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Divergence Measure of Neutrosophic Sets and Applications

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Abstract: In this paper, we first propose the concept of divergence measure on neutrosophic sets. We also provide some formulas for the divergence measure for neutrosophic sets. After that, we investigate the properties of proposed neutrosophic divergence measure. Finally, we also apply these formulas in medical problem and the classification problem.

Keywords: neutrosophic set, divergence measure, classification problem.

1 Introduction

The neutrosophic set [25] was first introduced by Smarandache as an extension of intuitionistic fuzzy set [1] and fuzzy set [36]. It is a useful mathematical tool for dealing with ambiguous and inaccurate problems [4-6, 10, 24, 26-35, 37]. So far, many theoretical and applied results have been exploited on neutrosophic sets as the similarity/distance measures of neutrosophic sets [7-9, 11, 17-19, 22]. Neutrosophic set is applied in the multi-criteria decision making (MCDM) problem [4-6, 10-16, 23]. A special case of neutrosophic set is Single valued neutrosophic set (SVNS) which introduced by Wang et al [29]. In 2014, Ye proposed distance-based similarity measures of single valued neutrosophic sets and their multiple attribute group decision making method [32]. In 2017, Ye studied cotangent similarity measures for single-valued neutrosophic sets and applied it in the MCDM problem and in the fault diagnosis of steam turbine [34].

In the study of the applications of fuzzy set theory, the measurements are focused heavily on research. Measurements are often used to measure the degree of similarity or dissimilarity between objects. One of the dissimilarity measures of fuzzy sets/intuitionistic fuzzy sets was recently investigated by investigators as a measure of the divergence of fuzzy sets [3, 12, 20, 21]. Divergence measures also have many applications in practical problem classes and give us interesting results [3, 12, 20, 21]. Some authors have applied divergence measure to determine the relationship between the patient and the treatment regimen based on symptoms, thereby selecting the most appropriate treatment regimen for each patient [3]. Divergence measure is also used in multi-criterion decision problems [3, 12, 20, 21].

In this paper, we introduce the concept of divergence measure of neutrosophic sets, called neutrosophic divergence measure. We also give some expressions that define the neutrosophic divergence measures. After that, we investigate the properties of them. Finally, we use these neutrosophic divergence measure to identify appropriate treatment regimens for each patient and use them in the sample recognition problem.

The article is organized as follows: In section 2, we recall the knowledge related to neutrosophic sets. In section 3, we introduce the concept of neutrosophic divergence measure and investigate their properties. We show some applications of neutrosophic divergence measures in section 4. In section 5, we give conclusion on neutrosophic divergence measure and its some development direction.

2 Preliminary

Definition 1. Neutrosophic set (NS) [28]:

$$A=\{(x,T_A(x),I_A(x),F_A(x))\,|\,x\in U\} \quad (1)$$

where $T_A(x)\in[0,1]$ is a trust membership function, $I_A(x)\in[0,1]$ is indeterminacy membership function, $F_A(x)\in[0,1]$ is falsity-membership function of A.

We denote $NS(U)$ is a collection of neutrosophic set on U. In which

$$U=\{(u,1,1,0)\,|\,u\in U\}$$

and

$$\varnothing=\{(u,0,0,1)\,|\,u\in U\}$$

For two set $A,B\in NS(U)$ we have:

- Union of A and B:

$$A\cup B=\{(x,T_{A\cup B}(x),I_{A\cup B}(x),F_{A\cup B}(x))\}$$

where

$T_{A\cup B}(x)=\max(T_A(x),T_B(x))$,

$I_{A\cup B}(x)=\min(I_A(x),I_B(x))$

and

$F_{A\cup B}(x)=\min(F_A(x),F_B(x))$

for all $x\in X$.

- Intersection of A and B:

$A\cap B=\{(x,T_{A\cap B}(x),I_{A\cap B}(x),F_{A\cap B}(x))\}$

where

$T_{A\cap B}(x)=\min(T_A(x),T_B(x))$,

$I_{A\cap B}(x)=\max(I_A(x),I_B(x))$

and

$F_{A\cap B}(x)=\max(F_A(x),F_B(x))$

for all $x\in X$.

- Subset: $A\subseteq B$ if only if

$T_A(x)\le T_B(x), I_A(x)\ge I_B(x), F_A(x)\ge F_B(x)$

for all $x\in X$.

- Equal set: $A=B$ if only if $A\subseteq B$ and $B\subseteq A$.
- Complement of A:

$A^C=\{(x,F_A(x),1-I_A(x),T_A(x))\,|\,x\in U\}$

3 Divergence measures of neutrosophic sets

Definition 2. Let A and B be two neutrosophic sets on U. A function $D:NS(U)\times NS(U)\to R$ is a divergence measure of neutrosophic sets if it satisfies the following conditions:

Div1. $D(A,B)=D(B,A)$,

Div2. $D(A,B)=0$ iff $A=B$

Div3. $D(A \cap C, B \cap C) \leq D(A,B)$ for all $C \in NS(U)$,

Div4. $D(A \cup C, B \cup C) \leq D(A,B)$ for all $C \in NS(U)$.

We can easily verify that the divergence measures of neutrosophic sets are non-negative. Because, if we choose $C = \varnothing$ then conditions Div2 and Div3 in definition 2, then we have

$$D(A,B) \geq D(A \cap C, B \cap C) = D(\varnothing, \varnothing) = 0.$$

Now we give some divergence measures of Neutrosophic sets and their properties.

Definition 3. Let A and B be two neutrosophic sets on $U = \{u_1, u_2, ..., u_n\}$. A function $D: NS(U) \times NS(U) \to R$ is defined as follows

$$D(A,B) = \frac{1}{n}\sum_{i=1}^{n}[D_T^i(A,B) + D_I^i(A,B) + D_F^i(A,B)] \tag{2}$$

where

$$D_T^i(A,B) = T_A(u_i)\ln\frac{2T_A(u_i)}{T_A(u_i)+T_B(u_i)} + T_B(u_i)\ln\frac{2T_B(u_i)}{T_A(u_i)+T_B(u_i)} \tag{3}$$

$$D_I^i(A,B) = I_A(u_i)\ln\frac{2I_A(u_i)}{I_A(u_i)+I_B(u_i)} + I_B(u_i)\ln\frac{2I_B(u_i)}{I_A(u_i)+I_B(u_i)} \tag{4}$$

and

$$D_F^i(A,B) = F_A(u_i)\ln\frac{2F_A(u_i)}{F_A(u_i)+F_B(u_i)} + F_B(x_i)\ln\frac{2F_B(u_i)}{F_A(u_i)+F_B(u_i)}. \tag{5}$$

To proof that $D(A,B)$ is a divergence measure of neutrosophic sets we need some following lemma.

Lemma 1. Given $a \in (0,1]$. For all $z \in [0, 1-a]$ then

$$f(z) = a\ln 2a + (a+z)ln(2a+2z) - (2a+z)\ln(2a+z) \tag{6}$$

is a non-decreasing function and $f(z) \geq 0$.

Proof.

We obtain $\frac{\partial f(z)}{\partial z} = \ln(2a+2z) - \ln(2a+z) \geq 0$ for all $z \in [0, 1-a]$. □

Lemma 2. Given $b \in (0,1]$. For all $z \in (0, b]$ then

$$f(z) = b\ln 2b + z\ln 2z - (b+z)\ln(b+z) \tag{7}$$

is a non-increasing function and $f(z) \geq 0$.

Proof.

We have $\frac{\partial f(z)}{\partial z} = \ln 2z - \ln(b+z) \leq 0$ for all $z \in (0, b]$. □

Lemma 3. Given $a \in (0,1]$. For all $z \in [a, 1]$ then

$$f(z) = a\ln 2a - (a+z)\ln(a+z) + z\ln 2z \tag{8}$$

is a non-decreasing function and $f(z) \geq 0$.

Proof.

We have $\frac{\partial f(z)}{\partial z} = \ln 2z - \ln(a+z) \geq 0$ for all $z \in [a, 1]$. □

Theorem 1. The function $D(A,B)$ defined by eq (2, 3, 4, 5) (in definition 3) is a divergence measure of two Neutrosophic sets.

Proof.

We check the conditions of the definition. For two Neutrosophic sets A and B on U, we have:

- Div1: $D(A,B)=D(B,A)$,
- Div2:

+ If $A=B$ we have $D_T^i(A,B)=D_I^i(A,B)=D_F^i(A,B)=0$. So that $D(A,B)=0$.

+ Assume that

$$D(A,B)=\frac{1}{n}\sum_{i=1}^{n}[D_T^i(A,B)+D_I^i(A,B)+D_F^i(A,B)]=0$$

For each $u_i\in U$ we have $T_A(u_i)\le T_B(u_i)$ (or $T_A(u_i)\le T_B(u_i)$). So that, using Lemma 1 with $a=T_A(u_i), z=T_B(u_i)-T_A(u_i)$ (if $T_A(u_i)\le T_B(u_i)$) we have

$$\begin{aligned} f(z)&=a\ln 2a+(a+z)ln(2a+2z)-(2a+z)\ln(2a+z)\\ &=a\ln\frac{2a}{2a+z}+(a+z)\ln\frac{2(a+z)}{2a+z}\ge 0\end{aligned}$$

We obtain

$$D_T^i(A,B)=T_A(u_i)\ln\frac{2T_A(u_i)}{T_A(u_i)+T_B(u_i)}+T_B(u_i)\ln\frac{2T_B(u_i)}{T_A(u_i)+T_B(u_i)}\ge 0$$

and $D_T^i(A,B)=0$ if only if $z=T_B(u_i)-T_A(u_i)=0$ i.e. $T_B(u_i)=T_A(u_i)$.

By same way, we also obtain $D_I^i(A,B)\ge 0$ and $D_I^i(A,B)=0$ if only if $I_B(u_i)=I_A(u_i)$; $D_F^i(A,B)\ge 0$ and $D_F^i(A,B)=0$ if only if $F_B(u_i)=F_A(u_i)$. Those imply that $D(A,B)=0$ if only if $A=B$.

- Div3. For all $C\in NS(U)$ and for all $u_i\in U,(i=1,2,...,n)$. Because of the symmetry of divergence measures, we can consider the following cases:

- With falsity-membership function we have:

+ If $T_A(u_i)\le T_B(u_i)\le T_C(u_i)$ then $T_{A\cap C}(u_i)=T_A(u_i)$ and $T_{B\cap C}(u_i)=T_B(u_i)$ so that

$$\begin{aligned}&D_T^i(A\cap C,B\cap C)\\ &=T_{A\cap C}(u_i)\ln\frac{2T_{A\cap C}(u_i)}{T_{A\cap C}(u_i)+T_{B\cap C}(u_i)}+T_{B\cap C}(u_i)\ln\frac{T_{B\cap C}(u_i)}{T_{A\cap C}(u_i)+T_{B\cap C}(u_i)}\\ &=T_A(u_i)\ln\frac{2T_A(u_i)}{T_A(u_i)+T_B(u_i)}+T_B(u_i)\ln\frac{2T_B(u_i)}{T_A(u_i)+T_B(u_i)}\\ &=D_T^i(A,B)\end{aligned}$$

+ If $T_A(u_i)\le T_C(u_i)\le T_B(u_i)$ then $T_{A\cup C}(u_i)=T_C(u_i)$ and $T_{B\cup C}(u_i)=T_B(u_i)$. So that, according the lemma 3 with $a=T_A(u_i)$, we have

$$\begin{aligned}&D_T^i(A\cap C,B\cap C)\\ &=T_A(u_i)\ln\frac{2T_A(u_i)}{T_A(u_i)+T_C(u_i)}+T_C(u_i)\ln\frac{2T_C(u_i)}{T_A(u_i)+T_C(u_i)}\end{aligned}$$

$$\le T_A(u_i)\ln\frac{2T_A(u_i)}{T_A(u_i)+T_B(u_i)}+T_C(u_i)\ln\frac{2T_B(u_i)}{T_A(u_i)+T_B(u_i)}$$
$$=D_T^i(A,B)$$

+ If $T_C(u_i)\le T_A(u_i)\le T_B(u_i)$ then $T_{A\cap C}(u_i)=T_{B\cap C}(u_i)=T_C(u_i)$ and $T_B(u_i)=T_C(u_i)+z$ with $z\in[0,1-T_A(u_i)]$ so that according the lemma 1 we have

$$D_T^i(A\cap C,B\cap C)$$
$$=T_C(u_i)\ln\frac{2T_C(u_i)}{T_C(u_i)+T_C(u_i)}+T_C(u_i)\ln\frac{2T_C(u_i)}{T_C(u_i)+T_C(u_i)}=0$$
$$\le T_A(u_i)\ln\frac{2T_A(u_i)}{2T_A(u_i)+z}+T_B(u_i)\ln\frac{2T_A(u_i)+2z}{2T_A(u_i)+z}$$
$$=T_A(u_i)\ln\frac{2T_A(u_i)}{T_A(u_i)+T_B(u_i)}+T_B(u_i)\ln\frac{2T_B(u_i)}{T_A(u_i)+T_B(u_i)}$$
$$=D_T^i(A,B).$$

- With indeterminacy membership function: we prove similarly to the case of falsity-membership function.
- With falsity membership function, we have:

+ If $F_A(u_i)\le F_B(u_i)\le F_C(u_i)$ then $F_{A\cap C}(u_i)=F_C(u_i)$ and $F_{B\cap C}(u_i)=F_C(u_i)$ so that according lemma 1 we have

$$D_F^i(A\cap C,B\cap C)$$
$$=F_{A\cap C}(u_i)\ln\frac{2F_{A\cap C}(u_i)}{F_{A\cap C}(u_i)+F_{B\cap C}(u_i)}+F_{B\cap C}(u_i)\ln\frac{2F_{B\cap C}(u_i)}{F_{A\cap C}(u_i)+F_{B\cap C}(u_i)}$$
$$=F_C(u_i)\ln\frac{2F_C(u_i)}{F_C(u_i)+F_C(u_i)}+F_C(u_i)\ln\frac{2F_C(u_i)}{F_C(u_i)+F_C(u_i)}=0$$
$$\le F_A(u_i)\ln\frac{2F_B(u_i)}{F_A(u_i)+F_B(u_i)}+F_B(u_i)\ln\frac{2F_B(u_i)}{F_A(u_i)+F_B(u_i)}$$
$$=D_F^i(A,B)$$

+ If $F_A(u_i)\le F_C(u_i)\le F_B(u_i)$ then $F_{A\cup C}(u_i)=F_C(u_i)$ and $F_{B\cup C}(u_i)=F_B(u_i)$. So that, according the lemma 2 with $b=F_B(u_i)$ we have

$$D_F^i(A\cap C,B\cap C)$$
$$=F_C(u_i)\ln\frac{2F_C(u_i)}{F_C(u_i)+F_B(u_i)}+F_B(u_i)\ln\frac{2F_B(u_i)}{F_C(u_i)+F_B(u_i)}$$
$$\le F_A(u_i)\ln\frac{2F_A(u_i)}{F_A(u_i)+F_B(u_i)}+F_B(u_i)\ln\frac{2F_B(u_i)}{F_A(u_i)+F_B(u_i)}$$
$$=D_F^i(A,B).$$

+ If $F_C(u_i)\le F_A(u_i)\le F_B(u_i)$ then according the lemma 1 we have

$$D_F^i(A\cap C,B\cap C)$$
$$=F_A(u_i)\ln\frac{2F_A(u_i)}{F_A(u_i)+F_B(u_i)}+F_B(u_i)\ln\frac{2F_B(u_i)}{F_A(u_i)+F_B(u_i)}$$
$$=D_F^i(A,B).$$

Now, we add that with respect to the respective components we have

$$D(A\cap C,B\cap C)$$
$$=\frac{1}{n}\sum_{i=1}^{n}[D_T^i(A\cap C,B\cap C)+D_I^i(A\cap C,B\cap C)+D_F^i(A\cap C,B\cap C)]$$
$$\leq \frac{1}{n}\sum_{i=1}^{n}[D_T^i(A,B)+D_I^i(A,B)+D_F^i(A,B)]$$
$$=D(A,B)$$

- Div4. We perform as Div 3. □

Now we consider some properties of the divergence measures defined in definition 3.

Theorem 2. For all Neutrosophic set $A,B\in PFS(U)$. We have

(D1) For all $A\subseteq B$, or $B\subseteq A$ we have

$D(A\cap B,B)=D(A,A\cup B)\leq D(A,B),$

(D2) $D(A\cap B,A\cup B)=D(A,B)$,

(D3) For all $A\subseteq B\subseteq C$ we have

$D(A,B)\leq D(A,C),$

(D4) For all $A\subseteq B\subseteq C$ we have

$D(B,C)\leq D(A,C)$.

Proof.

(D1). If $A\subseteq B$ then $D(A\cap B,B)=D(A,B)$ so that, we have

$$D(A,A\cup B)=D(A,B).$$

If $B\subseteq A$ then $D(A\cap B,B)=D(B,B)=0$ so that, we have

$$D(A,A\cup B)=D(A,A)=0.$$

It means that if $A\subseteq B$, or $B\subseteq A$ we have

$$D(A\cap B,B)=D(A,A\cup B)\leq D(A,B).$$

(D2). Because of the symmetry of the divergence measure. We consider the cases:

+ If $T_A(u_i)\leq T_B(u_i)$ then we have

$$D_T^i(A\cup B,A\cap B)$$
$$=T_B(u_i)\ln\frac{2T_B(u_i)}{T_A(u_i)+T_B(u_i)}+T_A(u_i)\ln\frac{2T_A(u_i)}{T_A(u_i)+T_B(u_i)}$$
$$=D(A,B),$$

+ if $T_B(u_i)\leq T_A(u_i)$ then we have

$$D_T^i(A\cup B,A\cap B)$$
$$=T_A(u_i)\ln\frac{2T_A(u_i)}{T_A(u_i)+T_B(u_i)}+T_B(u_i)\ln\frac{2T_B(u_i)}{T_A(u_i)+T_B(u_i)}$$
$$=D(A,B).$$

By the same consideration for indeterminacy membership function and falsity membership function, we obtain

$$D(A\cap B,A\cup B)=D(A,B),$$

(D3). For all $A\subseteq B\subseteq C$ and for all $u_i\in U$ we have:

- With the falsity-membership function:

From condition $T_A(u_i) \le T_B(u_i) \le T_C(u_i)$ and lemma 2 we have:

$$D_T^i(A,B)$$
$$= T_A(u_i)\ln\frac{2T_A(u_i)}{T_A(u_i)+T_B(u_i)} + T_B(u_i)\ln\frac{2T_B(u_i)}{T_A(u_i)+T_B(u_i)}$$
$$= T_A(u_i)\ln\frac{2T_A(u_i)}{T_A(u_i)+T_C(u_i)} + T_C(u_i)\ln\frac{2T_C(u_i)}{T_C(u_i)+T_A(u_i)}$$
$$= D_T^i(A,C),$$

- With the indeterminacy membership function:

By the same way as falsity- membership function we have $D_I^i(A,B) \le D_I^i(A,C),$

- With the falsity- membership function:

From condition $F_A(u_i) \ge F_B(u_i) \ge F_C(u_i)$ and lemma 3 we have:

$$D_F^i(A,B)$$
$$= F_A(u_i)\ln\frac{2F_A(u_i)}{F_A(u_i)+F_B(u_i)} + F_B(u_i)\ln\frac{2F_B(u_i)}{F_A(u_i)+F_B(u_i)}$$
$$\le F_A(u_i)\ln\frac{2F_A(u_i)}{F_A(u_i)+F_C(u_i)} + F_C(u_i)\ln\frac{2F_C(u_i)}{F_A(u_i)+F_C(u_i)}$$
$$= D_F^i(A,C).$$

So that, we obtain the result $D(A,B) \le D(A,C).$

(D4). By the same way as (D4) using lemma 1, lemma 2 and lemma 3, it is easy to derive these results when considering specific cases. □

4 Applications of divergence measure of Neutrosophic set

In this section we apply the Neutrosophic divergence measures in the medical diagnosis and classification problems.

4.1 In the medical diagnosis

Now, we applied the Neutrosophic divergence measure for obtaining a proper diagnosis for the data given in Table 1 and Table 2. This data was modified from the data that introduced in [2]. Usage of diagnostic methods D = {Viral fever (A_1), Malaria (A_2), Typhoid (A_3), Stomach problem (A_4), Chest problem(A_5)} for patients with given values of symptoms S = {temperature (s_1), headache (s_2), stomach pain (s_3), cough (s_4), chest pain (s_5)}. In this case, the neutrosophic set is useful to handle them. Here, for each $A_k \in D, (k = 1, 2, ..., 5)$, is expressed in form that is a neutrosophic set on the universal set $S = \{s_1, s_2, s_3, s_4, s_5\}$, see Table 1. The information of symptoms characteristic for the considered patients is given in Table 2. In which, for each patient $B_j (j = 1, 2, 3, 4)$ is a neutrosophic set in the universal set $S = \{s_1, s_2, s_3, s_4, s_5\}$.

To select the appropriate diagnostic method we calculate the divergence measure between each patient and each diagnosis. After that, we chose the smallest value of them. This will be to give us the best diagnosis for each patient (Table 3).

The divergence measure of a diagnosis $A_k \in D(k = 1, 2, ..., 5)$ for each patient $B_j (j = 1, 2, 3, 4)$ is computed by using the Eq.(2), Eq.(3), Eq.(4), Eq.(5) as follows:

$$D(A_k, B_j) = \frac{1}{n}\sum_{i=1}^{n}[D_T^i(A_k, B_j) + D_I^i(A_k, B_j) + D_F^i(A_k, B_j)]$$

where

$$D_T^i(A_k,B_j)=T_{A_k}(u_i)\ln\frac{2T_{A_k}(u_i)}{T_{A_k}(u_i)+T_{B_j}(u_i)}+T_{B_j}(u_i)\ln\frac{2T_{B_j}(u_i)}{T_{A_k}(u_i)+T_{B_j}(u_i)}$$

$$D_I^i(A_k,B_j)=I_{A_k}(u_i)\ln\frac{2I_{A_k}(u_i)}{I_{A_k}(u_i)+I_{B_j}(u_i)}+I_{B_j}(u_i)\ln\frac{2I_{B_j}(u_i)}{I_{A_k}(u_i)+I_{B_j}(u_i)}$$

and

$$D_F^i(A_k,B_j)=F_{A_k}(u_i)\ln\frac{2F_{A_k}(u_i)}{F_{A_k}(u_i)+F_{B_j}(u_i)}+F_{B_j}(u_i)\ln\frac{2F_{B_j}(u_i)}{F_{A_k}(u_i)+F_{B_j}(u_i)}.$$

Table 1. Symptoms Characteristics for the Diagnosis

	Viral fever	Malaria	Typhoid	Stomach Problem	Chest Problem
Temperature	(0.7,0.5,0.6)	(0.7,0.9,0.1)	(0.3,0.7,0.2)	(0.1,0.6,0.7)	(0.1,0.9,0.8)
Headache	(0.8,0.2,0.9)	(0.4,0.5, 0.5)	(0.6,0.9,0.2)	(0.7,0.4,0.3)	(0.1,0.6,0.7)
Somach pain	(0.8,1,0.1)	(0.5,0.9,0.2)	(0.2,0.5,0.5)	(0.7,0.7,0.8)	(0.5,0.7,0.6)
Cough	(0.45,0.8,0.7)	(0.7,0.8,0.6)	(0.2,0.5,0.5)	(0.2,0.8,0.65)	(0.2,0.8,0.6)
Chest pain	(0.2,0.6,0.5)	(0.1,0.6,0.8)	(0.1,0.8,0.8)	(0.5,0.8,0.6)	(0.8,0.8,0.2)

Table 2. Symptoms Characteristics for the Patients

	Temperature	Headache	Stomach pain	Cough	Chest pain
Al (B_1))	(0.7,0.6,0.5)	(0.6,0.3,0. 5)	(0. 5,0. 5,0.75)	(0.8,0.75,0.5)	(0.7,0.2,0.6)
Bob (B_2)	(0.7,0.3,0.5)	(0.5,0.5,0.8)	(0.6,0. 5,0. 5)	(0.65,0.4,0.75)	(0. 2,0.85,0.65)
Joe (B_3)	(0.75,0.5,0.5)	(0.2,0.85,0.7)	(0.7,0.6,0.4)	(0.7,0.55,0. 5)	(0. 5,0. 9,0.64)
Ted (B_4)	(0.4,0.7,0.6)	(0.7,0.5,0.7)	(0.6,0.7,0.5)	(0.5,0.9,0.65)	(0.6,0.5,0.85)

The computed results of the divergence measures are listed in Table 3. From the results, we see that Al and Ted should use diagnostic methods corresponding to Stomach Problem, Bob use a Viral fever, Joe use a Malaria.

Table 3. Diagnosis results for the divergence measure using eq. (2)

	Viral fever	Malaria	Typhoid	Stomach Problem	Chest Problem
Al	0.81614	0.82946	1.14558	**0.75326**	1.10798
Bob	**0.49750**	0.59104	0.73430	0.79456	1.14038
Joe	0.75011	**0.60603**	0.89659	0.88206	0.79920
Ted	0.48722	0.61785	0.81009	**0.36199**	0.72614

4.2 In the classification problem

Assume that, we have m pattern $\{A_1, A_2, ..., A_m\}$, in which each pattern is a Neutrosophic set on universal set $U = \{u_1, u_2, ..., u_n\}$. Suppose that, we have a sample B with the given feature information. Our goal is to classify sample B into which sample. To solve this, we calculate the divergence measure of B with each pattern $A_i (i = 1, 2, ..., m)$. Then we choose the smallest value. It gives us the class that B belongs to.

Example 1. Assume that three are three Neutrosophic patterns in $U = \{u_1, u_2, u_3\}$ as following

$A_1 = \{(u_1, 0.7, 0.7, 0.2), (u_2, 0.7, 0.8, 0.4), (u_3, 0.6, 0.8, 0.2)\}$

$A_2 = \{(u_1, 0.5, 0.7, 0.3), (u_2, 0.7, 0.7, 0.5), (u_3, 0.8, 0.6, 0.1)\}$

$A_3 = \{(u_1, 0.9, 0.5, 0.1), (u_2, 0.7, 0.6, 0.4), (u_3, 0.8, 0.5, 0.2)\}$

Assume that a sample

$B = \{(u_1, 0.7, 0.8, 0.4), (u_2, 0.8, 0.5, 0.3), (u_3, 0.5, 0.8, 0.5)\}$

Using the divergence measure in Eq.(2) we have $D(A_1, B) = 0.15372$, $D(A_2, B) = 0.26741$ $D(A_3, B) = 0.29516$.

So that we can classifies that B belongs to class A_1.

5 Conclusion

Neutrosophic set theory is more and more interested by researches. There are many theoretical and applied results on Neutrosophic sets that are built and developed. In this paper, we study the divergence measure of Neutrosophic sets. Along with that, we offer some divergence formulas on Neutrosophic sets and give some properties of these measurements. Finally we apply the proposed measures in some cases.

In the future, we will continue to study this measure and offer some of their applications in other areas such as image segmentation or multi-criteria decision making.

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