Distribution, taxation and employment in an open economy

van de Klundert, T.C.M.J.

Published in:
De Economist

Publication date:
1982

DE ECONOMIST 130, NR. 1, 1982

DISTRIBUTION, TAXATION AND EMPLOYMENT IN AN OPEN ECONOMY**

BY

THEO VAN DE KLUNDERT*

1 INTRODUCTION

Even in the prosperous years immediately before the first oil crisis, employment in the private sector of the Dutch economy showed a downward trend. Inspection of the macroeconomic data revealed that this rather unfavourable development could not be attributed to demand factors. Standard Keynesian explanation of unemployment was not appropriate. The recognition of this fact led Den Hartog and Tjan [7] to a proper specification and estimation of a macroeconomic production function, indicating that the decline in employment could be related to an excessive rise of the product wage. The production function applied was based on the idea of embodied labour saving technical progress, without ex-ante substitution possibilities, the so-called clay-clay vintage model.

The next step was to develop a more complete macroeconomic model for the Netherlands by adding empirical relations for demand components, prices and of course wages. This research programme resulted in the presentation of the 'Vintaf' model by Den Hartog, Van de Klundert and Tjan [8]. A revised version of this model is nowadays used by the Central Planning Bureau to analyse medium-term developments of the economy. The demand factors in the model are explained along familiar lines, which need no comment here.

Changes in prices are based on *ad hoc* reasoning. It is assumed that competition by foreigners and the degree of capacity utilization are important determinants of domestic prices. However, in a proper setting, decisions by entrepreneurs with regard to output and prices should be interrelated. Profit-

* Professor of Economics, Tilburg University, Faculty of Economics.
** Acknowledgement is due for his helpful comment to S.K. Kuipers

maximising entrepreneurs determine the optimum price and the corresponding planned output in accordance with the conditions of demand. As shown by Bruno [3], an excess demand element could then still appear in the price equation if the forecast on the firm's demand curve has not been accurate.

Wages in the 'Vintaf' model depend on consumption prices, labour productivity, the rate of unemployment, and a variable for the increase in the burden of taxation on labour income and social security premiums. The latter variable allows for the fact that changes in the rate of taxation (including premiums for the sake of convenience) are to a certain extent passed on in wages. Such a behaviour by workers has important consequences for fiscal policy. An increase in public spending financed by raising taxes may under these circumstances lead to wrong results. Or, to put it more succintly, the balanced budget multiplier may be negative!

An empirical model such as 'Vintaf' is constructed for the purpose of prediction and evaluation of policy alternatives. In economic theory the main issue is understanding or generalization. The latter approach will be followed here. The problems raised will nevertheless be the same as in the debate around the 'Vintaf' model. More particularly, attention will be paid to the consequences of a wage push and the options of fiscal policy in a small open economy. The theoretical model used resembles the empirical one in many aspects. There is, however, one important difference, which should be mentioned from the outset. It is assumed that the goods market always clears instantaneously, because prices are sufficiently flexible in the short-run. However, wages may deviate from their full employment level.[1] As a result, there could be classical or structural unemployment if real wages are too high. The opposite case of wages being too low – although not formally excluded – will not be considered as interesting.

The plan of the paper is as follows: In section 2 the basic assumptions of the model are set forth. The equations are presented in a linear form. Analytical solutions for the short- and long-run are given in section 3. This makes it possible to formulate a number of important conclusions. To facilitate comprehension, numerical examples are added in section 4. The parameter values chosen for the simulation runs are more or less representative for a small open economy, like the Netherlands for instance. In section 5 the implications of a gradual adjustment of the real wage rate are studied. For this purpose the model is extended with a Phillips curve. Finally, in section 6

1 As suggested by Malinvaud [12] and earlier also by Williamson [16] this may be thought of the situation Keynes had in mind when he wrote the 'General Theory.'

some limitations of the model along with needs for further development are noted. The linearization procedure applied in this paper is elaborated on Appendix A. The analytical solutions referred to above are derived in Appendix B. A list of symbols is included at the end.

2 A MODEL FOR AN OPEN ECONOMY

In this section a linearized version of a flexible-output-price model for a small open economy shall be presented. The model is dynamic, because account shall be taken of capital accumulation. A linear approximation is obtained by logarithmic differentiation of the original model. In addition, it is assumed that in the initial situation the economy follows a path of balanced growth in accordance with neoclassical traditions.[2] This implies that the growth rate is equal to the natural rate and that all relevant quantity ratios and relative prices are constant. These ratios may appear as constant parameters in the linear system. To facilitate understanding, the original equations with their linear counterpart are given in Appendix A.

Models for an open economy with a flexible output price are applied by Branson and Rotemberg [2] as well as Bruno and Sachs [4], [5], among others. Characteristic of these models is that the system may be reduced to macroeconomic demand and supply curves to demonstrate the market-clearing role of the relative price of the final good. The relative price of the home-produced good is defined as the price of this good in relation to the price of the foreign good. The rate of exchange is assumed to be constant. Alternatively, the rate of exchange could be made endogenous by introducing a monetary sector and stipulating perfect mobility of financial assets. This extension of the analysis is demonstrated in Bruno and Sachs [4], Sachs [14] and Van de Klundert [10].

Of course, equilibrium on the market for goods does not necessarily imply that unemployment always equals its natural rate. A different result will be obtained if the wage rate deviates for some reason from its equilibrium value. One of the factors responsible for such a deviation could be the burden of taxation. The hypothesis that direct taxes on wages are passed on in part or completely is supported by empirical studies for different countries. The consequence of this behaviour may be 'supply-crowding-out,' as it is coined

2 This method was introduced by Schouten in [15]. For a presentation of the method in English, reference can be made to a recent paper by Van den Goorbergh and Schouten [6].

by Bacon in [1]. Higher public spending financed by higher tax rates shifts the total supply curve to the left, which leads to a lower level of GNP. To put it differently: the balanced budget multiplier is negative under these circumstances.

Apart from some minor points with regard to the specification of the demand equations, the main differences of the present study *vis-á-vis* the ones mentioned above relate to the role of the public sector and the process of capital accumulation. The implications of these extensions will be brought out clearly by solving the model analytically. Such a solution is made easier by the linearization procedure to which we referred in the opening sentences of this section.

Domestic production of the final good (y^s) is a linear homogenous function of labour (ℓ) and capital (k), which can be written in a linearized form as[3]:

$$y^s = \lambda \ell + (1-\lambda)k \qquad (2.1)$$

where λ is the production elasticity of labour. The variables y, ℓ and k are expressed as relative or percentage deviations from a path of balanced growth. The value of λ corresponds to the relevant equilibrium ratio along this path.

The amount of capital is given in the short-run, because it takes time to accumulate investment goods (k=0). The demand for labour is obtained from the familiar equality of the real product wage and the marginal product of labour. If the production function is characterised by a constant elasticity of substitution (σ), this leads to:

$$\ell - y = -\sigma(p_\ell - p_y) \qquad (2.2)$$

Here p_ℓ stands for the nominal wage rate, whereas p_y relates to the price of the domestic good.

Total demand for the domestic product (y^d) includes four components, namely consumption of home-produced goods (c_h), investment of home-produced goods (i_h), real government expenditure falling entirely on the domestic product (g), and finally foreign demand or exports (b). Applying the appropriate weights we therefore state:

$$y^d = \gamma_c c_h + \gamma_i i_h + \gamma_g g + \gamma_b b \qquad (2.3)$$

with $\Sigma\gamma_j = 1$ ($j = c, i, g, b$)

3 Time subscripts are omitted. Lagged variables will be indicated by the subscript -1.

The volume of consumption (c) and the volume of investment (i) consist of home- and foreign-produced or imported goods. The latter will be indicated by the suffix m. For the sake of convenience the share of the domestic good in total nominal expenditure (μ) is assumed to be the same with regard to both categories. Therefore, we have:

$$(c + p_c) = \mu(c_h + p_y) + (1 - \mu)(c_m + \underline{p_y}^*) \tag{2.4}$$

$$(i + p_i) = \mu(i_h + p_y) + (1 - \mu)(i_m + \underline{p_y}^*) \tag{2.5}$$

The price index of private domestic expenditure can now be defined as:

$$p_x (= p_c = p_i) = \mu p_y + (1 - \mu)\underline{p_y}^* \tag{2.6}$$

The foreign good is an imperfect substitute for the domestic product in consumption and investment. The constant elasticity of substitution on the demand side is symbolized by φ. The price of imported goods ($\underline{p_y^*}$) is an exogenous variable. In our notation exogenous variables are barred. Recall that the rate of exchange is held constant. Import substitution then follows from the equations:

$$c_h - c_m = -\varphi(p_y - \underline{p_y^*}) \tag{2.7}$$

$$i_h - i_m = -\varphi(p_y - \underline{p_y^*}) \tag{2.8}$$

The consumption volume can be expressed as a function of the real disposable wage sum, neglecting consumption out of non-wage income or profits:

$$c = \ell + p_\ell - \frac{1}{1-\tau} t_\ell - p_x \tag{2.9}$$

In this equation the variable t_ℓ indicates a change in the direct tax rate on labour income. To simplify the exposition, two further assumptions are made. First, in the initial situation all incomes are taxed at the same rate (τ). Second, changes in the rate of taxation apply to labour income only. There is no incremental tax on profits ($t_r = 0$).

Investment behaviour is rather capricious. Nevertheless, it could be maintained that profitability, sales prospects, and the cost of capital are important factors with regard to decisions about the level of capacity. To simplify, we assume that the influence of the first and second factors is captured by (disposable) real profits (real non-labour income). There is no need to introduce a measure for the cost of capital, because the model does not contain a monetary sector. Moreover, in a small open economy under static money-exchange-rate expectations, the money rate of interest is determined by the level abroad. Therefore, the investment equation of the model comes down to:

$$i = Y_r - p_x \tag{2.10}$$

Nominal profits (Y_r) are deflated by the price of private domestic expenditure (p_x) to obtain real profits.

Under neo-classical conditions the share of labour in GNP is equal to the production elasticity of labour (λ). Nominal profits can then be found by applying the definition:

$$Y_r = \frac{1}{1-\lambda}(y + p_y) - \frac{\lambda}{1-\lambda}(\ell + p_\ell) \tag{2.11}$$

Export demand for the domestic good is a function of world trade (m^*) and the terms of trade:

$$b = \underline{m}^* - \varphi(p_y - \underline{p}_y^*) \tag{2.12}$$

As appears from (2.12) foreigners hold the same view with regard to the substitutability of both goods as economic subjects in the domestic country. As will we shown below, this does not mean that the elasticity of exports and that of imports are equal.

Real public expenditure increases with GNP but depends also on a policy variable ($\underline{g}$):

$$g = y + \underline{g} \tag{2.13}$$

In principle, government expenditure can be financed by taxation or borrowing. In the latter case there will be a deficit on account of the public sector. In the initial situation of balanced growth the deficit is zero by assumption. Deviations of this equilibrium position (positive in case of a surplus and negative in the other) are now expressed as a percentage of GNP and indicated by f. The balance on government account is considered to be a policy variable. Applying the definition of this variable and linearizing around the initial growth path we arrive at:

$$\underline{f} = \lambda\, t_{\ell} - \gamma_g (g - y) \tag{2.14}$$

It should be observed that because of a zero deficit in the initial situation we have ($\tau_{\ell} = \tau_r = \tau = \gamma_g$). Moreover, it should be recalled in this connection that no incremental taxes are imposed on profits ($t_r = 0$).

The output price p_y is obtained by equating supply and demand:

$$y^s = y^d \tag{2.15}$$

The labour market does not clear. Therefore, the wage rate appears as an exogenous variable. However, endogenous factors may play a supplementary role because of price indexation and bargaining on basis of real disposable income. In the latter case an increase in tax rates is passed on in wages, as observed before. These considerations lead to the following wage equation:

$$p_{\ell} = \underline{p}_{\ell} + \psi\, p_x + \omega\, t_{\ell} \tag{2.16}$$

Indexation is based on the price of private domestic expenditure. In general the following conditions must hold: $0 \leqslant \psi \leqslant 1$ and $0 \leqslant \omega \leqslant 1$. If $\psi = 1$ wages are fully protected against inflation. If $\omega = 1$ disposable labour income is not affected by changes in the rate of taxation.

Capital accumulation depends on gross investment in the preceding period and on the rate of technical obsolesence (δ). In the initial situation of balanced growth the ratio of net investment and the stock of capital equals the natural rate of growth (π). On the basis of these properties the following linear expression for the rate of accumulation can be derived[4]:

$$k = k_{-1} + \frac{\pi + \delta}{\pi + 1} (i_{-1} - k_{-1}) \tag{2.17}$$

For a given value of k the equations (2.1) – (2.16) may be used to solve for the following 15 endogenous variables: y, ℓ, c, c_d, c_m, i, i_d, i_m, g, b, t_{ℓ}, p_{ℓ}, p_y, p_x, Y_r. The analytical solution for the short-run will be presented in the next section. Substitution of the outcome for the rate of investment (i) in equation (2.17) makes it possible to trace the time path of the endogenous variables. The next thing to do is then to investigate whether the model is stable. In addition, the long-run solution for the state variable k will be derived.

4 The deriviation of (2.17) is given in Appendix A. Mathematically (2.17) looks like the revised Harrod formula derived by Hicks in [9].

Although the analytical solutions contain all the information there is, a numerical example will be added in section 4 to illustrate the working of the model. For this purpose, parameter values will be assigned to the equations which are more or less representative for a small open economy like the Netherlands.

3 ANALYTICAL SOLUTIONS

The short-run solution of the model for a given value of the capital stock (k=0) follows familiar lines. First we derive a supply and a demand equation on the macroeconomic level. Next, equilibrium values for output (y) and price (p_y) are obtained by equating demand and supply.

To find a relationship for total demand, one needs the equations (2.3) – (2.14) and (2.16). As shown in Appendix B the resulting formula can be simplified substantially by introducing two additional assumptions. These are: (I) full indexation of wages, ψ=1 and (II) equality of the propensities to spend out wages and profits with respect to domestic goods, $\frac{\gamma_c}{\lambda} = \frac{\gamma_i}{1-\lambda}$. Both propositions seem reasonable approximations of reality, at least in the case of the Dutch economy. In the numerical example of section 5 we shall adhere to these assumptions.

For the demand schedule in its simplified form we then have:

$$y^d = \frac{1}{\Gamma}[-\{(\eta_m+\eta_b) - \frac{\gamma_i(1-\mu)}{1-\lambda}\}(p_y-\underline{p}_y^*) - \frac{\gamma_c}{\lambda(1-\tau)}(\underline{f}+\gamma_g \underline{g}) + \gamma_g \underline{g} + \gamma_b \underline{m}^*] \quad (3.1)$$

The following short-hand notation is applied:

$$\Gamma = 1 - \frac{\gamma_c}{\lambda} - \gamma_g > 0$$

$$\eta_m = [\gamma_c(1-\mu) + \gamma_i(1-\mu)]\varphi > 0$$

$$\eta_b = \gamma_b \; \varphi > 0$$

The interpretation of (3.1) is straightforward. The factor $^1/\Gamma$ symbolizes the well-known Keynesian multiplier. The impact of the relative price variable (p_y-$\underline{p}_y^*$) consists of two terms. The negative impact of the substitution effect on the demand side is measured by the sum of the import elasticity (η_m) and the export elasticity (η_b). On the other hand, we have a positive effect due to an improvement in the terms of trade. Because wages are fully indexed (ψ=1), a change in the terms of trade has no influence on real labour income. Therefore, the total effect falls on real profits. The consequences for investment are clearly demonstrated by the following formula, derived in Appendix B:

$$i = k + \frac{1}{1-\lambda}[(1-\mu)(p_y - \underline{p}_y^*) - \lambda \underline{p}_\ell - \omega(\underline{f} + \gamma_g \underline{g})] \tag{3.2}$$

For a sufficient large value of φ the substitution effect outweighs the terms-of-trade effect, in which case the demand curve has the usual negative slope. In the remaining part of this paper we suppose this to be typical.

The impact of an autonomous change in public spending depends on the method of finance. In case of deficit spending we have $\underline{f} = -\gamma_g \underline{g}$. The impact of $\underline{g}$ is then equal to the weight of public spending in total demand for the domestic good (γ_g). If additional public spending is financed by an increase in the tax rate on labour income the impact amounts to: $[1 - \frac{\gamma_c}{\lambda(1-\tau)}]\gamma_g$. If the initial tariff is not too high the expression between brackets is positive. This implies that tax-financed extra public spending has a positive, but of course smaller, effect than deficit spending. The impact of changes in world trade depends on the weight of exports in total demand for the domestic product (γ_b).

The supply function can be found from equations (2.1), (2.2), (2.6), (2.14) and (2.16). After some manipulations the following result is obtained:

$$y^s = \frac{\sigma}{1-\lambda}[\lambda(1-\mu)(p_y - \underline{p}_y^*) - \lambda \underline{p}_\ell - \omega(\underline{f} + \gamma_g \underline{g})] + k \tag{3.3}$$

The supply schedule has a positive slope, as is immediately obvious from (3.3). The elasticity of supply depends of course on the elasticity of factor substitution. Easy substitution leads to a relatively flat curve. An autonomous increase in wages has a negative impact on supply. The same holds for an increase in the tax rate on wages, provided $\omega > 0$. For, in such a situation, taxes are to some extent passed on in wages. In both cases the impact will be greater the higher the elasticity of factor substitution. The relation between the stock of capital and supply of course needs no explanation. However, in the short-run we have $k=0$, because it takes time to accumulate additional capital. It should be noted that there is also a multiplier on the supply side of the model. An increase in output leads to additional demand for labour, which in combination with a given capital stock leads to additional supply. The relevant multiplier is then equal to $\frac{1}{1-\lambda}$.

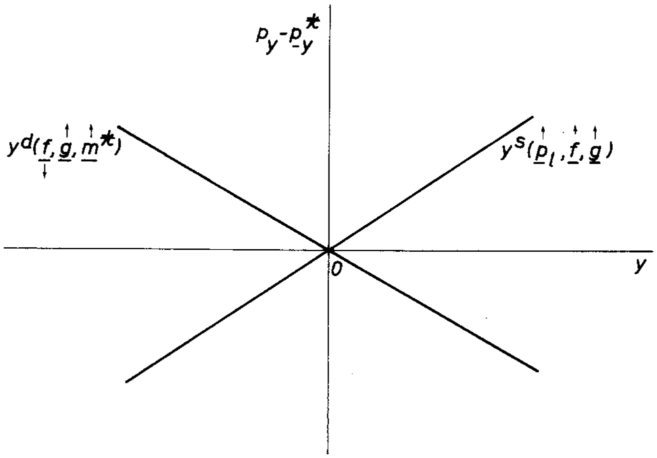

Figure 1

Demand and supply curves are drawn in Figure 1. In the initial situation the exogenous variables are zero, so both curves pass through the origin. The exogenous variables relevant for demand and supply are shown in parenthesis with an arrow indicating the direction in which the curve is shifted in case of a positive impulse. An autonomous wage increase ($\underline{p}_\ell > 0$) shifts the supply curve upward, which leads to a lower level of output and a higher price. A rise in world trade ($\underline{m}^* > 0$) shifts the demand curve upward, resulting in a higher output and price. Changes in fiscal policy are more difficult to analyse because both curves are affected. A tax-financed increase in public spending ($\underline{g} > 0, \underline{f} = 0$) moves both curves upwards. The price level will be higher, but the effect on output will be indeterminate. The opposite conclusion holds if the tax rate on wages is raised ($\underline{f} > 0$). In this case, the demand curve moves downward, whereas the supply curve shifts upward. As a result, output will be lower, but no unambiguous statement is possible with regard to the price level.

Short-run equilibrium solutions are readily obtained by solving (3.1) and (3.3) algebraically. After some rearrangements equating y^s and y^d gives:

$$p_y = \underline{p}_y^* + \frac{1}{\Lambda}[\Gamma\sigma\lambda\underline{p}_\ell + (\Gamma\sigma\omega - \frac{\gamma_c}{1-\tau}\frac{1-\lambda}{\lambda})(\underline{f}+\gamma_g\underline{g}) + (1-\lambda)(\gamma_g g + \gamma_b\underline{m}^* - \Gamma k)] \tag{3.4}$$

Substitution of (3.4) in (3.3) leads finally to the following solution for output:

$$y = \frac{1}{\Lambda}\Big[-\frac{\sigma\lambda}{1-\lambda}\chi_1 \underline{p}_\ell - \Big\{\frac{\sigma\omega}{1-\lambda}\chi_1 + \frac{\gamma_c}{1-\tau}\sigma(1-\mu)\Big\}(\underline{f}+\gamma_g \underline{g}) + \sigma\lambda(1-\mu)(\gamma_g \underline{g} + \gamma_b \underline{m}^*) + \chi_1 k\Big] \tag{3.5}$$

The parametric abbreviations introduced in these equations are given by:

$$\Lambda = (1-\mu)(\Gamma\sigma\lambda - \gamma_i) + (1-\lambda)(\eta_m + \eta_b) > 0$$

$$\chi_1 = \Lambda - \Gamma\sigma\lambda(1-\mu) > 0$$

The expressions for Λ and χ are positive for reasonable values of the parameters. More particularly, a sufficient high value of φ will do the job.

The qualitative conclusions found by analysing shifts in the demand and supply curves are of course confirmed by the solutions (3.4) and (3.5). A wage push ($\underline{p}_\ell > 0$) causes stagflation ($y < 0, p_y > 0$), while a recovery of world trade ($\underline{m}^* > 0$) leads to Keynesian results ($y > 0$, $p_y > 0$). Deficit spending ($\underline{g} > 0, \underline{f} = -\gamma_g \underline{g}$) also induces inflationary outcomes ($y>0, p_y > 0$) as may be expected. Tax-financed public spending ($\underline{g} > 0, \underline{f} = 0$) results in a price increase ($p_y > 0$) if the initial tariff is not too high. The condition required is the same as mentioned in connection with the demand schedule, namely $\tau < 1 - \frac{\gamma_c}{\lambda}$. The effect on y depends on ω, among other factors. It can be shown that output will decrease ($y < 0$) if the following condition holds:

$$\omega > \frac{\lambda - \frac{\gamma_c}{1-\tau}}{\frac{\eta_m + \eta_b}{1-\mu} - \frac{\gamma_i}{1-\lambda}} \tag{3.6}$$

Even if changes in the tax rate on wages are not fully passed on ($0 < \omega < 1$), there is a good chance that the balanced budget multiplier will be negative. The chance is greater *ceteris paribus* the easier substitution between the domestic and the foreign good. For a relatively high value of φ will be reflected in the import and export elasticity (respectively η_m and η_b). Changes in the wage rate will have no impact in case of factor complementary ($\sigma = 0$). This rather obvious conclusion is confirmed by the formulas (3.4) and (3.5).

There are two additional observations to be made with respect to the short-run solutions. First, a foreign inflationary impulse ($\underline{p}_y^* > 0$) has no repercussions on the real side of the model. The real-nominal dichotomy crucially depends on our assumption that wages are fully indexed ($\psi = 1$). Second, we note that an increase in k has a favourable impact on the economy ($y > 0$, $p_y < 0$). In the short-run we have $k=0$, whereas in the medium- and long-run k is an endogenous variable.

The solution for k over time is easily found by substitution of (3.2) and (3.4) in (2.17). The result is a first order difference equation in the stock of capital, which reads as follows:

$$k = [1 - \frac{\pi+\delta}{\pi+1} \frac{\Gamma(1-\mu)}{\Lambda}] k_{-1} + \frac{\pi+\delta}{\pi+1} \frac{1}{\Lambda} [- \frac{\lambda}{1-\lambda} \chi_2 \underline{p}_{\ell_{-1}} - (\frac{\omega}{1-\lambda} \chi_2 + \frac{1-\mu}{\lambda} \frac{\gamma_c}{1-\tau})$$

$$(\underline{f}_{-1} + \gamma_g \underline{g}_{-1}) + (1-\mu)(\gamma_g \underline{g}_{-1} + \gamma_b \underline{m}^*_{-1})] \tag{3.7}$$

where

$$\chi_2 = \Lambda - \Gamma\sigma(1-\mu) > 0$$

Stability of the dynamic process in (3.7) requires:

$$-1 < 1 - \frac{\pi+\delta}{\pi+1} \frac{\Gamma(1-\mu)}{\Lambda} < 1 \tag{3.8}$$

If we exclude oscillations, this comes down to the condition:

$$\frac{\pi+\delta}{\pi+1} < \frac{\Lambda}{\Gamma(1-\mu)} \tag{3.9}$$

This inequality will hold for realistic values of the natural rate of growth (π), the rate of depreciation (δ), and other relevant parameters.

The long-run solution for k can be derived from (3.7) by setting $k = k_{-1}$. This leads to the expression:

$$k = \frac{1}{\Gamma} [- \frac{\lambda}{1-\lambda} \frac{\chi_2}{1-\mu} \underline{p}_\ell - (\frac{\omega}{1-\lambda} \frac{\chi_2}{1-\mu} + \frac{1}{\lambda} \frac{\gamma_c}{1-\tau})(\underline{f} + \gamma_g \underline{g}) + \gamma_g \underline{g} + \gamma_b \underline{m}^*] \tag{3.10}$$

A wage push ($\underline{p}_\ell > 0$) induces a lower level of the capital stock on long term. This result is caused by the negative impact of an autonomous wage increase on profits and accumulation. In case of factor complementarity ($\sigma = 0$) there will also be a negative influence on capital accumulation. Pure demand shocks ($\underline{f} = -\gamma_g \underline{g} > 0$ and $\underline{m}^* > 0$) have an opposite effect, because the induced price increase leads to higher profits. The long-run impact of a balanced budget policy will be negative or positive depending on the value of the behavioural parameter ω. For a negative result we must have:

$$\omega > \frac{\lambda - \frac{\gamma_c}{1-\tau}}{\lambda(\frac{\eta_m+\eta_b}{1-\mu} - \frac{\gamma_i}{1-\lambda} - \Gamma\sigma)} \tag{3.11}$$

As can be seen by comparing the inequalities (3.6) and (3.11), the long-run condition is slightly different from that derived for the short-run. It will be obvious that if condition (3.11) is satisfied, condition (3.6) will be as well.

The long-run solution for production and the price level can be found by straightforward substitution from (3.4), (3.5) and (3.10). However, it is not necessary to do this, because the conclusions are clear. The long-run movements reinforce the pattern obtained for the short-run. In the next section an illustration will be given in the form of a numerical example based on globally realistic parameter values.

4 A NUMERICAL EXAMPLE

The parameter values chosen for the simulation runs are:

$$\lambda = \frac{2}{3}, \sigma = 0.3, \varphi = 3, \mu = \frac{1}{2}, \tau = 0.1, \omega = 1, \psi = 1$$

$$\gamma_c = 0.3, \gamma_i = 0.15, \gamma_g = 0.1, \gamma_b = 0.45, \pi = 0.05, \delta = 0.1075.$$

These figures imply: $\frac{\gamma_c}{\lambda} = \frac{\gamma_i}{1-\lambda} = 0.45$. The parametric abbreviations introduced in section 3 can readily be computed:

$$\eta_b = 1.35, \eta_m = 0.675, \Gamma = 0.45, \Lambda = 0.645, \chi_1 = 0.591, \chi_2 = 0.5775.$$

The RHS of (3.11) is approximately equal to 0.14, which means that this condition is satisfied. Further we have:

$$\frac{\pi+\delta}{\pi+1} = 0.15 \text{ and } \frac{\Lambda}{\Gamma(1-\mu)} = 2.87.$$

Therefore condition (3.9) is also satisfied, which means that the model is stable.

The parameters for the relative shares in domestic output (γ) are representative for a small open economy. The elasticity of factor substitution (σ) is well below unity, as it should be according to a number of empirical studies.

The elasticity of substitution with respect to final goods (φ) is more uncertain. The value chosen here implies reasonable outcomes for the import and export elasticities (η_m and η_b). The value for λ corresponds to the share of labour in gross production. The equilibrium growth rate is set at 5%, which may be a little too high, but the influence of this parameter on the result of the computations is small. As said before, throughout the paper we assume that wages are fully indexed ($\psi = 1$).

The assumption that taxes are also fully passed on in wages ($\omega = 1$) may seem rather strong. On the other hand, it should be observed that transfer payments and corresponding social security premiums are neglected in the model. This is legitimate in case changes in the burden of social security premiums are not passed on in wages. Moreover, as appears from inequality (3.11), there is a wide margin with respect to the value of ω giving the same qualitative results in case of an increase in the tax rate.

The Keynesian multiplier amounts to $\frac{1}{\Gamma} = 2.22$. This may seem somewhat high for a small open economy, but the share of consumption of the domestic product in total domestic output looks acceptable.

The numerical values assumed lead to the following specifications of the short-run demand schedule (3.1) and the short-run supply function (3.3):

$$y^d = -4(p_y - \underline{p}_y^*) - 1.11(\underline{f} + 0.1\underline{g}) + 2.22 \times 0.1\underline{g} + \underline{m}^* \tag{4.1}$$

$$y^s = 0.36(p_y - \underline{p}_y^*) - 0.72\, \underline{p}_\ell - 0.9(\underline{f} + 0.1\underline{g}) + k \tag{4.2}$$

It may be concluded that the demand curve has a much flatter slope than the supply curve. The slopes of these curves of course reflect the substitution possibilities on both sides of the market. The curves are drawn in Figure 2. The difference in slope has important consequences when both curves shift under the impact of an impulse. As remarked before, this will be the case if a balanced budget policy rule is adhered to. A tax-financed increase in public spending shifts the demand curve to the right, whereas the supply curve moves to the left. Inspection of Figure 2 shows that the supply curve dominates the result, despite the fact that the size of the shift of the demand curve is somewhat larger than that of the supply schedule.

A numerical specification of the model enables computation of time paths of the endogenous variables. In Table 1, results for the short-run (t=1), medium-run (t=5) and long-run ($t \to \infty$) are presented for two autonomous changes, *i.e.* a wage push of 1% ($\underline{p}_\ell = 1$, $\forall t$) and a tax-financed increase in

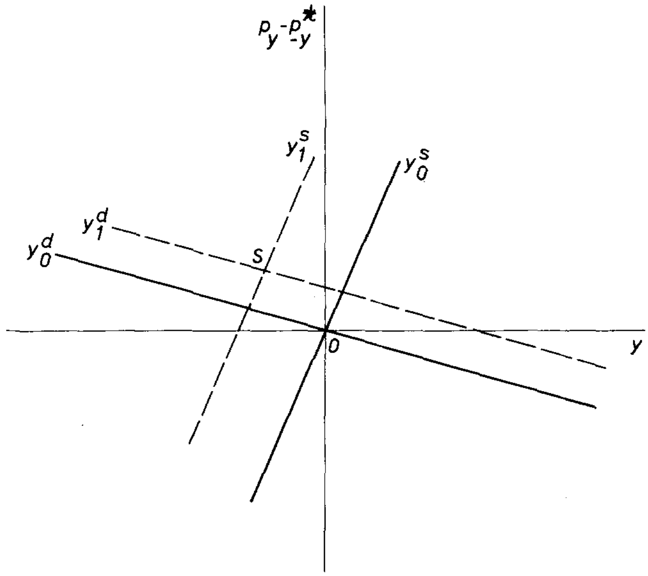

Figure 2

public spending of $6\frac{2}{3}\%$ ($\underline{g} = 6\frac{2}{3}$, $\underline{f} = 0, \forall t$). Exogenous variables are always kept constant over time, which implies that the impulses have a permanent character.

A wage push causes an increase in the product wage $(p_\ell - p_y)$ in the first period.[5] The demand for labour declines and production decreases. On the demand side there is extra consumption, but the rate of investment slows down under the impact of a fall in profits. Rising domestic prices lead to a lower level of exports and higher import substitution. The lower investment rate depresses capital accumulation in the following period. As a result of this, the pattern exhibited in the first period will be reinforced. The volume variables decline still further, while prices rise to a higher level. The picture of a *stagflating* economy emerges by comparing medium-run (t=5) as well as long-run ($t \to \infty$) results with the corresponding figures for the short-run.

Long-term equilibrium requires that the quotient of the macroeconomic

5 Although all figures relate to percentage deviations from a path of balanced growth, for the sake of convenience we shall speak of an increase if the deviation is positive and of a decrease if the deviation is negative irrespective of the movements of the variables in the initial situation.

TABLE 1 – SUPPLY SHOCKS

Impulse	$\underline{p}_\ell = 1, \forall t$			$\underline{g} = 6\frac{2}{3}, \forall t$		
Variable \ Period	1	5	∞	1	5	∞
y	−0.56	−1.48	−5.33	−0.51	−1.30	−4.59
c	0.16	−0.73	−4.43	−0.87	−1.63	−4.80
c_h	−0.05	−1.28	−6.43	−1.34	−2.40	−6.80
c_m	0.37	−0.17	−2.43	−0.40	−0.87	−2.80
i	−1.79	−2.44	−5.13	−1.53	−2.09	−4.39
i_h	−2.00	−2.99	−7.13	−2.00	−2.85	−6.39
i_m	−1.58	−1.88	−3.13	−1.06	−1.32	−2.39
b	−0.42	−1.11	−4.00	−0.94	−1.53	−4.00
p_y	0.14	0.37	1.33	0.31	0.51	1.33
p_x	0.07	0.19	0.67	0.16	0.25	0.67
p_ℓ	1.07	1.19	1.67	1.16	1.25	1.67
Y_r	−1.72	−2.25	−4.47	−1.38	−1.83	−3.73
ℓ	−0.84	−1.73	−5.43	−0.76	−1.52	−4.69
k	0	−0.99	−5.13	0	−0.85	−4.39

investment ratio and the capital coefficient (capital-output ratio) is constant. For $t \to \infty$ we have: $(i-y) - (k-y) = 0.2 - 0.2 = 0$. The capital coefficient will be higher in the new equilibrium situation, because of the rise in the product wage. The increase in the capital-output ratio is matched by an increase in the share of investment in total output. The share of profits in the value of production has fallen, but the improvement in the terms of trade leads to a higher share of profits in national income $[Y_r - (y+p_y) + (p_y-p_x)]$.

In the present model, excess supply of labour takes the form of structural unemployment as observed in section 1. Excessive real wages lead to substitution of labour and less accumulation of capital. With the passage of time the accumulation effect becomes more important compared with the substitution effect. The final result is a substantial reduction of employment. The second part of Table 1 shows the effects of an increase in public expenditure financed by extra taxation on wages. Here again the global scene is one of stagflation. The tax rate on wage income (not given in the table) rises initially at a rate of 1% ($\underline{g} = 6\frac{2}{3}$ and $\underline{f} = 0$ imply $t_\ell = 1$). Wage earners are able to shift such an increase in the burden of taxation. Because ω=1 in our exercises, the initial increase in real wages will also be 1%, which is the same amount as in the example of a wage push. This facilitates the comparison of both cases.

In period 1 employment and production both decline under influence of a rise in the product wage. The negative results are somewhat smaller in absolute value than in the wage push variant. Extra public spending, whether or not financed by additional taxation, induces an upward shift of the demand curve. The situation now corresponds to the picture presented in Figure 2. The demand shift has a positive impact on prices, which can be seen by comparing both variants. As a result of this shift, the increase in the product wage is somewhat smaller in case of an increase in public spending. But this does not mean very much. Again, labour is substituted and profits are squeezed from the very beginning. It appears that the shift in the supply curve dominates the scene. Therefore, in this case too, one may speak of a supply shock that has to be dealt with by the economy.

Another remarkable difference between the two situations relates to private consumption. A wage push induces an increase in real private consumption in the first period, whereas the opposite holds in case of a tax financed rise in public expenditure. The reason is of course that changes in tax rates influence real disposable incomes.

There is not much to say about the results for the medium-run, because the same differences as mentioned above are still visible. It should nevertheless be observed that the slow-down of capital accumulation is larger in the example of a wage push. It follows that prices now rise a little faster. This explains that in the long-run the same relative deviations for prices appear in both variants (at least up to two decimal points).

The effects of pure demand shocks are presented in Table 2. The left part of the table shows the result of an increase in public spending of $6\frac{2}{3}\%$ financed by running a deficit of $\frac{2}{3}$ ($\underline{g} = 6\frac{2}{3}, \underline{f} = -\frac{2}{3}$). The size of the demand pull is the same as before, which makes it easier to compare situations, which differ with respect to financing. The right part of Table 2 is reserved for the effects of an increase in world trade of 1.48% ($\underline{m}^* = 1.48$). Here again the size of the impulse is chosen in a way to facilitate comparison – this time between the two demand shocks, which have the same direct impact on y with the figures chosen.

A demand shock induces a rise in the price of production in period 1. The increase in the price of domestic expenditure is only half as much. Then the same holds for the wage rate. The consequence is a decline in the product wage, leading to an increase in the demand for labour. In the examples presented in Table 2 employment and output rise, which is only possible if there is some unemployment in the initial situation of balanced growth.

TABLE 2 – DEMAND SHOCKS

Impulse	$\underline{g} = 6\frac{2}{3}, \underline{f} = -\frac{2}{3}, \forall t$			$\underline{m}^* = 1.48, \forall t$		
Variable \ Period	1	5	∞	1	5	∞
y	0.10	0.37	1.48	0.10	0.37	1.48
c	0.16	0.41	1.48	0.16	0.41	1.48
c_h	−0.36	−0.01	1.48	−0.36	−0.01	1.48
c_m	0.67	0.83	1.48	0.67	0.83	1.48
i	0.52	0.70	1.48	0.52	0.70	1.48
i_h	0	0.29	1.48	0	0.29	1.48
i_m	1.03	1.12	1.48	1.03	1.12	1.48
b	−1.03	−0.83	0	0.45	0.65	1.48
p_y	0.34	0.28	0	0.34	0.28	0
p_x	0.17	0.14	0	0.17	0.14	0
p_ℓ	0.17	0.14	0	0.17	0.14	0
Y_r	0.69	0.84	1.48	0.69	0.84	1.48
ℓ	0.16	0.41	1.48	0.16	0.41	1.48
k	0	0.29	1.48	0	0.29	1.48

Otherwise fixed labour supply acts as a bottleneck.[6]

The fall in the product wage is favourable for profits, which induces an acceleration of capital accumulation. Production rises gradually. Absorption follows the same pattern under the provision that prices decline sufficiently. In the long-run, output, consumption, investment and factor inputs are about 1.5% above their initial value. Prices on the contrary exhibit no change at all.

So much for the similarities between the two variants. The main difference is the effect on the balance of payments on current account. A demand shock originating abroad favours exports, whereas the opposite holds for a demand impulse from within. The present model does not take account of possible feed-back effects of changes in foreign assets on demand, as analysed by Sachs [14] en Van de Klundert [10].

6 If the supply of labour depends on the consumption wage and if wages are only partially indexed the supply curve for output has a positive slope. In our analyses we have assumed that wages are fully indexed, which implies that labour supply is elastic.

5 THE PHILLIPS CURVE: AN EXTENSION

Hitherto the analysis is asymmetrical with respect to the working of both markets. The market for goods is cleared instantaneously, because the price of output is perfectly flexible. On the other hand, the wage rate is determined without any reference to the labour market. It seems reasonable to assume that the labour market functions less smoothly than the goods market. The wage rate may nevertheless adjust to its market equilibrium value slowly and at a lag. For the sake of convenience we shall assume here that the labour market is in equilibrium in the initial situation. If the supply of labour is fixed the relative deviation of employment (ℓ) equals the rate of unemployment. The Phillips curve implies a relationship between the percentage change in wages and the rate of unemployment. If this relationship is appended to equation (2.16) we can write:

$$p_\ell - p_{\ell_{-1}} = \underline{p}_\ell - \underline{p}_{\ell_{-1}} + \psi(p_x - p_{x_{-1}}) + \omega(t_\ell - t_{\ell_{-1}}) + \beta\,\ell_{-1} \qquad (5.1)$$

The analytical solution of the extended model obtained by substitution of (2.16) for (5.1) is somewhat tedious. For this reason we have taken recourse to a numerical example to show how the economy works if wages adjust to their long-run equilibrium value. The additional parameter β is set equal to 0.5 and the effects of a wage push of 1% ($\underline{p}_\ell = 1$) are again determined.

Computation of the characteristic roots shows that the model is stable in this case too, but the variables oscillate over time. In fact, the dynamic movement is determined by a pair of complex roots with modulus 0.82.

In Table 3 the results are summarized for a number of periods. In this way an impression can be given of the oscillations and the speed of adjustment. Because of the time lag introduced in equation (5.1) the results in the first period are equal to those in the previous section. This can be seen by comparing Tables 1 and 3. In the second period the wage rate declines under the impact of unemployment realized in the preceding period. As a result of this market conform behaviour unemployment decreases. However, as long as there is still some unemployment left, the downward pressure on the wage rate continues. Eventually, the wage rate falls below the equilibrium value of the initial situation ($p_\ell = 0$). Such a situation is reached in period 4. The point is that lower wages lead to a higher level of employment but at a lag. Substitution of labour occurs immediately, but it takes time to accumulate capital goods. The decline in the stock of capital is checked in period 5. As time passes, the relative deviations of employment and capital may even become positive. This is illustrated by the figures for period $t = 10$. Despite

TABLE 3 – AN ILLUSTRATION OF THE PHILLIPS CURVE ($p_{\ell} = 1, \forall t$)

Variable \ Period	1	2	3	4	5	10	15
y	−0.56	−0.57	−0.50	−0.38	−0.25	0.11	0.01
c	0.16	−0.15	−0.33	−0.41	−0.40	0	0.06
c_h	−0.05	−0.36	−0.52	−0.55	−0.50	0.04	0.06
c_m	0.37	0.07	−0.14	−0.27	−0.31	−0.04	0.05
i	−1.79	−1.22	−0.66	−0.19	0.15	0.28	−0.08
i_h	−2.00	−1.43	−0.85	−0.33	0.06	0.32	−0.07
i_m	−1.58	−1.00	−0.47	−0.04	0.25	0.24	−0.08
b	−0.42	−0.43	−0.38	−0.29	−0.19	0.08	0.01
p_y	0.14	0.14	0.13	0.10	0.06	−0.03	0
p_x	0.07	0.07	0.06	0.05	0.03	−0.01	0
p_{ℓ}	1.07	0.65	0.28	−0.01	−0.20	−0.16	0.06
Y_r	−1.72	−1.14	−0.59	−0.14	0.18	0.27	−0.08
ℓ	−0.84	−0.73	−0.55	−0.35	−0.17	0.15	−0.01
k	0	−0.27	−0.41	−0.45	−0.41	0.03	0.05

a favourable employment situation in this period the wage rate is still lower than its equilibrium value. As a result, employment will increase further until the movement is reversed by subsequent increases in the wage rate. The interaction of a dynamic wage adjustment mechanism and the process of capital accumulation thus leads to oscillations. However, as said, the model is stable. It follows that all variables return to their initial values in the long-run.

The Phillips curve will of course also have a dampening effect on the results in case of other shocks. This does not remove the possibility that a tax-financed increase in public spending may cause substantial unemployment effects on the short- and medium-run if the rise in tax rates is passed on in wages.

6 SOME COMMENTS ON THE MODEL

The comments which are in order here relate to the aspect of generality and the degree of realistic-ness of the model. The generality of the analysis could be improved in the following two ways:

(a) We have assumed that wages are fully indexed ($\psi = 1$). Undoubtedly, something could be gained by allowing for partial indexation ($0 < \psi < 1$).

(b) The analytical solutions of the model are determined without taking account of the Phillips curve. By applying the backward shift operator E and proper substitution this defect could eventually be removed.

Theoretical models can be made more realistic at the expense of increasing complexity. Analytical solutions may then be out of reach. Recent examples of this development towards more realistic but complex models are the studies by Malinvaud [13] and Kuipers [11]. In the analysis of Malinvaud markets are cleared in the short-run by non-price rationing. The behaviour of wages and prices over time depends on the prevailing regime (Keynesian Unemployment, Classical Unemployment or Repressed Inflation). In the medium-run the economy can switch from one regime to the other and the time paths of the variables must be explored by taking recourse to numerical examples.

The model of Kuipers is even more complex. Whereas Malinvaud distinguishes the short- and medium-run, Kuipers also introduces the long-run. In the short-run capacities and prices are given. The same holds for expected prices and expected sales. In the medium-run these variables may change, but the price expectations forming the basis of the choice of production techniques are still fixed. It therefore takes a long-run view to trace tendencies towards the neo-classical situation of balanced growth. Kuipers concludes that numerical analysis is inescapable for the examination of the properties of the models involved.

Both authors analyse a closed economy. Rationing models for an open economy may be even more difficult. The basic question is of course how markets in a small open economy where domestic entrepreneurs have to cope with foreign competition are cleared. In the present article we assumed that output prices are fully flexible, which simplifies the analysis to a large extent.

APPENDIX A

THE LINEARIZATION PROCEDURE

The variables in the main text relate to percentage deviations with respect to a path of balanced growth in the initial situation. This path is disturbed by shocks, if the exogenous variables take a different value in certain periods. The model in terms of relative or percentage deviations is a linear one. Starting from the original model the linearization procedure is explained in this appendix.

The procedure takes two steps. First, the original equations are differentiated totally around the balanced growth path. Second, all differentials are divided by the value of the corresponding variable on the steady growth path. Therefore, the resulting ratios can be considered as constants. To distinguish between the relative deviation of a variable and the absolute value we shall write the latter with an 'accent circonflex' (ˆ). The initial situation will be marked by the suffix 0.

To illustrate the technique let us take the standard neoclassical production function.

$$\hat{y} = f(\hat{\ell}, \hat{k})$$

Total differentiation leads to[7]:

$$d\hat{y} = \frac{\partial \hat{y}_0}{\partial \hat{\ell}_0} d\hat{\ell} + \frac{\partial \hat{y}_0}{\partial \hat{k}_0} d\hat{k}$$

If the differentials are divided by the value of the variables in the initial situation we get:

$$\frac{d\hat{y}}{\hat{y}_0} = \frac{\partial \hat{y}_0}{\partial \hat{\ell}_0} \frac{\hat{\ell}_0}{\hat{y}_0} \frac{d\hat{\ell}}{\hat{\ell}_0} + \frac{\partial \hat{y}_0}{\partial \hat{k}_0} \frac{\hat{k}_0}{\hat{y}_0} \frac{d\hat{k}}{\hat{k}_0}$$

In our short-hand notation this equation can be rewritten as:

$$y = \lambda \ell + (1-\lambda)k$$

where $\lambda = \dfrac{\partial \hat{y}_0}{\partial \hat{\ell}_0} \dfrac{\hat{\ell}_0}{\hat{y}_0}$ stands for the production elasticity of labour. Along a path of balanced growth the production elasticities are constant. It should be observed that we assumed constant returns to scale, so that both elasticities add up to one. Under neoclassical conditions the production elasticities are equal to the income shares of the production factors.

The procedure outlined above can be repeated for all equations. Below we present the original equations along with their linearized versions.

$$\hat{y} = f(\hat{\ell}, \hat{k}) \qquad y = \lambda \ell + (1-\lambda)k \qquad \text{(A1)}$$

$$\frac{\partial \hat{y}}{\partial \hat{\ell}} = \frac{\hat{p}_\ell}{\hat{p}_y} \qquad \ell - y = -\sigma(p_\ell - p_y) \qquad \text{(A2)}$$

7 There may be technical change. However, the equations are not differentiated with respect to time.

$$\hat{y} = \hat{c}_d + \hat{i}_d + \hat{g} + \hat{b} \qquad y = \gamma_c c_d + \gamma_i i_d + \gamma_g g + \gamma_b b \qquad \text{(A3)}$$

$$\hat{c} = \hat{c}_d + \hat{c}_m \qquad c = \mu c_d + (1-\mu) c_m \qquad \text{(A4)}$$

$$\hat{i} = \hat{i}_d + \hat{i}_m \qquad i = \mu i_d + (1-\mu) i_m \qquad \text{(A5)}$$

$$\frac{\hat{c}_h}{\hat{c}_m} = \nu \left(\frac{\hat{p}_y}{\hat{\underline{p}}_y^*}\right)^{-\varphi} \qquad c_h - c_m = -\varphi (p_y - \underline{p}_y^*) \qquad \text{(A6)}$$

$$\frac{\hat{i}_h}{\hat{i}_m} = \nu \left(\frac{\hat{p}_y}{\hat{\underline{p}}_y^*}\right)^{-\varphi} \qquad i_h - i_m = -\varphi (p_y - \underline{p}_y^*) \qquad \text{(A7)}$$

$$\hat{c}\hat{p}_x = \alpha\, \hat{\ell}\hat{p}_\ell\, (1-\tau_\ell) \qquad c = \ell + p_\ell - \frac{1}{1-\tau} t_\ell - p_x \qquad \text{(A8)}$$

$$\hat{p}_x = \hat{p}_y^{\mu}\, \hat{\underline{p}}_y^{*\,(1-\mu)} \qquad p_x = \mu p_y + (1-\mu) \underline{p}_y^* \qquad \text{(A9)}$$

$$\hat{i}\hat{p}_x = \iota\, \hat{Y}_r\, (1-\tau_r) \qquad i = Y_r - p_x \qquad \text{(A10)}$$

$$\hat{Y}_r = \hat{y}\hat{p}_y - \hat{\ell}\hat{p}_\ell \qquad Y_r = \frac{1}{1-\lambda}(y + p_y) - \frac{\lambda}{1-\lambda}(\ell + p_\ell) \qquad \text{(A11)}$$

$$\hat{b} = \epsilon\, \hat{\underline{m}}^* \left(\frac{\hat{p}_y}{\hat{\underline{p}}_y^*}\right)^{-\varphi} \qquad b = \underline{m}^* - \varphi (p_y - \underline{p}_y^*) \qquad \text{(A12)}$$

$$\hat{g} = \hat{\underline{g}}\, \hat{y} \qquad g = y + \underline{g} \qquad \text{(A13)}$$

$$\hat{\underline{f}} = \tau_\ell\, \hat{\ell}\hat{p}_\ell + \tau_r \hat{Y}_r - \hat{g}\hat{p}_y \qquad \underline{f} = \lambda t_\ell - \gamma_g (g - y) \qquad \text{(A14)}$$

$$\hat{p}_\ell = \hat{\underline{p}}_\ell\, \hat{p}_x^{\psi}\, \tau_\ell^{\theta} \qquad p_\ell = \underline{p}_\ell + \psi p_x + \omega t_\ell \qquad \text{(A15)}$$

$$\hat{k} = \hat{k}_{-1} + \hat{i}_{-1} - \delta \hat{k}_{-1} \qquad k = k_{-1} + \frac{\pi + \delta}{\pi + 1}(i_{-1} - k_{-1}) \qquad \text{(A16)}$$

Some transformations need additional explanation:

(A2) : The linearization is based on the assumption of a CES production function. In this case we have: $\frac{\partial \hat{y}}{\partial \hat{\ell}} = \zeta \left(\frac{\hat{y}}{\hat{\ell}}\right)^{1/\sigma}$

(A3) : The coefficients add up to one: $\gamma_c + \gamma_i + \gamma_g + \gamma_b = 1$

(A8) : The variable t_ℓ indicates a change in the tax rate on labour income: $t_\ell = d\tau_\ell$. Moreover it is assumed that the tax rates on factor incomes in the initial situation are equal $\tau_\ell = \tau_r = \tau$.

(A10) : It is assumed that the tax rate on profits is constant $t_r = d\tau_r = 0$.

(A14) : The variable $\hat{\underline{f}}$ is in the linear version expressed as a percentage of national income: $\underline{f} = \dfrac{d\underline{f}}{\hat{y}_0 \hat{p}_{y0}}$.

(A15) : Linearization implies: $\omega = \dfrac{\theta}{\tau_\ell}$.

(A16) : The two step procedure gives in this case:

$$\frac{\hat{k}_0}{\hat{k}_{0_{-1}}} k = k_{-1} + \frac{\hat{i}_{0_{-1}}}{\hat{k}_{0_{-1}}} i_{-1} - \delta\, k_{-1}$$

If the natural rate of growth is equal to π we may write:

$$\frac{\hat{k}_0}{\hat{k}_{0_{-1}}} = 1 + \pi \text{ and } \frac{\hat{i}_{0_{-1}}}{\hat{k}_{0_{-1}}} = \pi + \delta$$

APPENDIX B

SOLUTION OF THE MODEL

In short-run demand schedule can be derived in the following way. From (2.4), (2.6) and (2.7) we have:

$$c_h = c - (1-\mu)\varphi p_y + (1-\mu)\varphi\, \underline{p}_y^* \tag{B1}$$

Combination of (2.6), (2.9), (2.13), (2.14) and (2.16) leads to:

$$c = \ell + \underline{p}_\ell + (\psi-1)\mu p_y + (\psi-1)(1-\mu)\underline{p}_y^* + (\omega - \frac{1}{1-\tau})\frac{\underline{f} + \gamma_g \underline{g}}{\lambda} \tag{B2}$$

A similar procedure is applied with regard to investment. From (2.5), (2.6) and (2.8) we have:

$$i_h = i - (1-\mu)\varphi p_y + (1-\mu)\varphi\, \underline{p}_y^* \tag{B3}$$

Combination of (2.6), (2.10), (2.11), (2.13), (2.14) and (2.16) leads to:

$$i = \frac{1}{1-\lambda}[y - \lambda \ell + \left\{1-\mu(1-\lambda)-\lambda\psi\mu\right\}p_y - (1-\mu)\left\{1+\lambda(\psi-1)\right\}\underline{p}_y^* - \lambda\underline{p}_\ell$$
$$-\omega(\underline{f} + \gamma_g\underline{g})] \tag{B4}$$

The equations (B1) and (B2) provide an expression for c_h. The equations (B3) and (B4) do so for i_h. Substitution of these expressions along with (2.12) and (2.13) and taking account of (2.1) results after some manipulations in:

$$[1 - \frac{\gamma_c}{\lambda} - \gamma_g]y = -[\frac{\gamma_c(1-\lambda)}{\lambda} - \gamma_i]k + [\gamma_c - \gamma_i\frac{\lambda}{1-\lambda}]\underline{p}_\ell - [\left\{\gamma_c(1-\mu) +\right.$$
$$\left.\gamma_i(1-\mu) + \gamma_b\right\}\varphi - \frac{\gamma_i}{1-\lambda}\left\{(1-\mu) - \lambda\mu(\psi-1)\right\} - \gamma_c(\psi-1)\mu]p_y +$$
$$[\left\{\gamma_c(1-\mu) + \gamma_i(1-\mu) + \gamma_b\right\}\varphi + \gamma_c(\psi-1)(1-\mu) - \frac{\gamma_i(1-\mu)}{1-\lambda}\left\{1+\lambda(\psi-1)\right\}]\underline{p}_y^* +$$
$$\gamma_b\underline{m}^* + [\frac{\gamma_c}{\lambda}(\omega - \frac{1}{1-\tau}) - \frac{\gamma_i\omega}{1-\lambda} + 1]\gamma_g\underline{g} + [\frac{\gamma_c}{\lambda}(\omega - \frac{1}{1-\tau}) - \frac{\gamma_i\omega}{1-\lambda}]\underline{f} \tag{B5}$$

The demand equation (B5) can be simplified substantially by assuming $\frac{\gamma_c}{\lambda} = \frac{\gamma_i}{1-\lambda}$ and $\psi = 1$. Applying short-hand notation we finally get:

$$y = \frac{1}{\Gamma}[-\left\{(\eta_b+\eta_m) - \frac{\gamma_i(1-\mu)}{1-\lambda}\right\}(p_y-\underline{p}_y^*) + \gamma_b\underline{m}^* + \gamma_g\underline{g} - \frac{\gamma_c}{\lambda(1-\tau)}(\underline{f}+\gamma_g\underline{g})] \tag{B6}$$

where

$$\Gamma = 1 - \frac{\gamma_c}{\lambda} - \gamma_g$$

$$\eta_b = \gamma_b\varphi$$

$$\eta_m = \left\{\gamma_c(1-\mu) + \gamma_i(1-\mu)\right\}\varphi$$

Next, we turn to the short-run supply curve. Combination of (2.1) and (2.2) gives:

$$y = \lambda y - \sigma\lambda(p_\ell - p_y) + (1 - \lambda)k \tag{B7}$$

Substitution of (2.6), (2.13), (2.14), (2.16) and rearranging results in the supply equation:

$$y = \frac{\sigma}{1-\lambda}[\lambda(1-\psi\mu)p_y - \lambda\psi(1-\mu)\underline{p}_y^* - \lambda\underline{p}_\ell - \omega(\underline{f} + \gamma_g\underline{g})] + k \tag{B8}$$

Again we assume: $\psi = 1$. Therefore (B8) changes into:

$$y = \frac{\sigma}{1-\lambda}[\lambda(1-\mu)(p_y-\underline{p}_y^*) - \lambda\underline{p}_\ell - \omega(\underline{f} + \gamma_g\underline{g})] + k \tag{B9}$$

The solution for the relative price $(p_y-\underline{p}_y^*)$ can be found by equating (B6) and (B9). Multiplying through by $(1-\lambda)$ and Γ and introducing short-hand notation for the coefficient of the relative price variable in the equation we deduce:

$$p_y - \underline{p}_y^* = \frac{1}{\Lambda}[\Gamma\sigma\lambda\underline{p}_\ell + \left\{\Gamma\sigma\omega - \frac{\gamma_c}{1-\tau}\frac{1-\lambda}{\lambda}\right\}(\underline{f}+\gamma_g\underline{g}) + (1-\lambda)(\gamma_g\underline{g}+\gamma_b\underline{m}^* - \Gamma k)] \tag{B10}$$

where

$$\Lambda = (1-\mu)(\Gamma\sigma\lambda - \gamma_i) + (1-\lambda)(\eta_b + \eta_m)$$

The solution for output can readily be obtained from (B9) and (B10):

$$y = \frac{1}{\Lambda}[-\frac{\sigma\lambda}{1-\lambda}\chi_1\underline{p}_\ell - \left\{\frac{\sigma\omega}{1-\lambda}\chi_1 + \frac{\gamma_c}{1-\tau}\sigma(1-\mu)\right\}(\underline{f}+\gamma_g\underline{g}) + \sigma\lambda(1-\mu)(\gamma_g\underline{g}$$
$$+\gamma_b\underline{m}^*) + \chi_1 k] \tag{B11}$$

where

$$\chi_1 = \Lambda - \Gamma\sigma\lambda(1-\mu)$$

The time path of the state variable k can be determined by substitution of the solution for investment (i) in the accumulation equation (2.17). The solution of i for ψ=1 follows from (B4), (B10) and (2.1):

$$i = [1 - \frac{\Gamma(1-\mu)}{\Lambda}]k - \frac{1}{\Lambda}[\frac{\lambda}{1-\lambda}\chi_2\underline{p}_\ell + \left\{\frac{\omega}{1-\lambda}\chi_2 + \frac{\gamma_c(1-\mu)}{\lambda(1-\tau)}\right\}(\underline{f} + \gamma_g\underline{g})$$
$$-(1-\mu)(\gamma_g\underline{g} + \gamma_b\underline{m}^*)] \tag{B12}$$

where

$$\chi_2 = \Lambda - \Gamma\sigma(1-\mu)$$

Substitution of (B12) in (2.17) gives:

$$\Delta k = \frac{\pi+\delta}{\pi+1}\,\frac{1}{\Lambda}\Big[-\Gamma(1-\mu)k_{-1} - \frac{\lambda}{1-\lambda}\,\chi_2 \underline{p}_{\ell_{-1}} - \Big\{\frac{\omega}{1-\lambda}\,\chi_2 + \frac{\gamma_c(1-\mu)}{\lambda(1-\tau\;)}\Big\}$$

$$-(\underline{f}_{-1}+\gamma_g \underline{g}_{-1}) + (1-\mu)\,(\gamma_g \underline{g}_{-1} + \gamma_b \underline{m}^*_{-1})\Big] \tag{B13}$$

To obtain the long-run solution for k we must have $\Delta k = 0$, which results in:

$$k = \frac{1}{\Gamma}\Big[-\frac{\lambda}{1-\lambda}\,\frac{\chi_2}{1-\mu}\,\underline{p}_\ell - \Big\{\frac{\omega}{1-\lambda}\,\frac{\chi_2}{1-\mu} + \frac{\gamma_c}{\lambda(1-\tau)}\Big\}(\underline{f} + \gamma_g \underline{g}) + \gamma_g \underline{g} + \gamma_b \underline{m}^*\Big] \tag{B14}$$

LIST OF MAIN SYMBOLS

ENDOGENOUS VARIABLES

b : volume of exports
c : volume of total consumption
c_h : consumption of domestic output
c_m : imported consumption
g : volume of public spending
i : volume of total investment
i_h : investment of domestic output
i_m : imported investment
k : stock of capital
l : level of employment
p_l : nominal wage rate
p_x : price of domestic expenditure
p_y : price of domestic production
t_l : tax rate on labour income
t_r : tax rate on profits
y : level of production
Y_r : nominal level of profits

EXOGENOUS VARIABLES

$\underline{f}$: real public savings
$\underline{g}$: autonomous public spending (in real terms)
$\underline{m}^*$: level of world trade
$\underline{p}_\ell$: autonomous nominal wage level (wage push)
$\underline{p}_y^*$: price level of foreign output

PARAMETERS

β : elasticity of wages with respect to unemployment
γ_c : share of consumption of domestic output in total domestic output
γ_b : share of exports in total domestic output
γ_g : share of public spending in total domestic output
γ_i : share of investment of domestic output in total domestic output
δ : rate of depreciation of the capital stock
η_b : elasticity of exports
η_m : elasticity of imports
λ : production elasticity of labour
μ : share of domestic output in total consumption and investment
π : natural rate of growth
σ : elasticity of factor substitution
τ : tax rate on factor incomes
φ : elasticity of substitution with respect to demand for output
ψ : elasticity of wages with respect to the price of domestic expenditure
ω : elasticity of wages with respect to the tax rate on labour income

REFERENCES

[1] Bacon, Robert W., *The Effects of Public Employment and other Government Spending on the Rate of Wage Inflation (with special reference to U.K. experience)*, Paper prepared for a Congress on 'Public Finance and Public Employment.' International Institute of Public Finance, Jerusalem, 1980.

[2] Branson, William H. and Julio J. Rotemberg, 'International Adjustment with Wage Rigidity,' *European Economic Review*, 1980

[3] Bruno, Michael, 'Price and Output Adjustment: Micro Foundations and Macro Theory,' *Journal of Monetary Economics*, No. 2, 1979.

[4] Bruno, Michael, and Jeffrey Sachs, 'Macro-economic Adjustment with Import Price Shocks: Real and Monetary Aspects,' The Maurice Falk Institute for Economic Research in Israel, *Discussion Paper No. 793*, January 1979.

[5] Bruno, Michael, and Jeffrey Sachs, 'Supply versus Demand Approaches to the Problem of Stagflation,' The Maurice Falk Institute for Economic Research in Israel, *Discussion Paper No. 796*, May 1979.

[6] Goorbergh, W.M. van den, and D.B.J. Schouten, 'The Case of Labour Time Reduction Without Wage Compensation: More or Less Employment?,' *De Economist*, No. 4, 1980.

[7] Hartog, H. den, and H.S. Tjan, 'Investments, Wages, Prices and Demand For Labour,' *De Economist*, No. 1/2, 1976 [originally published in Dutch as an Occassional Paper of the Central Planning Bureau, 1974].

[8] Hartog, H. den, Theo van de Klundert, and H.S. Tjan, 'De structurele ontwikkeling van de werkgelegenheid in macro-economisch perspectief,' in *Werkloosheid*, Preadviezen van de Vereniging voor de Staathuishoudkunde, The Hague, 1975 (Dutch; translation available).

[9] Hicks, John, *Capital and Growth*, Oxford, 1965.
[10] Klundert, Theo van de, 'Flexibele wisselkoersen in een kleine open economie,' *Maandschrift Economie* (May 1981; Dutch).
[11] Kuipers, S.K., 'Keynesian and Neo-Classical Growth Models: A Sequential Analytical Approach,' *De Economist*, No. 1, 1981.
[12] Malinvaud, Edmond, *The Theory of Unemployment Reconsidered*, Oxford, 1977.
[13] Malinvaud, Edmond, *Profitability and Unemployment*, Cambridge, 1980.
[14] Sachs, Jeffrey, 'Wage Indexation, Flexible Exchange Rates, and Macro-economic Policy,' *Quarterly Journal of Economics*, June 1980.
[15] Schouten, D.B.J., *Dynamische Macro-economie*, Leiden, 1967 (Dutch).
[16] Williamson, J., 'A Simple Neo-Keynesian Growth Model,' *Review of Economic Studies*, 1970.

Summary

DISTRIBUTION, TAXATION AND EMPLOYMENT IN AN OPEN ECONOMY

If taxes on labour income are passed on in wages the balanced budget multiplier may be negative. The present paper analyses this problem from a theoretical point of view applying a linearized version of a model for the small open economy. The model is dynamic, because account is taken of capital accumulation. Short-run and long-run solutions are expressed in terms of fiscal policy variables, a wage push variable, the price level abroad and world demand. Numerical examples are supplemented to illustrate the analytical results.