Distributed Space–Time-Coded Protocols for Exploiting Cooperative Diversity in Wireless Networks

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***Abstract*—We develop and analyze space–time coded cooperative diversity protocols for combating multipath fading across multiple protocol layers in a wireless network. The protocols exploit spatial diversity available among a collection of distributed terminals that relay messages for one another in such a manner that the destination terminal can average the fading, even though it is unknown *a priori* which terminals will be involved. In particular, a source initiates transmission to its destination, and many relays potentially receive the transmission. Those terminals that can fully decode the transmission utilize a space-time code to cooperatively relay to the destination. We demonstrate that these protocols achieve full spatial diversity in the number of cooperating terminals, not just the number of decoding relays, and can be used effectively for higher spectral efficiencies than repetition-based schemes. We discuss issues related to space–time code design for these protocols, emphasizing codes that readily allow for appealing distributed versions.**

***Index Terms*—Diversity techniques, fading channels, outage probability, relay channel, user cooperation, wireless networks.**

I. Introduction

IN wireless networks, signal fading arising from multipath propagation is a particularly severe form of interference that can be mitigated through the use of *diversity*—transmission of redundant signals over essentially independent channel realizations in conjunction with suitable receiver combining to average the channel effects. Space, or multiple-antenna, diversity techniques are particularly attractive as they can be readily combined with other forms of diversity, e.g., time and frequency diversity, and still offer dramatic performance gains when other forms of diversity are unavailable. In contrast to the more conventional forms of single-user space diversity with physical arrays—co-located antenna elements connected via high-bandwidth cabling—this work builds upon the classical relay channel model [1] and examines the problem of creating and exploiting space diversity using a collection of distributed antennas belonging to multiple users, each with their own information to transmit. We refer to this form of space diversity as *cooperative diversity* (cf. *user cooperation diversity* of [2]) because the terminals share their antennas and other resources to create a "virtual array" through distributed transmission and signal processing.

In [3], [4], we develop various cooperative diversity algorithms for a pair of terminals based upon relays amplifying their received signals or fully decoding and repeating information. We refer to these algorithms as *amplify-and-forward* and *decode-and-forward*, respectively. In this paper, we extend these algorithms to combat multipath fading in larger networks. Full spatial diversity benefits of these *repetition-based cooperative diversity algorithms*, as we refer to them throughout this paper, come at a price of decreasing bandwidth efficiency with the number of cooperating terminals, because each relay requires its own subchannel for repetition. As in [3], [4], limited feedback from the destination terminal provides one means of overcoming such bandwidth inefficiencies, but we do not repeat the analysis here. Instead, we develop in this paper an alternative approach to improving bandwidth efficiency of the algorithms based upon space–time codes that allow all relays to transmit on the same subchannel. Requiring more computational complexity in the terminals, we will see these *space–time-coded cooperative diversity algorithms* also offer full spatial diversity benefits without requiring feedback. Both repetition-based and space–time-coded cooperative diversity are amenable to distributed implementation.

We consider a wireless network with a set of transmitting terminals denoted $\mathcal{M} = \{1, 2, \ldots, m\}$. Each transmitting source terminal $s \in \mathcal{M}$ has information to transmit to a single destination terminal, denoted $d(s) \notin \mathcal{M}$, potentially using terminals $\mathcal{M} - \{s\}$ as relays. Thus, there are m cooperating terminals communicating to $d(s)$. For algorithms in which we require the relays to fully decode the source message, we define the *decoding set* $\mathcal{D}(s)$ to be the set of relays that can decode the message of source s. In the case of amplify-and-forward cooperative diversity, we take $\mathcal{D}(s) = \mathcal{M} - \{s\}$. Although throughout the paper we point out the algorithmic differences between decode-and-forward and amplify-and-forward protocols that are repetition-based, we note that the analysis to follow focuses on decode-and-forward. Using the tools developed in [5], together

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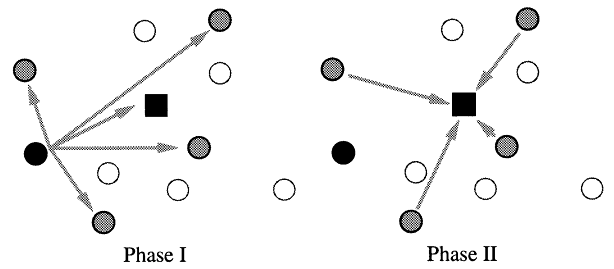

Fig. 1. Illustration of the two phases of repetition-based and space–time-coded cooperative diversity algorithms. In the first phase, the source broadcasts to the destination as well as potential relays. Decoding relays are shaded. In the second phase, the decoding relays either repeat on orthogonal subchannels or utilize a space–time code to simultaneously transmit to the destination.

with the results of [3], [4], the analysis can be extended to amplify-and-forward, for which similar performance characteristics can be obtained.

Both classes of algorithms consist of two transmission phases, as in [3], [4]. Fig. 1 illustrates these two phases, and allows us to point out the similarities and differences between the algorithms. In the first phase, the source broadcasts to its destination and all potential relays. During the second phase of the algorithms, the other terminals relay to the destination, either on orthogonal subchannels in the case of repetition-based cooperative diversity, or simultaneously on the same subchannel in the case of space–time-coded cooperative diversity.

To summarize our results, we show the outage probability performance of repetition-based cooperative diversity decays asymptotically in $\mathtt{SNR}$ proportional to $1/\mathtt{SNR}^{m(1-m\mathtt{R}_{\text{norm}})}$, where $\mathtt{SNR}$ corresponds to the average signal-to-noise ratio (SNR) between terminals, and $0 < \mathtt{R}_{\text{norm}} < 1/m$ corresponds to a suitably-normalized spectral efficiency of the protocol. In this context, full diversity refers to the fact that, as $\mathtt{R}_{\text{norm}} \to 0$, the outage probability decays proportional to $1/\mathtt{SNR}^m$. By contrast, the outage probability performance of noncooperative transmission decays asymptotically as $1/\mathtt{SNR}^{(1-\mathtt{R}_{\text{norm}})}$, where $0 < \mathtt{R}_{\text{norm}} < 1$ is allowed, and as $1/\mathtt{SNR}$ as $\mathtt{R}_{\text{norm}} \to 0$. Thus, while the outage probability performance of cooperative diversity can decay faster, it does so only for small $\mathtt{R}_{\text{norm}}$, in particular, for $\mathtt{R}_{\text{norm}} < 1/(m+1)$. For $\mathtt{R}_{\text{norm}} > 1/(m+1)$, the inherent bandwidth inefficiency of repetition-based cooperative diversity outweighs the benefits of diversity gains, so that noncooperative transmission is preferable in this regime.

Of course, there are more general forms of decode-and-forward transmission than repetition, just as there are more general forms of space–time codes. Space–time-coded cooperative transmission leads to schemes for which outage probability performance decays asymptotically as $1/\mathtt{SNR}^{m(1-2\mathtt{R}_{\text{norm}})}$. Thus, they a) achieve full spatial diversity order m as $\mathtt{R}_{\text{norm}} \to 0$, b) have larger diversity order than repetition-based algorithms for all $\mathtt{R}_{\text{norm}}$, and c) are preferable to noncooperative transmission if $\mathtt{R}_{\text{norm}} < (m-1)/(2m-1)$. Moreover, we will see that these protocols may be readily implemented in a distributed fashion, because they only require the relays to estimate the SNR of their received signals, decode them if the SNR is sufficiently high, re-encode with the appropriate waveform from a space–time code, and retransmit in the same subchannel.

II. System Model

This section highlights the system model that we employ to develop extensions of the repetition-based algorithms in [3], [4] as well as the space–time-coded cooperative diversity algorithms. Differences between the model employed here and the one employed in [3], [4] include a larger number of terminals and different medium-access control protocols for repetition-based and space–time-coded cooperative diversity. As a result, in this section, we only summarize the fundamental elements of the system model.

Narrow-band transmissions suffer the effects of frequency nonselective Rayleigh fading and additive white Gaussian noise (AWGN). We consider the scenario in which the receivers can accurately measure the realized fading coefficients in their received signals, but the transmitters either do not possess or do not exploit knowledge of the realized fading coefficients. As in [3], [4], we focus on the case of slow fading and measure performance by outage probability to isolate the benefits of space diversity. We utilize a baseband-equivalent, discrete-time channel model for the continuous-time channel.

A. Medium-Access Control

For medium-access control, terminals transmit on essentially orthogonal channels as in many current wireless networks. As a baseline for comparison, Fig. 2 illustrates example channel allocations for noncooperative transmission, in which each transmitting terminal utilizes a fraction $1/m$ of the total degrees of freedom in the channel.

For cooperative diversity transmission, the medium-access control protocol also manages orthogonal relaying to ensure that terminals satisfy the half-duplex constraint and do not transmit and receive simultaneously on the same subchannel. Note that these are the same basic restrictions on medium-access control protocols described in [3], [4]. We now describe how the medium-access control protocol differs under repetition-based and space–time-coded cooperative diversity.

Fig. 3 illustrates example channel and subchannel allocations for repetition-based cooperative diversity, in which relays either amplify what they receive or fully decode and repeat the source signal, as in [3], [4]. In order for the destination to combine these signals and achieve diversity gains, the repetitions must occur on essentially orthogonal subchannels. For simplicity, Fig. 3 shows channel allocations for different source terminals across frequency, and subchannel allocations for different relays across time. More generally, for a given source s and destination $d(s)$, the relays $\mathcal{M} - \{s\}$ can repeat in any predetermined order. Arbitrary permutations of these allocations in time and frequency do not alter the conclusions to follow, as long as causality is preserved and each of the subchannels contains a fraction $1/m^2$ of the total degrees of freedom in the channel. As in noncooperative transmission, transmission between source s and destination $d(s)$ utilizes a fraction $1/m$ of the total degrees of freedom in the channel. Similarly, each cooperating terminal transmits in a fraction $1/m$ of the total degrees of freedom.

Fig. 4 illustrates example channel and subchannel allocations for space–time-coded cooperative diversity, in which relays

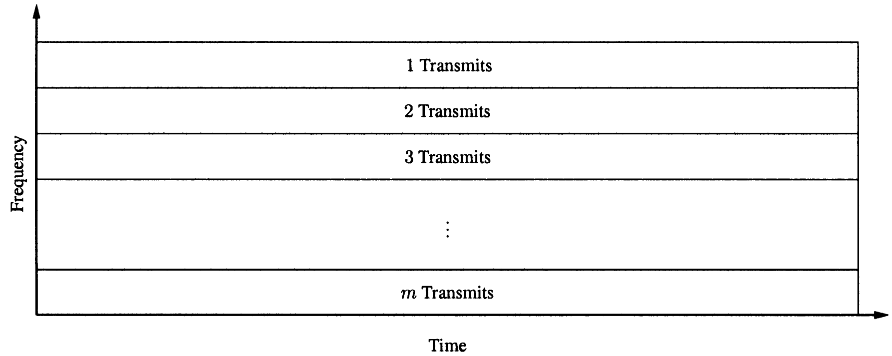

Fig. 2. Noncooperative medium-access control. Example source allocations among m transmitting terminals across orthogonal frequency channels.

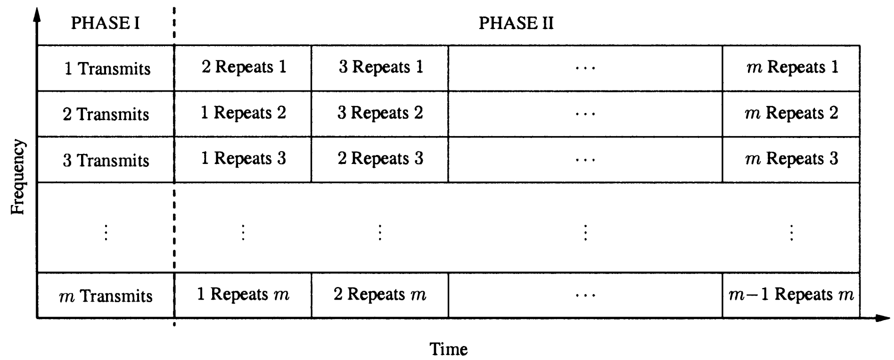

Fig. 3. Repetition-based medium-access control. Example source channel allocations across frequency and relay subchannel allocations across time for repetition-based cooperative diversity among m terminals.

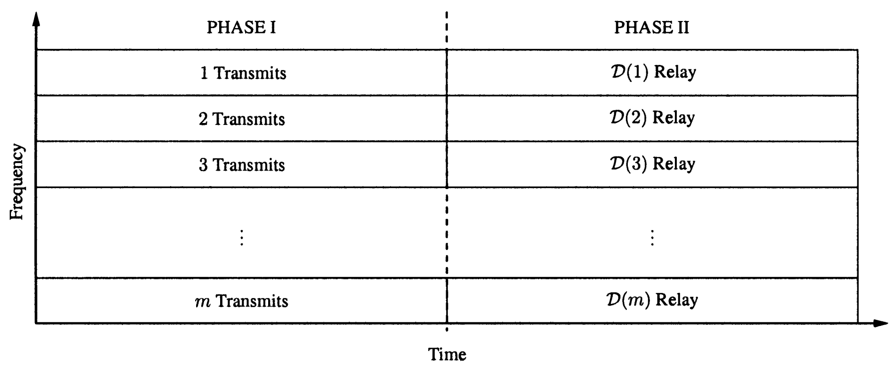

Fig. 4. Space–time-coded medium-access control. Example channel allocations across frequency and time for m transmitting terminals. For source s, $\mathcal{D}(s)$ denotes the set of decoding relays participating in a space–time code during the second phase.

utilize a suitable space–time code in the second phase and therefore can transmit simultaneously on the same subchannel. Again, transmission between source s and destination $d(s)$ utilizes $1/m$ of the total degrees of freedom in the channel. However, in contrast to noncooperative transmission and repetition-based cooperative diversity transmission, each terminal employing space–time-coded cooperative diversity transmits in $1/2$ the total degrees of freedom in the channel. It is important to keep track of these ratios when normalizing power and bandwidth in the sequel.

B. Equivalent Channel Models

Under the above orthogonality constraints, we can now conveniently, and without loss of generality, characterize our channel models. Due to symmetry of the channel allocations, we focus on transmission of a message from source s to its destination $d(s)$ using terminals $\mathcal{M} - \{s\}$ as relays.

During the first phase, each potential relay $r \in \mathcal{M} - \{s\}$ receives

$$\mathsf{y}_r[n] = \mathsf{a}_{s,r}\mathsf{x}_s[n] + \mathsf{z}_r[n] \tag{1}$$

in the appropriate subchannel, where $\mathsf{x}_s[n]$ is the source transmitted signal and $\mathsf{y}_r[n]$ is the received signal at r. For decode-and-forward transmission, if the SNR is sufficiently large for r to decode the source transmission, then r serves as a decoding relay for the source s, so that $r \in \mathcal{D}(s)$. Again, for amplify-and-forward transmission, we can think of $\mathcal{D}(s)$ as being the entire set of relays for source s, i.e., $\mathcal{D}(s) = \mathcal{M} - \{s\}$.

The destination receives signals during both phases. During the first phase, we model the received signal at $d(s)$ as

$$\mathsf{y}_{d(s)}[n] = \mathsf{a}_{s,\, d(s)}\mathsf{x}_s[n] + \mathsf{z}_{d(s)}[n] \tag{2}$$

in the appropriate subchannel. During the second phase, the equivalent channel models are different for repetition-based and space–time-coded cooperative diversity. For repetition-based cooperative diversity, the destination receives separate retransmissions from each of the relays, i.e., for $r \in \mathcal{M} - \{s\}$, we model the received signal at $d(s)$ as

$$\mathsf{y}_{d(s)}[n] = \mathsf{a}_{r,\, d(s)}\mathsf{x}_r[n] + \mathsf{z}_{d(s)}[n] \tag{3}$$

in the appropriate subchannel, where $\mathsf{x}_r[n]$ is the transmitted signal of relay r. For space–time-coded cooperative diversity, all of the relay transmissions occur in the same subchannel and superimpose at the destination, so that

$$\mathsf{y}_{d(s)}[n] = \sum_{r \in \mathcal{D}(s)} \mathsf{a}_{r,\, d(s)}\mathsf{x}_r[n] + \mathsf{z}_{d(s)} \tag{4}$$

in the appropriate subchannel.

In (1)–(4), $\mathsf{a}_{i,\,j}$ captures the effects of path loss, shadowing, and frequency nonselective fading, and $\mathsf{z}_j[n]$ captures the effects of receiver noise and other forms of interference in the system. Note that all the fading coefficients are constant over the example time and frequency axes shown in Figs. 2–4. We focus on the scenario in which the fading coefficients are known to, i.e., accurately measured by, the appropriate receivers, but not fully known to, or not exploited by, the transmitters. Statistically, we model $\mathsf{a}_{i,\,j}$ as zero-mean, independent, circularly symmetric complex Gaussian random variables with variances $1/\lambda_{i,\,j}$, so that the magnitudes $|\mathsf{a}_{i,\,j}|$ are Rayleigh distributed ($|\mathsf{a}_{i,\,j}|^2$ are exponentially distributed with parameter $\lambda_{i,\,j}$) and the phases $\measuredangle \mathsf{a}_{i,\,j}$ are uniformly distributed on $[0,\, 2\pi)$. Furthermore, we model $\mathsf{z}_j[n]$ as zero-mean mutually independent, circularly symmetric, complex Gaussian random sequences with variance N_0.

C. Parameterizations

Two important parameters of the system are the transmit SNR $\mathtt{SNR}$ and the spectral efficiency $\mathtt{R}$. It is natural to define these parameters in terms of standard parameters in the continuous-time channel with noncooperative transmission (cf. Fig. 2) as a baseline.

For a continuous-time channel with total bandwidth W hertz available for transmission, the discrete-time model contains W two-dimensional symbols per second (2-D/s). If the transmitting terminals have an average power constraint in the continuous-time channel model of P_c joules per second (J/s), we see that this translates into a discrete-time power constraint of $P = mP_c/W$ J/2-D, since each terminal transmits in a fraction $1/m$ of the available degrees of freedom for noncooperative transmission (cf. Fig. 2) and repetition-based cooperative diversity (cf. Fig. 3). Thus, the channel model is parameterized by the SNR random variables $\mathtt{SNR}|\mathsf{a}_{i,\,j}|^2$, where

$$\mathtt{SNR} = \frac{mP_c}{N_0 W} = \frac{P}{N_0} \tag{5}$$

is the SNR without fading. For space–time-coded cooperative diversity (cf. Fig. 4), the terminals transmit in half the available degrees of freedom, so the discrete-time power constraint becomes $2P/m$.

In addition to SNR, transmission schemes are further parameterized by the spectral efficiency $\mathtt{R}$ bits per second per hertz (b/s/Hz) attempted by the transmitting terminals. Note that throughout this paper, $\mathtt{R}$ is the transmission rate normalized by the number of degrees of freedom utilized by each terminal under noncooperative transmission, not by the total number of degrees of freedom in the channel.

Nominally, one could parameterize the system by the pair $(\mathtt{SNR},\, \mathtt{R})$; however, our results lend additional insight when we parameterize the system by the pair $(\mathtt{SNR},\, \mathtt{R}_{\text{norm}})$, where[1]

$$\mathtt{R}_{\text{norm}} = \frac{\mathtt{R}}{\log\left(1 + \mathtt{SNR}\,\sigma^2_{s,\, d(s)}\right)}. \tag{6}$$

For an AWGN channel with bandwidth (W/m) and SNR given by $\mathtt{SNR}\,\sigma^2_{s,\,d(s)}$, $\mathtt{R}_{\text{norm}} < 1$ is the spectral efficiency normalized by the *maximum* achievable spectral efficiency, i.e., channel capacity. In our setting with fading, as we increase $\mathtt{SNR}$, the two parameterizations yield tradeoffs between different aspects of system performance: results under $(\mathtt{SNR},\, \mathtt{R})$ exhibit a tradeoff between the normalized SNR gain and spectral efficiency of a protocol, while results under $(\mathtt{SNR},\, \mathtt{R}_{\text{norm}})$ exhibit a tradeoff between the diversity order and normalized spectral efficiency of a protocol [3], [4]. The latter tradeoff, called the *diversity–multiplexing* tradeoff, was developed originally in the context of multiple-antenna systems in [6], [7].

III. Repetition-Based Cooperative Diversity

In this section, we analyze performance of a repetition decode-and-forward cooperative diversity algorithm for more than two terminals. Such protocols consist of the source broadcasting its transmission to its destination and potential relays. Potential relays that can decode the transmission become decoding relays and participate in the second phase of the protocol by repeating the source message on orthogonal subchannels. Although the set of decoding relays $\mathcal{D}(s)$ is a random set, we will see that protocols of this form offer full spatial diversity in the number of cooperating terminals, not just the number of decoding relays participating in the second phase. Interestingly, potential relays that cannot decode contribute as much to the performance of the protocol as the decoding relays, just as in the selection decode-and-forward algorithm developed for two terminals in [3], [4]. We note that similar high-SNR results can be obtained for amplify-and-forward transmission using the appropriate results in [3]–[5].

A. Mutual Information and Outage Probability

Since the channel average mutual information I_{rep} is a function of, e.g., the coding scheme, the rule for including potential relays into the decoding set $\mathcal{D}(s)$, and the fading coefficients of the channel, it too is a random variable. As in [3], [4], the event $\mathsf{I}_{\text{rep}} < \mathtt{R}$ that this mutual information random variable falls

[1] Unless otherwise indicated, logarithms in this paper are taken to base 2.

below some fixed spectral efficiency $\mathtt{R}$ is referred to as an outage event, and the probability of an outage event, $\Pr[\mathsf{I}_{\text{rep}} < \mathtt{R}]$, is referred to as the outage probability of the channel [8].

Since $\mathcal{D}(s)$ is a random set, we utilize the total probability law and write

$$\Pr[\mathsf{I}_{\text{rep}} < \mathtt{R}] = \sum_{\mathcal{D}(s)} \Pr[\mathcal{D}(s)] \Pr[\mathsf{I}_{\text{rep}} < \mathtt{R} | \mathcal{D}(s)]. \tag{7}$$

1) Outage Conditioned on the Decoding Set: For repetition coding, the random codebook at the source is generated independent and identically distributed (i.i.d.) circularly symmetric, complex Gaussian; each of the relays employs the exact same codebook as the source. Conditioned on $\mathcal{D}(s)$ being the decoding set, the mutual information between s and $d(s)$ is

$$\mathsf{I}_{\text{rep}} = \frac{1}{m} \log\left(1 + \mathtt{SNR}|\mathsf{a}_{s,\,d(s)}|^2 + \mathtt{SNR} \sum_{r \in \mathcal{D}(s)} |\mathsf{a}_{r,\,d(s)}|^2\right). \tag{8}$$

Thus, $\Pr[\mathsf{I}_{\text{rep}} < \mathtt{R}|\mathcal{D}(s)]$ involves[2] $|\mathcal{D}(s)| + 1$ independent fading coefficients, so we expect it to decay asymptotically proportional to $1/\mathtt{SNR}^{|\mathcal{D}(s)|+1}$. Indeed, we develop the following high-$\mathtt{SNR}$ approximation[3] in the Appendix:

$$\Pr[\mathsf{I}_{\text{rep}} < \mathtt{R}|\mathcal{D}(s)] \sim \left[\frac{2^{m\mathtt{R}} - 1}{\mathtt{SNR}}\right]^{|\mathcal{D}(s)|+1} \times \lambda_{s,\,d(s)} \prod_{r \in \mathcal{D}(s)} \lambda_{r,\,d(s)} \times \frac{1}{(|\mathcal{D}(s)| + 1)!}. \tag{9}$$

Note that we have expressed (9) in such a way that the first term captures the dependence upon $\mathtt{SNR}$ and the second term captures the dependence upon $\{\lambda_{i,\,j}\}$.

More generally, with the channel allocation illustrated in Fig. 3, the relays could employ independently generated codebooks, corresponding to utilizing parallel channels. In this case, the mutual information would become a sum of logarithmic terms

$$\mathsf{I}_{\text{par}} = \frac{1}{m} \sum_{r \in \{s\} \cup \mathcal{D}(s)} \log\left(1 + \mathtt{SNR}|\mathsf{a}_{r,\,d(s)}|^2\right) \tag{10}$$

instead of the log-sum in (8). By Jensen's inequality, clearly (10) is larger than (8), which means that parallel channel coding is more bandwidth efficient than repetition coding, as we might expect. Although the analysis can be extended to the parallel channel case [3], [4], we focus in this paper on repetition coding because of its low complexity and on space–time-coded cooperative diversity because it offers even larger mutual informations, and therefore enhanced bandwidth efficiency, when compared to (10).

2) Decoding Set Probability: Next, we consider the term $\Pr[\mathcal{D}(s)]$, the probability of a particular decoding set. As one rule for selecting from the potential relays, we can require that a potential relay fully decode the source message in order to participate in the second phase. Indeed, full decoding is required in order for the mutual information expression (8) to be correct; however, nothing prevents us from imposing additional restrictions on the members of the set $\mathcal{D}(s)$. For example, we might require that a potential relay fully decode *and* experience a realized SNR some factor larger than its average, to either the source, the destination, or both.

Since the realized mutual information between s and r for i.i.d. complex Gaussian codebooks is given by

$$\frac{1}{m} \log\left(1 + \mathtt{SNR}|\mathsf{a}_{s,\,r}|^2\right)$$

under this rule we have

$$\begin{aligned}\Pr[r \in \mathcal{D}(s)] &= \Pr[|\mathsf{a}_{s,\,r}|^2 > (2^{m\mathtt{R}} - 1)/\mathtt{SNR}] \\ &= \exp[-\lambda_{s,\,r}(2^{m\mathtt{R}} - 1)/\mathtt{SNR}].\end{aligned}$$

Moreover, since each potential relay makes its decision independently under the above restrictions, and the fading coefficients are independent in our model, we have

$$\begin{aligned}\Pr[\mathcal{D}(s)] &= \prod_{r \in \mathcal{D}(s)} \exp[-\lambda_{s,\,r}(2^{m\mathtt{R}} - 1)/\mathtt{SNR}] \\ &\quad \times \prod_{r \notin \mathcal{D}(s)} (1 - \exp[-\lambda_{s,\,r}(2^{m\mathtt{R}} - 1)/\mathtt{SNR}]) \\ &\sim \left[\frac{2^{m\mathtt{R}} - 1}{\mathtt{SNR}}\right]^{m-|\mathcal{D}(s)|-1} \times \prod_{r \notin \mathcal{D}(s)} \lambda_{s,\,r}.\end{aligned} \tag{11}$$

Note that any selection means by which $\Pr[r \in \mathcal{D}(s)] \sim 1$ and $(1 - \Pr[r \in \mathcal{D}(s)]) \propto 1/\mathtt{SNR}$, for $\mathtt{SNR}$ large, independently for each r, will result in similar asymptotic behavior for $\Pr[\mathcal{D}(s)]$.

Combining (9) and (11) into (7), we obtain

$$\begin{aligned}\Pr[\mathsf{I}_{\text{rep}} < \mathtt{R}] &\sim \left[\frac{2^{m\mathtt{R}} - 1}{\mathtt{SNR}}\right]^m \\ &\quad \times \sum_{\mathcal{D}(s)} \lambda_{s,\,d(s)} \times \prod_{r \in \mathcal{D}(s)} \lambda_{r,\,d(s)} \prod_{r \notin \mathcal{D}(s)} \lambda_{s,\,r} \\ &\quad \times \frac{1}{(|\mathcal{D}(s)| + 1)!}.\end{aligned} \tag{12}$$

Fig. 5 compares the results of numeric integration of the actual outage probability to computing the approximation (12), for an increasing number of terminals with $\lambda_{i,\,j} = 1$. As the result (12) and Fig. 5 indicate, repetition decode-and-forward cooperative diversity achieves full spatial diversity of order m, the number of cooperating terminals, for sufficiently large SNR. However, the SNR loss due to bandwidth inefficiency is exponential in m.

B. Convenient Bounds

While the approximation given in (12) is quite general and can be numerically evaluated to determine performance, it is not very convenient for further analysis. Its complexity results from dependence upon $\{\lambda_{i,\,j}\}$. In this subsection, we develop upper and lower bounds for (12) that we exploit in the sequel.

Our objective is to simplify the summation in (12). To this end, we note that for a given decoding set $\mathcal{D}(s)$, either $r \in \mathcal{D}(s)$, in which case $\lambda_{r,\,d(s)}$ appears in the corresponding term in (12), or $r \notin \mathcal{D}(s)$, in which case $\lambda_{s,\,r}$ appears in the corresponding term in (12). We, therefore, define

$$\underline{\lambda}_r = \min\{\lambda_{r,\,d(s)}, \lambda_{s,\,r}\}, \quad \overline{\lambda}_r = \max\{\lambda_{r,\,d(s)}, \lambda_{s,\,r}\} \tag{13}$$

[2]For a set $\mathcal{S}$, $|\mathcal{S}|$ denotes the cardinality of the set. This should not be confused with the usual notation $|x|$ for absolute value of a variable x.

[3]The approximation $f(\mathtt{SNR}) \sim g(\mathtt{SNR})$ is in the sense of $f(\mathtt{SNR})/g(\mathtt{SNR}) \to 1$ as $\mathtt{SNR} \to \infty$.

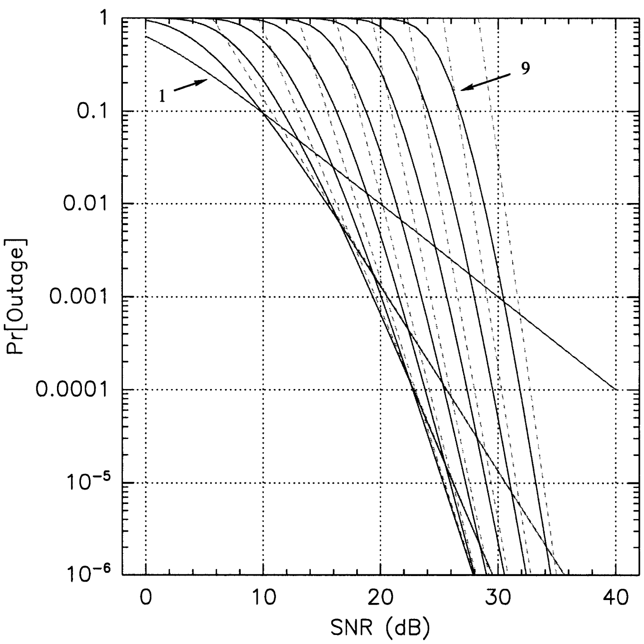

Fig. 5. Outage probabilities for repetition-based cooperative diversity. Comparison of numeric integration of the outage probability (solid lines) to calculation of the outage probability approximation (12) (dashed lines) versus SNR for different network sizes $m = 1, 2, \ldots, 9$. Successive solid curves from left to right at high outage probability correspond to larger networks. For simplicity of exposition, we have plotted the case of $\mathtt{R} = 1$ b/s/Hz and $\lambda_{i,j} = 1$; more generally, the results can be readily updated to incorporate a model of the network geometry.

and $\overline{\lambda}_s = \underline{\lambda}_s = \lambda_{s,d(s)}$. Then the product dependent upon $\{\lambda_{i,j}\}$ is bounded by

$$\underline{\lambda}^m \leq \lambda_{s,d(s)} \prod_{r\in\mathcal{D}(s)} \lambda_{r,d(s)} \prod_{r\notin\mathcal{D}(s)} \lambda_{s,r} \leq \overline{\lambda}^m \tag{14}$$

where $\underline{\lambda}$ is the geometric mean of the $\underline{\lambda}_i$ and $\overline{\lambda}$ is the geometric mean of the $\overline{\lambda}_i$, for $i \in \mathcal{M}$. We note that the upper and lower bounds in (14) are independent of $\mathcal{D}(s)$. We also note that the bounds in (14) coincide, i.e., $\overline{\lambda} = \underline{\lambda}$, if, though not only if, $\overline{\lambda}_i = \underline{\lambda}_i$ for all $i \in \mathcal{M}$. Viewing $\lambda_{i,j}$ as a measure of distance between terminals i and j, the class of planar network geometries that satisfy this condition are those in which all the relays lie with arbitrary spacing along the perpendicular bisector between the source and destination. A complete study of the effects of network geometry on performance is warranted, but beyond the scope of this paper.

Substituting (14) into (12), we arrive at the following simplified asymptotic bounds for outage probability:

$$\Pr[\mathsf{I}_{\text{rep}} < \mathtt{R}] \gtrsim \left[\frac{2^{m\mathtt{R}}-1}{\mathtt{SNR}/\underline{\lambda}}\right]^m \sum_{\mathcal{D}(s)} \frac{1}{(|\mathcal{D}(s)|+1)!} \tag{15}$$

$$\Pr[\mathsf{I}_{\text{rep}} < \mathtt{R}] \lesssim \left[\frac{2^{m\mathtt{R}}-1}{\mathtt{SNR}/\overline{\lambda}}\right]^m \sum_{\mathcal{D}(s)} \frac{1}{(|\mathcal{D}(s)|+1)!}. \tag{16}$$

IV. Space–Time-Coded Cooperative Diversity

In this section, we analyze performance of a decode-and-forward space–time cooperative diversity algorithm. Such protocols operate in similar fashion to the repetition decode-and-forward cooperative diversity algorithm analyzed in the previous section, except that all the relays transmit simultaneously on the same subchannel using a suitable space–time code. Again, we will see that protocols of this form offer full spatial diversity in the number of cooperating terminals, not just the number of decoding relays participating in the second phase. In addition, these algorithms have superior bandwidth efficiency to repetition-based algorithms.

A. Mutual Information and Outage Probability

As before, we utilize the total probability law to write

$$\Pr[\mathsf{I}_{\text{stc}} < \mathtt{R}] = \sum_{\mathcal{D}(s)} \Pr[\mathcal{D}(s)] \Pr[\mathsf{I}_{\text{stc}} < \mathtt{R} | \mathcal{D}(s)] \tag{17}$$

and examine each term in the summation.

1) Outage Conditioned on the Decoding Set: Conditioned on $\mathcal{D}(s)$ being the decoding set, the mutual information between s and $d(s)$ for random codebooks generated i.i.d. circularly symmetric, complex Gaussian at the source and all potential relays can be shown to be

$$\mathsf{I}_{\text{stc}} = \frac{1}{2}\log\left(1 + \frac{2}{m}\mathtt{SNR}|\mathsf{a}_{s,d(s)}|^2\right) + \frac{1}{2}\log\left(1 + \frac{2}{m}\mathtt{SNR}\sum_{r\in\mathcal{D}(s)} |\mathsf{a}_{r,d(s)}|^2\right) \tag{18}$$

the sum of the mutual informations for two "parallel" channels, one from the source to the destination, and one from the set of decoding relays to the destination. Again, $\Pr[\mathsf{I}_{\text{stc}} < \mathtt{R}|\mathcal{D}(s)]$ involves $|\mathcal{D}(s)|+1$ independent fading coefficients, so we expect it to decay asymptotically proportional to $1/\mathtt{SNR}^{|\mathcal{D}(s)|+1}$. We develop the following high-SNR approximation in the Appendix:

$$\Pr[\mathsf{I}_{\text{stc}} < \mathtt{R}|\mathcal{D}(s)] \sim \left[\frac{2^{2\mathtt{R}}-1}{2\mathtt{SNR}/m}\right]^{|\mathcal{D}(s)|+1} \times \lambda_{s,d(s)} \prod_{r\in\mathcal{D}(s)} \lambda_{r,d(s)} \times A_{|\mathcal{D}(s)|}(2^{2\mathtt{R}}-1) \tag{19}$$

where

$$A_n(t) = \frac{1}{(n-1)!}\int_0^1 \frac{w^{(n-1)}(1-w)}{(1+tw)}\,dw, \qquad n > 0 \tag{20}$$

and $A_0(t) = 1$. Note that we have expressed (19) in such a way that the first term captures the dependence upon $\mathtt{SNR}$ and the second term captures the dependence upon $\{\lambda_{i,j}\}$.

2) Decoding Set Probability: Next, we consider the term $\Pr[\mathcal{D}(s)]$, the probability of a particular decoding set. As before, we require that a potential relay fully decode the source message in order to participate in the second phase, a necessary condition for the mutual information expression (18) to be correct.

Since the realized mutual information between s and r for i.i.d. complex Gaussian codebooks is given by

$$\frac{1}{2}\log\left(1 + \frac{2}{m}\mathtt{SNR}|\mathsf{a}_{s,r}|^2\right)$$

under this rule we have

$$\Pr[r \in \mathcal{D}(s)] = \Pr\left[|\mathsf{a}_{s,r}|^2 > \frac{2^{2\mathtt{R}}-1}{2\mathtt{SNR}/m}\right] = \exp\left[-\lambda_{s,r}\,\frac{2^{2\mathtt{R}}-1}{2\mathtt{SNR}/m}\right].$$

Moreover, since each potential relay makes its decision independently, and the fading coefficients are independent in our model, we have

$$\Pr[\mathcal{D}(s)] = \prod_{r\in\mathcal{D}(s)} \exp\left[-\lambda_{s,r}\,\frac{2^{2\mathtt{R}}-1}{2\mathtt{SNR}/m}\right] \times \prod_{r\notin\mathcal{D}(s)}\left(1-\exp\left[-\lambda_{s,r}\,\frac{2^{2\mathtt{R}}-1}{2\mathtt{SNR}/m}\right]\right) \sim \left[\frac{2^{2\mathtt{R}}-1}{2\mathtt{SNR}/m}\right]^{m-|\mathcal{D}(s)|-1} \times \prod_{r\notin\mathcal{D}(s)} \lambda_{s,r}. \quad (21)$$

Combining (19) and (21) into (17), we obtain

$$\Pr[\mathsf{I}_{\rm stc} < \mathtt{R}] \sim \left[\frac{2^{2\mathtt{R}}-1}{2\mathtt{SNR}/m}\right]^m \times \sum_{\mathcal{D}(s)} \lambda_{s,d(s)} \times \prod_{r\in\mathcal{D}(s)} \lambda_{r,d(s)} \prod_{r\notin\mathcal{D}(s)} \lambda_{s,r} \times A_{|\mathcal{D}(s)|}(2^{2\mathtt{R}}-1). \quad (22)$$

Fig. 6 compares the results of numeric integration of the actual outage probability to computing the approximation (22), for an increasing number of terminals with $\lambda_{i,j} = 1$. As the result (22) and Fig. 6 indicate, space–time-coded cooperative diversity achieves full spatial diversity of order m, the number of cooperating terminals, for sufficiently large SNR. In contrast to repetition-based algorithms, the SNR loss for space–time-coded cooperative diversity is only linear in m.

B. Convenient Bounds

Again, although the approximation given in (22) is quite general and can be numerically evaluated to determine performance, it is not very convenient for further analysis. There are two factors contributing to its complexity: dependence upon $\{\lambda_{i,j}\}$, and the involved closed-form expression for $A_n(t)$ as n grows. In this subsection, we develop upper and lower bounds for (22) that we exploit in the sequel.

Our objective is to simplify the summation in (22). The product dependent upon $\{\lambda_{i,j}\}$ can again be bounded as in (14). To avoid dealing with (20), we exploit the bounds

$$\frac{1}{(n+1)!(1+t)} \le A_n(t) \le \frac{1}{n!}. \quad (23)$$

Combining (14) and (23) into (22), we arrive at the following simplified asymptotic bounds for outage probability:

$$\Pr[\mathsf{I}_{\rm stc} < \mathtt{R}] \gtrsim \left[\frac{2^{2\mathtt{R}}-1}{2\mathtt{SNR}/(m\underline{\lambda})}\right]^m 2^{-2\mathtt{R}} \sum_{\mathcal{D}(s)} \frac{1}{(|\mathcal{D}(s)|+1)!} \quad (24)$$

$$\Pr[\mathsf{I}_{\rm stc} < \mathtt{R}] \lesssim \left[\frac{2^{2\mathtt{R}}-1}{2\mathtt{SNR}/(m\overline{\lambda})}\right]^m \sum_{\mathcal{D}(s)} \frac{1}{|\mathcal{D}(s)|!}. \quad (25)$$

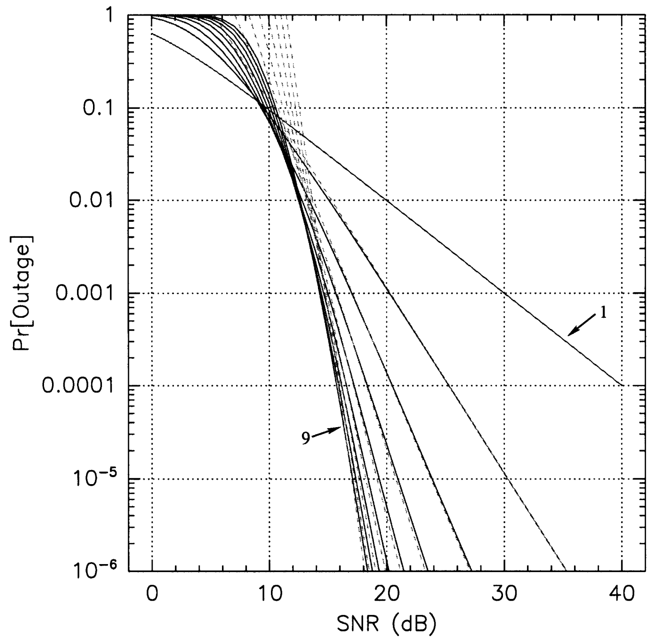

Fig. 6. Outage probability of space–time-coded cooperative diversity. Comparison of numeric integration of the outage probability (solid lines) to calculation of the outage probability approximation (22) (dashed lines) versus SNR for different network sizes $m = 1, 2, \ldots, 9$. Successive solid curves from right to left at low outage probability correspond to larger networks. For simplicity of exposition, we have plotted the case of $\mathtt{R} = 1$ b/s/Hz and $\lambda_{i,j} = 1$; more generally, the results can be readily updated to incorporate a model of the network geometry.

C. Practical Issues

1) Space–Time Code Design: The outage analysis in Section IV relies on a random coding argument, and demonstrates that full spatial diversity can be achieved using such a rich set of codes. In practice, one may wonder whether or not there exist space–time codes for which the number of participating antennas is not known *a priori* and yet full diversity can be achieved. More specifically, if we design a space–time code for a maximum of N transmit antennas, but only a randomly selected subset of n of those antennas actually transmit, can the space–time code offer diversity n? It turns out that the class of space–time block codes based upon orthogonal designs [9], [10] have this property [11]. Essentially, these codes have orthogonal waveforms emitted from each antenna, corresponding to columns in a code matrix. Absence of an antenna corresponds to deletion of a column in the matrix, analogous to that antenna experiencing a deep fade, but the columns remain orthogonal, allowing the code to maintain its residual diversity benefits. Thus, space–time-coded cooperative diversity protocols may be readily deployed in practice using such codes.

We note that there are issues with space–time codes based upon orthogonal designs, specifically nonexistence of "rate-one" codes for more than two antennas, limited capacity with multiple receive antennas, and so forth [9], [10], [12]. On the other hand, orthogonal designs do offer full diversity benefits given a single receive antenna, which is the scenario of interest in this paper, and have the useful property that they maintain full residual diversity benefits as discussed above. It may be that other practical space–time codes have this property as well. Our purpose in adding some discussion on these issues is to point out the potential for leveraging existing space–time code designs as well as to spark interest in code design for cooperative problems.

2) Distributed Implementation: Given a suitably designed space–time code, space–time-coded cooperative diversity reduces to a simple, distributed network protocol. The network must possess a means for distributing columns from the code matrix to the terminals, as well as coordinating the medium-access control. With these elements in place, when each terminal transmits its message, the other terminals receive and potentially decode, requiring only an SNR measurement. If a relay can decode, it transmits the information in the second phase using its column from the space–time code matrix. Because the destination receiver can measure the fading, it can determine which relays are involved in the second phase and adapt its decoding rule appropriately. Although certainly the terminals could exchange more information in order to adapt power to the network geometry, for example, such overhead is not required in order to obtain full diversity.

One of the key challenges to implementing such protocols could be block and symbol synchronization of the cooperating terminals. Such synchronization might be obtained through periodic transmission of known synchronization prefixes, as proposed in current wireless local-area network (LAN) standards [13]. A detailed study of issues involved with synchronization is beyond the scope of this paper.

V. Diversity–Multiplexing Tradeoff

In Sections III and IV, we parameterize performance by the pair $(\mathtt{SNR}, \mathtt{R})$. We interpreted the results for fixed $\mathtt{R}$ and growing $\mathtt{SNR}$, as is typically done from a communication-theoretic view for a fixed-rate system operating over a variety of channel conditions. Since the mutual information generally increases with increasing $\mathtt{SNR}$, another possibility is to increase $\mathtt{R}$ with $\mathtt{SNR}$. It turns out that increasing the rate according to

$$\mathtt{R} = \mathtt{R}_{\text{norm}} \log\left(1 + \mathtt{SNR}\, \sigma^2_{s,\, d(s)}\right) \tag{26}$$

where $0 < \mathtt{R}_{\text{norm}} < 1$ is a constant, leads to an illuminating tradeoff between the reliability with which data can be received and the ability to transmit more data. This tradeoff as a function of $\mathtt{R}_{\text{norm}}$ has been called the *diversity–multiplexing* tradeoff for multiple-antenna systems [6], [7]. In this section, we extend the diversity–multiplexing tradeoff to repetition-based and space–time-coded cooperative diversity.

Specifically, as in [6], [7], we approximate the outage probability as

$$\Pr[\mathtt{I} < \mathtt{R}] \doteq \mathtt{SNR}^{-\Delta(\mathtt{R}_{\text{norm}})}$$

i.e., in the sense of equality to first order in the exponent, where

$$\Delta(\mathtt{R}_{\text{norm}}) = \lim_{\mathtt{SNR}\to\infty} -\frac{\log(\Pr[\mathtt{I} < \mathtt{R}])}{\log(\mathtt{SNR})}. \tag{27}$$

with $\mathtt{R}$ given by (26). A tradeoff between diversity and multiplexing results because, as we will see, increasing $\mathtt{R}_{\text{norm}}$ decreases Δ.

Utilizing the lower and upper bounds (15) and (16) in (27) yields diversity order

$$\Delta_{\text{rep}}(\mathtt{R}_{\text{norm}}) = m(1 - m\mathtt{R}_{\text{norm}}) \tag{28}$$

for repetition decode-and-forward cooperative diversity. Similarly, utilizing the lower and upper bounds (24) and (25) in (27) yields upper and lower bounds, respectively, on the diversity order

$$m(1 - 2\mathtt{R}_{\text{norm}}) \le \Delta_{\text{stc}}(\mathtt{R}_{\text{norm}}) \le m\left(1 - \left[\frac{m-1}{m}\right] 2\mathtt{R}_{\text{norm}}\right) \tag{29}$$

for space–time-coded cooperative diversity.

Fig. 7 compares the diversity exponents, along with the corresponding tradeoff for noncooperative transmission, $\Delta_{\text{dir}}(\mathtt{R}_{\text{norm}}) = 1 - \mathtt{R}_{\text{norm}}$, from [3], [4]. Both repetition-based and space–time-coded cooperative diversity offer full diversity m as $\mathtt{R}_{\text{norm}} \to 0$. Clearly, space–time-coded cooperative diversity offers larger diversity order than repetition-based algorithms and can be effectively utilized for higher spectral efficiencies than repetition-based schemes.

VI. Conclusion

As we have developed in this paper, cooperative diversity, and particularly space–time-coded cooperative diversity, provides an effective way for a collection of wireless terminals to relay signals for one another in order to exploit spatial diversity in the channel. Extending our earlier results for two cooperating terminals [3], [4], we have analyzed repetition-based and space–time-coded cooperative diversity in nonergodic settings using outage probability as a performance measure. In both cases, we showed how the algorithms provide full spatial diversity in the number of cooperating terminals, and characterized the effective coding or SNR gain/loss as a function the interterminal average SNRs. We also characterized the diversity–multiplexing tradeoff for these algorithms, and demonstrated the extent to which space–time-coded cooperative diversity achieves higher diversity order than repetition-based schemes for larger spectral efficiencies.

Cooperative diversity relates to two topics of active research in resource-efficient wireless communication and networks. From one perspective, cooperative diversity mimics the performance advantages of multiple-antenna, or multiple-input, multiple-output (MIMO), systems by exploiting the spatial richness of the wireless channel. From another perspective, cooperative diversity corresponds to a particular form of network coding that explicitly models the multipath fading, noise, and interference effects of the wireless channel. In either case, existing insights and code designs can be leveraged and tailored to the distributed nature of the cooperative problem.

We believe several areas of future research on cooperative diversity will be fruitful. First, practical coding schemes can be designed, building upon insights obtained from information-theoretic treatments. Some progress toward this objective has occurred in [14], [15]. Second, algorithms and protocols for forming cooperating groups of terminals will be necessary. The effects of network geometry on the coding or SNR gain/loss of the protocols will be particularly important here. Third, integra-

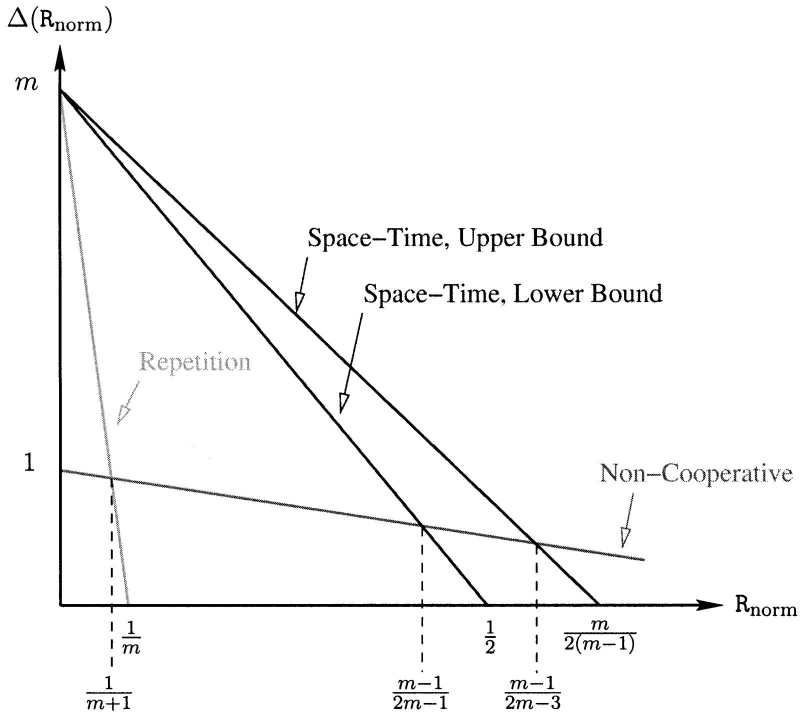

Fig. 7. Diversity order $\Delta(\mathtt{R}_{\rm norm})$ for noncooperative transmission, repetition-based cooperative diversity, and space–time-coded cooperative diversity. As $\mathtt{R}_{\rm norm} \to 0$, all cooperative diversity protocols provide full spatial diversity order m, the number of cooperating terminals. Relative to direct transmission, space–time-coded cooperative diversity can be effectively utilized for a much broader range of $\mathtt{R}_{\rm norm}$ than repetition-coded cooperative diversity, especially as m becomes large.

tion and interaction with higher layer network protocols, such as routing, can be explored.

APPENDIX
HIGH-SNR RESULTS

We gather in this appendix the analytical results for Sections III-A and IV-A, in order to focus the body of the paper on discussion and interpretation of the results. We begin in subsection A of the appendix by developing a general result about asymptotic properties of the cumulative distribution function (CDF) of a sum of independent random variables. We then apply this result to obtain large-SNR approximations for repetition decode-and-forward cooperative diversity in subsection B and for space–time-coded cooperative diversity in subsection C of the appendix.

A. The Basic Result

Both of the arguments later in this appendix rely upon the following result, which is a generalization of [4, Fact 2] to several random variables with fairly general probability density functions (PDFs).

Claim 1: Let u_k, $k = 1, 2, \ldots, m$, be positive, independent random variables with

$$\liminf_{\epsilon \to 0} p_{\mathsf{u}_k}(\epsilon u) \geq \lambda_k \tag{30}$$

and

$$p_{\mathsf{u}_k}(\epsilon u) \leq \lambda_k. \tag{31}$$

Then

$$\lim_{\epsilon \to 0} \frac{1}{\epsilon^m} \Pr\left[\sum_{k=1}^{m} \mathsf{u}_k < \epsilon\right] = \frac{1}{m!} \prod_{k=1}^{m} \lambda_k. \tag{32}$$

Before proving Claim 1, we note that the exponential distribution satisfies both requirements (30) and (31). More generally, however, this result suggests that many of our results hold for a much larger class of PDFs, and, in particular, depend mainly upon properties of the PDFs near the origin.

Proof: Let $\mathsf{s}_n = \sum_{k=1}^{n} \mathsf{u}_k$, $n \leq m$. Then

$$\begin{aligned}\Pr\left[\sum_{k=1}^{m} \mathsf{u}_k < \epsilon\right] &= \Pr[\mathsf{s}_m < \epsilon] \\ &= \int_0^{\epsilon} p_{\mathsf{s}_m}(s)\, ds \qquad (33)\\ &= \epsilon \int_0^1 p_{\mathsf{s}_m}(\epsilon w)\, dw \qquad (34)\end{aligned}$$

where the last equality results from the change of variables $w = s/\epsilon$. Thus, it is sufficient for us to compute the limit

$$\lim_{\epsilon \to 0} \frac{1}{\epsilon^{(m-1)}} \int_0^1 p_{\mathsf{s}_m}(\epsilon w)\, dw. \tag{35}$$

To lower-bound the $\liminf$, we exploit Fatou's lemma [16] to obtain

$$\begin{aligned}&\liminf_{\epsilon \to 0} \frac{1}{\epsilon^{(m-1)}} \int_0^1 p_{\mathsf{s}_m}(\epsilon w)\, dw \\ &\qquad \geq \int_0^1 \left\{\liminf_{\epsilon \to 0} \frac{1}{\epsilon^{(m-1)}} p_{\mathsf{s}_m}(\epsilon w)\right\} dw. \qquad (36)\end{aligned}$$

Now, $\mathsf{s}_m = \mathsf{s}_{m-1} + \mathsf{u}_m$, and by independence the PDF of s_m is the convolution of the PDFs of s_{m-1} and u_m. Specifically, since

u_m is positive, we have

$$\begin{aligned} p_{\mathsf{s}_m}(s) &= \int_0^s p_{\mathsf{s}_{m-1}}(s-r) p_{\mathsf{u}_m}(r)\, dr \\ &= s\int_0^1 p_{\mathsf{s}_{m-1}}(s(1-y)) p_{\mathsf{u}_m}(sy)\, dy \end{aligned} \tag{37}$$

where the last equality results from the change of variables $y = r/s$.

Letting

$$A_m(w) = \liminf_{\epsilon\to 0} \frac{1}{\epsilon^{(m-1)}}\, p_{\mathsf{s}_m}(\epsilon w) \tag{38}$$

substituting into (37), and again exploiting Fatou's lemma, we obtain the recursion

$$\begin{aligned} A_m(w) &= \liminf_{\epsilon\to 0} \frac{1}{\epsilon^{(m-1)}} p_{\mathsf{s}_m}(\epsilon w)\, dw \\ &\geq w\int_0^1 \left\{ \liminf_{\epsilon\to 0} \frac{1}{\epsilon^{(m-2)}}\, p_{\mathsf{s}_{m-1}}(\epsilon w(1-y)) \right\} \\ &\quad \cdot \left\{ \liminf_{\epsilon\to 0} p_{\mathsf{u}_m}(\epsilon wy) \right\} dy \\ &\geq \lambda_m w \int_0^1 A_{(m-1)}(w(1-y))\, dy \end{aligned} \tag{39}$$

where the last inequality follows from (30) and substitution of $A_{(m-1)}(w(1-y))$. Beginning with $A_1(w) \geq \lambda_1$ from (30), the recursion (39) yields

$$A_m(w) \geq \frac{1}{(m-1)!}\, w^{(m-1)} \prod_{k=1}^m \lambda_k. \tag{40}$$

As a result, (36) with (38) and (40) yields

$$\liminf_{\epsilon\to 0} \frac{1}{\epsilon^m} \Pr[\mathsf{s}_m < \epsilon] \geq \frac{1}{m!} \prod_{k=1}^m \lambda_k. \tag{41}$$

To upper-bound the $\limsup$, we obtain a recursive upper bound for the PDF of s_m similar to the lower bound developed above. Specifically, letting

$$B_m(w, \epsilon) = p_{\mathsf{s}_m}(\epsilon w) \tag{42}$$

we have

$$\begin{aligned} B_m(w, \epsilon) &= \epsilon w \int_0^1 p_{\mathsf{s}_{m-1}}(\epsilon w(1-y)) p_{\mathsf{u}_m}(\epsilon wy)\, dy \\ &\leq \epsilon\lambda_m w \int_0^1 B_{m-1}(w(1-y), \epsilon)\, dy \end{aligned} \tag{43}$$

where the equality comes from the convolution (37), and the inequality follows from (31) and substitution of $B_{m-1}(w(1-y))$. Beginning with $B_1(w, \epsilon) \leq \lambda_1$ from (31), (43) yields an upper bound very similar to the lower bound in (40), namely

$$B_m(w, \epsilon) \leq \epsilon^{(m-1)} w^{(m-1)} \frac{1}{(m-1)!} \prod_{k=1}^m \lambda_k. \tag{44}$$

Then

$$\begin{aligned} &\limsup_{\epsilon\to 0} \frac{1}{\epsilon^{(m-1)}} \int_0^1 p_{\mathsf{s}_m}(\epsilon w)\, dw \\ &\quad \leq \limsup_{\epsilon\to 0} \frac{1}{\epsilon^{(m-1)}} \int_0^1 B_m(w, \epsilon)\, dw \\ &\quad \leq \frac{1}{m!} \prod_{k=1}^m \lambda_k. \end{aligned} \tag{45}$$

Together with the fact that, in general $\liminf \leq \limsup$, (41), and (45) yield the desired result (32). □

B. Repetition Decode-and-Forward Cooperative Diversity

In this subsection, we utilize the result of Claim 1 to obtain a large $\mathtt{SNR}$ approximation for $\Pr[\mathsf{I}_{\text{rep}} < \mathtt{R}|\mathcal{D}(s)]$, the conditional outage probability for repetition decode-and-forward cooperative diversity for source s given a set of decoding relays $\mathcal{D}(s)$. As in (8), I_{rep} is of the form

$$\mathsf{I}_{\text{rep}} = \frac{1}{m} \log\left(1 + \mathtt{SNR} \sum_{k=1}^m \mathsf{u}_k\right) \tag{46}$$

where u_k are independent exponential random variables with parameters λ_k, $k = 1, 2, \ldots, m$.

After some algebraic manipulations, the outage probability reduces to exactly the same form as in Claim 1

$$\Pr[\mathsf{I}_{\text{rep}} < \mathtt{R}|\mathcal{D}(s)] = \Pr\left[\sum_{k=1}^m \mathsf{u}_k < \epsilon\right] \tag{47}$$

with $\epsilon = (2^{m\mathtt{R}} - 1)/\mathtt{SNR} \to 0$ as $\mathtt{SNR} \to \infty$. Thus, Claim 1 and continuity yield the approximation

$$\Pr[\mathsf{I}_{\text{rep}} < \mathtt{R}|\mathcal{D}(s)] \sim \left[\frac{2^{m\mathtt{R}} - 1}{\mathtt{SNR}}\right]^m \frac{1}{m!} \prod_{k=1}^m \lambda_k \tag{48}$$

for large $\mathtt{SNR}$.

C. Space–Time-Coded Cooperative Diversity

In this subsection, we compute a large-$\mathtt{SNR}$ approximation for $\Pr[\mathsf{I}_{\text{stc}} < \mathtt{R}|\mathcal{D}(s)]$, the conditional outage probability for space–time-coded cooperative diversity for source s given a set of decoding relays $\mathcal{D}(s)$. As in (18), I_{stc} is of the form

$$\mathsf{I}_{\text{stc}} = \frac{1}{2}\log\left(1 + \frac{2}{m}\mathtt{SNR}\,\mathsf{u}_m\right) + \frac{1}{2}\log\left(1 + \frac{2}{m}\,\mathtt{SNR} \sum_{k=1}^{m-1} \mathsf{u}_k\right) \tag{49}$$

where again u_k are independent exponential random variables with parameters λ_k, $k = 1, 2, \ldots, m$.

Let

$$\mathsf{s}_{m-1} = \sum_{k=1}^{m-1} \mathsf{u}_k$$

$t = (2^{2\mathtt{R}} - 1)$, and $\epsilon = (2^{2\mathtt{R}} - 1)/(2\mathtt{SNR}/m)$. Then

$$\begin{aligned} &\Pr[\mathsf{I}_{\text{stc}} < \mathtt{R}|\mathcal{D}(s)] \\ &\quad = \Pr\left[\mathsf{u}_m + \mathsf{s}_{m-1} + \frac{2}{m}\,\mathtt{SNR}\mathsf{u}_m\mathsf{s}_{m-1} < \epsilon\right] \\ &\quad = \int_0^\epsilon \Pr\left[\mathsf{u}_m < \frac{\epsilon - s}{1 + (2\mathtt{SNR}/m)s}\right] p_{\mathsf{s}_{m-1}}(s)\, ds \\ &\quad = \epsilon\int_0^1 \Pr\left[\mathsf{u}_m < \frac{\epsilon(1-w)}{1+tw}\right] p_{\mathsf{s}_{m-1}}(\epsilon w)\, dw \\ &\quad = \epsilon\int_0^1 \left[1 - \exp\left(-\lambda_m \frac{\epsilon(1-w)}{1+tw}\right)\right] p_{\mathsf{s}_{m-1}}(\epsilon w)\, dw. \end{aligned} \tag{50}$$

Note the penultimate equality in (50) follows from the change of variables $w = s/\epsilon$, and the last equality follows from substituting the CDF for u_m.

We now compute the limit

$$\lim_{\epsilon\to 0}\frac{1}{\epsilon^m}\Pr[\mathsf{I}_{\rm stc}<\mathtt{R}|\mathcal{D}(s)] = \frac{1}{(m-2)!}\prod_{k=1}^{m}\lambda_m\int_0^1\left[\frac{1-w}{1+tw}\right]w^{(m-2)}\,dw \quad (51)$$

that, along with continuity, provides the large-SNR approximation

$$\Pr[\mathsf{I}_{\rm stc}<\mathtt{R}|\mathcal{D}(s)]\sim\left[\frac{2^{2\mathtt{R}}-1}{2\mathtt{SNR}/m}\right]^m\frac{1}{(m-2)!}\prod_{k=1}^{m}\lambda_m \times\int_0^1\left[\frac{1-w}{1+(2^{2\mathtt{R}}-1)w}\right]w^{(m-2)}\,dw. \quad (52)$$

To lower-bound the $\liminf$, we exploit Fatou's lemma in (50) to obtain

$$\begin{aligned}\liminf_{\epsilon\to 0}\frac{1}{\epsilon^m}\Pr[\mathsf{I}_{\rm stc}<\mathtt{R}|\mathcal{D}(s)] &\geq \int_0^1\left\{\liminf_{\epsilon\to 0}\frac{1}{\epsilon}\left[1-\exp\left(-\lambda_m\frac{\epsilon(1-w)}{1+tw}\right)\right]\right\}\\ &\quad\cdot\left\{\liminf_{\epsilon\to 0}\frac{1}{\epsilon^{(m-2)}}p_{\mathsf{s}_{m-1}}(\epsilon w)\right\}dw\\ &=\int_0^1\lambda_m\left[\frac{1-w}{1+tw}\right]A_{m-1}(w)\,dw\\ &\geq\frac{1}{(m-2)!}\prod_{k=1}^{m}\lambda_k\int_0^1\left[\frac{1-w}{1+tw}\right]w^{(m-2)}\,dw \quad (53)\end{aligned}$$

where the first equality follows from properties of exponentials and substitution of $A_{m-1}(w)$ from (38), and the second equality follows from the result (40) in the proof of Claim 1.

To upper-bound the $\limsup$, we derive

$$\begin{aligned}\limsup_{\epsilon\to 0}\frac{1}{\epsilon^m}\Pr[\mathsf{I}_{\rm stc}<\mathtt{R}|\mathcal{D}(s)] &\leq\limsup_{\epsilon\to 0}\int_0^1\left\{\frac{1}{\epsilon}\left[1-\exp\left(-\lambda_m\frac{\epsilon(1-w)}{1+tw}\right)\right]\right\}\\ &\quad\cdot\left\{\frac{1}{\epsilon^{(m-2)}}B_{m-1}(w,\epsilon)\right\}dw\\ &\leq\limsup_{\epsilon\to 0}\int_0^1\left\{\lambda_m\left[\frac{(1-w)}{1+tw}\right]\right\}\\ &\quad\cdot\left\{\frac{1}{\epsilon^{(m-2)}}B_{m-1}(w,\epsilon)\right\}dw\\ &\leq\limsup_{\epsilon\to 0}\int_0^1\left\{\lambda_m\left[\frac{(1-w)}{1+tw}\right]\right\}\\ &\quad\cdot\left\{\frac{1}{(m-2)!}w^{(m-2)}\prod_{k=1}^{m-1}\lambda_k\right\}dw\\ &=\frac{1}{(m-2)!}\prod_{k=1}^{m}\lambda_k\int_0^1\left[\frac{(1-w)}{1+tw}\right]w^{(m-2)}\,dw \quad (54)\end{aligned}$$

where the first inequality follows from substitution of $B_{m-1}(w,\epsilon)$ from (42), the second inequality follows from the fact that $1-\exp(-x)\leq x$ for all $x\geq 0$, and the third inequality follows from the result (44) in Claim 1.

Taken together with the fact that $\liminf\leq\limsup$, (53) and (54) yield the desired result (51).

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