Distributing and Delivering Vessels of the Human Heart

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ABSTRACT   The branching characteristics of the right coronary artery, acute marginal, posterior descending, left anterior descending, circumflex, and obtuse marginal arteries are compared with those of diagonal branches, left and right ventricular branches, septal, and higher-order branches, to test a newly proposed functional classification of the coronary arteries in which the first group rank as distributing vessels and the second as delivering vessels. According to this classification, the function of the first type is merely to convey blood to the borders of myocardial zones, while the function of the second is to implement the actual delivery of blood into these zones. This functional difference is important in the hemodynamic analysis of coronary heart disease, as it provides an assessment of the role of a vessel within the coronary network and hence an assessment of the functional importance of that vessel in a particular heart. Measurements from casts of human coronary arteries are used to examine the relevant characteristics of these vessels and hence to test the basis of this classification.

INTRODUCTION

Variability in the morphological pattern of coronary arteries and their major branches is an important factor in the assessment and treatment of coronary heart disease. At present, hearts are categorized as being left dominant or right dominant, to indicate the relative extents of the left and right sides of the coronary network (McAlpine, 1975), but there is a whole continuum of possible variations from one extreme to another that these two simple categories do not adequately describe. The left circumflex artery in one heart may carry a substantial amount of blood flow to the posterior wall of the left ventricle, it may give rise to the posterior descending artery and thereby supply the interventricular septum, or it may do neither. Instead, it may terminate at the obtuse margin of the heart in the form of a relatively small branch. The right coronary artery may supply only the right side of the heart, it may give rise to the posterior descending artery and thereby supply the interventricular septum, and it may go far beyond the crux to supply a large part of the posterior left

ventricular wall. The detailed arrangement of branches arising from these main vessels is also highly variable. The lateral wall of the left ventricle is usually supplied by one or more diagonal branches of the left anterior descending artery and one or more left ventricular branches of the left circumflex artery, but the combination of these branches in different hearts is highly variable.

These features of the coronary network make the hemodynamic analysis of coronary heart disease somewhat more complicated. To determine the severity of stenosis of a coronary artery, it is necessary to assess not only the stenosis itself but also the functional significance of that stenosis. The latter requires an assessment of the role of the stenosed vessel within the coronary network of that particular heart. To repair a stenosed coronary artery, whether by means of a bypass or by angioplasty, requires a prior assessment of the functional importance of that vessel in the particular heart in question. Furthermore, in recent years it has become apparent that individual vessels within the coronary network supply individual regions of the myocardium and that collateral pathways within the network are not as abundant as they were believed to be in the past (DiDio and Rodrigues, 1983; Elayda et al., 1985; Hearse et al., 1986; DeWood et al., 1986). This makes it more crucial to identify the role of individual vessels within the coronary network, and to do so individually for each heart.

The difficulties involved in implementing such a scheme are obvious. The scheme would require a fairly detailed map of the coronary arteries and their major branches for each individual heart. While such maps can be constructed from routine angiography (Gensini, 1975), the task is far too tedious and may not be possible in many cases. An important aid in this process would be a model of the underlying design of the coronary network, which can serve as a reference with which the many variations in the network can be compared. A model of this kind was proposed recently, based on a study of casts of the coronary networks of human hearts (Zamir and Silver, 1985). This study indicated that, from the point of view of blood supply, the myocardium of the human heart is divided into six zones, and the morphological arrangement of the coronary arteries is based on the particular scheme by which these zones are served. A principal feature of this scheme is that each zone is circled by major "distributing" vessels, which run along its borders but do not enter the zone. Branches from these vessels enter the zones and implement the delivery of blood. The latter were named "delivering" vessels. An immediate implication of this model is that a delivering vessel is associated with blood supply to only a small region of the myocardium, while a distributing vessel affects the supply to an entire zone, or even two zones. This can be a valuable aid in mapping the coronary arteries of individual hearts, since it assigns a degree of "functional importance" to a vessel by determining whether it is a delivering or distributing vessel.

The purpose of this article is to test this model of the human coronary network by examining in a critical manner the implied functional difference between these two types of coronary arteries. The model has so far been based on mainly anatomical observations relating to the positions where these vessels were actually found in a series of human hearts (Zamir and Silver, 1985). It remains to be demonstrated that these vessels actually have distinguishing features that are consistent with their different functional roles. Since the main function of distributing vessels is to convey blood to borders of myocardial zones, it would be expected that they do not branch as profusely as do delivering vessels, and that their caliber does not diminish as rap-

idly along their course. We propose a scheme by which these differences can be tested, based on actual measurements from human coronary arteries.

MATERIALS AND METHODS

In order to examine the rate of branching of a particular coronary artery, it is necessary to have a sequence of diameter measurements along its course, taken before and after every branching site along that course. Such measurements can be obtained accurately from corrosion casts of the coronary arteries. Full details of the technique have been described elsewhere (Gannon, 1978). A series of casts was produced recently for a study of the general architecture of the human coronary network (Zamir and Silver, 1985). The hearts were obtained at autopsy from cases that did not involve cardiovascular disease. A cast that offered the greatest amount of detail of the coronary arteries and their branches was chosen for the present study. These details were carefully mapped and the cast was then broken into smaller pieces, as illustrated in Fig. 1, in order to bring all segments of each vessel into view on the stage of the measuring instrument. The diameter of each segment was measured in several places in order to produce an average diameter for that particular segment. A total of 278 segments were surveyed.

FIGURE 1. Branching sites along the course of a vessel were numbered consecutively, as shown in this sample. The diameter of each segment of the vessel was measured, usually in several places, to produce an average for that segment.

ANALYSIS OF BRANCHING RATE

It has generally been established that when a blood artery gives rise to a branch, its diameter diminishes by a certain amount, which depends on the diameter of the branch (Murray, 1926a; Kamiya and Togawa, 1972; Zamir, 1976, 1978). The amount of change is large when the branch diameter is large, as in the case of a symmetrical
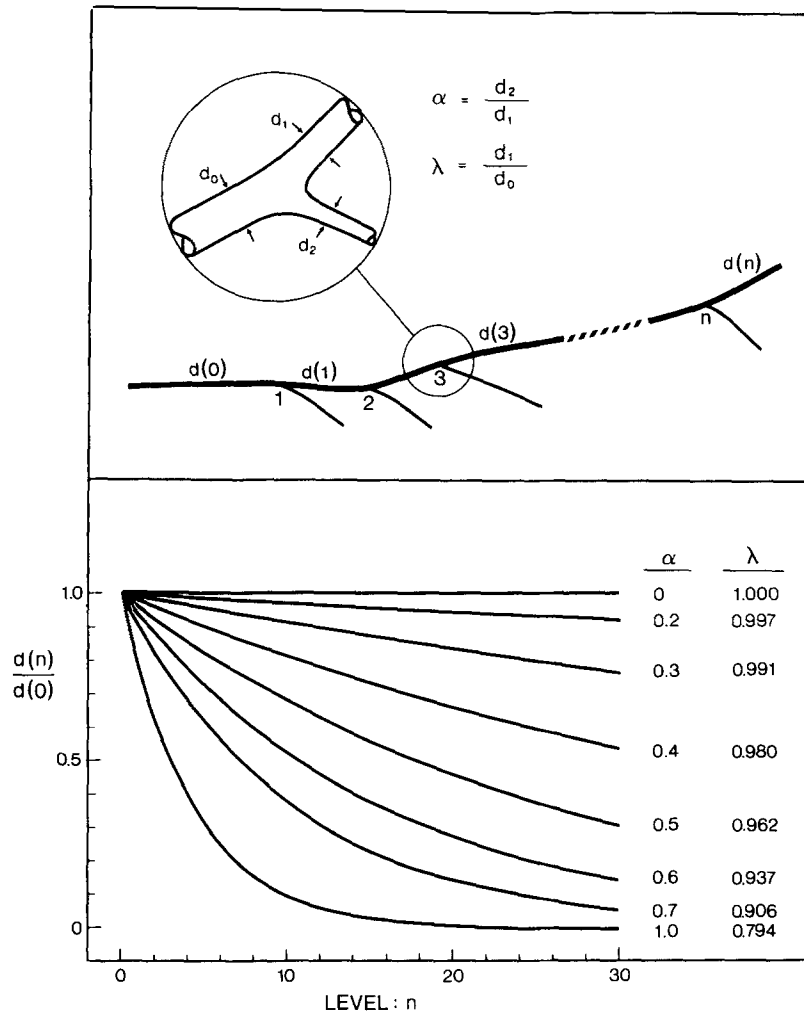

FIGURE 2. The rate of branching of a coronary artery is the rate at which its diameter decreases as it progresses along its course, that course being measured not in terms of distance along the vessel but in terms of the number of branching sites. At each site, the amount λ by which the parent diameter diminishes depends on the bifurcation index α, as defined in the top part of the figure. Each curve represents the course of a vessel for a particular value of α, and hence of λ. The diameter, $d(n)$, of the vessel following n branching sites along its course is expressed as a fraction of its initial diameter, $d(0)$. Along the upper curves, the diameter of the vessel decreases slowly and the branching rate is low; along the lower curves, the reverse is true. The figure thus presents a framework in which the branching rate of an actual artery can be examined by comparing its course with the above curves.

bifurcation, where a parent artery divides into two branches of equal calibers. On the other hand, the change may be imperceptibly small when the branch diameter is very small, as in the case of a small side branch, where the parent artery appears to continue unchanged. Thus, if a coronary artery undergoes a succession of mainly
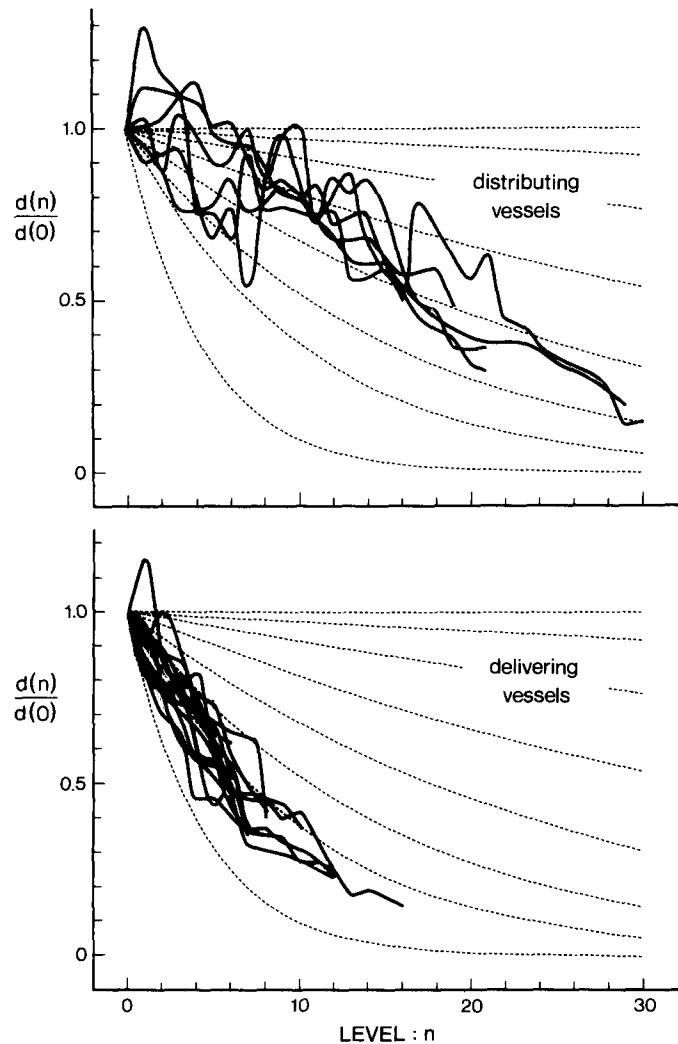

FIGURE 3.   Diameter measurements from distributing and delivering vessels, compared in the framework of Fig. 2. The distributing group comprises the right coronary artery, acute marginal, posterior descending, left anterior descending, circumflex, and obtuse marginal arteries. The delivering group comprises diagonal branches, left and right ventricular, septal, and higher-order branches.

symmetrical bifurcations, while another artery gives rise to a succession of mostly small side branches, the diameter of the first will decrease much more rapidly than that of the second. We define the "rate of branching" of an artery as the rate at which its diameter decreases as it progresses along its course, that course being measured

not in terms of distance along the vessel but in terms of the number of branching sites.

In a more rigorous view of the branching process, all branching sites of the dichotomous type can be regarded as arterial bifurcations. At an arterial bifurcation, a stream of blood divides into two separate streams, and a parent artery of diameter d_0 divides into two daughter branches of diameters d_1 and d_2. If d_1 is taken to be the larger of the two diameters, then a bifurcation "index" is defined by

$$\alpha = d_2/d_1.$$

The entire range of dichotomous branching sites can be described in terms of the value of this index. A value near zero corresponds to the site of a side branch and a value near 1.0 corresponds to that of a nearly symmetrical bifurcation. Thus, the branching rate of an artery can now be said to depend on the values of α at the
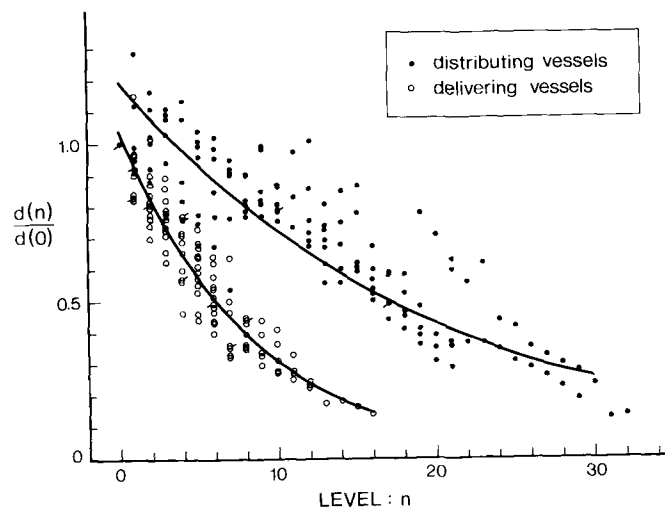

FIGURE 4. Diameter measurements from the same group of vessels as in Fig. 3, but here the data points are fitted collectively by statistical curves. A small dash indicates more than one data point in the same position.

branching sites along its course. A succession of values near 1.0 will indicate a very rapid branching rate, while a succession of values near zero will indicate a slow branching rate.

It has also been argued on theoretical grounds (Murray, 1926a, b; Rodbard, 1975; Zamir, 1977), and adequately confirmed by actual measurements (Hutchins et al., 1976; Zamir and Medeiros, 1982; Zamir and Brown, 1982; Mayrovitz and Roy, 1983; Zamir and Chee, 1985), that an optimum relation exists between the three diameters involved at an arterial bifurcation, namely

$$d_0^3 = d_1^3 + d_2^3.$$

If this relation is expressed in terms of the bifurcation index as defined above, it then reads

$$\frac{d_1}{d_0} = (1 + \alpha^3)^{-1/3} = \lambda.$$

Since d_1 is the diameter of the larger of the two branches at the bifurcation site, and if this branch is regarded as the "continuation" of the parent artery, then the quantity λ above represents the fraction by which the diameter of the parent artery decreases through a bifurcation site of index α. If the artery undergoes a succession of n such branching sites, then the ratio of its final diameter, $d(n)$, to its initial diameter, $d(0)$,
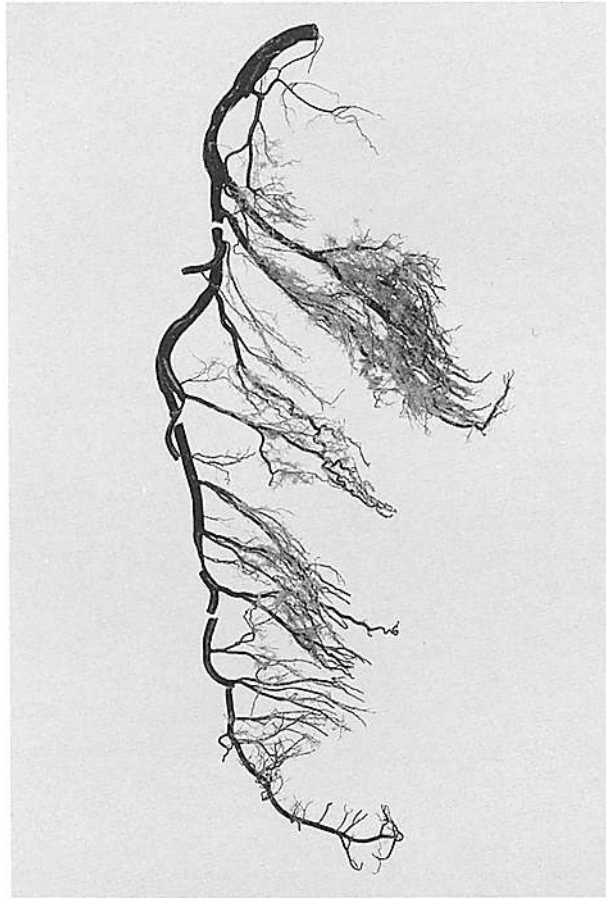

FIGURE 5.   A cast of a left anterior descending artery and its branches, which illustrates the difference between the rates of branching of the two types of vessels. The branches, being delivering vessels, typically divide more profusely and terminate more rapidly than the feeding distributing vessel, which in this case is the left anterior descending.

will be given by

$$\frac{d(n)}{d(0)} = \lambda^n = (1 + \alpha^3)^{-n/3}.$$

For a given value of α, and hence λ, the gradual change in diameter from $d(0)$ to $d(n)$ can be represented by a descending curve along which the value of the fraction $d(n)/d(0)$ is given at increasing levels n. The degree of descent of the curve represents the rate of branching. Different values of α give different curves, which represent

different rates of branching as illustrated in Fig. 2. This figure thus represents a framework on which the measured segment diameters of a coronary artery can be plotted and in which the rate of branching of that artery can be evaluated by comparison with these standard curves or with the curves of other arteries.

FIGURE 6. A partial cast of a right posterior descending artery and its branches, again illustrating the difference between the rates of branching of distributing and delivering vessels. The distributing vessel in this case is the right posterior descending artery, a segment of which lies horizontally at the top of this photograph.

RESULTS

Diameter measurements along the individual courses of the right coronary artery, acute marginal, posterior descending, left anterior descending, circumflex, and obtuse marginal arteries are shown in Fig. 3. Measurements from diagonal branches, left and right ventricular branches, septal, and higher-order branches are shown separately in the same figure. The vessels were divided in this way since, according to
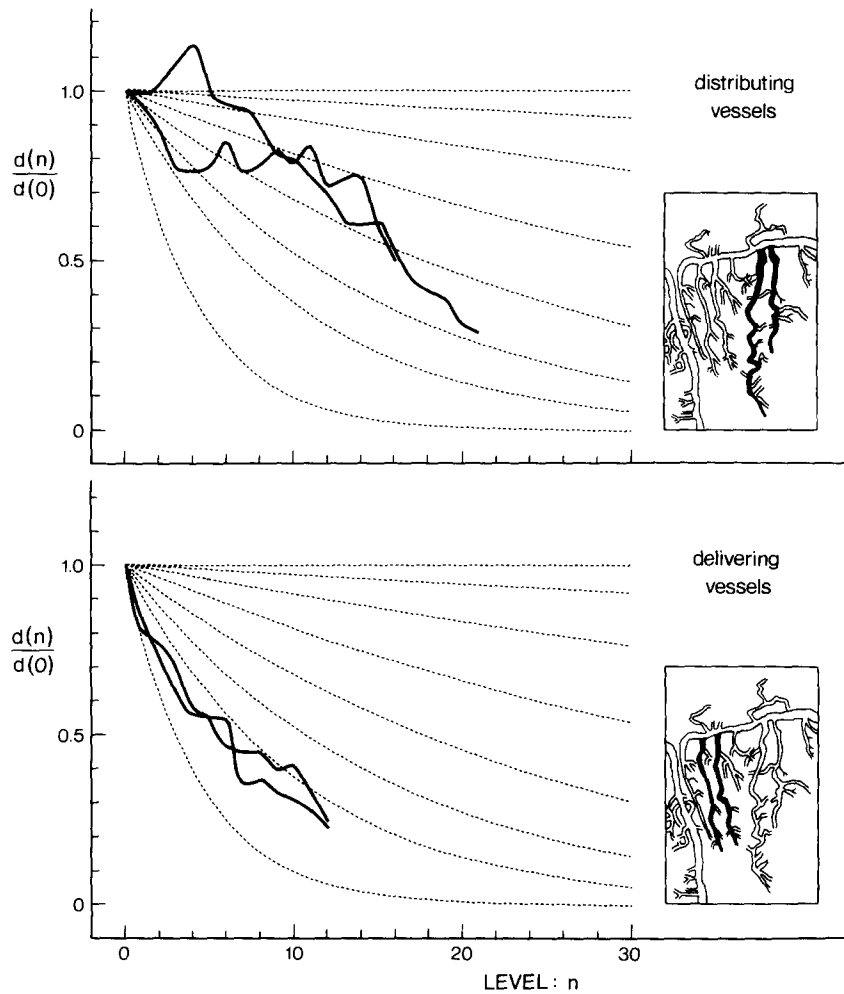
Figure 7. While distributing vessels usually have larger calibers than delivering vessels, caliber alone does not determine the role of a vessel in the coronary network. The four vessels shown here appear comparable in size but their rates of branching are clearly different. Two of the vessels (top) are right posterior descending, while the other two (bottom) are left ventricular branches of the right coronary artery. Their closely neighboring positions within the coronary network are shown in the insets. The difference in their rates of branching is consistent with the first pair being distributing vessels and the second being delivering vessels.

the newly proposed classification of coronary arteries (Zamir and Silver, 1985), the first group of these vessels would be classed as distributing vessels and the second as delivering vessels. It is important to note in this comparison that the diameter of each vessel at its various stages is here expressed as a fraction of its initial diameter; thus, all vessels start out with a normalized diameter of 1.0 regardless of their actual diameter. The comparison therefore refers not to the diameter of a vessel but to the rate of change of its (normalized) diameter with the number of branching sites along its

course. Data from the same two groups of vessels are also shown in a statistical format in Fig. 4, with curves of best fit for comparison.

DISCUSSION AND CONCLUSION

According to the proposed classification of coronary arteries into distributing and delivering vessels (Zamir and Silver, 1985), the function of the first type is merely to convey blood to the borders of myocardial zones, while the function of the second is to implement the actual delivery of blood into these zones. On the basis of this functional difference, it would be expected that distributing vessels will have a lower branching rate than delivering vessels, and the results of Figs. 3 and 4 certainly confirm this expectation. While there is some meandering in the diameter course of individual vessels, their tendency to clump together into two distinct groups supports the notion of a functional difference between them. Furthermore, when viewed as two groups, the branching rates of the right coronary artery, acute marginal, posterior descending, left anterior descending, circumflex, and obtuse marginal arteries are distinctly lower than those of diagonal branches, left and right ventricular branches, septal, and higher-order branches. This difference is consistent with the classification of these coronary arteries into distributing and delivering vessels, respectively.

The difference in the branching rates of different coronary arteries can be observed directly from individual casts of these vessels. Two such casts are shown in Figs. 5 and 6, relating to the left anterior descending and right posterior descending arteries. In both cases, the branches, being delivering vessels, are seen to divide more profusely and terminate more rapidly than the feeding distributing vessel.

Finally, while distributing vessels usually have larger calibers than delivering vessels, caliber alone does not determine the role of a vessel in the coronary network. In Fig. 7, four coronary arteries of similar calibers are compared with regard to their branching rates. Two of the vessels are right posterior descending, while the other two are left ventricular branches of the right coronary artery. There is a distinct difference between the rates of branching of the two pairs, consistent with the first pair being distributing vessels and the second being delivering vessels.

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