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Distributed Model Predictive Control for Multi-Vehicle Formation with Collision Avoidance Constraints

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Abstract— **This paper presents a distributed model predictive control (MPC) method for unicycles in formation with collision avoidance constraints. The proposed method first stabilizes the system by using a feedback linearization, and then a collision avoidance method based on MPC is applied to the linearized system. One of the features of the proposed method is that each vehicle sequentially solves its optimal control problem at different time step. Unlike other MPC collision avoidance methods in which all vehicles solve optimal control problems at every time step, only one vehicle can solve its optimization problem at one time step. We derive a condition for the proposed method to ensure the feasibility of the optimization method and stability of the closed-loop system. The effectiveness of the method is also investigated by experiments.**

I. Introduction

Coordinated control of multiple vehicles has been a significant field of research in recent years. Many researchers have worked on formation control problems, in which multiple vehicles aim at regulating their relative positions (see *e.g.* [1], [2], [3], [4]). One of the challenging issues in formation control is collision avoidance between vehicles. Since the collision avoidance constraints make the formation control problems much more complicated, we might need to develop new design tools suitable for formation control with collision avoidance.

One of the prospective design tools for control problems with constraints is model predictive control (MPC), which determines on-line the control input by solving a finite horizon open-loop control optimization problem (see *e.g.* [5], [6]). Model predictive control with collision avoidance constraints have been studied in [7], [8]. Since the collision avoidance constraints can be described by linear inequality constraints including integers, on-line optimal control problems are solved by using mixed-integer programming[9], [10]. One limitation of these methods for application to multi-vehicle formation control is that the optimal control problems are "centralized". In other words, all vehicles determine their control inputs by solving a common optimal control problem. To prevent the computation time for optimization from growing rapidly as the number of vehicles, "distributed MPC", in which vehicles solve individual optimization problems, is more compatible with multi-vehicle formation control. Some recent works have studied distributed MPC for a group of point-mass type vehicles[11], [12].

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In [11], the computation time is reduced compared with the centralized MPC method, since each vehicle solves the independent optimal control problem based on other vehicles' plans transmitted. However, this method could still have a large time delay before the determined control inputs are applied, in the case of a large number of vehicles, since the determined control inputs are applied after all vehicles finish their optimizations. In other words, all vehicles need to solve the optimization problems at each time step.

In the distributed MPC method in [12], each vehicle solves an individual optimal control problem and applies the control input as soon as it is computed. The stability analysis is more difficult in this case. The asymptotical stability of a multi-vehicle formation system without collision avoidance constraints is guaranteed by requiring that each distributed optimal control does not deviate too far from the previous optimal control. However, the formation control with collision avoidance using this method is still an open problem.

In this paper, we propose a different type of distributed MPC method for a group of unicycles with collision avoidance constraints. This method first stabilizes the system by using feedback linearization, and then a collision avoidance method based on MPC is applied to the linearized system. One of the features of the proposed method is that each vehicle sequentially solves its optimal control problem at different time step. Unlike other MPC collision avoidance methods in which all vehicles solve optimal control problems at every time step, only one vehicle can solve its optimization problem at each time. We derive a condition for the proposed method to guarantee the feasibility of the optimization method and stability of the closed-loop system. The effectiveness of the method is also investigated by experiments.

II. Problem formulation

We consider a group of n unicycles indexed by $i = 1, \cdots, n$:

$$\dot{x}_i = v_i \cos\theta_i, \quad \dot{y}_i = v_i \sin\theta_i, \quad \dot{\theta}_i = \omega_i, \tag{1}$$

where v_i and ω_i are the linear and angular velocities of the vehicle i respectively, and (x_i, y_i, θ_i) denotes the measurable coordinate with respect to a global frame (see Fig. 1).

We also define a leader vehicle described as:

$$\dot{x}_r = v_r \cos\theta_r, \quad \dot{y}_r = v_r \sin\theta_r, \quad \dot{\theta}_r = \omega_r, \tag{2}$$

where v_r and ω_r are the linear and angular velocities respectively, and (x_r, y_r, θ_r) denotes the measurable coordinate with respect to the global frame.

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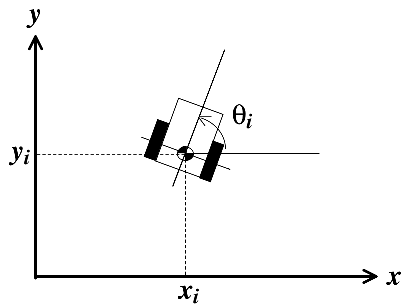

Fig. 1. Wheeled vehicle

The reference position of the vehicle i in (1) is given as a constant vector (r_i, l_i) in a local frame on the leader vehicle in (2) (see Fig. 2). In other words, the reference trajectory for the vehicle i is given with respect to the global frame as

$$z_i^d := \begin{bmatrix} x_r + r_i \sin\theta_r + l_i \cos\theta_r \\ y_r - r_i \cos\theta_r + l_i \sin\theta_r \end{bmatrix}. \quad (3)$$

We refer to the vehicle $i(= 1, \cdots, n)$ in (1) as the "follower i", and the leader vehicle in (2) as the "leader".

Our goal is to control the each follower's position with a given offset d, defined as

$$z_i := \begin{bmatrix} x_{vi} \\ y_{vi} \end{bmatrix} = \begin{bmatrix} x_i + d\cos\theta_i \\ y_i + d\sin\theta_i \end{bmatrix}, \quad (4)$$

to the reference trajectory z_i^d in (3) without collisions. We assume that a sufficient condition for collision avoidance between the followers i and j is given from the size of the vehicles as follows:

$$\|z_i - z_j\|_\infty \geq \psi, \quad \forall j \neq i. \quad (5)$$

Note that it is known in the literature that the collision avoidance constraint in (5) can be written as the following linear constraints[9], [10]

$$\begin{array}{rcl} x_{vi} - x_{vj} & \leq & \Psi\kappa_{ij1} - \psi \\ y_{vi} - y_{vj} & \leq & \Psi\kappa_{ij2} - \psi \\ -x_{vi} + x_{vj} & \leq & \Psi\kappa_{ij3} - \psi \\ -y_{vi} + y_{vj} & \leq & \Psi\kappa_{ij4} - \psi \\ \sum_{p=1}^{4} \kappa_{ijp} & \leq & 3 \end{array} \quad (6)$$

including binary variables κ_{ijp} (= 0 or 1), where Ψ is a positive number much larger than the possible values of z_i. In the same way, it is possible to take account of the collisions with the leader. However, we do not incorporate the collision avoidance with the leader into the control problem, since the leader could be given only virtually in some applications.

In this paper, we propose an algorithm for the followers to determine (v_i, ω_i) steering z_i to z_i^d without collisions. This method first stabilizes the system by using feedback linearization as described in Section III-A, and then a collision avoidance method based on a distributed MPC in Section III-B is applied to the linearized system. For implementation of this method, we assume that the vehicles can communicate necessary information on the future trajectories predicted in their optimization problems, as in the existing works [11], [12]. Note that we also need to assume that the followers know the future trajectory of $\theta_r(t)$ ($t \leq \tau \leq t + T$) for a given constant T, when the collision avoidance method is applied.

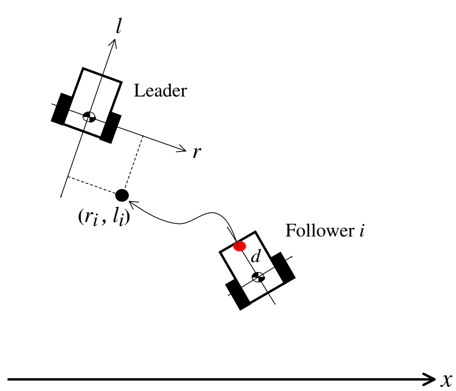

Fig. 2. Leader and follower

III. Formation control method

A. Feedback linearization

In this section, we describe the feedback linearization method. The basic idea is similar to the one in [1]. However, we need to modify the method in [1] to take account of the collision avoidance between the followers.

From (2) and (3), we have

$$\dot{z}_i^d = F_i u_r, \quad (7)$$

where

$$F_i := \begin{bmatrix} \cos\theta_r & r_i\cos\theta_r - l_i \sin\theta_r \\ \sin\theta_r & r_i \sin\theta_r + l_i\cos\theta_r \end{bmatrix}, \quad u_r := \begin{bmatrix} v_r \\ \omega_r \end{bmatrix} \quad (8)$$

It is also seen from (1) and (4)

$$\dot{z}_i = G_i u_i, \quad (9)$$

where

$$G_i := \begin{bmatrix} \cos\theta_i & -d\sin\theta_i \\ \sin\theta_i & d\cos\theta_i \end{bmatrix}, \quad u_i := \begin{bmatrix} v_i \\ \omega_i \end{bmatrix}. \quad (10)$$

The tracking error $e_i := z_i - z_i^d$ is now described as

$$\dot{e}_i = G_i u_i - F_i u_r. \quad (11)$$

By applying

$$u_i = G_i^{-1}(-\lambda e_i + F_i u_r + \alpha_i) \quad (12)$$

to (11) we have

$$\dot{e}_i = -\lambda e_i + \alpha_i, \tag{13}$$

where $\lambda > 0$ is a design parameter. Note that G_i is always invertible for $d > 0$, since

$$\det G_i = d\cos^2\theta_i + d\sin^2\theta_i = d.$$

B. Model predictive collision avoidance

In the proposed method, each follower sequentially solves an optimal control problem at every update interval δ. More precisely, the follower i solves the optimal control problem at $t = k_i\delta$ for $k_i := sn + i - 1$ $(s = 0, 1, \cdots)$, and apply the optimal control trajectory α_i^* until its next update time $t = (k_i + n)\delta$. The predicted value of $e_i(\tau)$ at $t = k\delta$ is defined as $\widehat{e}_i(\tau|k)$, which is transmitted to all other followers as soon as the optimal control problem at $t = k\delta$ is solved. The vehicle i uses the predicted trajectories $\widehat{e}_j$ transmitted from other vehicles j $(j \neq i)$ to take account of collision avoidance.

The algorithm for the vehicle i is described as follows:

Algorithm for the follower i:

Step 0: At the initial time $t = 0$, define

$$k := 0 \tag{14}$$
$$\widehat{\alpha}_i(\tau|0) := \lambda e_i(0), \qquad 0 \le \tau \le T \tag{15}$$
$$\widehat{e}_j(\tau|0) := e_j(0), \qquad 0 \le \tau \le T, \tag{16}$$

for each $j \neq i$.

Step 1: At $t = k\delta$,

- If $k = i - 1$ (mod n), solve the optimization described below, update $\widehat{\alpha}_i$ and $\widehat{e}_i$ as

$$\widehat{\alpha}_i(\tau|k) = \alpha_i^*(\tau|k) \tag{17}$$
$$\widehat{e}_i(\tau|k) = e_i^*(\tau|k)$$
$$t \le \tau \le t + T, \tag{18}$$

  and transmit $\widehat{e}_i(\tau|k)$, where $\alpha_i^*(\tau|k)$ and $e_i^*(\tau|k)$ denote the optimal trajectories obtained from the optimization below.
- Otherwise, receive $\widehat{e}_p(\tau|k)$ from the vehicle p, where $p = k$ (mod n).

Step 2: Apply $u_i(\tau)$ in (12) for $t \le \tau \le t + \delta$ using

$$\alpha_i(\tau) = \widehat{\alpha}_i(\tau|k). \tag{19}$$

Step 3: Update k, $\widehat{\alpha}_i$, $\widehat{e}_j$ as

$$k = k + 1 \tag{20}$$
$$\widehat{\alpha}_i(\tau|k) = \widehat{\alpha}_i(\tau|k-1) \tag{21}$$
$$\widehat{e}_j(\tau|k) = \widehat{e}_j(\tau|k-1), \quad t + \delta \le \tau \le t + T \tag{22}$$
$$\widehat{e}_j(\tau|k) = \exp(\lambda(t + T - \tau))\, \widehat{e}_j(\tau|k-1),$$
$$t + T \le \tau \le t + T + \delta \tag{23}$$

and go to Step 1.

Note that, in (23), the predicted value of e_j is computed by assuming

$$\alpha_j(\tau) = 0, \quad t + T \le \tau \le t + T + \delta, \tag{24}$$

since the vehicle j does not have the optimal control trajectory for $\alpha_j(\tau)$ at $t+T \le \tau \le t+T+\delta$. It is also important to note that the prediction horizon T needs to be chosen such that $T \ge n\delta$ to achieve the collision avoidance.

The optimal control problem for the vehicle i at $k = i-1$ (mod n) is described as follows:

Optimization at $t = k\delta$:

$$\min_{\widehat{\alpha}_i} \int_t^{t+T} \widehat{\alpha}_i(\tau|k)^T R \widehat{\alpha}_i(\tau|k) d\tau \tag{25}$$

subject to

$$\dot{\widehat{e}}_i = -\lambda\widehat{e}_i + \widehat{\alpha}_i, \quad \widehat{e}_i(t|k) = e_i(t) \tag{26}$$
$$\|\widehat{e}_i(\tau|k) - \widehat{e}_j(\tau|k) + \mu_i - \mu_j\|_\infty \ge \psi \tag{27}$$
$$\| -\lambda\widehat{e}_i(\tau|k) + \widehat{\alpha}_i(\tau|k)\|_\infty \le \eta \tag{28}$$
$$\|\widehat{e}_i(t + T|k)\|_\infty \le \gamma_i \tag{29}$$
$$\forall j \neq i, \quad t \le \tau \le t + T,$$

where

$$\mu_i := \begin{bmatrix} \sin\theta_r & \cos\theta_r \\ -\cos\theta_r & \sin\theta_r \end{bmatrix} \begin{bmatrix} r_i \\ l_i \end{bmatrix} \tag{30}$$

and R is a given symmetric positive definite matrix. Note that this optimization needs to be approximated by a discretized problem for implementation.

This problem determines $\widehat{e}_i(\tau|k)$ and $\widehat{\alpha}_i(\tau|k)$ as mentioned in Step 1 above, while $\widehat{e}_j(\tau|k)$ is given in advance in Step 3. The equality constraint in (26) is a prediction model for $\widehat{e}_i$ based on (13). The inequality in (27) is the collision avoidance constraint in (5).

The inequality constraint in (28) is introduced to constrain the control input u_i in (12), where η is a design parameter chosen as a positive number. The terminal constraint in (29) is introduced to guarantee the feasibility of the optimal control problem at each time and the asymptotic stability of the closed-loop system. Conditions, which γ_i needs to satisfy for the feasibility and stability, are given in the next section.

IV. Feasibility and Stability Results

As mentioned in the previous section, we need the following assumption prescribing upper bounds for γ_i to ensure that the optimal control problem in (25) is feasible at each time.

Assumption 1: Each γ_i $(= 1, \cdots, n)$ in (29) satisfies the following conditions:

$$\left\| \begin{array}{c} r_i - r_j \\ l_i - l_j \end{array} \right\|_\infty \ge \sqrt{2}(\psi + \gamma_i + \gamma_j), \quad \forall j \neq i \tag{31}$$
$$\lambda\gamma_i \le \eta. \tag{32}$$

We need the following lemma to derive the feasibility result.

Lemma 1: If $\|e_i\|_\infty \le \gamma_i$ and $\|e_j\|_\infty \le \gamma_j$ $(i \neq j)$ for γ_i and γ_j satisfying Assumption 1, then it is satisfied that

$$\|e_i - e_j + \mu_i - \mu_j\|_\infty \ge \psi \tag{33}$$
$$\| -\lambda e_i + \alpha_i\|_\infty \le \eta \tag{34}$$

for $\alpha_i = 0$ and any $\theta_r \in [0, 2\pi]$.

Proof: From Assumption 1, we have

$$\frac{1}{\sqrt{2}} \left\| \begin{array}{c} r_i - r_j \\ l_i - l_j \end{array} \right\|_\infty - \gamma_i - \gamma_j \ge \psi, \quad \forall j \neq i. \tag{35}$$

This implies from $\|e_i\|_\infty \le \gamma_i$ and $\|e_j\|_\infty \le \gamma_j$ that

$$\frac{1}{\sqrt{2}}\left\| \begin{array}{c} r_i - r_j \\ l_i - l_j \end{array} \right\|_\infty - \|e_i\|_\infty - \|e_j\|_\infty \ge \psi, \quad \forall j \ne i. \quad (36)$$

Since

$$\mu_i - \mu_j = \left[\begin{array}{cc} \sin\theta_r & \cos\theta_r \\ -\cos\theta_r & \sin\theta_r \end{array} \right] \left[\begin{array}{c} r_i - r_j \\ l_i - l_j \end{array} \right], \quad (37)$$

we have

$$\begin{aligned} \|\mu_i - \mu_j\|_\infty &\ge \frac{1}{\sqrt{2}}\|\mu_i - \mu_j\|_2 \\ &= \frac{1}{\sqrt{2}}\left\| \begin{array}{c} r_i - r_j \\ l_i - l_j \end{array} \right\|_2 \ge \frac{1}{\sqrt{2}}\left\| \begin{array}{c} r_i - r_j \\ l_i - l_j \end{array} \right\|_\infty \end{aligned} \quad (38)$$

From (36) and (38), we have

$$\begin{aligned} \psi &\le \|\mu_i - \mu_j\|_\infty - \|e_i\|_\infty - \|e_j\|_\infty & (39) \\ &\le \|\mu_i - \mu_j + e_i - e_j\|_\infty, & (40) \end{aligned}$$

which shows (33). Since we have from (32) that

$$\| - \lambda e_i + \alpha\|_\infty \le \lambda\|e_i\|_\infty \le \lambda\gamma_i \le \eta \quad (41)$$

for $\alpha = 0$, we have (34). ■

The feasibility property is now described in the following theorem.

Theorem 1: Assume the optimization in (25) is feasible at the initial update time $t = (i-1)\delta$ of each follower i ($i = 1, \cdots, n$) for γ_i, which satisfies Assumption 1. Then, the optimization in (25) is feasible at each update time $t \ge n\delta$.

Proof: The proof uses mathematical induction. We assume that every vehicle i $(= 1, \cdots, n)$ has the optimal solution at $t = k_i\delta$, where $k_i = sn + i - 1$, for some positive integer s. This implies from the algorithm that the follower i $(= 2, \cdots, n)$ determines $\alpha_i^*(\tau|k_i)$ $(k_i\delta \le \tau \le k_i\delta + T)$ by assuming that the vehicle 1 will apply the following input:

$$\widehat{\alpha}_1(\tau|k_i) = \left\{ \begin{array}{cc} \alpha_1^*(\tau|k_1), & k_i\delta \le \tau \le k_1\delta + T \\ 0, & k_1\delta + T < \tau \le k_i\delta + T. \end{array} \right. \quad (42)$$

Thus, if the vehicle i $(= 2, \cdots, n)$ applies $\alpha_i^*(\tau|k_i)$, it arrives at its terminal set

$$\Omega_i := \{z_i : \ \|z_i - z_i^d\|_\infty \le \gamma_i\} \quad (43)$$

at $t = k_i\delta + T$ without collision with the vehicle 1. Therefore, at $t = (k_1 + n)\delta$, if the vehicle 1 applies the input

$$\begin{aligned} &\widehat{\alpha}_1(\tau|k_1 + n) \\ &= \left\{ \begin{array}{cc} \alpha_1^*(\tau|k_1), & (k_1 + n)\delta \le \tau \le k_1\delta + T \\ 0, & k_1\delta + T < \tau \le (k_1 + n)\delta + T, \end{array} \right. \end{aligned} \quad (44)$$

it does not have collision at $(k_1 + n)\delta \le \tau \le k_i\delta + T$. We can also see from Lemma 1 that the collision avoidance constraint (27) is satisfied at $k_i\delta + T \le \tau \le (k_1 + n)\delta + T$, since both the vehicle 1 and i are in their terminal sets. Thus the vehicle 1 has at least one feasible solution in (44) at $t = (k_1 + n)\delta$. Likewise, it is easily seen from Lemma 1 that the input $\widehat{\alpha}_1$ in (44) satisfies the input constraint in (28).

In the same way, at $t = (k_i + n)\delta$, if the vehicle i $(= 2, \cdots, n)$ applies the input

$$\begin{aligned} &\alpha_i(\tau|k_i + n) \\ &= \left\{ \begin{array}{cc} \alpha_i^*(\tau|k_i), & (k_i + n)\delta \le \tau \le k_i\delta + T \\ 0, & k_i\delta + T < \tau \le (k_i + n)\delta + T, \end{array} \right. \end{aligned} \quad (45)$$

it has no collision with Vehicle j $(= i + 1, \cdots, n)$ for $(k_i + n)\delta \le \tau \le k_j\delta + T$. We can also see that there is no collision with the vehicle j for $k_j\delta + T < \tau \le (k_i + n)\delta + T$, since both the vehicle i and j are in their terminal sets.

Therefore, if every vehicle i $(= 1, \cdots, n)$ has the optimal solution at $k_i = sn + i - 1$ for some positive integer s, then the every vehicle also has the feasible solution at k_i $(i = 1, \cdots, n)$ for $s + 1$. Since the feasibility for $s = 0$ is assumed, the proof is completed by induction with respect to s. ■

Theorem 2: If assumptions in Theorem 1 are satisfied, all followers converge to the reference positions, that is

$$\lim_{t\to\infty} e_i(t) = 0, \quad i = 1, \cdots, n, \quad (46)$$

without collisions.

Proof: See Appendix. ■

Note that the control problem in Section II does not consider constraints for the amplitude of the control input u_i explicitly, although there usually exist such constraints in practical situations. Therefore, in those cases, the design parameter η in (28) needs to be selected such that the constraints for u_i are also satisfied, as shown in the next section. Estimation of proper values of η is one of the future issues.

V. EXPERIMENTS

The formation control method in Section III is applied to the experimental vehicle systems developed based on Tamiya radio-controlled model tanks (see Fig. 3). The coordinates of the vehicles (x_i, y_i, θ_i) are measured by a camera located on the ceiling. The control inputs for all vehicles are computed by one PC to reduce the cost and size of the experimental system. The collision avoidance control is implemented using a MATLAB algorithm[14] for solving Mixed Integer Quadratic Programs and MATLAB compiler 4[15].

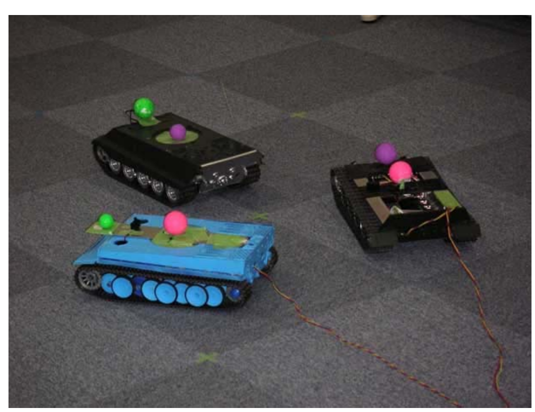

Fig. 3. Vehicles

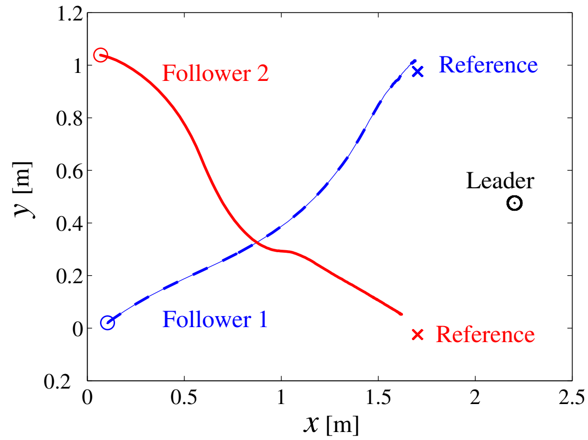

Fig. 4. x-y plot of followers

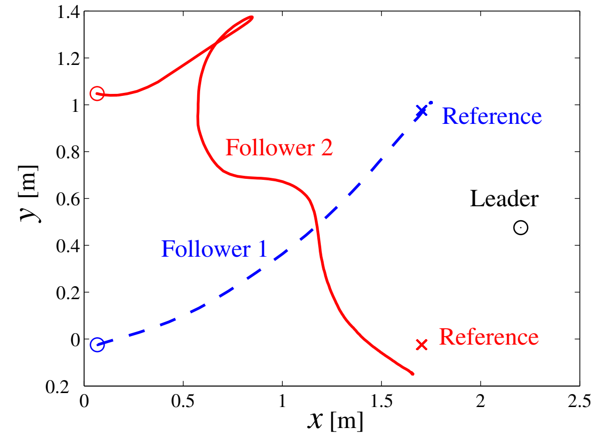

Fig. 6. x-y plot of followers without input constraints

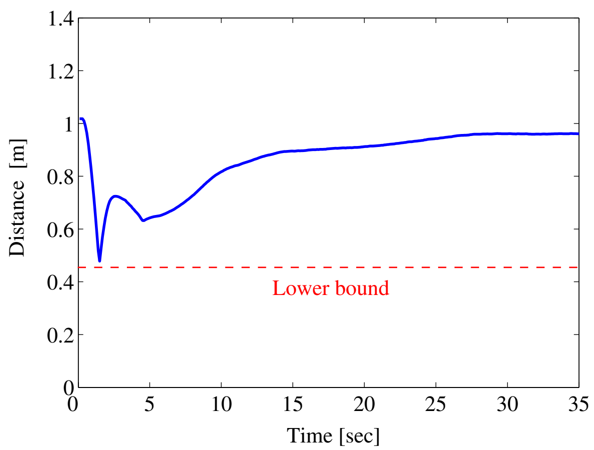

Fig. 5. Minimum distance between followers

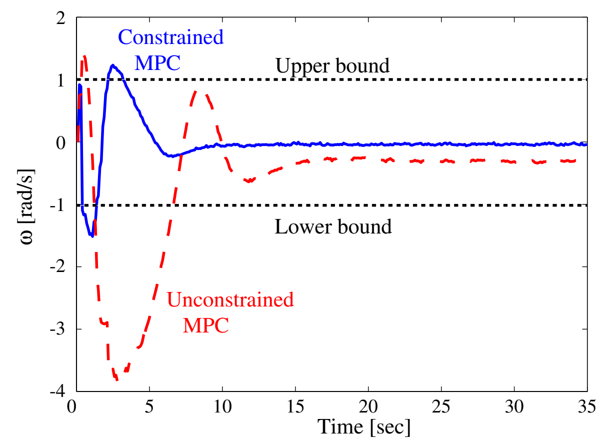

Fig. 7. ω_i of the follower 2

The offset d in (4), the distance bound ψ, and a large number Ψ in (6) are given as $d = 0.15$, $\psi = 0.45$ and $\Psi = 30$, respectively. The prediction horizon T and update time δ in the control algorithm are given as $T = 3$ and $\delta = 0.1$. We set $\lambda = 0.3$ in (12), $\gamma_i = 0.25$ in (29) and $\eta = 0.6$ in (28). The initial coordinates of the follower 1 and 2 are $(0.5, -2.0, 3\pi/4)$ and $(-0.5, -2.0, \pi/4)$, while the reference positions are $(-0.5, -0.5)$ and $(0.5, -0.5)$ in the global frame. The leader does not move in this example due to the restriction of the space for experiments. The follower 1 and 2 collide in this situation, if the collision avoidance method in Section III-B is not applied (*i.e.* $\alpha = 0$).

We apply the collision avoidance method by discretizing the problem in (25) with sampling interval 1.0[sec]. Fig. 4 and Fig. 5 show the x-y plots of the followers' trajectories in the global frame and the minimum distance between the followers, *i.e.* $\min_{i,j} \|z_i - z_j\|_\infty$, respectively. From these figures, it is seen that the vehicles track the reference positions without violating collision avoidance constraints.

Note that the constraint in (28) plays a significant role in this example, since the vehicle system has an input constraint $|\omega_i| \leq 1$. Fig. 6 and Fig. 7 show the x-y plots of the vehicles without applying the constraints in (28) and the time plot of a input ω_2 for the follower 2, respectively. These figures show that the follower 2 has an erratic trajectory without the constraint in (28), since the constraint $|\omega_i| \leq 1$ is largely violated.

VI. Conclusion

In this paper, we have proposed a distributed MPC method for unicycles in formation with collision avoidance constraints. One of the features of the proposed method is that each vehicle sequentially solves its optimal control problem at different time step. Unlike other MPC collision avoidance methods in which all vehicles solve optimal control problems at every time step, only one vehicle can solve its optimization problem at each time. We have derived a condition for the proposed method to guarantee the feasibility of the optimization method and stability of the closed-loop system. Furthermore, the effectiveness of the method has been investigated by experiments. One of the future works is development of a collision avoidance control algorithm with less computational burden. Also, more studies are necessary on how to determine the design parameter η in the control input constraint in (28).

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Appendix

A. Proof of Theorem 2

We first show that the optimal cost

$$J_i^*(s) := \int_{k_i\delta}^{k_i\delta+T} \alpha_i^*(\tau|k_i)^T R \alpha_i^*(\tau|k_i) d\tau \tag{47}$$

is nonincreasing with respect to s. At the update time $t = (k_i+n)\delta$, the feasible solution in (45)

$$\overline{J}_i(s+1) := \int_{(k_i+n)\delta}^{(k_i+n)\delta+T} \widehat{\alpha}_i(\tau|k_i+n)^T R \widehat{\alpha}_i(\tau|k_i+n) d\tau \tag{48}$$

satisfies

$$\overline{J}_i(s+1) \leq J_i^*(s), \tag{49}$$

since

$$\begin{aligned} &\overline{J}_i(s+1) - J_i^*(s) \qquad (50)\\ &= \int_{(k_i+n)\delta}^{(k_i+n)\delta+T} \widehat{\alpha}_i(\tau|k_i+n)^T R \widehat{\alpha}_i(\tau|k_i+n) d\tau \\ &\quad - \int_{k_i\delta}^{k_i\delta+T} \alpha_i^*(\tau|k_i)^T R \alpha_i^*(\tau|k_i) d\tau \\ &= -\int_{k_i\delta}^{(k_i+n)\delta} \alpha_i^*(\tau|k_i)^T R \alpha_i^*(\tau|k_i) d\tau \leq 0. \qquad (51) \end{aligned}$$

It is also satisfied that

$$J_i^*(s+1) \leq \overline{J}_i(s+1), \tag{52}$$

from the optimality of $\alpha_i^*(\cdot|k_i+n)$. From (49) and (52), the optimal cost is nonincreasing, that is $J_i^*(s+1) \leq J_i^*(s)$. Since the optimal cost is nonincreasing and bounded by 0 from below, it satisfies $J_i^*(s) \to c$ as $s \to \infty$ for a constant $c \geq 0$. This implies that, for each $\epsilon > 0$, there exists $s_1 > 0$ such that

$$0 \leq J_i^*(s) - J_i^*(s+1) < \epsilon, \quad \forall s \geq s_1. \tag{53}$$

But, from (49), (51) and (52), we have

$$\begin{aligned} &\int_t^{t+\delta} \alpha^*(\tau|k)^T R \alpha^*(\tau|k) d\tau \\ &= J_i^*(s) - \overline{J}_i(s+1) \leq J_i^*(s) - J_i^*(s+1). \qquad (54) \end{aligned}$$

Since $R > 0$, (53) and (54) imply

$$\lim_{s\to\infty} \alpha_i^*(\tau|k_i) \to 0, \quad k_i\delta \leq \tau < (k_i+n)\delta. \tag{55}$$

Thus, we can see from (17) and (19) that

$$\lim_{t\to\infty} \alpha_i(t) = 0. \tag{56}$$

This implies that for a given constant $\epsilon_1 > 0$ which satisfies

$$\epsilon_1 < \lambda \min\{\gamma_{i1}, \gamma_{i2}\}, \tag{57}$$

we can choose t_1 such that

$$\|\alpha_i(t)\|_\infty \leq \epsilon_1, \quad \forall t \geq t_1. \tag{58}$$

Note that γ_{i1} and γ_{i2} in (57) denote the first and second elements of γ_i, respectively. From (58), it follows that $V(\cdot) := |\cdot|$ satisfies

$$\begin{aligned} \dot{V}(e_{i1}(t)) &= \frac{e_{i1}\dot{e}_{i1}}{|e_{il}|} = -\lambda|e_{i1}| + \frac{e_{i1}}{|e_{i1}|}\alpha_{i1} \\ &\leq -\lambda V(e_{i1}(t)) + |\alpha_{i1}^*(t)| \\ &\leq -\lambda V(e_{i1}(t)) + \epsilon_1, \quad \forall t \geq t_1. \end{aligned}$$

Therefore, by the comparison principle[13],

$$\begin{aligned} V(e_{i1}(\tau)) &\leq e^{-\lambda(\tau-t_1)} V(e_{i1}(t_1)) + \epsilon_1 \int_0^{\tau-t_1} e^{-\lambda s} ds \\ &= e^{-\lambda(\tau-t_1)} \left(V(e_{i1}(t_1)) - \frac{\epsilon_1}{\lambda}\right) + \frac{\epsilon_1}{\lambda}, \qquad (59) \end{aligned}$$

and the right-hand side of (59) converges to ϵ_1/λ as $\tau \to \infty$. We first consider the case where $V(e_{i1}(t_1)) > \gamma_{i1}$. Since (57) implies $\epsilon_1/\lambda < \gamma_{i1}$, there exists a finite time $t_c(> t_1)$ which satisfies

$$e^{-\lambda(t_c-t_1)} \left(V(e_{i1}(t_1)) - \frac{\epsilon_1}{\lambda}\right) + \frac{\epsilon_1}{\lambda} = \gamma_{i1}.$$

Therefore,

$$V(e_{i1}(t)) \leq \gamma_{i1}, \quad \forall t \geq t_c. \tag{60}$$

On the other hand, if $V(e_{i1}(t_1)) \leq \gamma_{i1}$, we have (60) for $t_c = t_1$, since (59) and (57) show that $V(e_{i1}(t))$ cannot be greater than γ_{i1} at any $t \geq t_1$. In the same way, it can be shown that $V(e_{i2}(t)) \leq \gamma_{i2}, \forall t \geq t_c$. Thus, since the optimal solution for $t \geq t_c$ is always $\alpha_i^*(\tau) = 0$, we have $e_i(t) \to 0$ as $t \to \infty$ from (13).