University of Groningen

Distributed Optimal Load Frequency Control with Non-Passive Dynamics

Trip, Sebastian; De Persis, Claudio

IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.

Document Version
Publisher's PDF, also known as Version of record

[Link to publication in University of Groningen/UMCG research database](#)

Distributed Optimal Load Frequency Control with Non-Passive Dynamics

Sebastian Trip and Claudio De Persis

Abstract—Motivated by an increase of renewable energy sources, we propose a distributed *optimal* load frequency control scheme achieving frequency regulation and economic dispatch. Based on an energy function of the power network, we derive an incremental passivity property for a well-known nonlinear structure preserving network model, differentiating between generator and load buses. Exploiting this property, we design *distributed* controllers that adjust the power generation. Notably, we explicitly include the turbine-governor dynamics, where first-order and the widely used second-order dynamics are analyzed in a unifying way. Due to the non-passive nature of the second-order turbine-governor dynamics, incorporating them is challenging, and we develop a suitable dissipation inequality for the interconnected generator and turbine-governor. This allows us to include the generator side more realistically in the stability analysis of optimal load frequency control than was previously possible.

Index Terms—Consensus, economic dispatch, incremental passivity, load frequency control (LFC), turbine-governor dynamics.

I. INTRODUCTION

WHENEVER there is an imbalance between generation and load, the frequency in the power network deviates from its nominal value. This makes frequency regulation, or "load frequency control" (LFC), a critical task to maintain the stability of the network. Whereas primary droop control is utilized to act fast on smaller fluctuations to prevent destabilization, the frequency in the power network is conventionally regulated by "automatic generation control" (AGC) that acts on the reference setting of the governors. To do so, each control area determines its "area control error" (ACE) and changes the set points accordingly to compensate for local load changes and to maintain the scheduled tie-line power flows between different areas [3], [4]. However, due to an ever-increasing penetration of renewable energy, it is uncertain if the current AGC implementations are still adequate [5]. The use of smart grids,

computer-based control, and communication networks offers on the other hand possibilities to improve the current practices [6], [7]. Various solutions have been proposed to improve the performance of the AGC [8]–[12]. Specifically, the effect of a large share of volatile renewable energy sources has been investigated [13], [14]. Economic efficiency over slower timescales is achieved by a tertiary optimization layer, commonly called the economic dispatch, that is outside of the conventional LFC loop.

Since the AGC was designed to be completely decentralized where each control area only reacts to its own ACE, there is loss of economic efficiency on the fast timescales of LFC. Instead of enforcing a predefined power flow over tie-lines, it is cost effective to coordinate the various regulation units within the whole system. This becomes especially relevant with a larger share of renewable energy sources where generation cannot be as accurately predicted as in the past. It is, therefore, desirable to further merge the secondary LFC and the tertiary optimization layer, which we call "*optimal* load frequency control" (OLFC). The proposed solutions to obtain OLFC can be roughly divided into two approaches. The first approach formulates the Lagrangian dual of the economic dispatch problem and solves the optimization problem based on a distributed primal-dual gradient algorithm that runs in parallel with the network dynamics [15]–[27]. The advantage of this approach is that capacity constraints and convex cost functions can be straightforwardly incorporated. A drawback is, however, that information on the amount of uncontrollable generation and load needs to be available, which is generally unknown in LFC where only the frequency is used as a proxy for the imbalance. This issue is alleviated by the second approach, realizing that, in the unconstrained case, the marginal costs of the various generation units are identical at a cost-effective coordination. In this approach, optimality is achieved by employing a distributed consensus algorithm that converges to a state of identical marginal costs [28]–[39]. Although OLFC has been proposed as a viable alternative to the conventional AGC, it poses the fundamental question if incorporating the economic dispatch into the LFC deteriorates the stability of the power network [40].

Main Contributions: This work continues and extends the study of the closed-loop stability of OLFC and the power network. Specifically, on the generation side, there are still remaining challenges to include realistic models required in the study of frequency regulation. Recent advances in the analysis of OLFC in closed loop with the power network enable stability studies in the presence of detailed generator models [29], [41] and improved network representations [30]. However,

including the important turbine-governor dynamics is less understood. We notice that indeed all of the referred studies on AGC [8]–[14] include a second-order model for the turbine-governor dynamics, whereas none of analytical studies on the stability of OLFC include such dynamics and are generally restricted to at most a first-order model. This paper makes the noteworthy extension toward closing this gap and incorporates the second-order turbine-governor dynamics in the stability analysis of the OLFC. We do this by establishing an incremental passivity property [28], [29] for a well-studied structure-preserving network that represents various relevant power network configurations [42]. This crucial passivity property of the power network is then exploited to incorporate first-order and second-order turbine-governor models in a unifying way. Including the second-order turbine-governor dynamics is especially challenging as they are non-passive, and we cannot rely on the standard methodology for interconnecting passive systems. Instead, we develop a suitable dissipation inequality for the interconnected generator and turbine-governor. Due to the advantage of reduced generation and demand information requirements, we focus in this work on a distributed consensus-based controller, where information on marginal costs is exchanged among neighboring buses. Nevertheless, we provide some guidelines how the higher order turbine-governor dynamics can be included in primal-dual-based approaches as well. Along the stability analysis for the second-order turbine-governor model, we establish a locally verifiable range of acceptable droop constants that allows us to infer frequency regulation. A case study confirms that a disregarding this range of droop constants in the controller design can lead to instability. We, therefore, argue that the design of an OLFC algorithm needs to carefully incorporate the effect of the turbine-governor dynamics. As a result of the distributed and modular design of the controllers, the proposed solution permits to straightforwardly include load control along the generation control, and we provide a brief discussion on this topic.

The remainder of this paper is organized as follows. In Section II, we introduce the dynamic model of the power network, which we will study throughout this work. In Section III, we discuss the steady state of the power network and introduce an optimality criterium. In Section IV, we prove an incremental cyclo-passivity property of the power network that is essential to the controller design. In Section V, we introduce the turbine-governor dynamics and propose distributed controllers that ensure frequency regulation and achieve economic dispatch. In Section VI, we test our controllers in an academic case study using simulations. In Section VII, conclusions and directions for future research are given.

II. Power Network Model

We consider the nonlinear structure-preserving model of the power network proposed in [42] that we will extend in the later sections to include turbine-governor and load dynamics. The network consists of n_g generator buses and n_l load buses. Each bus is assumed to be either a generator or a load bus, such that the total number of buses in the network is $n_g + n_l = n$. The network is represented by a connected and undirected graph $\mathscr{G} = (\mathcal{V}_g \cup \mathcal{V}_l, \mathcal{E})$, where $\mathcal{V}_g = \{1, \ldots, n_g\}$ is the set of generator

TABLE I
DESCRIPTION OF THE VARIABLES AND PARAMETERS APPEARING
IN THE POWER NETWORK MODEL

	State variables
δ_i	Voltage angle
ω_{gi}	Frequency deviation at the generator bus
ω_{li}	Frequency deviation at the load bus
	Parameters
M_i	Moment of inertia
D_{gi}	Damping constant of the generator
D_{li}	Damping constant of the load
B_{ij}	Susceptance of the transmission line
V_i	Voltage
	Controllable input
P_{mi}	Mechanical power
	Uncontrollable input
P_{li}	Unknown constant power demand

buses, $\mathcal{V}_l = \{n_g + 1, \ldots, n\}$ is the set of load buses, and $\mathcal{E} = \{1, \ldots, m\}$ is the set of transmission lines connecting the buses. The network structure can be represented by its corresponding incidence matrix $\mathcal{B} \in \mathbb{R}^{n \times m}$. The ends of transmission line k are arbitrarily labeled with a "+" and a "−." The incidence matrix is then given by

$$\mathcal{B}_{ik} = \begin{cases} +1, & \text{if } i \text{ is the positive end of } k \\ -1, & \text{if } i \text{ is the negative end of } k \\ 0, & \text{otherwise.} \end{cases}$$

Following [42], generator bus $i \in \mathcal{V}_g$ is modeled as

$$\dot{\delta}_i = \omega_{gi}$$
$$M_i \dot{\omega}_{gi} = -D_{gi}\omega_{gi}$$
$$- \sum_{j \in \mathcal{N}_i} V_i V_j B_{ij} \sin(\delta_i - \delta_j) + P_{mi} \qquad (1)$$

where $\mathcal{N}_i$ is the set of buses connected to bus i. In high-voltage transmission networks considered here, the conductance is close to zero and, therefore, neglected, i.e., we assume the network to be lossless. The uncontrollable loads[1] are assumed [42], [43] to consist of a constant and a frequency-dependent component. We model a load bus for $i \in \mathcal{V}_l$, therefore, as

$$\dot{\delta}_i = \omega_{li}$$
$$0 = -D_{li}\omega_{li}$$
$$- \sum_{j \in \mathcal{N}_i} V_i V_j B_{ij} \sin(\delta_i - \delta_j) - P_{li}. \qquad (2)$$

An overview of the used symbols is provided in Table I.

Since the power flows are determined by the differences in voltage angles, it is convenient to introduce $\eta_k = \delta_i - \delta_j$, where η_k is the difference of voltage angles across line k joining buses i and j. For all buses, the dynamics of the power network are

---

[1]Controllable loads can be incorporated as well. The discussion on this topic is postponed to Remark 10 to facilitate a concise treatment.

written as

$$\dot{\eta} = \mathcal{B}^T \omega$$

$$M\dot{\omega}_g = -D_g\omega_g - \mathcal{B}_g\Gamma\sin(\eta) + P_m$$

$$\mathbf{0} = -D_l\omega_l - \mathcal{B}_l\Gamma\sin(\eta) - P_l \qquad (3)$$

where $\omega = (\omega_g^T, \omega_l^T)^T$, $\eta = \mathcal{B}^T\delta$, and $\Gamma = \mathrm{diag}\{\gamma_1, \ldots, \gamma_m\}$, with $\gamma_k = V_iV_jB_{ij} = V_jV_iB_{ji}$ and the index k denoting the line $\{i, j\}$. The matrices $\mathcal{B}_g \in \mathbb{R}^{n_g \times m}$ and $\mathcal{B}_l \in \mathbb{R}^{n_l \times m}$ are obtained by collecting from $\mathcal{B}$ the rows indexed by $\mathcal{V}_g$ and $\mathcal{V}_l$, respectively. The remaining symbols follow straightforwardly from the node dynamics and are diagonal matrices or vectors of suitable dimensions. It is possible to eliminate ω_l in (3) by exploiting the identity $\omega_l = D_l^{-1}(-\mathcal{B}_l\Gamma\sin(\eta) - P_l)$ and realizing that $\mathcal{B}^T\omega = \mathcal{B}_g^T\omega_g + \mathcal{B}_l^T\omega_l$ [44]. As a result, we can write (3) equivalently as

$$\dot{\eta} = \mathcal{B}_g^T\omega_g + \mathcal{B}_l^T D_l^{-1}(-\mathcal{B}_l\Gamma\sin(\eta) - P_l)$$

$$M\dot{\omega}_g = -D_g\omega_g - \mathcal{B}_g\Gamma\sin(\eta) + P_m. \qquad (4)$$

We will, however, keep ω_l when it enhances the readability of this paper.

Remark 1 (Control areas): In the absence of load buses, the considered model appears in the study of AGC of control areas, where a control area is described by an equivalent generator. A control area is then typically modeled as

$$\dot{\delta}_i = \omega_{gi}$$

$$M_i\dot{\omega}_{gi} = -D_{gi}\omega_{gi}$$

$$- \sum_{j \in \mathcal{N}_i} V_iV_jB_{ij}\sin(\delta_i - \delta_j) + P_{mi} - P_{li} \qquad (5)$$

where the loads are collocated at the equivalent generator. All results in this paper also hold for this particular case.

Remark 2 (Microgrids): Besides modeling high-voltage power networks, system (3) has also been used to model (Kron reduced) microgrids [45]–[50]. Smaller synchronous machines and inverters are then represented by (1) and (2), respectively.

Remark 3 (Detailed network models): To stress the contribution of this work, we focus on a basic structure preserving model of the power network. The voltages in this paper are considered constant, which is a common assumption in models tailored to study frequency regulation, since the voltage dynamics are (generally) fast compared to the frequency dynamics [51], [52]. As becomes clear in the subsequent sections, our analysis depends mostly on the existence of an energy function for the considered model. These energy functions have been developed for more realistic network models that, e.g., include voltage dynamics, exciter dynamics, and that distinguish between internal and terminal generator buses [29], [41], [53], [54]. Commonly, these energy functions include a kinetic term $\frac{1}{2}\omega_g^T M\omega_g$, which in our work is essential to derive the passivity property that we exploit in the controller design. It is, therefore, expected that the proposed design can be extended to more complex network dynamics as well. Specifically, the passivity property derived for the model at hand (see Lemma 3) has explicitly been established for generators including voltage dynamics in [29] and [41].

III. STEADY STATE AND OPTIMALITY

Before addressing the turbine-governor dynamics that adjust P_m, we discuss the steady-state frequency deviation under constant generation $\overline{P}_m$. In particular, we study the optimal value of $\overline{P}_m$ that allows for a zero frequency deviation at the steady state, i.e., $\overline{\omega} = \mathbf{0}$. The steady state $(\overline{\eta}, \overline{\omega}, \overline{P}_m)$ of (3) necessarily satisfies

$$\mathbf{0} = \mathcal{B}^T\overline{\omega}$$

$$\mathbf{0} = -D_g\overline{\omega}_g - \mathcal{B}_g\Gamma\sin(\overline{\eta}) + \overline{P}_m$$

$$\mathbf{0} = -D_l\overline{\omega}_l - \mathcal{B}_l\Gamma\sin(\overline{\eta}) - P_l. \qquad (6)$$

We make the natural assumption that a, possibly nonunique, solution to (6) exists, which corresponds to the ability of the network to transfer the required power at the steady state.

Assumption 1 (Solvability): For a given $P_l \in \mathbb{R}^{n_l}$ and $\overline{P}_m \in \mathbb{R}^{n_g}$, there exist $\overline{\eta} \in \mathrm{Im}(\mathcal{B}^T)$, $\overline{\omega} \in \mathrm{Ker}(\mathcal{B}^T)$ such that (6) is satisfied.

From algebraic manipulations of (6), we can derive the following lemma that makes the frequency deviation at steady state $\overline{\omega}$ explicit.

Lemma 1 (Steady-state frequency): Let Assumption 1 hold; then, necessarily $\overline{\omega} = \mathbb{1}_n\omega_*$, with

$$\omega_* = \frac{\mathbb{1}_{n_g}^T \overline{P}_m - \mathbb{1}_{n_l}^T P^l}{\mathbb{1}_{n_g}^T D_g \mathbb{1}_{n_g} + \mathbb{1}_{n_l}^T D_l \mathbb{1}_{n_l}} \qquad (7)$$

where $\mathbb{1}_n \in \mathbb{R}^n$ is the vector consisting of all ones.

We recover, therefore, the well-known fact that the total generation needs to be equal to the total load in order to have a zero frequency deviation in a lossless network. As we only require the total generation to be equal to the total load, it is natural to wonder if we can distribute the generation in an optimal manner. To this end, we assign to every generator a strictly convex linear-quadratic cost function that relates the generated power P_{mi} to the generation costs $C_i(P_{mi})$, typically expressed in $/MWh, i.e.,

$$C_i(P_{mi}) = \frac{1}{2}q_iP_{mi}^2 + r_iP_{mi} + s_i. \qquad (8)$$

To formalize the notion of optimality in this work, we pose the following optimization problem:

$$\min_{P_m} C(P_m)$$

$$\text{s.t.} \quad 0 = \mathbb{1}_{n_g}^T \overline{P}_m - \mathbb{1}_{n_l}^T P^l \qquad (9)$$

where $C(P_m) = \sum_{i \in \mathcal{V}_g} C_i(P_{mi})$. Defining furthermore $Q = \mathrm{diag}(q_1, \ldots, q_{n_g})$, $R = (r_1, \ldots, r_{n_g})^T$, and $S = (s_1, \ldots, s_{n_g})^T$, we can compactly write

$$C(P_m) = \frac{1}{2}P_m^T Q P_m + R^T P_m + \mathbb{1}_{n_g}^T S. \qquad (10)$$

From the discussion of Lemma 1, we note that satisfying the equality constraint in (9) implies $\overline{\omega} = \mathbf{0}$. The solution to (9), indicated by the superscript *opt*, therefore satisfies [29,

Lemma 4]

$$0 = \mathcal{B}^T \mathbf{0}$$

$$0 = -D_g \mathbf{0} - \mathcal{B}_g \Gamma \sin(\overline{\eta}) + \overline{P}_m^{opt}$$

$$0 = -D_l \mathbf{0} - \mathcal{B}_l \Gamma \sin(\overline{\eta}) - P_l. \quad (11)$$

It is possible to explicitly characterize the solution to (9).

Lemma 2 (Optimal generation): The solution $\overline{P}_m^{opt}$ to (9) satisfies

$$\overline{P}_m^{opt} = Q^{-1}(\overline{\lambda}^{opt} - R) \quad (12)$$

where

$$\overline{\lambda}^{opt} = \frac{\mathbb{1}_{n_g} (\mathbb{1}_{n_l}^T P_l + \mathbb{1}_{n_g}^T Q^{-1} R)}{\mathbb{1}_{n_g}^T Q^{-1} \mathbb{1}_{n_g}}. \quad (13)$$

The first derivative of the cost function is commonly called the "marginal cost function." From (12) and (13), it is then immediate to see that

$$Q\overline{P}_m^{opt} + R = \overline{\lambda}^{opt} \in \mathrm{Im}(\mathbb{1}_{n_g}) \quad (14)$$

which implies that at the solution to (9), all marginal costs are identical.

Remark 4 (Information requirements): Solving (9) explicitly requires the knowledge of the total load $\mathbb{1}_{n_l}^T P_l$. A popular approach to solve (9) in a distributed fashion is based on primal-dual gradient dynamics [15]–[24]. Commonly, these approaches do, however, require knowledge of the loads or power flows. A remarkable feature of our work is that the proposed distributed controllers, that will be discussed in the remaining of this paper, solve (9) without such measurements at the cost of the restriction to linear-quadratic cost functions and the absence of generation and power flow constraints.

The focus of this section was the characterization of the (optimal) steady state of the power network under constant power generation. In the next section, we establish a passivity property of the power network that will be useful to design controllers that dynamically adjust P_m, ensuring that P_m converges to the optimal steady state $\overline{P}_m^{opt}$.

IV. INCREMENTAL PASSIVITY PROPERTY OF THE POWER NETWORK

We now establish a passivity property for the considered power network model, that is essential to the stability analysis in the following section. Being more specific, we show that (3) is output strictly incrementally cyclo-passive [55]–[57] with respect to its steady-state solution, when we consider P_m as the input and ω_g as the output. We first recall the following definition (with some abuse of terminology[2]):

Definition 1 (Incremental cyclo-passivity): System

$$\dot{x} = f(x, u)$$

$$y = h(x) \quad (15)$$

$x \in \mathcal{X}$, $\mathcal{X}$ the state space, $u, y \in \mathbb{R}^n$, is incrementally cyclo-passive with respect to a constant triplet $(\overline{x}, \overline{u}, \overline{y})$ satisfying

$$0 = f(\overline{x}, \overline{u})$$

$$\overline{y} = h(\overline{x}) \quad (16)$$

if there exists a continuously differentiable function $\mathcal{S} : \mathcal{X} \to \mathbb{R}$, such that for all $x \in \mathcal{X}$, $u \in \mathbb{R}^m$ and $y = h(x), \overline{y} = h(\overline{x})$

$$\dot{\mathcal{S}} = \frac{\partial \mathcal{S}}{\partial x} f(x, u) + \frac{\partial \mathcal{S}}{\partial \overline{x}} f(\overline{x}, \overline{u}) \leq -\|y - \overline{y}\|_W^2$$

$$+ (y - \overline{y})^T (u - \overline{u})$$

where $\|y - \overline{y}\|_W^2 = (y - \overline{y})^T W (y - \overline{y})$. If $W > \mathbf{0}$, the system is output strictly incrementally cyclo-passive.

We remark that the definition above differs from the ordinary definition of incremental passivity in that it includes the prefix "*cyclo*" indicating that $\mathcal{S}$ is not required to be positive definite nor to be bounded from below. If $\mathcal{S}$ is positive definite, we call the system incrementally passive. We now show that the power network satisfies Definition 1 above.

Lemma 3 [Incremental cyclo-passivity of (4)]: Let Assumption 1 hold. System (4) with input P_m and output ω_g is an output strictly incrementally cyclo-passive system, with respect to $(\overline{\eta}, \overline{\omega}_g)$ satisfying

$$0 = \mathcal{B}_g^T \overline{\omega}_g + \mathcal{B}_l^T D_l^{-1} (-\mathcal{B}_l \Gamma \sin(\overline{\eta}) - P_l)$$

$$0 = -D_g \overline{\omega}_g - \mathcal{B}_g \Gamma \sin(\overline{\eta}) + \overline{P}_m. \quad (17)$$

Namely, there exists a storage function $U(\eta, \overline{\eta}, \omega_g, \overline{\omega}_g)$, which satisfies the following incremental dissipation inequality:

$$\dot{U} = -\|\omega_g - \overline{\omega}_g\|_{D_g}^2 - \|\omega_l - \overline{\omega}_l\|_{D_l}^2$$

$$+ (\omega_g - \overline{\omega}_g)^T (P_m - \overline{P}_m) \quad (18)$$

where $\dot{U}$ represents the derivative of $U(\eta, \overline{\eta}, \omega_g, \overline{\omega}_g)$ along the solutions to (4).

Proof: Consider the incremental storage function

$$U(\eta, \overline{\eta}, \omega_g, \overline{\omega}_g) = \frac{1}{2}(\omega_g - \overline{\omega}_g)^T M(\omega_g - \overline{\omega}_g)$$

$$- \mathbb{1}^T \Gamma \cos(\eta) + \mathbb{1}^T \Gamma \cos(\overline{\eta})$$

$$- (\Gamma \sin(\overline{\eta}))^T (\eta - \overline{\eta}). \quad (19)$$

We have that $U(\eta, \overline{\eta}, \omega_g, \overline{\omega}_g)$ satisfies

$$
\begin{aligned}
\dot{U} = {} & (\omega_g - \overline{\omega}_g)^T (-D_g \omega_g - \mathcal{B}_g \Gamma \sin(\eta) + P_m) \\
& + (\Gamma \sin(\eta) - \Gamma \sin(\overline{\eta}))^T \\
& \quad \cdot (\mathcal{B}_g^T \omega_g + \mathcal{B}_l^T D_l^{-1} (-\mathcal{B}_l \Gamma \sin(\eta) - P_l)) \\
= {} & -\|\omega_g - \overline{\omega}_g\|_{D_g}^2 + (\omega_g - \overline{\omega}_g)^T (P_m - \overline{P}_m) \\
& + (\Gamma \sin(\eta) - \Gamma \sin(\overline{\eta}))^T \mathcal{B}_l^T (\omega_l - \overline{\omega}_l) \\
= {} & -\|\omega_g - \overline{\omega}_g\|_{D_g}^2 + (\omega_g - \overline{\omega}_g)^T (P_m - \overline{P}_m) \\
& + (\Gamma \sin(\eta) \mathcal{B}_l + P_l)^T D_l^{-1} D_l (\omega_l - \overline{\omega}_l) \\
& - (\Gamma \sin(\overline{\eta}) \mathcal{B}_l + P_l)^T D_l^{-1} D_l (\omega_l - \overline{\omega}_l) \\
= {} & -\|\omega_g - \overline{\omega}_g\|_{D_g}^2 - \|\omega_l - \overline{\omega}_l\|_{D_l}^2 \\
& + (\omega_g - \overline{\omega}_g)^T (P_m - \overline{P}_m) \qquad\qquad (20)
\end{aligned}
$$

along the solutions to (4), where we exploit identity (17) in the second equation. ∎

Note that the result of Lemma 3 holds in particular if we take $\overline{\omega} = \mathbf{0}$ and $\overline{P}_m = \overline{P}_m^{opt}$. We now consider what conditions ensure that storage function (19) has a local minimum at a steady state satisfying (17).

Assumption 2 (Steady-state angle differences): The differences in voltage angles $\overline{\eta}$ in (6) satisfy $\overline{\eta}_k \in (\frac{-\pi}{2}, \frac{\pi}{2}) \quad \forall k \in \mathcal{E}$.

Note that Assumption 2 is generally satisfied under normal operating conditions of the power network, where a small difference in voltage angle is also referred to as phase cohesiveness [58] and is preferred to avoid instability after perturbations [59].

Lemma 4 [Local minimum of (19)]: Let Assumption 2 hold. Then, the storage function (19) has a local minimum at $(\overline{\eta}, \overline{\omega}_g)$.

Proof: We first recall the definition of a Bregman distance [60]. Let $F : \mathcal{X} \to \mathbb{R}$ be a continuously differentiable and strictly convex function defined on a closed convex set $\mathcal{X}$. The Bregman distance associated with F for the points $x, \overline{x}$ is defined as

$$
D_F(x, \overline{x}) = F(x) - F(\overline{x}) - \nabla F(\overline{x})^T (x - \overline{x}). \qquad (21)
$$

A useful property of D_F is that it is positive definite in its first argument, due to the strict convexity of F. Lemma 4 then follows from (19) being the Bregman distance associated with the function $F(\eta, \omega_g) = \frac{1}{2} \omega_g^T M \omega_g - \mathbb{1}_m^T \Gamma \mathbf{cos}(\eta)$, which is strictly convex at the point $(\overline{\eta}, \overline{\omega}_g)$ under Assumption 2. ∎

Remark 5 (Boundedness of solutions): In the proof of Theorem 9, we require Assumption 2 and subsequently Lemma 4 to ensure that there exists a compact forward invariant set around an equilibrium of (3). This allows us to apply LaSalle's invariance principle in the stability analysis.

In this section, we have established that the power network model (3) is an output strictly incrementally cyclo-passive system. Furthermore, we have shown that under Assumption 2, the incremental storage function U has a local minimum at its steady state. These results turn out the be essential to the design of the distributed controllers in the next section and to prove asymptotic stability of the obtained closed-loop system.

TABLE II
DESCRIPTION OF THE VARIABLES AND PARAMETERS APPEARING
IN THE TURBINE-GOVERNOR DYNAMICS

State variables	
P_{si}	Steam power
P_{mi}	Mechanical power
Parameters	
T_{si}	Governor time constant
T_{mi}	Turbine time constant
K_i	Droop constant
Controllable input	
θ_i	Power generation control

V. OPTIMAL TURBINE-GOVERNOR CONTROL

The generated power P_{mi} at generator i is the output of the turbine-governor system. Various turbine-governor models appear in the literature. We consider two of the most widely used models that have fundamentally different properties. We, therefore, partition the set of generators $\mathcal{V}_g = \mathcal{V}_{g1} \cup \mathcal{V}_{g2}$ into the sets $\mathcal{V}_{g1}$ and $\mathcal{V}_{g2}$, where the turbine-governor dynamics are described by first-order and second-order dynamics, respectively. Being able to incorporate both types in a single framework unifies the various modeling assumptions appearing in conventional AGC and OLPC studies and increases the modeling flexibility.

The first-order and second-order turbine-governor dynamics will be discussed separately and controllers are proposed that achieve frequency regulation. To facilitate the controller design using only local information, we write (20), taking therein and in the remainder of this work $\overline{\omega} = \mathbf{0}$ and $\overline{P}_m = \overline{P}_m^{opt}$, as

$$
\begin{aligned}
\dot{U} = {} & -\|\omega_g - \mathbf{0}\|_{D_g}^2 - \|\omega_l - \mathbf{0}\|_{D_l}^2 \\
& + (\omega_g - \overline{\omega}_g)^T (P_m - \overline{P}_m^{opt}) \\
= {} & \sum_{i \in \mathcal{V}_g} \dot{U}_{gi}(\omega_{gi}, P_{mi}, \overline{P}_{mi}^{opt}) + \sum_{i \in \mathcal{V}_l} \dot{U}_{li}(\omega_{li}) \qquad (22)
\end{aligned}
$$

where we define with a slight abuse of notation

$$
\dot{U}_{gi}(\omega_{gi}, P_{mi}, \overline{P}_{mi}^{opt}) = -D_{gi} \omega_{gi}^2 + \omega_{gi}(P_{mi} - \overline{P}_{mi}^{opt})
$$

$$
\dot{U}_{li}(\omega_{li}) = -D_{li} \omega_{li}^2. \qquad (23)
$$

For the sake of exposition, we only consider decentralized controllers in Sections V-A and V-B that guarantee frequency regulation without achieving optimality. These results are then instrumental to Section V-C, where a distributed control architecture is proposed with controllers that exchange information on their marginal costs with their neighbors over a communication network to achieve optimality.

A. First-Order Turbine-Governor Dynamics

We start with the first-order turbine-governor dynamics of a single generator $i \in \mathcal{V}_{g1}$. The dynamics are given by

$$
T_{mi} \dot{P}_{mi} = -P_{mi} - K_i^{-1} \omega_{gi} + \theta_i \qquad (24)
$$

where θ_i is an additional control input to be designed. An overview of the used symbols is provided in Table II. Consider

the following controller at bus i:

$$T_{\theta i}\dot{\theta}_i = -\theta_i + P_{mi} \tag{25}$$

where the controller time constant T_{θ_i} can be chosen to obtain a desirable rate of change of the control input θ_i. As explained before, an additional communication term will be added to controller (25) in Section V-C to enforce optimality at the steady state. The following lemma provides an intermediate result that is useful later on.

Lemma 5 [Incremental passivity of (24) and (25)]: System (24), (25) with input $-\omega_{gi}$ and output P_{mi} is an incrementally passive system, with respect to $(\overline{P}_{mi}^{opt}, \overline{\theta}_i)$ satisfying

$$0 = -\overline{P}_{mi}^{opt} - K_i^{-1}0 + \overline{\theta}_i^{opt}$$
$$0 = -\overline{\theta}_i^{opt} + \overline{P}_{mi}^{opt} \tag{26}$$

Namely, there exists a positive-definite storage function $Z_{1i}(P_{mi}, \overline{P}_{mi}^{opt}, \theta_i, \overline{\theta}_i^{opt})$, which satisfies the following incremental dissipation inequality:

$$\dot{Z}_{1i} = -K_i(\theta_i - P_{mi})^2 - \omega_{gi}(P_{mi} - \overline{P}_{mi}^{opt}) \tag{27}$$

where $\dot{Z}_{1i}$ represents the derivative of $Z_{1i}(\theta_i, \overline{\theta}_i^{opt}, P_{mi}, \overline{P}_{mi}^{opt})$ along the solutions to (24) and (25).

Proof: Consider the incremental storage function

$$Z_{1i} = \frac{T_{\theta i}K_i}{2}(\theta_i - \overline{\theta}_i^{opt})^2 + \frac{T_{mi}K_i}{2}(P_{mi} - \overline{P}_{mi}^{opt})^2. \tag{28}$$

We note that Z_{1i} satisfies along the solutions to (24) and (25)

$$\begin{aligned}\dot{Z}_{1i} = &-(\theta_i - \overline{\theta}_i^{opt})K_i\theta_i + (\theta_i - \overline{\theta}_i^{opt})K_iP_{mi} \\ &-(P_{mi} - \overline{P}_{mi}^{opt})K_iP_{mi} + (P_{mi} - \overline{P}_{mi}^{opt})K_i\theta_i \\ &-(P_{mi} - \overline{P}_{mi}^{opt})\omega_{gi} \\ = &-K_i(\theta_i - P_{mi})^2 - (P_{mi} - \overline{P}_{mi}^{opt})\omega_{gi} \tag{29}\end{aligned}$$

where we exploit identity (26) in the second equation. ∎

The interconnection of generator dynamics (3) and turbine-governor dynamics (24) including controller (25) can be understood as a feedback interconnection of two incrementally passive systems. The following corollary is then an immediate result from this observation.

Corollary 6 (Passive interconnection): Along the solutions to (3), (24), and (25), $Z_{1i}(\theta_i, \overline{\theta}_i^{opt}, P_{mi}, \overline{P}_{mi}^{opt})$ satisfies

$$\dot{U}_{gi} + \dot{Z}_{1i} = -D_{gi}\omega_{gi}^2 - K_i(\theta_i - P_{mi})^2 \leq 0 \tag{30}$$

where $\dot{U}_{gi}$ and $\dot{Z}_{1i}$ are given in (23) and (27) respectively.

We now perform a similar analysis for the second-order turbine-governor dynamics.

B. Second-Order Turbine-Governor Dynamics

Consider the second-order turbine-governor dynamics of a single generator $i \in \mathcal{V}_{g2}$. The dynamics are given by

$$\begin{aligned}T_{si}\dot{P}_{si} &= -P_{si} - K_i^{-1}\omega_{gi} + \theta_i \\ T_{mi}\dot{P}_{mi} &= -P_{mi} + P_{si} \tag{31}\end{aligned}$$

where θ_i is again an additional control input to be designed. In contrast to the first-order dynamics, the second-order dynamics do not possess a useful passivity property. This can be readily concluded from the observation that system (31) with input ω_{gi} and output P_{mi} has relative degree 2. We now propose a different controller than (25) to accommodate the higher order turbine-governor model, namely

$$T_{\theta i}\dot{\theta}_i = -\theta_i + P_{si} - (1 - K_i^{-1})\omega_{gi} \tag{32}$$

where K^{-1} is the droop constant appearing in (31). Similar to (25), we postpone adding an additional communication term until the next subsection.

Lemma 7 (Storage function for second-order dynamics): There exists a positive-definite storage function $Z_{2i}(P_{si}, \overline{P}_{si}, P_{mi}, \overline{P}_{mi}^{opt}, \theta_i, \overline{\theta}_i^{opt})$, which satisfies along the solutions to (3), (31), and (32)

$$\dot{U}_{gi} + \dot{Z}_{2i} = \begin{bmatrix} \omega_{gi} \\ P_{si} - P_{mi} \\ P_{si} - \theta_i \end{bmatrix}^T W_i \begin{bmatrix} \omega_{gi} \\ P_{si} - P_{mi} \\ P_{si} - \theta_i \end{bmatrix} \tag{33}$$

with

$$W_i = \begin{bmatrix} -D_{gi} & -\frac{1}{2}K_i^{-1} - \frac{1}{2} & -\frac{1}{2}K_i^{-1} + \frac{1}{2} \\ -\frac{1}{2}K_i^{-1} - \frac{1}{2} & -T_{si}T_{mi}^{-1} & -\frac{1}{2} \\ -\frac{1}{2}K_i^{-1} + \frac{1}{2} & -\frac{1}{2} & -1 \end{bmatrix}. \tag{34}$$

Proof: Consider the incremental storage function

$$\begin{aligned}Z_{2i} =\ & \frac{T_{\theta i}}{2}(\theta_i - \overline{\theta}_i^{opt})^2 + \frac{T_{si}}{2}(P_{si} - \overline{P}_{si}^{opt})^2 \\ &+ \frac{T_{si}}{2}(P_{mi} - P_{si})^2 \\ =\ & \frac{T_{\theta i}}{2}(\theta_i - \overline{\theta}_i^{opt})^2 + T_{si}(P_{si} - \overline{P}_{si}^{opt})^2 \\ &+ \frac{T_{si}}{2}(P_{mi} - \overline{P}_{mi}^{opt})^2 - T_{si}(P_{si} - \overline{P}_{si}^{opt})(P_{mi} - \overline{P}_{mi}^{opt}). \tag{35}\end{aligned}$$

It can be readily confirmed that Z_{2i} is positive definite. We have that $Z_{2i}(P_{si}, \overline{P}_{si}, P_{mi}, \overline{P}_{mi}^{opt}, \theta_i, \overline{\theta}_i^{opt})$ satisfies along the

solutions to (31) and (32)

$$
\begin{aligned}
\dot{Z}_{2i} = & (\theta_i - \overline{\theta}_i^{opt})(-\theta_i + P_{si} - (1 - K_i^{-1})\omega_{gi}) \\
& + 2(P_{si} - \overline{P}_{si}^{opt})(-P_{si} - K_i^{-1}\omega_{gi} + \theta_i) \\
& + T_{si}T_{mi}^{-1}(P_{mi} - \overline{P}_{mi}^{opt})(-P_{mi} + P_{si}) \\
& - T_{si}T_{mi}^{-1}(P_{si} - \overline{P}_{si}^{opt})(-P_{mi} + P_{si}) \\
& - (P_{mi} - \overline{P}_{mi}^{opt})(-P_{si} - K_i^{-1}\omega_{gi} + \theta_i) \\
= & - T_{si}T_{mi}^{-1}(P_{si} - P_{mi})^2 - (P_{si} - \theta_i)^2 \\
& - K_i^{-1}(P_{si} - P_{mi})\omega_{gi} - K_i^{-1}(P_{si} - \theta_i)\omega_{gi} \\
& - (P_{si} - P_{mi})(P_{si} - \theta_i) \\
& - (\theta_i - \overline{\theta}_i^{opt})\omega_{gi} \quad\quad\quad\quad\quad\quad (36)
\end{aligned}
$$

where we exploited in the second identity the fact that at the steady state

$$
\begin{aligned}
0 &= -\overline{P}_{si}^{opt} - K_i^{-1}0 + \overline{\theta}_i^{opt} \\
0 &= -\overline{P}_{mi}^{opt} + \overline{P}_{si}^{opt} \\
0 &= -\overline{\theta}_i^{opt} + \overline{P}_{si}^{opt} - (1 - K_i^{-1})0 \quad\quad (37)
\end{aligned}
$$

holds. We recall that $\dot{U}_{gi} = -D_{gi}\omega_{gi}^2 + \omega_{gi}(P_{mi} - \overline{P}_{mi}^{opt})$ and notice that

$$
\begin{aligned}
& \omega_{gi}(P_{mi} - \overline{P}_{mi}^{opt}) - (\theta_i - \overline{\theta}_i^{opt})\omega_{gi} \\
& = \omega_{gi}(P_{mi} - \theta_i) \\
& = \omega_{gi}(P_{si} - \theta_i) - \omega_{gi}(P_{si} - P_{mi}). \quad (38)
\end{aligned}
$$

The expression for W_i then follows from writing $\dot{U}_{gi} + \dot{Z}_{2i}$ as a quadratic form. ∎

We now address under what conditions W_i is negative definite, which is important for the stability analysis in the next subsection.

Assumption 3 (Conditions on K_i^{-1}): Let the permanent droop constant K_i be such that the following inequalities hold:

$$
1 - \frac{T_{mi}}{T_{si}} - \sqrt{\alpha_i} < K_i^{-1} < 1 - \frac{T_{mi}}{T_{si}} + \sqrt{\alpha_i} \quad (39)
$$

where

$$
\alpha_i = T_{mi}^2 T_{si}^{-2}(4T_{si}T_{mi}^{-1} - 1)(D_{gi}T_{si}T_{mi}^{-1} - 1). \quad (40)
$$

Additionally, let $D_{gi}, T_{si},$ and T_{mi} be such that

$$
\begin{aligned}
4T_{si}T_{mi}^{-1} &> 1 \\
D_{gi}T_{si}T_{mi}^{-1} &> 1 \quad\quad\quad\quad\quad\quad (41)
\end{aligned}
$$

are satisfied.

Remark 6 (Locally verifiable): The power network generally consists of many generators. It is, therefore, important to note that the validity of Assumption 3 can be checked at each generator using only information that is locally available.

Lemma 7 (Negative definiteness of W_i): Let Assumption 3 hold. Then, $W_i < \mathbf{0}$.

Proof: Inequality (41) guarantees that

$$
X_i = \begin{bmatrix} -T_{si}T_{mi}^{-1} & -\frac{1}{2} \\ -\frac{1}{2} & -1 \end{bmatrix} < \mathbf{0}. \quad (42)
$$

It follows that $W_i < \mathbf{0}$ if and only if the Schur complement of X_i in W_i is negative definite. This Schur complement is given by

$$
S_i = -D_{gi} - \begin{bmatrix} -\frac{1}{2}K_i^{-1} - \frac{1}{2} \\ -\frac{1}{2}K_i^{-1} + \frac{1}{2} \end{bmatrix}^T X_i^{-1} \begin{bmatrix} -\frac{1}{2}K_i^{-1} - \frac{1}{2} \\ -\frac{1}{2}K_i^{-1} + \frac{1}{2} \end{bmatrix} \quad (43)
$$

and is quadratic in K_i^{-1}. By Cramer's rule, we have

$$
X_i^{-1} = \frac{1}{T_{si}T_{mi}^{-1} - \frac{1}{4}} \begin{bmatrix} -1 & \frac{1}{2} \\ \frac{1}{2} & -T_{si}T_{mi}^{-1} \end{bmatrix} \quad (44)
$$

and a straightforward calculation yields

$$
\begin{aligned}
S_i = & -D_{gi} \\
& + \frac{\frac{1}{4}T_{si}T_{mi}^{-1}K_i^{-2} + (\frac{1}{2} - \frac{1}{2}T_{si}T_{mi}^{-1})K_i^{-1} + \frac{1}{2} + \frac{1}{4}T_{si}T_{mi}^{-1}}{T_{si}T_{mi}^{-1} - \frac{1}{4}}.
\end{aligned}
$$
$$
(45)
$$

The solution to $S_i = 0$ is given by the quadratic formula resulting in

$$
K_i^{-1} = \frac{-b_i}{2a_i} \pm \sqrt{\frac{b_i^2 - 4a_ic_i}{4a_i^2}} \quad (46)
$$

with

$$
\begin{aligned}
a_i &= \frac{1}{4}T_{si}T_{mi}^{-1} \\
b_i &= \frac{1}{2} - \frac{1}{2}T_{si}T_{mi}^{-1} \\
c_i &= -D_{gi}(T_{si}T_{mi}^{-1} - \frac{1}{4}) + \frac{1}{2} + \frac{1}{4}T_{si}T_{mi}^{-1}. \quad (47)
\end{aligned}
$$

Algebraic manipulations then yield

$$
\begin{aligned}
\frac{-b_i}{2a_i} &= 1 - \frac{T_{mi}}{T_{si}} \\
\frac{b_i^2 - 4a_ic_i}{4a_i^2} &= T_{mi}^2 T_{si}^{-2} - T_{mi}T_{si}^{-1}(4 + D_{gi}) + 4D_{gi} \\
&= T_{mi}^2 T_{si}^{-2}(4T_{si}T_{mi}^{-1} - 1)(D_{gi}T_{si}T_{mi}^{-1} - 1) \\
&= \alpha_i. \quad\quad\quad\quad\quad\quad\quad\quad (48)
\end{aligned}
$$

It can now be readily confirmed that $S_i < 0$ when (39) holds, where $\sqrt{\alpha_i}$ is real as a result of inequality (41). ∎

C. Stability Analysis and Optimal Distributed Control

Having discussed the separate control of the various turbine-governors, we now turn our attention to the question of how the different controllers in the network can cooperate to ensure minimization of the generation costs at the steady state. To this end, we add an additional communication term to controllers (25) and (32) representing the exchange of information on the

marginal costs among the controllers

$$T_{\theta i}\dot{\theta}_i = -\theta_i + P_{mi}$$
$$-K_i^{-1}q_i \sum_{j\in\mathcal{N}_i^{com}} (q_i\theta_i + r_i - (q_j\theta_j + r_j)) \quad \forall i\in\mathcal{V}_{g1}$$
(49)

$$T_{\theta i}\dot{\theta}_i = -\theta_i + P_{si} - (1 - K_i^{-1})\omega_{gi}$$
$$-q_i \sum_{j\in\mathcal{N}_i^{com}} (q_i\theta_i + r_i - (q_j\theta_j + r_j)) \quad \forall i\in\mathcal{V}_{g2}$$
(50)

where $\mathcal{N}_i^{com}$ is the set of buses connected via a communication link to bus i. The additional communication term can be interpreted as a consensus algorithm, where generator i compares its marginal cost with the marginal costs of connected generators, such that the overall network converges to the state where there is consensus in the marginal costs (see Theorem 9). Due to the modified dynamics of the controller state θ_i, the derivatives of Z_{1i} and Z_{21} along the solutions to (49) and (50) need to be reevaluated. We exploit the result in the proof of Theorem 9, but is discussed separately for the sake of readability.

Remark 7 (Communication induced modifications): As a result of the additional communication term in (49) and (50), the expressions for $\dot{Z}_{1i}$ and $\dot{Z}_{2i}$ given in, respectively, (27) and (36) need to be modified. Notice that

$$q_i \sum_{j\in\mathcal{N}_i^{com}} (q_i\theta_i + r_i - (q_j\theta_j + r_j)) = \left(QL^{com}(Q\theta + R)\right)_i$$
(51)

where L^{com} is the Laplacian matrix reflecting the topology of the communication network. Therefore, we add the following term to $\dot{Z}_{1i}$ and $\dot{Z}_{2i}$:

$$-(\theta_i - \overline{\theta}_i^{opt})\left(QL^{com}(Q\theta + R)\right)_i.$$
(52)

Summing over all buses $i\in\mathcal{V}_g$ then yields

$$-\sum_{i\in\mathcal{V}_g}(\theta_i - \overline{\theta}_i^{opt})\left(QL^{com}(Q\theta + R)\right)_i$$
$$= -(\theta - \overline{\theta}^{opt})^T QL^{com}(Q\theta + R)$$
$$= -(Q\theta + R - (Q\overline{\theta}^{opt} + R))^T L^{com}$$
$$\cdot (Q\theta + R - (Q\overline{\theta}^{opt} + R))$$
(53)

where we exploited

$$L^{com}(Q\overline{\theta}^{opt} + R) = \mathbf{0}$$
(54)

which is a result of $\overline{\theta}^{opt} = \overline{P}_m^{opt}$, $Q\overline{\theta}^{opt} + R \in \text{Im}(\mathbb{1}_{n_g})$, and $\text{Ker}(L^{com}) = \text{Im}(\mathbb{1}_{n_g})$.

The communication network is utilized to ensure that all marginal costs converge to the same value throughout the network (see the proof of Theorem 9), leading to the following assumption.

Assumption 4 (Connectivity): The graph reflecting the topology of information exchange among the controllers is undirected and connected, but can differ from the topology of the power network.

We are now ready to state the main result of this work.

Theorem 1 (Distributed optimal LFC): Let Assumptions 1–4 hold. Consider the power network (3), turbine-governor dynamics (24), (31), and the distributed controllers (49), (50). Then, solutions that start sufficiently close to $(\overline{\eta}, \overline{\omega} = \mathbf{0}, \overline{P}_m^{opt}, \overline{P}_s^{opt}, \overline{\theta}^{opt})$ converge to the set where we have frequency regulation and where the power generation solves optimization problem (9), i.e., $\overline{\omega} = \mathbf{0}$ and $\overline{P}_m = \overline{P}_m^{opt}$.

Proof: As a result of Lemma 3, Corollary 6, Lemma 7, and Remark 7, we have that $U + \sum_{i\in\mathcal{V}_{g1}} Z_{1i} + \sum_{i\in\mathcal{V}_{g2}} Z_{2i}$ satisfies

$$\dot{U} + \sum_{i\in\mathcal{V}_{g1}}\dot{Z}_{1i} + \sum_{i\in\mathcal{V}_{g2}}\dot{Z}_{2i}$$
$$= -\|\omega_l\|_{D_l}^2 + \sum_{i\in\mathcal{V}_{g1}}\left(-D_{gi}\omega_{gi}^2 - K_i(\theta_i - P_{mi})^2\right)$$
$$+ \sum_{i\in\mathcal{V}_{g2}} \begin{bmatrix} \omega_{gi} \\ P_{si} - P_{mi} \\ P_{si} - \theta_i \end{bmatrix}^T W_i \begin{bmatrix} \omega_{gi} \\ P_{si} - P_{mi} \\ P_{si} - \theta_i \end{bmatrix} \qquad (55)$$
$$- (Q\theta + R - (Q\overline{\theta}^{opt} + R))^T L^{com}(Q\theta + R - (Q\overline{\theta}^{opt} + R))$$
$$\leq 0$$

along the solutions to the power network (3), turbine-governor dynamics (24), (31), and the distributed controllers (49), (50). Particularly, it follows from Assumption 3 that $W_i < 0$. Since $(\overline{\eta}, \overline{\omega} = \mathbf{0}, \overline{P}_m^{opt}, \overline{P}_s^{opt}, \overline{\theta}^{opt})$ is a strict local minimum of $U + \sum_{i\in\mathcal{V}_{g1}} Z_{1i} + \sum_{i\in\mathcal{V}_{g2}} Z_{2i}$ as a consequence of Assumption 2, there exists a compact level set Υ around $(\overline{\eta}, \overline{\omega} = \mathbf{0}, \overline{P}_m^{opt}, \overline{P}_s^{opt}, \overline{\theta}^{opt})$, which is forward invariant. By LaSalle's invariance principle, any solution starting in Υ asymptotically converges to the largest invariant set contained in

$$\Upsilon \cap \{(\eta, \omega, P_m, P_s, \theta):$$
$$\omega = \mathbf{0},\ P_m = \theta,\ Q\theta + R = Q\overline{\theta}^{opt} + R + c\mathbb{1}_{n_g}\} \qquad (56)$$

where $c : \mathbb{R}_{\geq 0} \to \mathbb{R}$ is a scalar, and $Q\theta + R = Q\overline{\theta}^{opt} + R + c\mathbb{1}_{n_g}$ follows from the connectedness of the communication graph. Since $P_m = \theta = \overline{\theta}^{opt} + cQ^{-1}\mathbb{1}_{n_g} = \overline{P}_m^{opt} + cQ^{-1}\mathbb{1}_{n_g}$, the power network satisfies on this invariant set

$$\dot{\eta} = \mathcal{B}^T\mathbf{0}$$
$$\mathbf{0} = -D_g\mathbf{0} - \mathcal{B}_g\Gamma\sin(\eta) + \overline{P}_m^{opt} + cQ^{-1}\mathbb{1}_{n_g}$$
$$\mathbf{0} = -D_l\mathbf{0} - \mathcal{B}_l\Gamma\sin(\eta) - P_l. \qquad (57)$$

Premultiplying the second and third lines of (57) with $\mathbb{1}_n^T$, we have

$$\mathbb{1}_n^T \begin{bmatrix} -D_g\mathbf{0} - \mathcal{B}_g\Gamma\sin(\eta) + \overline{P}_m^{opt} + cQ^{-1}\mathbb{1}_{n_g} \\ -D_l\mathbf{0} - \mathcal{B}_l\Gamma\sin(\eta) - P_l. \end{bmatrix} = 0. \quad (58)$$

Since $\mathbb{1}_n^T \begin{bmatrix} \mathcal{B}_g \\ \mathcal{B}_l \end{bmatrix} = \mathbf{0}$, $\mathbb{1}_{n_g}^T \overline{P}_m^{opt} - \mathbb{1}_{n_l}^T P_l = 0$, and Q^{-1} is a diagonal matrix with only positive elements, it follows that necessarily $c = 0$ and therefore $\theta = \overline{\theta}^{opt}$. We can conclude that the

system indeed converges to the set, where $\omega = \mathbf{0}$ and $P_m = \overline{P}_m^{opt}$, characterized in Lemma 2.   ∎

Remark 8 (Region of attraction): The local nature of our result is a consequence of the considered incremental storage function having a local minimum at the desired steady state. Nevertheless, the provided results are helpful to further characterize various sublevel sets of the incremental storage function (see [49], [61], and [62]), for instance by numerically assessing the sublevel sets that are compact. We leave a thorough analysis of the region of attraction as an interesting future direction.

Remark 9 (Primal-dual based approaches): A popular alternative to the consensus-based algorithm (49), (50) is a primal-dual gradient-based approach. To obtain a distributed solution, optimization problem (9) is replaced[3] by

$$\min_{P_m} C(P_m)$$

$$\text{s.t.} \quad \mathbf{0} = -\mathcal{B}v + \begin{bmatrix} P_m \\ -P_l \end{bmatrix}. \tag{59}$$

The associated Lagrangian function is given by

$$L(P_m, \lambda) = C(P_m) + \lambda^T \left(-\mathcal{B}v + \begin{bmatrix} P_m \\ -P_l \end{bmatrix} \right) \tag{60}$$

where λ is called the Lagrange multiplier. Under convexity of (59), strong duality holds and the solution to (59) is equivalent [63] to the solution to

$$\max_{\lambda} \min_{P_m} L(P_m, \lambda). \tag{61}$$

Following [15]–[17], a continuous primal-dual algorithm can be exploited to solve (61). However, since the evolution of P_m is described by the turbine dynamics, we cannot design its dynamics. Bearing in mind that controller (25) and (32) enforce a steady state where $P_m = \theta$, we solve instead

$$\max_{\lambda} \min_{\theta} L(\theta, \lambda) \tag{62}$$

where the dynamics of θ can be freely adjusted. Inspired by the results in [15]–[17], we replace the communication term in (49) and (50)

$$-q_i \sum_{j \in \mathcal{N}_i^{com}} (q_i \theta_i + r_i - (q_j \theta_j + r_j)) \tag{63}$$

by

$$\frac{\partial L}{\partial \theta_i} = -\nabla C_i(\theta_i) + \lambda_i \tag{64}$$

yielding the modified controllers

$$T_{\theta i} \dot{\theta}_i = -\theta_i + P_{mi} - K_i^{-1}(\nabla C_i(\theta_i) - \lambda_i) \quad \forall i \in \mathcal{V}_{g1} \tag{65}$$

$$T_{\theta i} \dot{\theta}_i = -\theta_i + P_{si} - (1 - K_i^{-1})\omega_{gi} \\ - (\nabla C_i(\theta_i) - \lambda_i). \quad \forall i \in \mathcal{V}_{g2} \tag{66}$$

The variables v and λ evolve according to

$$\dot{v} = \frac{\partial L}{\partial v} = -\mathcal{B}^T \lambda$$

$$\dot{\lambda} = -\frac{\partial L}{\partial \lambda} = \mathcal{B}v - \begin{bmatrix} \theta \\ -P_l \end{bmatrix}. \tag{67}$$

The analysis of Theorem 9 can now be repeated with the additional storage term

$$Z_3 = \frac{1}{2}(v - \overline{v})^T (v - \overline{v}) + \frac{1}{2}(\lambda - \overline{\lambda})^T (\lambda - \overline{\lambda}). \tag{68}$$

We notice that in this case only convexity of $C(\cdot)$ is required and that the load P_l appears in (67).

Remark 10 (Load control): Incorporating load control in the LFC has been recently studied in, e.g., [64]–[66] and can be incorporated within the presented framework with minor modifications with respect to the previous discussion. To do so, we modify the dynamics at the load buses $i \in \mathcal{V}_l$ to become

$$\dot{\delta}_i = \omega_{li}$$

$$0 = -D_{li}\omega_{li} \tag{69}$$

$$\quad - \sum_{j \in \mathcal{N}_i} V_i V_j B_{ij} \sin(\delta_i - \delta_j) - P_{li} - u_{li}$$

where u_{li} is the additional controllable load. Associated with every controllable load is a strictly concave benefit function of the form

$$C_i^B(u_{li}) = \frac{1}{2} q_i u_{li}^2 + r_i u_{li} + s_i \tag{70}$$

which is a common approach to quantify the benefit of the consumed power. Instead of minimizing the total generation costs as in (9), we now aim at maximizing the so-called social welfare [67], [68],

$$\max_{u_l, P_m} C^B(u_l) - C(P_m)$$

$$\text{s.t.} \quad 0 = \mathbb{1}_{n_g}^T \overline{P}_m - \mathbb{1}_{n_l}^T (P_l + u_l) \tag{71}$$

where $C^B(u_l) - C(P_m) = \sum_{i \in \mathcal{V}_l} C_i^B(u_{li}) - \sum_{i \in \mathcal{V}_g} C_i(P_{mi})$. Notice that (71) is equivalent to (9) in the absence of controllable loads. A straightforward but remarkable extension of Lemma 3 is that $U(\eta, \overline{\eta}, \omega_g, \overline{\omega}_g)$ as in (19) now satisfies along the solutions to (3) and (69)

$$\dot{U} = -\|\omega_g - \overline{\omega}_g\|_{D_g}^2 - \|\omega_l - \overline{\omega}_l\|_{D_l}^2$$

$$+ (\omega_g - \overline{\omega}_g)^T (P_m - \overline{P}_m)$$

$$- (\omega_l - \overline{\omega}_l)^T (u_l - \overline{u}_l) \tag{72}$$

i.e., the power network is also output strictly incrementally cyclo-passive with respect to the additional input–output pair $(u_l, -\omega_l)$. This property allows us to incorporate load control in the same manner as the generation control. A thorough discussion on all possible load dynamics is outside the scope of this paper, although the considered turbine-governor dynamics can be straightforwardly adapted. In the case there are no restrictions on the design, a possible load controller at the load buses

---

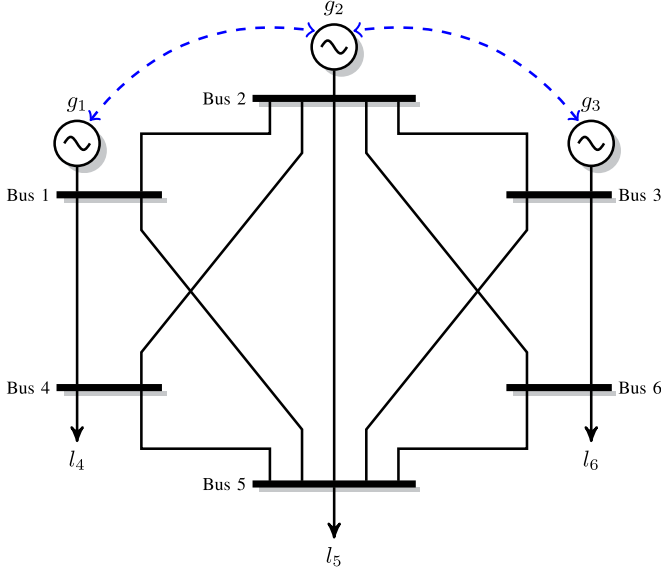[3]See [29, Lemma 4] for a discussion on the equivalence of (9) and (59).

Fig. 1. Diagram for a six-bus power network, consisting of three generator and three load buses. The turbine-governor dynamics of generators are represented by a second-order model. The communication links are represented by the dashed lines.

TABLE III
NUMERICAL VALUES OF THE GENERATOR AND LOAD PARAMETERS

	Bus 1	Bus 2	Bus 3	Bus 4	Bus 5	Bus 6
M_i	4.6	6.2	5.1	–	–	–
D_{gi}	3.4	3.0	4.2	–	–	–
D_{li}	–	–	–	1.0	1.6	1.2
V_i	1.05	0.98	1.04	1.01	1.03	1.00
T_{si}	4.0	4.6	5.0	–	–	–
T_{mi}	5.0	6.7	10.0	–	–	–
K_i	0.5	0.5	0.5	–	–	–
$T_{\theta i}$	0.1	0.1	0.1	–	–	–
q_i	2.4	3.8	3.4	–	–	–
r_i	10.5	5.7	8.9	–	–	–
s_i	9.1	14.4	13.2	–	–	–

TABLE IV
SUSCEPTANCE B_{ij} OF THE TRANSMISSION LINE CONNECTING BUS i AND BUS j

B_{ij} (pu)	1	2	3	4	5	6	j
1	–	−4.0	–	−4.7	−3.1	–	
2	−4.0	–	−3.8	−8.0	−3.0	−4.5	
3	–	−3.8	–	–	−3.2	−9.6	
4	−4.7	−8.0	–	–	−2.0	–	
5	−3.1	−3.0	−3.2	−2.0	–	−3.0	
6	–	−4.5	−9.6	–	−3.0	–	
i							

$i \in \mathcal{V}_l$ is given by

$$T_{\theta i} \dot{\theta}_i = \omega_{li} - q_i \sum_{j \in \mathcal{N}_i^{com}} (q_i \theta_i + r_i - (q_j \theta_j + r_j))$$

$$u_{li} = \theta_{li}. \tag{73}$$

The analysis of Theorem 9 can now be repeated with the additional storage term

$$Z_{3i} = \frac{1}{2} \sum_{i \in \mathcal{V}_l} (\theta_i - \overline{\theta}_i^{opt})^2. \tag{74}$$

Remark 11 (Time-varying loads): Theorem 1 above establishes frequency regulation under the assumption of a constant unknown load P_l. In a realistic setting, the (net) load, including uncontrollable renewable energy generation, is likely to change erratically. Although exact frequency regulation is not possible in that case, the results in this paper are useful to bound the resulting frequency deviation. If a varying load $Q_l(t)$ with finite $\mathcal{L}_2$-norm ($\int_0^\infty \|Q_l(\tau)\|^2 d\tau < \infty$) is added to the load bus, e.g., by taking $u_{li} = Q_{li}(t)$ in (69), it is possible, following [29, Remark 8], to derive from (55) the existence of a finite $\mathcal{L}_2$-to-$\mathcal{L}_\infty$ gain and a finite $\mathcal{L}_2$-to-$\mathcal{L}_2$ gain from the load (disturbance) Q_l to the frequency deviation ω [69].

VI. CASE STUDY

To illustrate the proposed control scheme, we adopt the six-bus system from [4]. Its topology is shown in Fig. 1. The relevant generator and load parameters are provided in Table III, whereas the transmission line parameters are provided in Table IV. The used numerical values are based on [4] and [70]. The turbine-governor dynamics are modeled by the second-order model (31). Every generator is equipped with the con-

troller presented in (50). The communication links between the controllers are also depicted in Fig. 1. The system is initially at the steady state with loads P_{l1}, P_{l2}, and P_{l3} being 1.01, 1.20, and 1.18 p.u., respectively (assuming a base power of 100 MVA). After 10 s, the loads are, respectively, increased to 1.15, 1.25, and 1.21 p.u. From Fig. 2, we can see how the controllers regulate the frequency deviation back to zero. The total generation is shared optimally among the different generators such that (9) is solved.

A. Instability

We now show that a wrongly chosen value for the frequency gain $(1 - K_i^{-1})$ in controller (50) can lead to instability. To do so, we change the controller at generator 3 into

$$T_{\theta_3} \dot{\theta}_3 = -\theta_3 + P_{s3} - 5(1 - K_3^{-1})\omega_{g3}$$
$$- q_3 \sum_{j \in \mathcal{N}_3^{com}} (q_3 \theta_3 + r_3 - (q_j \theta_j + r_j)) \tag{75}$$

for $t > 5$. Leaving all other values identical to the previous simulation, we notice from Fig. 3 that this change at only one generator can cause instability throughout the whole network.

VII. CONCLUSION AND FUTURE RESEARCH

We presented the design of a distributed optimal LFC control architecture that regulates the frequency in the power network, while minimizing the generation costs (or maximizing the social welfare in the case of controllable loads). Based on an energy function of the power network, we derived an
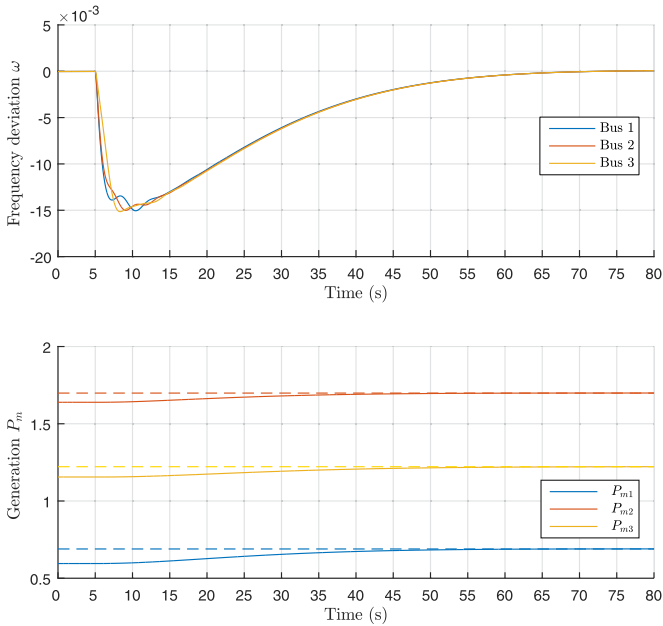
Fig. 2. Frequency response and generated power at the generator buses using the controllers (50). The load is increased at timestep 5, whereafter the frequency deviation is regulated back to zero and generation costs are minimized. The cost minimizing generation $\overline{P}_m^{opt}$ for $t > 5$, characterized in Lemma 2, is given by the dashed lines.
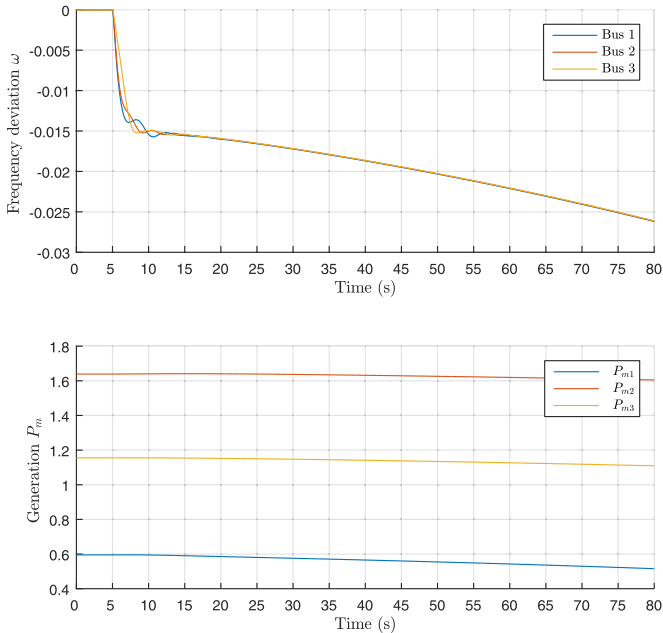

Fig. 3. Frequency response and generated power at the generator buses using the controllers (50). The load is increased at timestep 5 and the controller at generator 3 is replaced by (75). Both the frequency deviation and the power generation become unstable.

incremental passivity property for a well-known structure preserving network model. The passivity property then facilitates the design of distributed controllers that adjust the input to the turbine-governor and load. In this work, we have considered a first-order and a (non-passive) second-order model describing the turbine-governor dynamics. We establish a locally verifiable

range of acceptable droop constants for the second-order model that allows us to infer stability. The presented results provide the opportunity to include the important turbine-governor models in the stability analysis of optimal LFC in a more realistic manner than was previously possible.

There are various interesting extensions to the presented work. We briefly discuss a few. The distributed controllers (49) and (50) are shown to solve optimization problem (9). Minimizing general convex cost functions and satisfying transmission and generator constraints using consensus-based controllers is still an open problem. The distributed control architecture employs a communication network to obtain the desired optimality features. In this work, the communication is assumed to be continuous and instantaneous. It is desirable to relax these communication assumptions. Lyapunov arguments have been used to design distributed event-triggered control algorithms in, e.g., [71] and [72] within a hybrid system framework. A promising research direction is to adapt these results to the present setting. Furthermore, we note that Assumption 3 provides a sufficient condition on the parameters of the second-order turbine-governor model to infer stability of the overall network. Exploring the necessity of this condition, and potentially relaxing it, might offer additional insights on the role of turbine-governor dynamics within (optimal) LFC schemes.

REFERENCES

[1] S. Trip and C. De Persis, "Optimal frequency regulation in nonlinear structure preserving power networks including turbine dynamics: An incremental passivity approach," in *Proc. Amer. Control Conf.*, Boston, MA, USA, 2016, pp. 4132–4137.

[2] S. Trip and C. De Persis, "Optimal generation in structure-preserving power networks with second-order turbine-governor dynamics," in *Proc. 15th Eur. Control Conf.*, Aalborg, Denmark, 2016, pp. 916–921.

[3] J. Machowski, J. Bialek, and D. J. Bumby, *Power System Dynamics: Stability and Control*, 2nd ed. Hoboken, NJ, USA: Wiley, 2008.

[4] A. Wood and B. Wollenberg, *Power Generation, Operation, and Control*, 2nd ed. Hoboken, NJ, USA: Wiley, 1996.

[5] D. Apostolopoulou, A. D. Domnguez-Garca, and P. W. Sauer, "An assessment of the impact of uncertainty on automatic generation control systems," *IEEE Trans. Power Syst.*, vol. 31, no. 4, pp. 2657–2665, Jul. 2016.

[6] S. K. Pandey, S. R. Mohanty, and N. Kishor, "A literature survey on load frequency control for conventional and distribution generation power systems," *Renew. Sustain. Energy Rev.*, vol. 25, pp. 318–334, 2013.

[7] Ibraheem, P. Kumar, and D. P. Kothari, "Recent philosophies of automatic generation control strategies in power systems," *IEEE Trans. Power Syst.*, vol. 20, no. 1, pp. 346–357, Feb. 2005.

[8] C. T. Pan and C. M. Liaw, "An adaptive controller for power system load-frequency control," *IEEE Trans. Power Syst.*, vol. 4, no. 1, pp. 122–128, Feb. 1989.

[9] Q. Liu and M. D. Iliç, "Enhanced automatic generation control (E-AGC) for future electric energy systems," in *Proc. IEEE Power Energy Soc. Gen. Meeting*, 2012, pp. 1–8.

[10] C. K. Zhang, L. Jiang, Q. H. Wu, Y. He, and M. Wu, "Delay-dependent robust load frequency control for time delay power systems," *IEEE Trans. Power Syst.*, vol. 28, no. 3, pp. 2192–2201, Aug. 2013.

[11] Y. Mi, Y. Fu, C. Wang, and P. Wang, "Decentralized sliding mode load frequency control for multi-area power systems," *IEEE Trans. Power Syst.*, vol. 28, no. 4, pp. 4301–4309, Nov. 2013.

[12] H. A. Yousef, K. AL-Kharusi, M. H. Albadi, and N. Hosseinzadeh, "Load frequency control of a multi-area power system: An adaptive fuzzy logic approach," *IEEE Trans. Power Syst.*, vol. 29, no. 4, pp. 1822–1830, Jul. 2014.

[13] M. H. Variani and K. Tomsovic, "Distributed automatic generation control using flatness-based approach for high penetration of wind generation," *IEEE Trans. Power Syst.*, vol. 28, no. 3, pp. 3002–3009, Aug. 2013.

[14] Y. Xu, F. Li, Z. Jin, and M. H. Variani, "Dynamic gain-tuning control (DGTC) approach for AGC with effects of wind power," *IEEE Trans. Power Syst.*, vol. 31, no. 5, pp. 3339–3348, Sep. 2016.

[15] X. Zhang and A. Papachristodoulou, "A real-time control framework for smart power networks: Design methodology and stability," *Automatica*, vol. 58, pp. 43–50, 2015.

[16] N. Li, C. Zhao, and L. Chen, "Connecting automatic generation control and economic dispatch from an optimization view," *IEEE Trans. Control Netw. Syst.*, vol. 3, no. 3, pp. 254–264, Sep. 2016.

[17] T. Stegink, C. De Persis, and A. van der Schaft, "A unifying energy-based approach to stability of power grids with market dynamics," *IEEE Trans. Autom. Control*, 2016, to be published.

[18] S. You and L. Chen, "Reverse and forward engineering of frequency control in power networks," in *Proc. 53rd IEEE Conf. Decision Control*, 2014, pp. 191–198.

[19] A. Kasis, E. Devane, C. Spanias, and I. Lestas, "Primary frequency regulation with load-side participation—Part I: Stability and optimality," *IEEE Trans. Power Syst.*, 2016, to be published.

[20] A. Jokic, M. Lazar, and P. van den Bosch, "Real-time control of power systems using nodal prices," *Int. J. Elect. Power Energy Syst.*, vol. 31, no. 9, pp. 522–530, 2009.

[21] R. Mudumbai, S. Dasgupta, and B. B. Cho, "Distributed control for optimal economic dispatch of a network of heterogeneous power generators," *IEEE Trans. Power Syst.*, vol. 27, no. 4, pp. 1750–1760, Nov. 2012.

[22] Z. Miao and L. Fan, "Achieving economic operation and secondary frequency regulation simultaneously through local feedback control," *IEEE Trans. Power Syst.*, vol. 32, no. 1, pp. 85–93, Jan. 2017.

[23] D. Cai, E. Mallada, and A. Wierman, "Distributed optimization decomposition for joint economic dispatch and frequency regulation," in *Proc. 54th IEEE Conf. Decision Control*, 2015, pp. 15–22.

[24] D. Apostolopoulou, P. W. Sauer, and A. D. Domnguez-Garca, "Distributed optimal load frequency control and balancing authority area coordination," in *Proc. North Amer. Power Symp.*, 2015, pp. 1–5.

[25] P. Yi, Y. Hong, and F. Liu, "Distributed gradient algorithm for constrained optimization with application to load sharing in power systems," *Syst. Control Lett.*, vol. 83, pp. 45–52, 2015.

[26] A. Cherukuri and J. Cortés, "Initialization-free distributed coordination for economic dispatch under varying loads and generator commitment," *Automatica*, vol. 74, pp. 183–193, 2016.

[27] P. Yi, Y. Hong, and F. Liu, "Initialization-free distributed algorithms for optimal resource allocation with feasibility constraints and application to economic dispatch of power systems," *Automatica*, vol. 74, pp. 259–269, 2016.

[28] M. Bürger, C. De Persis, and S. Trip, "An internal model approach to (optimal) frequency regulation in power grids," in *Proc. 21th Int. Symp. Math. Theory Netw. Syst.*, Groningen, the Netherlands, 2014, pp. 577–583.

[29] S. Trip, M. Bürger, and C. De Persis, "An internal model approach to (optimal) frequency regulation in power grids with time-varying voltages," *Automatica*, vol. 64, pp. 240–253, 2016.

[30] J. Schiffer and F. Dörfler, "On stability of a distributed averaging pi frequency and active power controlled differential-algebraic power system model," in *Proc. 15th Eur. Control Conf*, Aalborg, Denmark, 2016, pp. 1487–1492.

[31] C. Zhao, E. Mallada, and F. Dörfler, "Distributed frequency control for stability and economic dispatch in power networks," in *Proc. Amer. Control Conf.*, 2015, pp. 2359–2364.

[32] N. Monshizadeh, C. De Persis, A. J. van der Schaft, and J. Scherpen, "A novel reduced model for electrical networks with constant power loads," arXiv preprint arXiv:1512.08250, 2016 (*abridged version in the Proc. 2016 Amer. Control Conf.*, pp. 3644–3649).

[33] M. Andreasson, D. V. Dimarogonas, K. H. Johansson, and H. Sandberg, "Distributed vs. centralized power systems frequency control," in *Proc. 12th Eur. Control Conf.*, 2013, pp. 3524–3529.

[34] S. Kar and G. Hug, "Distributed robust economic dispatch in power systems: A consensus innovations approach," in *Proc. IEEE Power Energy Soc. Gen. Meeting*, 2012, pp. 1–8.

[35] G. Binetti, A. Davoudi, F. L. Lewis, D. Naso, and B. Turchiano, "Distributed consensus-based economic dispatch with transmission losses," *IEEE Trans. Power Syst.*, vol. 29, no. 4, pp. 1711–1720, Jul. 2014.

[36] N. Rahbari-Asr, U. Ojha, Z. Zhang, and M. Y. Chow, "Incremental welfare consensus algorithm for cooperative distributed generation/demand response in smart grid," *IEEE Trans. Smart Grid*, vol. 5, no. 6, pp. 2836–2845, Nov. 2014.

[37] S. Yang, S. Tan, and J. X. Xu, "Consensus based approach for economic dispatch problem in a smart grid," *IEEE Trans. Power Syst.*, vol. 28, no. 4, pp. 4416–4426, Nov. 2013.

[38] T. Yang, D. Wu, Y. Sun, and J. Lian, "Minimum-time consensus-based approach for power system applications," *IEEE Trans. Ind. Electron.*, vol. 63, no. 2, pp. 1318–1328, Feb. 2016.

[39] Z. Zhang and M. Y. Chow, "Convergence analysis of the incremental cost consensus algorithm under different communication network topologies in a smart grid," *IEEE Trans. Power Syst.*, vol. 27, no. 4, pp. 1761–1768, Nov. 2012.

[40] F. Alvarado, J. Meng, C. DeMarco, and W. Mota, "Stability analysis of interconnected power systems coupled with market dynamics," *IEEE Trans. Power Syst.*, vol. 16, no. 4, pp. 695–701, Nov. 2001.

[41] T. Stegink, C. De Persis, and A. van der Schaft, "Optimal power dispatch in networks of high-dimensional models of synchronous machines," *arXiv preprint arXiv:1603.06688*, 2016.

[42] A. Bergen and D. Hill, "A structure preserving model for power system stability analysis," *IEEE Trans. Power App. Syst.*, vol. PAS-100, no. 1, pp. 25–35, Jan. 1981.

[43] P. Kundur, N. J. Balu, and M. G. Lauby, *Power System Stability and Control*, vol. 7. New York, NY, USA: McGraw-Hill, 1994.

[44] N. Monshizadeh and C. D. Persis, "Agreeing in networks: Unmatched disturbances, algebraic constraints and optimality," *Automatica*, vol. 75, pp. 63—74, 2017.

[45] J. Schiffer, "Stability and power sharing in microgrids," Ph.D. dissertation, Tech. Univ. Berlin, Berlin, Germany, 2015.

[46] S. Trip, M. Bürger, and C. De Persis, "An internal model approach to frequency regulation in inverter-based microgrids with time-varying voltages," in *Proc.. IEEE 53rd Conf. Dec. Control*, 2014, pp. 223–228.

[47] F. Dörfler, J. Simpson-Porco, and F. Bullo, "Breaking the hierarchy: Distributed control & economic optimality in microgrids," *IEEE Trans. Control Netw. Syst.*, vol. 3, no. 3, pp. 241–253, Sep. 2016.

[48] Q. Shafiee, J. M. Guerrero, and J. C. Vasquez, "Distributed secondary control for islanded microgrids; a novel approach," *IEEE Trans. Power Electron.*, vol. 29, no. 2, pp. 1018–1031, Feb. 2014.

[49] C. De Persis and N. Monshizadeh, "Bregman storage functions for microgrid control," *arXiv preprint arXiv:1510.05811*, 2016.

[50] J. Schiffer, R. Ortega, A. Astolfi, J. Raisch, and T. Sezi, "Conditions for stability of droop-controlled inverter-based microgrids," *Automatica*, vol. 50, no. 10, pp. 2457–2469, 2014.

[51] A. D. Dom ínguez-García, "Models for impact assessment of wind-based power generation on frequency control," in *Control and Optimization Methods for Electric Smart Grids*. Berlin, Germany: Springer, 2012, pp. 149–165.

[52] H. Bevrani, *Robust Power System Frequency Control*, vol. 85. Berlin, Germany: Springer, 2009.

[53] C.-C. Chu and H.-D. Chiang, "Constructing analytical energy functions for network-preserving power system models," *Circuits, Syst. Signal Process.*, vol. 24, no. 4, pp. 363–383, 2005.

[54] K. Padiyar, *Structure Preserving Energy Functions in Power Systems: Theory and Applications*. New York, NY, USA: Taylor & Francis, 2013.

[55] D. J. Hill and P. J. Moylan, "Dissipative dynamical systems: Basic input-output and state properties," *J. Franklin Inst.*, vol. 309, no. 5, pp. 327–357, 1980.

[56] J. Willems, "Dissipative dynamical systems," *Eur. J. Control*, vol. 13, pp. 134–151, 2007.

[57] A. Pavlov and L. Marconi, "Incremental passivity and output regulation," *Syst. Control Lett.*, vol. 57, pp. 400–409, 2008.

[58] F. Dörfler, M. Chertkov, and F. Bullo, "Synchronization in complex oscillator networks and smart grids," *Proc. Nat. Acad. Sci.*, vol. 110, no. 6, pp. 2005–2010, 2013.

[59] North American Electric Reliability Corporation (NERC), *Phase Angle Monitoring: Industry Experience Following the 2011 Pacific Southwest Outage Recommendation 27*, 2016.

[60] L. Bregman, "The relaxation method of finding the common point of convex sets and its application to the solution of problems in convex programming," *USSR Comput. Math. Math. Phys.*, vol. 7, no. 3, pp. 200–217, 1967.

[61] K. Dvijotham, S. Low, and M. Chertkov, "Convexity of energy-like functions: Theoretical results and applications to power system operations," *arXiv preprint arXiv:1501.04052*, 2015.

[62] T. L. Vu and K. Turitsyn, "Lyapunov functions family approach to transient stability assessment," *IEEE Trans. Power Syst.*, vol. 31, no. 2, pp. 1269–1277, Mar. 2016.

[63] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, U.K.: Cambridge Univ. Press, 2004.

[64] C. Zhao, U. Topcu, and S. H. Low, "Optimal load control via frequency measurement and neighborhood area communication," *IEEE Trans. Power Syst.*, vol. 28, no. 4, pp. 3576–3587, Nov. 2013.

[65] L. Chen, N. Li, L. Jiang, and S. H. Low, "Optimal demand response: Problem formulation and deterministic case," in *Control and Optimization Methods for Electric Smart Grids*, 2012, Berlin, Germany: Springer, pp. 63–85.

[66] S. Weckx, R. D'Hulst, and J. Driesen, "Primary and secondary frequency support by a multi-agent demand control system," *IEEE Trans. Power Syst.*, vol. 30, no. 3, pp. 1394–1404, May 2015.

[67] A. W. Berger and F. C. Schweppe, "Real time pricing to assist in load frequency control," *IEEE Trans. Power Syst.*, vol. 4, no. 3, pp. 920–926, Aug. 1989.

[68] A. Kiani and A. Annaswamy, "Wholesale energy market in a smart grid: Dynamic modeling and stability," in *Proc. 50th IEEE Conf. Decision Control/Eur. Control Conf.*, 2011, pp. 2202–2207.

[69] P. Kundur *et al.*, "Definition and classification of power system stability IEEE/CIGRE joint task force on stability terms and definitions," *IEEE Trans. Power Syst.*, vol. 19, no. 3, pp. 1387–1401, Aug. 2004.

[70] A. N. Venkat, I. A. Hiskens, J. B. Rawlings, and S. J. Wright, "Distributed MPC strategies with application to power system automatic generation control," *IEEE Trans. Control Syst. Technol.*, vol. 16, no. 6, pp. 1192–1206, Nov. 2008.

[71] C. De Persis and R. Postoyan, "A Lyapunov redesign of coordination algorithms for cyber-physical systems," *IEEE Trans. Autom. Control*, vol. 62, no. 2, pp. 808–823, Feb. 2016.

[72] R. Postoyan, P. Tabuada, D. Nešić, and A. Anta, "A framework for the event-triggered stabilization of nonlinear systems," *IEEE Trans. Autom. Control*, vol. 60, no. 4, pp. 982–996, Apr. 2015.

Sebastian Trip received the B.Sc. degree in physics and the M.Sc. degree in energy and environmental sciences from the University of Groningen, Groningen, The Netherlands, in 2012 and 2013, respectively, where he is working toward the Ph.D. degree in systems theory.

His main research interests include distributed control and optimization, with applications to energy and cyber-physical systems.

Claudio De Persis received the Ph.D. degree in system engineering from the University of Rome "La Sapienza," Rome, Italy, in 2000.

He is currently a Professor with the Engineering and Technology Institute, Faculty of Science and Engineering, University of Groningen, Groningen, The Netherlands. Previously, he was with the Department of Mechanical Automation and Mechatronics, University of Twente, Twente, and with the Department of Computer, Control, and Management Engineering, University of Rome "La Sapienza." He was a Research Associate at the Department of Systems Science and Mathematics, Washington University, St. Louis, MO, USA, in 2000–2001, and at the Department of Electrical Engineering, Yale University, New Haven, CT, USA, in 2001–2002. His main research interests include control theory, and his recent research focuses on dynamical networks, cyberphysical systems, smart grids, and resilient control.

Dr. De Persis was/has been an Editor of the *International Journal of Robust and Nonlinear Control* (2006–2013) and an Associate Editor of the IEEE TRANSACTIONS ON CONTROL SYSTEMS TECHNOLOGY (2010–2015), the IEEE TRANSACTIONS ON AUTOMATIC CONTROL (2012–2015), *Automatica* (since 2013), and the IEEE CONTROL SYSTEMS LETTERS (since 2017).