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Research Article

Distributed Synchronization Control to Trajectory Tracking of Multiple Robot Manipulators

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This paper investigates the issue of designing decentralized control laws to cooperatively command a team of general fully actuated manipulators. The purpose is to synchronize their movements while tracking a common desired trajectory. Based on the well-known consensus algorithm, the control strategy consists in synchronizing the joint position and the velocity of each robot in the network with respect to neighboring robots' joints and velocities. Modeled by an undirected graph, the cooperative robot network requires just local neighbor-to-neighbor information exchange between manipulators. So, it does not assume the existence of an explicit leader in the team. Based above all on combination of Lyapunov direct method and cross-coupling strategy, the proposed decentralized control law is extended to an adaptive synchronization control taking into account parameter uncertainties. To address the time delay problems in the network communication channels, the suggested synchronization control law robustly synchronizes robots to track a given trajectory. To this end, Krasovskii functional method has been used to deal with the delay-dependent stability problem. A real-time software simulator is developed to visualize the robot manipulators coordination.

1. Introduction

There has been a great research effort for synchronization problems, in distributed cooperative control strategies where robot control laws are coupled and each robot control is updated using local rule based on its own sensors and the states of its neighbors [1, 2]. Consensus algorithms, distributed coordination, and passivity-based output synchronization for networked Euler-Lagrange systems have been studied in [3–5], respectively. In this context, one recent representative work [6] shows that we can synchronize the multicomposed system when only position measurements are available. Synchronization framework that can be directly applied to cooperative control of multiagent systems in robotic manipulation and teleoperation has been presented in [1]. Specifically, the design based on the graph theory and the Laplacian matrix produces interesting results [7–9]. Adaptive control is an effective strategy used to address the synchronization problem [10, 11].

In the literature, most of earlier works on multiagent coordination and consensus [2, 8, 9, 12] mainly deal with very simple dynamic models such as linear systems and focuses on algorithm taking the form of first-order dynamics or second-order dynamics without nonlinear inertia matrices [13–15]. Most previous works on consensus and coordination of multiagent systems using the graph theory and the Laplacian [2, 8, 9, 16, 17] have presented a synchronization to the weighted average of initial conditions but they do not consider multiagent systems where there is a desired path to follow.

The objective of this paper is to develop a synchronized trajectory-tracking control of multiple robot manipulators. The proposed controller relies principally on a consensus algorithm for systems modeled by nonlinear second-order dynamics and applies the algorithm for the synchronization control problem by choosing appropriately information states on which consensus is reached. The concept key of the new synchronizing controller is the introduction of a state vector that quantifies the coordination degree between robot manipulator positions and different positions of its neighbors, using cross-coupling technique. Robot manipulators are widely used in production processes. In tasks that can not be fulfilled by a single robot, either because of the complexity of the task or the spatial and temporal limits of the robot, the use of cooperative robots proved to be a good determination [18]. The tasks being executed by each robot may terminate in different time due to nonhomogenity of robots or even electronics that drives motors of robots' joints. Despite the common trajectory robots have to track, it is not almost sure that the final running time of each robot will definitely be the same. So, in order to ensure this same final running time, robots have to have image of the current state of the neighboring robots as such to synchronize their movement together.

This paper presents a distributed control strategy. Through local interactions, the proposed approach can achieve more efficient performances, in particular in the presence of external disturbances. The proposed method based on a combination of Lyapunov direct method and concepts of cross-coupling, through a convenient mathematical manipulation, has converted the motion control problem of multiple systems into a stabilization problem for one single system. Compared to existing approaches, main contributions of this paper can be stated as follows.

(1) The present work deals with highly nonlinear systems. While [13–15], to name a few, deal with simple dynamic models.

(2) In contrast to [2, 8, 9, 16, 17] the proposed approach achieves not only global asymptotical synchronization of the configuration variables, but also global asymptotical convergence to the desired trajectory.

(3) The theory is extended to adaptive control and time delay control.

(4) In [10, 11, 19], the position synchronization error of each robot is defined as the differential position error between this robot and its two adjacent robots. However, the proposed design allows interconnections between all robots, such that all robots have direct influence in the combined dynamics, and consequently the synchronous behavior is the result of interactions between all robots. Moreover, it is straightforward to show that the particular choice of the tracking error surface in the proposed approach provides an exponential convergence of q_i to q_d if we have a constant tracking error surface.

(5) Using the Virtual Reality Modeling Language (VRML) a virtual world was developed to simulate the robot synchronization application in 3D scenes.

2. Preliminaries

The dynamic equation of a general rigid link manipulator having n degrees of freedom in a free space can be written as

$$M_i(q_i)\ddot{q}_i + C_i(q_i,\dot{q}_i)\dot{q}_i + g_i(q_i) = \tau_i, \qquad (1)$$

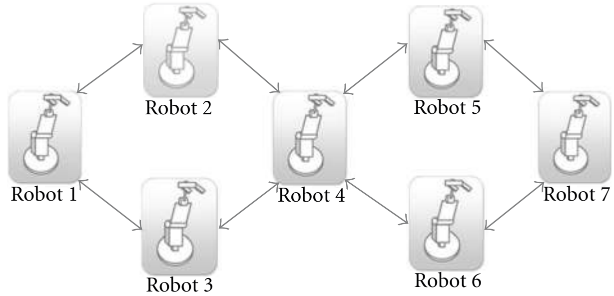


Figure 1: Multirobot system under mutual synchronization scheme.

where i $(1 \leq i \leq p)$ denotes the ith robot index in the network and p is the total number of the individual elements. In addition, $q_i \in \mathbb{R}^n$ denotes the vector of generalized displacements of the ith robot coordinates, and $\tau_i \in \mathbb{R}^n$ denotes the vector of generalized control input torques in robot coordinates; $M_i(q_i) \in \mathbb{R}^{n\times n}$ inertia matrix which is symmetric uniformly bounded and positive definite, $C_i(q_i,\dot{q}_i)\dot{q}_i \in \mathbb{R}^n$ is a vector function containing Coriolis and centrifugal forces, and $g_i(q_i) \in \mathbb{R}^n$ is a vector function consisting of gravitational forces. According to [20, 21], we have some fundamental properties of motion equations.

(i) The inertia matrix $M_i(q_i)$ is symmetric, positive definite, and uniformly bounded:

$$M_i(q_i) = M_i^T(q_i) > 0. \qquad (2)$$

(ii) Using a proper definition, $\dot{M}_i(q_i) - 2C_i(q_i,\dot{q}_i)$ is a skew symmetric matrix, satisfying

$$X^T(\dot{M}_i(q_i) - 2C_i(q,q))X = 0, \qquad (3)$$

where X^T is the transpose of a vector $X \in \mathbb{R}^n$.

(iii) The Euler-Lagrange equation (1) is linear with respect to the structural parameter θ, hence,

$$M_i(q_i)\ddot{q}_i + C_i(q_i,\dot{q}_i)\dot{q}_i + g_i(q_i) = Y(q_i,\dot{q}_i,\ddot{q}_i)\theta_i, \qquad (4)$$

where $Y \in \mathbb{R}^{n\times a}$ is the regressor matrix composed of known functions of q, $\dot{q}$ and $\ddot{q}$, $\theta_i \in \mathbb{R}^a$ is the vector of structural parameters of the manipulator, and a is the number of unknown parameters. In the present topology, the edge represents bidirectional communication links. This consists of a group of p manipulators interchanging information that can be viewed as an undirected graph (Figure 1).

3. Controller Design

We design decentralized control laws for p robot manipulators such that all joint positions mutually synchronize and track a common desired trajectory. The control objective of the proposed synchronization controller scheme is to synchronize the ith-joint position and velocity q_i, $\dot{q}_i$ to the state of any manipulator q_j, $\dot{q}_j$. Besides the controller is required to regulate the joint position q_i to track a desired trajectory q_d. Specifically, the control torque for the ith-robot is to control the tracking error to converge to zero and at the same time to synchronize its motion with respect to motions of the $p-1$ robots in the network, so that the synchronization error converges to zero. To this end, we define the tracking error surface of the ith manipulator as

$$\varepsilon_{1i}(t) = q_i(t) - q_d(t) + \int_{t_0}^{t} \Lambda_i[q_i(\lambda) - q_d(\lambda)]d\lambda, \quad (5)$$

where Λ_i is a diagonal positive-definite matrix. Information on the vector ε_{1i} will give insight on the convergence of the joint positions to the desired trajectory. However, it is straightforward to show that this particular choice of this tracking error surface provides an exponential convergence of q_i to q_d if we have a constant tracking error surface. But the defined sliding error only guarantees the trajectory tracking. Nevertheless, it is required to know the performance of the controller, that is, to know how the trajectory of each robot manipulator converges with respect to each other. There are various ways to choose the synchronization error. For example in [6], authors include the error information of all systems involved in the synchronization. In [10], authors use the cross-coupling concept to solve the synchronization problem. Our approach will make use of the cross-coupling to propose a feasible and efficient synchronization error, which consists on a measure of the synchronization for robot manipulators as defined as follows:

$$\varepsilon_{2i}(t) = \sum_{j \neq i}^{p} K_{ij}\left[(q_i - q_d) - \left(q_j - q_d\right)\right] = \sum_{j \neq i}^{p} K_{ij}\left(q_i - q_j\right), \quad (6)$$

where K_{ij} is a symmetric positive-definite matrix that gives insight on the communication quality between the ith and jth robot manipulators. We note that $K_{ij} \neq 0$ if and only if there is information exchange between robot manipulators i and j. Consequently, each robot is not necessarily aware of all other robots. Therefore, we define the synchronizing tracking error surface which encompasses both synchronization error and trajectory tracking error for manipulator i as:

$$e_i = \varepsilon_{1i} + \int_{t_0}^{t} \varepsilon_{2i}(\lambda)d\lambda. \quad (7)$$

The objective is to design a control law such that coupling errors, that is, position errors, velocity errors, and synchronization errors, all converge to zero. For each manipulator, the control law τ_i is defined as follows:

$$\begin{aligned}\tau_i &= C_i(q_i, \dot{q}_i)\dot{q}_i + g_i(q_i) \\ &\quad + M_i(q_i)\left[\ddot{q}_d - K_{pi}e_i - K_{di}\dot{e}_i - \Lambda_i(\dot{q}_i - \dot{q}_d)\right] \\ &\quad + M_i(q_i)\left[\sum_{j \neq i} K_{ij}\left(\varepsilon_{2i} - \varepsilon_{2j}\right) + \Lambda_i\varepsilon_{2i}\right],\end{aligned} \quad (8)$$

where q_d is a common trajectory reference to be tracked, which is a smooth time-varying trajectory and for which the first and the second derivative exist for all $t \geq 0$. K_{di} and K_{pi} are symmetric positive-definite matrices.

Theorem 1. *If $K_{di} > 2\sum_{j \neq i} K_{ij}$, the proposed synchronization tracking controller (8) guarantees asymptotic convergence to zero of position errors, velocity errors, and synchronization errors, that is, $e_i \to 0$ as $t \to \infty$.*

Remark 1. Note that the synchronization controller only requires local information of the joint position q_i, the joint velocity and the information about the desired trajectory. Also in order to ensure synchronization with other robots it needs information on the joint position from neighbors. It has to be noticed that the joint position of the neighboring robots are lumped into the error variable ε_{2j}.

Proof. Notice that the controller law (8) contains PD controller terms. However, these terms are premultiplied by the inertia matrix $M_i(q_i)$. Therefore it is clear that this is not a linear controller as the PD control law, since the position and velocity gains are not constant but they depend explicitly on the position error.

Substituting (8) in (1) yields:

$$\begin{aligned}M_i(q_i)\ddot{q}_i &= M_i(q_i)\left[\ddot{q}_d - K_{pi}e_i - K_{di}\dot{e}_i - \Lambda_i(\dot{q}_i - \dot{q}_d)\right] \\ &\quad + M_i(q_i)\left[\sum_{j \neq i} K_{ij}\left(\varepsilon_{2i} - \varepsilon_{2j}\right) + \Lambda_i\varepsilon_{2i}\right].\end{aligned} \quad (9)$$

Multiplying by M_i^{-1} in both sides yields

$$\begin{aligned}\ddot{q}_i &= \ddot{q}_d - K_{pi}e_i - K_{di}\dot{e}_i - \Lambda_i(\dot{q}_i - \dot{q}_d) \\ &\quad + \sum_{j \neq i} K_{ij}\left(\varepsilon_{2i} - \varepsilon_{2j}\right) + \Lambda_i\varepsilon_{2i}.\end{aligned} \quad (10)$$

Adding $\dot{\varepsilon}_{2i}$ in both sides yields

$$\begin{aligned}&\ddot{q}_i - \ddot{q}_d + \Lambda_i(\dot{q}_i - \dot{q}_d) + \dot{\varepsilon}_{2i} \\ &\quad = -K_{pi}e_i - K_{di}\dot{e}_i + \sum_{j \neq i} K_{ij}\left(\varepsilon_{2i} - \varepsilon_{2j}\right) + \Lambda_i\varepsilon_{2i} + \dot{\varepsilon}_{2i}.\end{aligned} \quad (11)$$

This results in

$$\ddot{e}_i = -K_{pi}e_i - K_{di}\dot{e}_i + \sum_{j \neq i} K_{ij}\left(\varepsilon_{2i} - \varepsilon_{2j}\right) + \Lambda_i\varepsilon_{2i} + \dot{\varepsilon}_{2i}. \quad (12)$$

Using the expression of the synchronization error ε_{2i} and its first derivative gives

$$\begin{aligned}\ddot{e}_i &= -K_{pi}e_i - K_{di}\dot{e}_i + \sum_{j \neq i} K_{ij}\left(\varepsilon_{2i} - \varepsilon_{2j}\right) \\ &\quad + \sum_{j \neq i}\left[K_{ij}(\dot{q}_i - \dot{q}_d) - \left(\dot{q}_j - \dot{q}_d\right)\right] \\ &\quad + \sum_{j \neq i} \Lambda_i K_{ij}\left[(q_i - q_d) - \left(q_j - q_d\right)\right].\end{aligned} \quad (13)$$

Further calculation will result in

$$\ddot{e}_i = -K_{pi}e_i - K_{di}\dot{e}_i + \sum_{j \neq i} K_{ij}\left(\dot{e}_i - \dot{e}_j\right). \quad (14)$$

Equation (14) represents the closed-loop synchronized system for the ith manipulator. In the sequel, we proceed to analyze the stability properties of the proposed synchronized control scheme and ultimately show that the control goals: the position error, velocity error, and synchronization error, all converge to zero. To prove stability of the overall synchronized system, we define $e^T = [e_1^T, e_2^T, \ldots, e_n^T]^T$.

Using (14) we obtain the synchronized error dynamics:

$$\ddot{e} = -K_p e - (K_d - K_c)\dot{e}, \tag{15}$$

where $K_p = \operatorname{diag}(K_{pi}), K_d = \operatorname{diag}(K_{di})$, and K_c is given by

$$\begin{pmatrix} \sum_{j\neq 1} K_{1j} & \cdots & -K_{1j} & \cdots & -K_{1n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ -K_{i1} & \cdots & \sum_{j\neq i} K_{ij} & \cdots & -K_{in} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ -K_{n1} & \cdots & -K_{nj} & \cdots & \sum_{j\neq n} K_{nj} \end{pmatrix}. \tag{16}$$

Note that K_c is symmetric and positive semidefinite matrix.

The synchronized error dynamics (15) is a linear time invariant system described by a second-order linear differential equation. A sufficient condition for the error dynamics to be exponentially stable is that the matrices K_p and $K_d - K_c$ are positive definite. In particular, matrices K_{di} can be diagonal satisfying $K_{di} > 2\sum_{i\neq j} K_{ij}$.

To analyze the stability properties of the closed-loop error dynamics (15), we take the following definite and radially unbounded Lyapunov function candidate:

$$V = \dot{e}^T\dot{e} + e^T K_p e. \tag{17}$$

Its derivative with respect to time can be expressed as

$$\dot{V} = -2\dot{e}^T(K_d - K_c)\dot{e} \leq 0. \tag{18}$$

It follows by direct application of LaSalle's invariance principle that the origin $(e, \dot{e}) = (0, 0)$ is globally asymptotically stable and $\lim \dot{e} \to 0$ for $t \to \infty$.

Referring to the expression of the global error (7)

$$e_i = q_i - q_d + \int_{t_0}^{t} \Lambda[q_i(\lambda) - q_d(\lambda)]d\lambda + \int_{t_0}^{t} \sum_{j\neq i} K_{ij}\left(q_i - q_j\right) \tag{19}$$

as $\dot{e} = 0$ we have

$$\dot{q}_i - \dot{q}_d = -\Lambda_i(q_i - q_d) - \sum_{j\neq i} K_{ij}\left(q_i - q_j\right). \tag{20}$$

We set $\varepsilon_i = q_i - q_d$. Then (20) can be written as

$$\dot{\varepsilon}_i = -\Lambda_i\varepsilon_i - \sum_{j\neq i} K_{ij}\left(\varepsilon_i - \varepsilon_j\right). \tag{21}$$

Our objective is to show that $\lim \varepsilon_i$ converges to zero as $t \mapsto \infty$. To this end, we define $\varepsilon = [\varepsilon_1^T \cdots \varepsilon_i^T \cdots \varepsilon_n^T]^T$ and $\Lambda = \operatorname{diag}(\Lambda_1 \cdots \Lambda_i \cdots \Lambda_n)$. Equation (21) can be written as

$$\dot{\varepsilon} = A \cdot \varepsilon, \tag{22}$$

where matrix A is given by

$$\begin{pmatrix} -\Lambda_1 - \sum_{j\neq 1} K_{1j} & \cdots & K_{1j} & \cdots & K_{1n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ K_{i1} & \cdots & -\Lambda_i - \sum_{j\neq i} K_{ij} & \cdots & K_{in} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ K_{n1} & \cdots & K_{nj} & \cdots & -\Lambda_n - \sum_{j\neq n} K_{nj} \end{pmatrix}. \tag{23}$$

Consider the following Lyapunov function:

$$v(t) = \varepsilon^T\varepsilon. \tag{24}$$

Differentiating $v(t)$ with respect to time yields

$$\begin{aligned} \dot{v} &= 2\sum_{i=1}^{n} \varepsilon_i^T\dot{\varepsilon}_i \\ &= 2\sum_{i=1}^{n} \varepsilon_i^T\left(-\Lambda_i\varepsilon_i - \sum_{j\neq i} K_{ij}\left(\varepsilon_i - \varepsilon_j\right)\right) \\ &= -2\sum_{i=1}^{n} \varepsilon_i^T\Lambda_i\varepsilon_i - 2\sum_{i=1}^{n}\sum_{j\neq i} \varepsilon_i^T K_{ij}\left(\varepsilon_i - \varepsilon_j\right) \\ &= -2\sum_{i=1}^{n} \varepsilon_i^T\Lambda_i\varepsilon_i - 2\sum_{i=1}^{n}\sum_{j\neq i} \varepsilon_i^T K_{ij}\varepsilon_i + 2\sum_{i=1}^{n}\sum_{j\neq i} \varepsilon_i^T K_{ij}\varepsilon_j. \end{aligned} \tag{25}$$

Knowing that

$$\sum_{i=1}^{n}\sum_{j\neq i} \varepsilon_i^T K_{ij}\varepsilon_i = \sum_{j=1}^{n}\sum_{i\neq j} \varepsilon_j^T K_{ji}\varepsilon_j, \tag{26}$$

consequently,

$$\dot{v} = -2\sum_{i=1}^{n} \varepsilon_i^T\Lambda_i\varepsilon_i - \sum_{i=1}^{n}\sum_{j\neq i} \left(\varepsilon_i - \varepsilon_j\right)^T K_{ij}\left(\varepsilon_i - \varepsilon_j\right) \leq 0. \tag{27}$$

It follows by direct application of LaSalle's invariance that the origin is globally asymptotically stable. Consequently we obtain $\lim \varepsilon_i(t) \to 0$ for $t \to \infty$. Then $q_i \to q_d$ and $\dot{q}_i \to \dot{q}_d$ for $t \to \infty$.

Referring to (19), we show that $q_i \to q_j$ for $t \to \infty$. □

4. Adaptive Synchronization

In this section, we consider an uncertainty in the model parameters. We propose to extend the previous decentralized control law to an adaptive version. For that, we consider the following adaptive control law, which has the similar local coupling structure as the proposed control law in (8):

$$\begin{aligned} \tau_i &= \widehat{C}_i(q_i, \dot{q}_i) + \widehat{g}_i(q_i) + \widehat{M}_i(q_i) \\ &\quad \times \left[\ddot{q}_d - K_{pi}e_i - K_{di}\dot{e}_i - \Lambda_i(\dot{q}_i - \dot{q}_d)\right] \\ &\quad + \widehat{M}_i(q_i)\left[\sum_{j\neq i} K_{ij}\left(\varepsilon_{2i} - \varepsilon_{2j}\right) + \Lambda_i\varepsilon_{2i}\right]. \end{aligned} \tag{28}$$

We recall that a similar relation to (1) holds when the estimate parameter vector $\hat{\theta}$ is used to replace the exact parameter vector θ:

$$\widehat{M}_i(q_i)\ddot{q}_i + \hat{C}_i(q_i, \dot{q}_i)\dot{q}_i + \hat{g}_i(q_i) = Y_i(q_i, \dot{q}_i, \ddot{q}_i)\hat{\theta}_i, \tag{29}$$

where $\theta_i \in \mathbb{R}^a$ is the vector of structural parameters of the manipulator and a is the number of unknown parameters. The estimated parameter $\hat{\theta}_i$ is subject to the adaptation law:

$$\dot{\hat{\theta}}_i = -\Gamma_i^{-1}\left(\widehat{M}_i^{-1} Y_i\right)^T \dot{e}_i, \tag{30}$$

where Γ_i is a diagonal positive-definite matrix. Since the value of the dynamic parameter θ_i is hard to be known exactly in practice, one defines $\hat{\theta}_i(t)$ as the estimate of θ_i. $\widehat{M}_i$, $\hat{C}_i$, and $\hat{g}_i$ are estimates of M_i, C_i, and g_i, respectively. $Y_i(q_i, \dot{q}_i, \ddot{q}_i)$ denotes a regression matrix. Define $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$ as a vector containing model estimation errors. Then the adaptation law (30) can be written as

$$\dot{\tilde{\theta}}_i = \Gamma_i^{-1}\left(\widehat{M}_i^{-1} Y_i\right)^T \dot{e}_i. \tag{31}$$

Theorem 2. *If $K_{di} > 2\sum_{j\neq i} K_{ij}$, the proposed adaptive coupling controllers* (28) *and* (31) *guarantee the asymptotic convergence to zero of the joint position, velocity, and synchronization errors.*

Proof. Substituting (28) into the dynamic model (1) leads to the following closed-loop dynamics:

$$\begin{aligned} &M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + g_i(q_i) \\ &\quad = \widehat{M}_i(q_i)\Bigg[\ddot{q}_d - K_{pi}e_i - K_{di}\dot{e}_i - \Lambda_i(\dot{q}_i - \dot{q}_d) \\ &\qquad + \sum_{j\neq i} K_{ij}\left(\varepsilon_{2i} - \varepsilon_{2j}\right) + \Lambda_i \varepsilon_{2i}\Bigg] \\ &\qquad + \hat{C}_i(q_i, \dot{q}_i)\dot{q}_i + \hat{g}_i(q_i). \end{aligned} \tag{32}$$

We define

$$\begin{aligned} \widetilde{M}_i(q_i) &= M_i(q_i) - \widehat{M}_i(q_i), \\ \tilde{C}_i(q_i, \dot{q}_i) &= C_i(q_i, \dot{q}_i) - \hat{C}_i(q_i, \dot{q}_i), \\ \tilde{g}_i(q_i) &= g_i(q_i) - \hat{g}_i(q_i). \end{aligned} \tag{33}$$

Taking into account (33), (32) gives:

$$\begin{aligned} &\widetilde{M}_i(q_i)\ddot{q}_i + \tilde{C}_i(q_i, \dot{q}_i)\dot{q}_i + \tilde{g}_i(q_i) + \widehat{M}_i(q_i)(\ddot{q}_i - \ddot{q}_d) \\ &\quad = \widehat{M}_i(q_i)\Bigg[-K_{pi}e_i - K_{di}\dot{e}_i - \Lambda_i(\dot{q}_i - \dot{q}_d) \\ &\qquad + \sum_{j\neq i} K_{ij}\left(\varepsilon_{2i} - \varepsilon_{2j}\right) + \Lambda_i \varepsilon_{2i}\Bigg]. \end{aligned} \tag{34}$$

Using (29), the dynamic (34) is written as

$$\begin{aligned} &Y_i(q_i, \dot{q}_i, \ddot{q}_i)\tilde{\theta}_i + \widehat{M}_i(q_i)(\ddot{q}_i - \ddot{q}_d) \\ &\quad = \widehat{M}_i(q_i)\Bigg[-K_{pi}e_i - K_{di}\dot{e}_i - \Lambda_i(\dot{q}_i - \dot{q}_d) \\ &\qquad + \sum_{j\neq i} K_{ij}\left(\varepsilon_{2i} - \varepsilon_{2j}\right) + \Lambda_i \varepsilon_{2i}\Bigg]. \end{aligned} \tag{35}$$

Consequently,

$$\begin{aligned} &\widehat{M}_i(q_i)\ddot{e}_i + Y_i(q_i, \dot{q}_i, \ddot{q}_i)\tilde{\theta}_i \\ &\quad = \widehat{M}_i(q_i)\Bigg[-K_{pi}e_i - K_{di}\dot{e}_i + \sum_{j\neq i} K_{ij}\left(\dot{e}_i - \dot{e}_j\right)\Bigg]. \end{aligned} \tag{36}$$

To prove the stability of the overall synchronized system, we define $Y^T = [Y_1^T, \ldots, Y_i^T, \ldots, Y_n^T]^T$, $\theta^T = [\theta_1^T, \ldots, \theta_i^T, \ldots, \theta_n^T]^T$, $\Gamma = \operatorname{diag}(\Gamma_1^T, \ldots, \Gamma_i^T, \ldots, \Gamma_n^T) K_d = \operatorname{diag}(K_{di})$, $K_p = \operatorname{diag}(K_{pi})$, $\widehat{M} = \operatorname{diag}(\widehat{M}_i)$.

Writing (36) in a compact form gives the following:

$$\widehat{M}\left[\ddot{e} + K_p e + K_d \dot{e} - K_c \dot{e}\right] = -Y(q_i, \dot{q}_i, \ddot{q}_i)\tilde{\theta}. \tag{37}$$

Multiplying by $\widehat{M}^{-1}$ both sides yields

$$\ddot{e} = -K_p e - K_d \dot{e} + K_c \dot{e} - \widehat{M}^{-1} Y(q_i, \dot{q}_i, \ddot{q}_i)\tilde{\theta}. \tag{38}$$

Define the Lyapunov function candidate as

$$v = \frac{1}{2}\left(\dot{e}^T \dot{e} + e^T K_p e + \tilde{\theta}^T \Gamma \tilde{\theta}\right). \tag{39}$$

Differentiating $v(t)$ with respect to time yields

$$\dot{v} = \ddot{e}^T \dot{e} + e^T K_p \dot{e} + \tilde{\theta}^T \Gamma \dot{\tilde{\theta}}. \tag{40}$$

Substituting (38) into (40)

$$\dot{v} = \left(-K_p e - K_d \dot{e} + K_c \dot{e} - \widehat{M}^{-1} Y \tilde{\theta}\right)^T \dot{e} + e^T k_p \dot{e} + \tilde{\theta}^T \Gamma \dot{\tilde{\theta}}, \tag{41}$$

consequently

$$\dot{v} = -\dot{e}^T (K_d - K_c)\dot{e} + \tilde{\theta}^T\left[\Gamma \dot{\tilde{\theta}} - \left(\widehat{M}^{-1} Y\right)^T \dot{e}\right]. \tag{42}$$

Using the adaptation law (31) gives

$$\dot{v} = -\dot{e}^T (K_d - K_c)\dot{e}. \tag{43}$$

Since $K_{di} > \sum_{j\neq i} K_{ij}$, we have $\dot{v}(t) < 0$, and this yields that $v(t) < v(0)$, which gives that e, $\dot{e}$, and $\widehat{M}^{-1} Y \tilde{\theta}^T$ are bounded.

Differentiating $\dot{v}(t)$ with respect to time yields

$$\ddot{v} = 2\left(K_p + (K_d - K_c)\dot{e} + \widehat{M}^{-1} Y \tilde{\theta}\right)^T (K_d - K_c)\dot{e}. \tag{44}$$

Using Barbalat's lemma, $\ddot{v}$ is bounded because e, $\dot{e}$, and $\widehat{M}^{-1} Y \tilde{\theta}$ are bounded. This implies $\dot{v}(t) \to 0$ for $t \to \infty$ and hence $\dot{e} \to 0$ for $t \to \infty$. The proof pursued the same line reasoning as the proof of Section 3. Consequently, we show that position errors, velocity errors and synchronization errors, converge asymptotically to zero. □

5. Synchronization with Time Delays

In this section, we study the coordination control problem taking into account time delays of communication channels. As a first assumption, we suppose that these delays can be justified by the fact that data information sent by the neighboring vehicles $j \neq i$ reaches vehicle i after a certain time delay due to the short-range communication channels. To take into account time delays produced during the communication among robots, we introduce, in the coordination error expression, a term τ which represents the same time delay due to the short-range communication channels. Therefore, a coordination error, in the time delay context, will be presented as the well-known classical time delay model of multiagent network [9]:

$$\varepsilon_{2i}(t) = \sum_{j \neq i} K_{ij}\left[q_i(t-\tau) - q_j(t-\tau)\right]. \tag{45}$$

It will be shown that the behavior of the coordinated system under the effect of time delay changes significantly. Consequently, the controller implanted in each lagrangian system among the network takes the following expression.

Theorem 3. *Implemented controller taking the following expressions* ($i = 1, \ldots, p$)*:*

$$\begin{aligned}\tau_i &= C_i(q_i, \dot{q}_i)\dot{q}_i + g_i(q_i) + M_i(q_i) \\ &\quad \times \left[\ddot{q}_d - K_{pi}e_i(t) - K_{di}\dot{e}_i(t) - \Lambda_i(\dot{q}_i - \dot{q}_d)\right] \\ &\quad + M_i(q_i)\left[\sum_{j \neq i} K_{ij}\left[\varepsilon_{2i}(t-\tau) - \varepsilon_{2j}(t-\tau)\right]\right. \\ &\quad \left. + \Lambda_i \varepsilon_{2i}(t-\tau)\right]\end{aligned} \tag{46}$$

stabilize the behavior of the robot network.

Proof. Substituting (46) into (1) yields

$$\begin{aligned}M_i(q_i)\ddot{q}_i &= M_i(q_i)\left[\ddot{q}_d - K_{pi}e_i - K_{di}\dot{e}_i - \Lambda_i(\dot{q}_i - \dot{q}_d)\right] \\ &\quad + M_i(q_i)\left[\sum_{j \neq i} K_{ij}\left[\varepsilon_{2i}(t-\tau) - \varepsilon_{2j}(t-\tau)\right]\right. \\ &\quad \left. + \Lambda_i \varepsilon_{2i}(t-\tau)\right].\end{aligned} \tag{47}$$

By further calculation, we obtain the synchronized error dynamics:

$$\ddot{e} = -K_p e - K_d \dot{e} + K_c \dot{e}(t-\tau), \tag{48}$$

where K_p, K_d, and K_c are the same matrices already defined (see Section 3). By the Leibnitz formula, we have

$$\dot{e} - \dot{e}(t-\tau) = \int_{t-\tau}^{t} \ddot{e}(\lambda)\,d\lambda. \tag{49}$$

Substituting (49) into (48) leads to

$$\ddot{e} = -K_p e - K_d \dot{e} + K_c\left(\dot{e} - \int_{t-\tau}^{t} \ddot{e}(\lambda)\,d\lambda\right). \tag{50}$$

Setting $\tilde{e} = [e^T, \dot{e}^T]^T$. Therefore (50) can be written as

$$\dot{\tilde{e}} = \begin{pmatrix} 0 & I \\ -K_p & -K_d + K_c \end{pmatrix}\tilde{e} - \begin{pmatrix} 0 & 0 \\ 0 & K_c \end{pmatrix}\int_{t-\tau}^{t} \dot{\tilde{e}}(\lambda)\,d\lambda. \tag{51}$$

This yields the following form:

$$\dot{\tilde{e}} = \beta_0 \tilde{e} - \beta_1 \int_{t-\tau}^{t} \dot{\tilde{e}}(\lambda)\,d\lambda \tag{52}$$

with $\beta_0 = \begin{pmatrix} 0 & I \\ -K_p & -K_d + K_c \end{pmatrix}$ and $\beta_1 = \begin{pmatrix} 0 & 0 \\ 0 & K_c \end{pmatrix}$.

To analyze the stability of the global system, we consider the following Lyapunov-Krasovskii functional (LKF):

$$\begin{aligned}v(t) &= v_1(t) + v_2(t) + v_3(t), \\ v_1(t) &= \tilde{e}^T(t)P\tilde{e}(t), \\ v_2(t) &= \int_{t-\tau}^{t} \tilde{e}^T(\lambda)R\tilde{e}(\lambda)\,d\lambda, \\ v_3(t) &= \int_{-\tau}^{0}\left(\int_{t+s}^{t} \dot{\tilde{e}}^T(\alpha)Z\dot{\tilde{e}}(\alpha)\,d\alpha\right)d\lambda,\end{aligned} \tag{53}$$

where $P = P^T > 0$, $R = R^T > 0$, $Z = Z^T > 0$ are weighting matrices of appropriate dimensions. A straightforward computation gives the time derivative of $v(t)$ along the solution of (53) as

$$\begin{aligned}\dot{v}(t) &= \delta^T\left[2N^T P M + N^T R N - Q^T R Q + \tau M^T Z M\right]\delta \\ &\quad - \int_{t-\tau}^{t} \dot{\tilde{e}}^T(\lambda)Z\dot{\tilde{e}}(\lambda)\,d\lambda,\end{aligned} \tag{54}$$

where $\delta = [\tilde{e}^T(t), \tilde{e}^T(t-\tau)]^T$, $N = [\beta_0, \beta_1]$, $M = [I, 0]$, $Q = [0, I]$. The Jensen's inequality gives a suitable bound for the last term of (54):

$$\begin{aligned}-\int_{t-\tau}^{t} \dot{\tilde{e}}^T(\lambda)Z\dot{\tilde{e}}(\lambda)\,d\lambda &\leq \int_{t-\tau}^{t} \dot{\tilde{e}}^T(\lambda)\,d\lambda\left(\frac{Z}{\tau}\right)\int_{t-\tau}^{t} \dot{\tilde{e}}(\lambda)\,d\lambda \\ &\leq -\tilde{e}^T T^T\left(\frac{Z}{\tau}\right)T\tilde{e}\end{aligned} \tag{55}$$

with $T = [I, -I]$. Thus, the time derivative of the LKF (53) can be bounded by $\dot{v}(t) \leq \delta^T \xi \delta$, where

$$\xi = 2N^T P M + N^T R N - Q^T R Q + \tau M^T Z M - \frac{1}{\tau T^T Z T}. \tag{56}$$

Then if the LMI $\xi < 0$ is satisfied, the derivative of the Lyapunov-Krasovskii functional is negative definite. To ensure matrix ξ is negative definite, we select appropriate control gains $K_p > K_p^*$ and $K_d - K_c > K^*$ throught processing MATLAB's LMI solver such that

$$2N^T P M + N^T R N - Q^T R Q + \tau M^T Z M - \frac{1}{\tau}T^T Z T < 0. \tag{57}$$

Then, if the LMI $\xi < 0$ is satisfied, the derivative of the Lyapunov-Krasovskii functional is therefore negative definite. In consequence, the origin $\tilde{e} = 0$ is asymptotically stable.

This results in $e \to 0$ for $t \to \infty$ and $\dot{e} \to 0$ for $t \to \infty$. The proof for the asymptotic convergence of the coordinated tracking error $\tilde{e}$ is not sufficient to prove the convergence to zero of both errors e_1 and e_2. Now, our concern is is to show that coordination is successfully realized for a specific time delay τ_c.

The proof pursued the same line reasoning as the proof of Section 3. Consequently, we obtain the following equation derived from the global error expression:

$$\dot{\varepsilon}_i = -\Lambda_i \varepsilon_i - \sum_{j \neq i} K_{ij} \left[\varepsilon_i(t-\tau) - \varepsilon_j(t-\tau) \right]. \tag{58}$$

Rewriting all states of (58) into a compact representation and applying the Laplace transform lead to

$$s\varepsilon(s) - \varepsilon(0) = -\Lambda\varepsilon(s) - e^{-\tau s} K_c \varepsilon(s). \tag{59}$$

This can be written as

$$\varepsilon(s) = \left(sI + \Lambda + e^{-\tau s} K_c\right)^{-1} \varepsilon(0). \tag{60}$$

If the characteristic equation $P(s,\tau)=\det|sI+\Lambda+e^{-\tau s}K_c| = 0$ has all its zeros in the left half complex plan, then the system is stable and one can easily conclude about the convergence of q_i to q_d. Since the ordinal system, free from time delay (i.e, $\tau = 0$), is a continuous function of τ, then using the D-decomposition, the minimal positive solution to the following equation:

$$\tau_i = \frac{1}{\sqrt{(K_{ci}^2 - \Lambda_i^2)}} \arccos\left(\frac{-\Lambda_i}{K_{ci}}\right) \tag{61}$$

which would make all the zeros of the characteristic equation in the left half complex plane. Therefore if we select $\tau \in [0, \tau_c]$, where $\tau_c = \sup\{\tau_i, \ \forall 1 \leq i \leq p\}$, therefore solutions of (60) converge to zero and consequently $q_i \to q_d, \dot{q}_i \to \dot{q}_d$, and $q_i \to q_j$ for $t \to \infty$. □

6. Simulation Results

To show the effectiveness of the proposed synchronizing controller, we run two types of simulation.

Simulation 1. Simulations are performed on MATLAB/Simulink. These simulations were performed on identical three manipulators with two degrees of freedom. Robots do not have the same starting positions, and they start their motion at $t_0 = 0$. First, we assume that no parameter uncertainty is present and that the transmission delay can be neglected. Figure 2 illustrates the robots synchronization tracking a common trajectory. This proves that tracking and synchronization objectives are attained by the proposed controller which explains how robots, while tracking the desired trajectory, synchronize their position with others. Next, we assume that the length and the mass of manipulators are unknown. Consequently, mass, Coriolis, and gravity matrices are ill known. Figure 3 plots an adaptive position tracking while in Figures 4 and 5 tracking errors and synchronizing errors are shown, respectively. Finally, we consider the time delay in communication. Figures 6 and 7 illustrate that the behavior of the coordinated system changes significantly, under the effect of time delays in communication channels. The speed

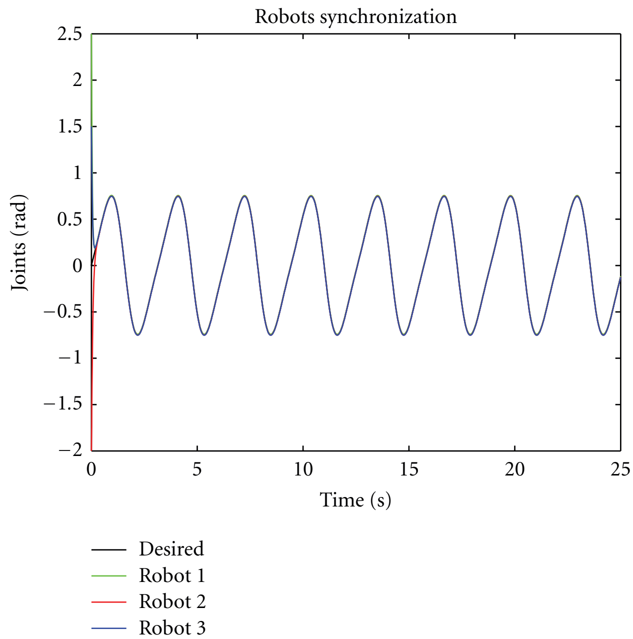


FIGURE 2: Synchronization of robots without uncertain parameters.

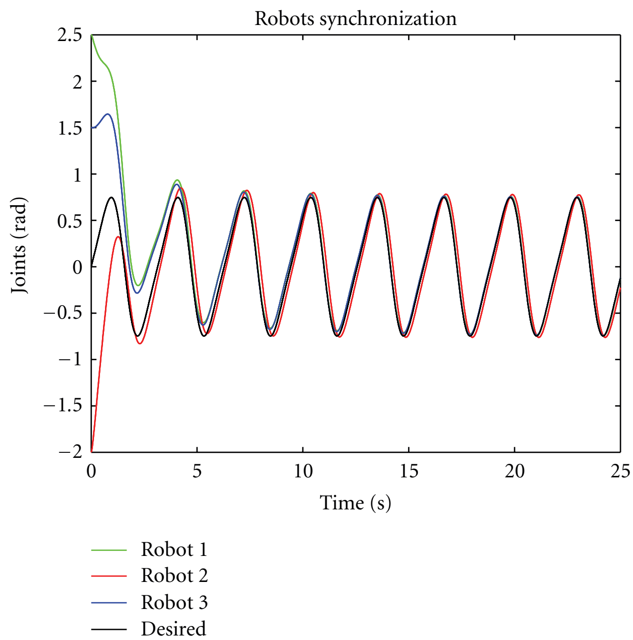


FIGURE 3: Synchronization of robots in the presence of uncertain parameters.

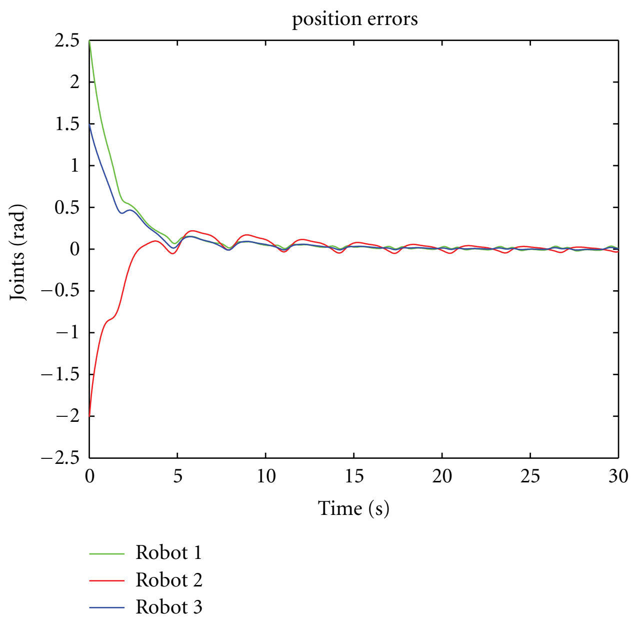


Figure 4: Position errors: adaptive synchronization.

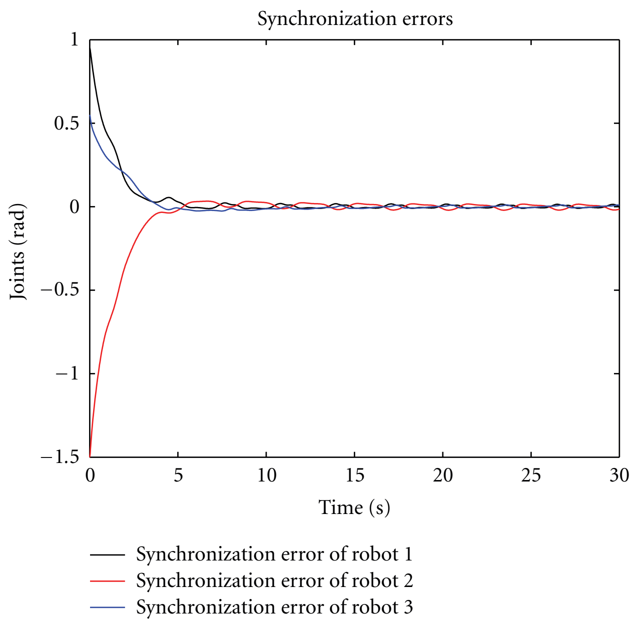


Figure 5: Synchronization errors: adaptive synchronization.

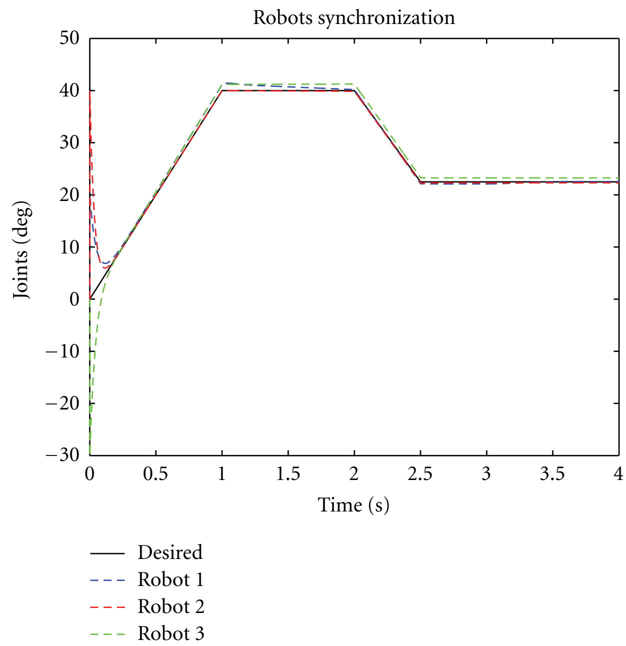


Figure 6: Robots synchronization without time delay.

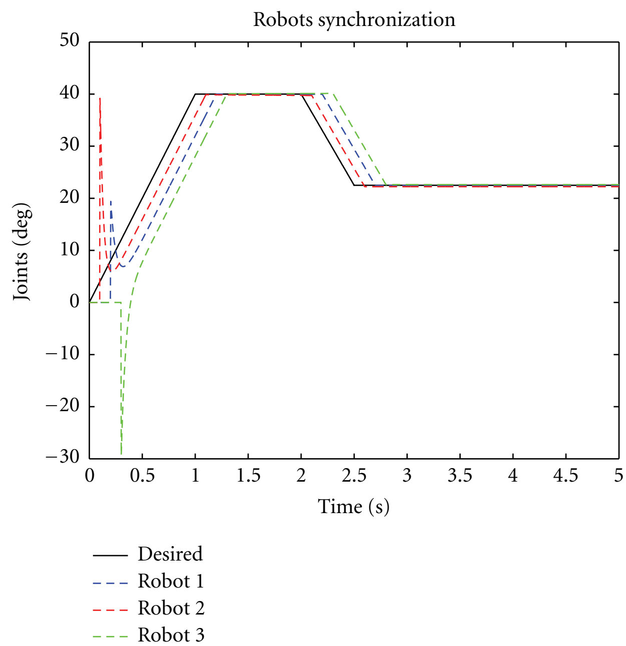
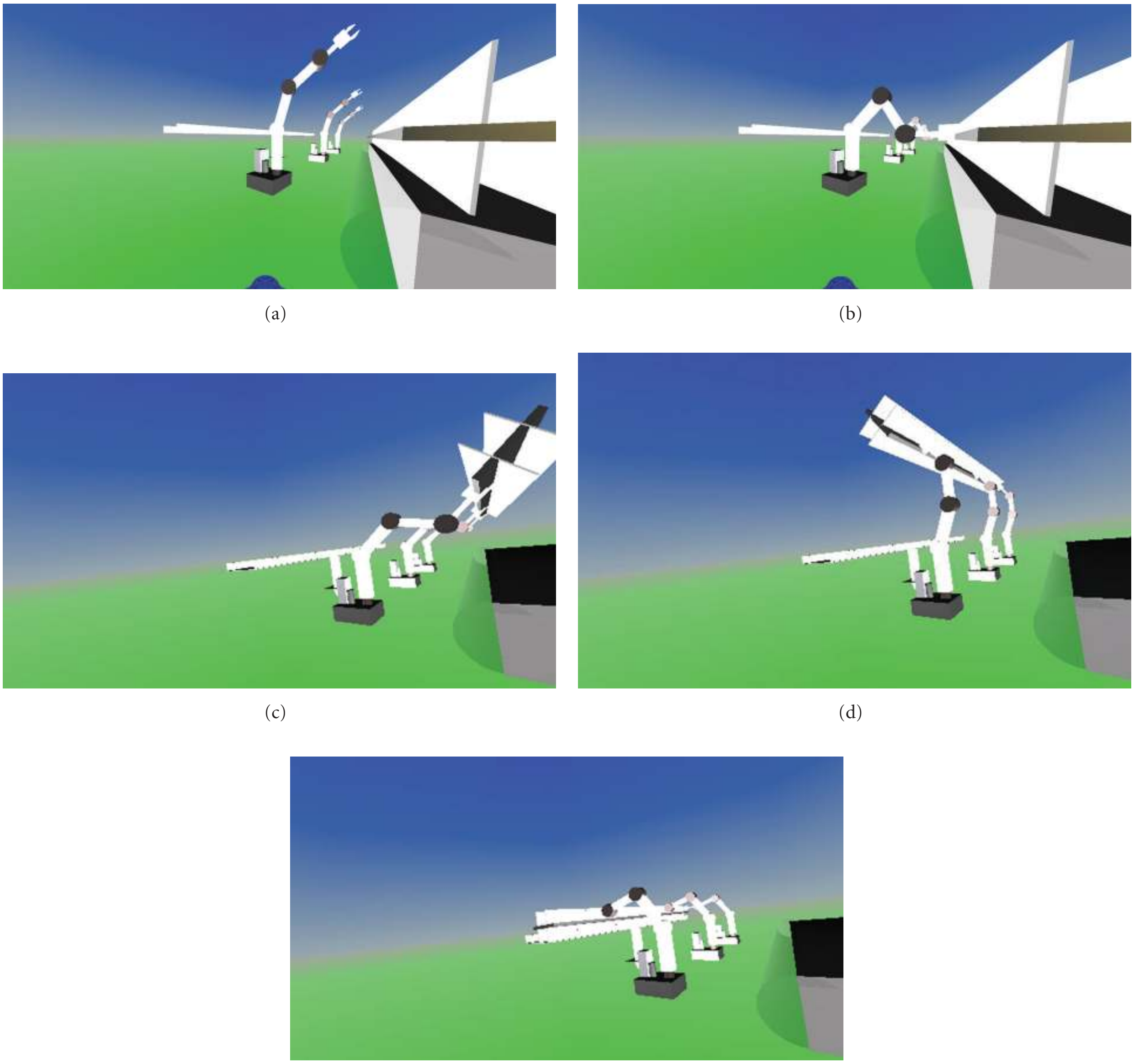


Figure 7: Robots synchronization in presence of time delay.

for achieving an agreement depends essentially on the time delay of communication channels.

Simulation 2. One effective solution is to develop a platform to simulate the complex system design before hardware implementation. To design this virtual environment, we apply several tools, such as the modeling package Cinema 4D, MATLAB VRML (Virtual Reality Modeling Language) toolbox, and MATLAB/Simulink. Robot CAD (Concept Architecture Design) models created using 4D cinema are extracted in VRML97 (.wrl) format. The VRML file is edited by V-Realm Builder 2.0 before being imported into MATLAB-Simulink by the VR (Virtual Reality) Toolbox. A VRML file uses a standard text format which can be read with any text editor. For MATLAB, the Virtual Reality Toolbox software includes the Ligos V-Realm Builder application as a native 3D editor. But the 3D editing tools remain the most efficient. In fact, the CAD packages offer the power and versatility to create many types of practical models and techniques. In the developed real-time software simulator we



(a) (b) (c) (d) (e)

Figure 8: The 3D virtual environment.

used the 4D Cinema as CAD package where the robots are designed. Next, the 3D objects are imported. Simulink model generates signal data which is used to control and animate the virtual world. Reality Toolbox block is added to the Simulink model in order to ensure communication with the virtual world. The 3D virtual reality environment is presented in Figure 8.

7. Conclusions

This paper has considered the synchronization problem in distributed Multirobot systems under cooperative schemes. The aim of this work is to find out a decentralized controller, applied to each manipulator; the synchronization is therefore met. It has been shown that the proposed strategy can coordinate manipulator articulations to track a given time-varying trajectory. As an extension, we moved to the step which entails the fact that mass, Coriolis, and gravity matrices are badly known. In this case, the proposed controller has encompassed an adaptive version. The obtained simulation results from Multirobot motion control system demonstrate the effectiveness of the synchronization approach. To deal with time delays problem in communication between robots, the proposed decentralized control guarantees that information variables of each robot reach agreement even in the presence of communication channels delays. A new package for simulation of coordinated robot manipulators is developed. Simulations which have been applied to an illustrative example have shown the effectiveness of the described strategy.

References

[1] S.-J. Chung and J.-J. E. Slotine, "Cooperative robot control and concurrent synchronization of lagrangian systems," *IEEE Transactions on Robotics*, vol. 25, no. 3, pp. 686–700, 2009.

[2] A. Jadbabaie, J. Lin, and A. S. Morse, "Coordination of groups of mobile autonomous agents using nearest neighbor rules," *IEEE Transactions on Automatic Control*, vol. 48, no. 6, pp. 988–1001, 2003.

[3] W. Ren, "Distributed leaderless consensus algorithms for networked Euler-Lagrange systems," *International Journal of Control*, vol. 82, no. 11, pp. 2137–2149, 2009.

[4] J. Mei, W. Ren, and G. Ma, "Distributed coordinated tracking with a dynamic leader for multiple euler-lagrange systems," *IEEE Transactions on Automatic Control*, vol. 56, no. 6, pp. 1415–1421, 2011.

[5] H. Yu and P. J. Antsaklis, "Passivity-based output synchronization of networked Euler-Lagrange systems subject to nonholonomic constraints," in *Proceedings of the American Control Conference (ACC '10)*, pp. 208–213, 2010.

[6] A. Rodriguez-Angeles and H. Nijmeijer, "Mutual synchronization of robots via estimated state feedback: a cooperative approach," *IEEE Transactions on Control Systems Technology*, vol. 12, no. 4, pp. 542–554, 2004.

[7] Y.-C. Liu and N. Chopra, "Synchronization of networked robotic systems on strongly connected graphs," in *Proceedings of the 49th IEEE Conference on Decision and Control (CDC '10)*, pp. 3194–3199, 2010.

[8] Z. Lin, M. Broucke, and B. Francis, "Local control strategies for groups of mobile autonomous agents," *IEEE Transactions on Automatic Control*, vol. 49, no. 4, pp. 622–629, 2004.

[9] R. Reza Olfati-Saber and R. M. Murray, "Consensus poblems in networks of agents with switching topology and time-delays," *IEEE Transactions on Automatic Control on Networked Control Systems*, vol. 49, no. 9, 2004.

[10] D. Sun, "Position synchronization of multiple motion axes with adaptive coupling control," *Automatica*, vol. 39, no. 6, pp. 997–1005, 2003.

[11] D. Sun and J. K. Mills, "Adaptive synchronized control for coordination of multirobot assembly tasks," *IEEE Transactions on Robotics and Automation*, vol. 18, no. 4, pp. 498–510, 2002.

[12] Z. Qu, J. Wang, and R. A. Hull, "Cooperative control of dynamical systems with application to autonomous vehicles," *IEEE Transactions on Automatic Control*, vol. 53, no. 4, pp. 894–911, 2008.

[13] J. R. Lawton and R. W. Beard, "Synchronized multiple spacecraft rotations," *Automatica*, vol. 38, no. 8, pp. 1359–1364, 2002.

[14] R. Olfati-Saber, J. A. Fax, and R. M. Murray, "Consensus and cooperation in networked multi-agent systems," *Proceedings of the IEEE*, vol. 95, no. 1, pp. 215–233, 2007.

[15] W. Ren and R. W. Beard, "Consensus seeking in multiagent systems under dynamically changing interaction topologies," *IEEE Transactions on Automatic Control*, vol. 50, no. 5, pp. 655–661, 2005.

[16] W. Ren, "Multi-vehicle consensus with a time-varying reference state," *Systems and Control Letters*, vol. 56, no. 7-8, pp. 474–483, 2007.

[17] W. Ren, R. W. Beard, and E. Atkins, "Information consensus in multivehicle cooperative control: collective group behavior through local interaction," *IEEE Control Systems Magazine*, vol. 27, no. 2, pp. 71–82, 2007.

[18] B. Yassine, G. Jawhar, and G. Poisson, "Adaptive multi-robots synchronization," in *Proceedings of the IEEE International Symposium on Industrial Electronics (ISIE '10)*, pp. 1797–1802, Bary, Italy, jully 2010.

[19] D. Sun, X. Shao, and G. Feng, "A model-free cross-coupled control for position synchronization of multi-axis motions: theory and experiments," *IEEE Transactions on Control Systems Technology*, vol. 15, no. 2, pp. 306–314, 2007.

[20] J. J. Craig, P. Hsu, and S. S. Sastry, "Adaptive control of mechanical manipulators," in *Proceedings of the IEEE International Conference on Robotics and Automation*, pp. 190–195, San Francisco, Calif, USA.

[21] J. J. E. Slotine and W. Li, "Adaptive manipulator control: a case study," *IEEE Transactions on Automatic Control*, vol. 33, no. 11, pp. 995–1003, 1988.