_j^d\right)^*\mathbf{E}_{kj}^*\mathbf{u}_k^l\\ &\quad+\left(\mathbf{u}_k^l\right)^*\mathbf{E}_{kj}\mathbf{v}_j^d\left(\mathbf{v}_j^d\right)^*\mathbf{H}_{kj}^*\mathbf{u}_k^l\\ &\quad+\left(\mathbf{u}_k^l\right)^*\mathbf{E}_{kj}\mathbf{v}_j^d\left(\mathbf{v}_j^d\right)^*\mathbf{E}_{kj}^*\mathbf{u}_k^l\Big)p_j^d\Bigg)\\ &\quad-k_1\Big(\left(\mathbf{u}_k^l\right)^*\mathbf{H}_{kk}\mathbf{v}_k^l\left(\mathbf{v}_k^l\right)^*\mathbf{E}_{kk}^*\mathbf{u}_k^l\\ &\quad+\left(\mathbf{u}_k^l\right)^*\mathbf{E}_{kk}\mathbf{v}_k^l\left(\mathbf{v}_k^l\right)^*\mathbf{H}_{kk}^*\mathbf{u}_k^l\\ &\quad+\left(\mathbf{u}_k^l\right)^*\mathbf{E}_{kk}\mathbf{v}_k^l\left(\mathbf{v}_k^l\right)^*\mathbf{E}_{kk}^*\mathbf{u}_k^l\Big)p_k^l\\ &\quad-k_1\sigma_e^2\left(\sum_{j=1}^{K}\sum_{d=1}^{d_j}p_j^d-p_k^l\right).\end{aligned} \tag{30}$$

We can write this equality in the following matrix form

$$\widehat{\mathbf{I}}(\mathbf{p})-\mathbf{I}(\mathbf{p})=\mathbf{B}\mathbf{p}. \tag{31}$$

where $\mathbf{B}$ is a $\left(\sum_{k=1}^{K}d_k\right)\times\left(\sum_{k=1}^{K}d_k\right)$ matrix. Using (31) we can verify the growing condition as follows:

$$\mathbb{E}\left[\left\|\widehat{\mathbf{I}}(\mathbf{p})-\mathbf{I}(\mathbf{p})\right\|^2\Big|\mathbf{p}\right]=\mathbb{E}\left[\|\mathbf{B}\mathbf{p}\|^2|\mathbf{p}\right]\overset{(a)}{\le}\mathbb{E}\left[|||\mathbf{B}|||\right]\|\mathbf{p}\|^2, \tag{32}$$

where the inequality $(a)$ follows [13, *Theorem* 5.6.2]. If we choose $K_2=\mathbb{E}\left[|||\mathbf{B}|||\right]$, the growing condition holds. Thus, the function in (24) is a standard stochastic interference function. This completes the proof. ■

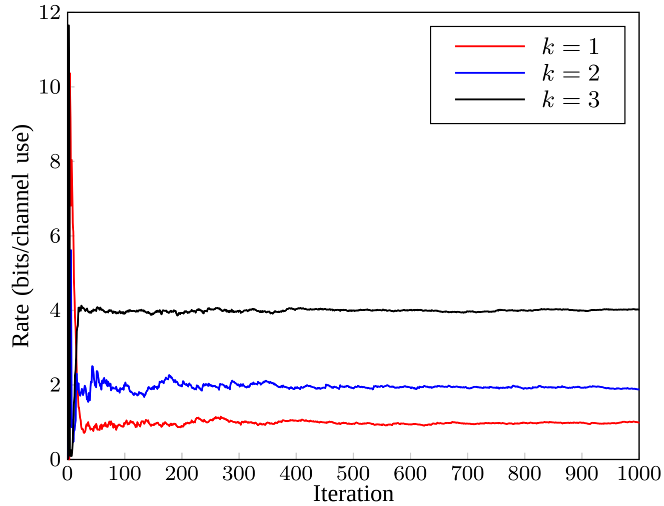


Fig. 2: Mutual information between $\mathrm{S}_k - \mathrm{D}_k$.

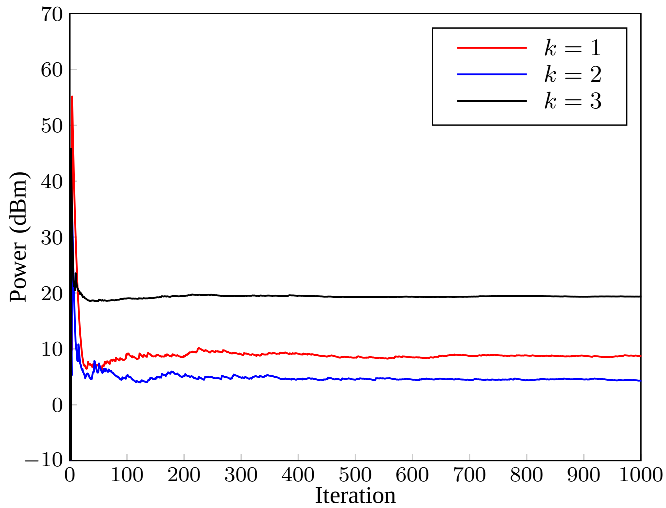


Fig. 3: Transmission power of user $k$.

## V. Performance Evaluation

In this section, we numerically evaluate the performance of the proposed iterative algorithm. We consider a three-user MIMO interference network in which each terminal is equipped with two antennas. Each source transmits one data stream ($d_1 = d_2 = d_3 = 1$). In the simulations, we set $\sigma_e^2 = 0.01$ and the transmissions rates are $R_1 = 1$, $R_2 = 2$, and $R_3 = 4$ (bits/channel use). We choose the step-size of the iterative algorithm as $\alpha(n) = 10/(10+n)$. Fig. 2 shows the achievable rate of each user as a function of the number of iterations. It is clear that for each user this quantity converges to the corresponding transmission rate. The simulations confirm that if the parameters of the algorithm are properly designed, then the iterative stochastic power control and transceiver design converges. Fig. 3 shows the transmission powers of different users as functions of the iterations of the algorithm. It is clear that as the number of iterations increases the power of each user converges to a certain value.

## Appendix A
## Calculation of $\mathbb{E}[\hat{\varphi}_k^l]$ in the Proof of Theorem 1

$$
\begin{aligned}
\mathbb{E}[\hat{\varphi}_k^l(\mathbf{p})] = \mathbb{E}\Bigg[ & \sum_{j=1}^{K}\sum_{d=1}^{d_j} \left| \left(\mathbf{u}_k^l\right)^* \widehat{\mathbf{H}}_{kj}\mathbf{v}_j^d \right|^2 p_j^d - \left| \left(\mathbf{u}_k^l\right)^* \widehat{\mathbf{H}}_{kk}\mathbf{v}_k^l \right|^2 p_k^l \\
& - \sigma_e^2 \left( \sum_{j=1}^{K}\sum_{d=1}^{d_j} p_j^d - p_k^{d_k} \right) \Bigg] \\
\overset{(a)}{=} & \varphi_k^l(\mathbf{p}) + \sum_{j=1}^{K}\sum_{d=1}^{d_j} \mathbb{E}\left[ \left(\mathbf{u}_k^l\right)^* \mathbf{E}_{kj}\mathbf{v}_j^d \left(\mathbf{v}_j^d\right)^* \mathbf{E}_{kj}^* \mathbf{u}_k^l \right] p_j^d \\
& - \mathbb{E}\left[ \left(\mathbf{u}_k^l\right)^* \mathbf{E}_{kk}\mathbf{v}_k^l \left(\mathbf{v}_k^l\right)^* \mathbf{E}_{kk}^* \mathbf{u}_k^l \right] p_k^l \\
& - \sigma_e^2 \left( \sum_{j=1}^{K}\sum_{d=1}^{d_j} p_j^d - p_k^l \right) \overset{(b)}{=} \varphi_k^l(\mathbf{p})
\end{aligned} \tag{33}
$$

where $(a)$ follows the fact that errors have zero mean; $(b)$ follows $\mathbb{E}\left[\mathbf{E}_{kj}\mathbf{v}_j^d \left(\mathbf{v}_j^d\right)^* \mathbf{E}_{kj}^*\right] = \sigma_e^2 \mathbf{I}_{n_D}$

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