Diversity Analysis of Space–Time Modulation Over Time-Correlated Rayleigh-Fading Channels

Weifeng Su, *Member, IEEE*, Zoltan Safar, *Member, IEEE*, and K. J. Ray Liu, *Fellow, IEEE*

***Abstract*—Most space–time codes in the literature were proposed based on two ideal channel conditions: either quasi-static or rapid fading. However, these codes may suffer performance degradation due to temporal correlation caused by the movement of the mobile terminal or imperfect interleaving. In this correspondence, we provide a novel analytical framework for the diversity analysis of space–time modulation in time-correlated fading environment. We show that the space–time signals of square size achieving full diversity in quasi-static fading channels also achieve full diversity in time-correlated fading channels, independently of the time correlation matrix. Consequently, various classes of space–time signals designed for quasi-static fading channels can also be used for full-diversity transmission over time-correlated fading channels. Moreover, we show that if the time correlation matrix is of full rank, the design criteria for time-correlated fading channels are the same as those for rapid fading channels. To illustrate the theoretical results, some simulations were also performed under various temporal fading conditions.**

***Index Terms*—Diversity, interleaving, multiple antennas, space–time modulation, time-correlated fading.**

I. Introduction

The challenges imposed by the wireless propagation environment and the need for high data rate communication links have started a quest for methods to exploit a new resource: the spatial dimension. The idea of equipping the transmitter and the receiver with multiple antennas and developing coding and modulation schemes to improve the performance of communication systems have gained increasing popularity, which is demonstrated by the abundant literature that has been published on this topic recently.

However, most results on space–time coding have been proposed based on two ideal channel conditions: either spatially independent quasi-static or rapid fading [1]–[16]. These codes may suffer performance degradation if there is spatial or temporal correlation existing in wireless channel. In addition, different mobile stations or a mobile moving through different geographical locations may experience different channel correlations. This motivates the development of *robust* coding and modulation methods that can guarantee good performance over various channel conditions.

For the quasi-static channel model, the authors of [17] investigated the achievable diversity order as a function of spatial correlation, taking into account some physical propagation parameters. The problem of code design for correlated fading channels was addressed in [18], [19], and general performance criteria were derived for space–time-correlated Rayleigh-fading channels. In [20], it was assumed that the channel stays constant for a number of channel symbol periods equal to the number of transmit antennas. The performance criteria were obtained for this channel model, and hand-crafted trellis codes were proposed combining multiple trellis coded modulation with Alamouti's scheme [4]. The general performance criteria of [18], [19] were further simplified in [21], assuming that the space–time correlation matrix is of full rank. In this case, the design criteria were simplified to those for rapid fading channels.

In [22], characterizing the performance of space–time codes over space–time-correlated Rayleigh-fading channels was also considered. The minimum diversity order achieved over all space–time correlation matrices of a given rank was defined as the measure of robustness. The relationship between the robustness (diversity) and the rank of the space–time correlation matrix was also established, and space–time codes designed for the independent fading channel model were proposed for communication over space–time-correlated fading channels.

In this correspondence, we analyze the performance of space–time modulation in time-correlated fading environment. We assume that the wireless channel exhibits temporal correlation, but there is no spatial correlation between the transmit and the receive antennas. This assumption is true if the transmit and receive antennas are placed far enough from each other. However, temporal correlation may be caused by the movement of the mobile terminal, or imperfect time interleaving due to the constraint of allowable decoding delay [23], [24]. Thus, it is of interest to design robust space–time signals that can provide good performance over all time-correlated fading conditions.

First, we derive the performance criteria for the time-correlated Rayleigh-fading channel model. Then, we show that the space–time signals of square size achieving full diversity in quasi-static fading channels can also achieve full diversity in time-correlated fading channels, irrespectively of the time correlation matrix. We find that in this case, the maximum achievable diversity is only a function of the number of the transmit and receive antennas, and is not limited by the rank of the correlation matrix, in contrast to the results of [22] obtained for spatially nonwhite channels. As a consequence, various classes of space–time signals designed for quasi-static fading channels, for example, cyclic codes [10], codes from orthogonal designs [4]–[7], parametric codes [13], Cayley codes [12], and so on, can be used for full-diversity transmission over time-correlated fading channels, providing robust performance over a wide range of channel conditions. We also show that if the time correlation matrix is of full rank, the design criteria for time-correlated fading channels are the same as those for rapid fading channels, in agreement with the results of [21].

The correspondence is organized as follows. Section II will introduce the channel model and briefly summarize the relevant results from previous work. The design criteria for the time-correlated fading channel model will be derived in Section III. Section IV will provide some simulation results under various temporal fading conditions. The conclusion will be given in Section V.

II. Channel Model and Background

We consider a wireless communication system with M transmit antennas and N receive antennas. The space–time modulator divides the input bit stream into b bit long blocks, and for each block, it selects

Manuscript received May 28, 2003; revised August 13, 2003. This work was supported in part by the U.S. Army Research Laboratory under Cooperative Agreement DAAD 190120011. The material in this correspondence was presented in part at the IEEE International Conference on Communications, Anchorage, AK, May 2003.

W. Su and K. J. R. Liu are with the Department of Electrical and Computer Engineering and Institute for Systems Research, University of Maryland, College Park, MD 20742 USA (e-mail: weifeng@eng.umd.edu; kjrliu@eng.umd.edu).

Z. Safar was with the Department of Electrical and Computer Engineering and Institute for Systems Research, University of Maryland, College Park, MD 20742 USA. He is now with the Department of Innovation, IT University of Copenhagen, Copenhagen, Denmark (e-mail: safar@itu.dk).

Communicated by D. N. C. Tse, Associate Editor for Communications.
Digital Object Identifier 10.1109/TIT.2004.831848

one space–time signal from the signal set of size $L = 2^b$. The selected signal is then transmitted through the channel over the M transmit antennas and T time slots. Each space–time signal can be expressed as a $T \times M$ matrix

$$C = \begin{bmatrix} c_1^1 & c_1^2 & \dots & c_1^M \\ c_2^1 & c_2^2 & \dots & c_2^M \\ \vdots & \vdots & \ddots & \vdots \\ c_T^1 & c_T^2 & \dots & c_T^M \end{bmatrix}_{T \times M} \tag{1}$$

where c_t^i denotes the channel symbol transmitted by transmit antenna $i, i = 1, 2, \dots, M$, at discrete time $t, t = 1, 2, \dots, T$. The space–time signals are assumed to satisfy the energy constraint

$$E\|C\|_F^2 = MT \tag{2}$$

where $\|C\|_F$ is the Frobenius norm [1] of C, and E stands for expectation.

The received signal y_t^j at receive antenna j at time t is given by

$$y_t^j = \sqrt{\frac{\rho}{M}} \sum_{i=1}^{M} c_t^i h_{i,j}(t) + z_t^j, \qquad t = 1, 2, \dots, T \tag{3}$$

where z_t^j is the complex additive white Gaussian noise (AWGN) at receive antenna j at time t with zero mean and unit variance, and $h_{i,j}(t)$ is the channel coefficient from transmit antenna i to receive antenna j at time t. If the system has an interleaver, the $h_{i,j}(t)$'s are the equivalent channel coefficients after interleaving. The channel coefficients are modeled as zero-mean complex Gaussian random variables with unit variance. These coefficients are assumed to be known at the receiver, but unknown at the transmitter. Moreover, we assume that the channel fading has only temporal correlation, i.e., the channel coefficients $h_{i,j}(t)$ are independent for different indexes (i, j) and dependent only in the temporal domain. The factor ρ in (3) is the average signal-to-noise ratio (SNR) per space–time signal at each receive antenna, and it is independent of the number of transmit antennas.

The received signal (3) can be rewritten in vector form as [18], [19]

$$\boldsymbol{Y} = \sqrt{\frac{\rho}{M}} \boldsymbol{D}\boldsymbol{H} + \boldsymbol{Z} \tag{4}$$

where $\boldsymbol{D}$ is an $NT \times MNT$ matrix constructed from the space–time signal matrix C as shown in (5) at the bottom of the page, in which

$$D_i = \text{diag}\left(c_1^i, c_2^i, \dots, c_T^i\right), \qquad i = 1, 2, \dots, M. \tag{6}$$

[1]The Frobenius norm of C is defined as

$$\|C\|_F^2 = \text{tr}(C^{\mathcal{H}}C) = \text{tr}(CC^{\mathcal{H}}) = \sum_{t=1}^{T}\sum_{i=1}^{M} |c_t^i|^2.$$

Each D_i in (6) is related to the ith column of the space–time signal matrix C. The channel vector $\boldsymbol{H}$ of size $MNT \times 1$ is formatted as (7) at the bottom of the page, where

$$\boldsymbol{h}_{i,j} = [\, h_{i,j}(1) \quad h_{i,j}(2) \quad \dots \quad h_{i,j}(T) \,]^{\mathcal{T}}.$$

The received signal vector $\boldsymbol{Y}$ of size $NT \times 1$ is given by

$$\boldsymbol{Y} = [\, y_1^1 \quad \dots \quad y_T^1 \quad y_1^2 \quad \dots \quad y_T^2 \quad \dots \quad y_1^N \quad \dots \quad y_T^N \,]^{\mathcal{T}} \tag{8}$$

and the noise vector $\boldsymbol{Z}$ has the form

$$\boldsymbol{Z} = [\, z_1^1 \quad \dots \quad z_T^1 \quad z_1^2 \quad \dots \quad z_T^2 \quad \dots \quad z_1^N \quad \dots \quad z_T^N \,]^{\mathcal{T}}. \tag{9}$$

Suppose that $\boldsymbol{D}$ and $\tilde{\boldsymbol{D}}$ are two different matrices related to two different space–time signals C and $\tilde{C}$, respectively. Then, the pairwise error probability between $\boldsymbol{D}$ and $\tilde{\boldsymbol{D}}$ can be upper-bounded as [18], [19]

$$P(\boldsymbol{D} \to \tilde{\boldsymbol{D}}) \le \binom{2K-1}{K} \left(\prod_{i=1}^{K} \gamma_i\right)^{-1} \left(\frac{\rho}{M}\right)^{-K} \tag{10}$$

where K is the rank of $(\boldsymbol{D} - \tilde{\boldsymbol{D}})\boldsymbol{R}(\boldsymbol{D} - \tilde{\boldsymbol{D}})^{\mathcal{H}}$, $\gamma_1, \gamma_2, \dots, \gamma_K$ are the nonzero eigenvalues of $(\boldsymbol{D} - \tilde{\boldsymbol{D}})\boldsymbol{R}(\boldsymbol{D} - \tilde{\boldsymbol{D}})^{\mathcal{H}}$, and $\boldsymbol{R} = E\{\boldsymbol{H}\boldsymbol{H}^{\mathcal{H}}\}$ is the correlation matrix of $\boldsymbol{H}$. The superscript $\mathcal{H}$ stands for the complex conjugate and transpose of a matrix.

Based on the upper bound on the pairwise error probability (10), a general code design criterion has been proposed in [18], [19]: the minimum rank of $(\boldsymbol{D} - \tilde{\boldsymbol{D}})\boldsymbol{R}(\boldsymbol{D} - \tilde{\boldsymbol{D}})^{\mathcal{H}}$ should be as large as possible, and the minimum value of the product $\prod_{i=1}^{K} \gamma_i$ should also be maximized. This criterion is consistent with the well-known criteria [1], [2] for two ideal cases: the quasi-static and the rapid-fading channel models which can be summarized as follows.

- For quasi-static fading channels: The minimum rank of

$$\Delta \triangleq (C - \tilde{C})(C - \tilde{C})^{\mathcal{H}} \tag{11}$$

over all pairs of distinct signals C and $\tilde{C}$ should be as large as possible. If Δ is of full rank for any pair of distinct signals C and $\tilde{C}$, then the diversity product [10], [11] is given by

$$\zeta_{\text{static}} = \frac{1}{2\sqrt{M}} \min_{C \neq \tilde{C}} |\det(\Delta)|^{\frac{1}{2T}} \tag{12}$$

which is related to the coding advantage and should also be maximized.

- For rapid-fading channels: The minimum number of nonzero rows of $C - \tilde{C}$ should be as large as possible for any pair of distinct signals C and $\tilde{C}$. If for any pair of distinct signals C and $\tilde{C}$, there is no zero row in $C - \tilde{C}$, then the diversity product, given by

$$\zeta_{\text{rapid}} = \frac{1}{2\sqrt{M}} \min_{C \neq \tilde{C}} \left(\prod_{t=1}^{T} \|\boldsymbol{c}_t - \tilde{\boldsymbol{c}}_t\|_F^2\right)^{\frac{1}{2T}} \tag{13}$$

should be maximized. In (13), $\boldsymbol{c}_t$ and $\tilde{\boldsymbol{c}}_t$ are the t-th rows of C and $\tilde{C}$, respectively.

$$\boldsymbol{D} = \begin{bmatrix} D_1 & D_2 & \dots & D_M & 0 & 0 & \dots & 0 & \dots & 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & D_1 & D_2 & \dots & D_M & \dots & 0 & 0 & \dots & 0 \\ & \vdots & & & & & \vdots & & \ddots & & & \vdots & \\ 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & \dots & D_1 & D_2 & \dots & D_M \end{bmatrix}_{NT \times MNT} \tag{5}$$

$$\boldsymbol{H} = [\, \boldsymbol{h}_{1,1}^{\mathcal{T}} \quad \dots \quad \boldsymbol{h}_{M,1}^{\mathcal{T}} \quad \boldsymbol{h}_{1,2}^{\mathcal{T}} \quad \dots \quad \boldsymbol{h}_{M,2}^{\mathcal{T}} \quad \dots \quad \boldsymbol{h}_{1,N}^{\mathcal{T}} \quad \dots \quad \boldsymbol{h}_{M,N}^{\mathcal{T}} \,]^{\mathcal{T}} \tag{7}$$

III. Design Criteria for Time-Correlated Fading Channels

In this section, we derive the design criteria and investigate the achievable performance limits of space–time coded communication systems assuming only time-domain correlation. In this case, the channel correlation matrix $\boldsymbol{R}$ of size $MNT \times MNT$ becomes

$$\boldsymbol{R} = E\{\boldsymbol{H}\boldsymbol{H}^{\mathcal{H}}\} = \mathrm{diag}\Big(R_{1,1}, \ldots, R_{M,1}, R_{1,2}, \ldots, R_{M,2}, \ldots\ldots, R_{1,N}, \ldots, R_{M,N}\Big)$$

where

$$R_{i,j} = E\left(\boldsymbol{h}_{i,j}\boldsymbol{h}_{i,j}^{\mathcal{H}}\right)$$

is the time correlation matrix of the channel coefficients from transmit antenna i to receive antenna j. Assuming that all of the time correlation matrices $R_{i,j}$ are the same, which is true for the Jakes fading model [26], the correlation matrix can be expressed as

$$\boldsymbol{R} = I_{MN} \otimes R \tag{14}$$

where $\otimes$ denotes the tensor product, I_{MN} is the identity matrix of size $MN \times MN$, and R is the time-correlation matrix, defined as

$$R \triangleq R_{i,j} = \begin{bmatrix} r_{1,1} & \ldots & r_{1,T} \\ \ldots & \ldots & \ldots \\ r_{T,1} & \ldots & r_{T,T} \end{bmatrix}_{T\times T}.$$

Using (5), (6), and (14), we obtain (15) at the bottom of the page, where $\circ$ denotes the Hadamard product,[2] and Δ is defined in (11). Substituting (15) into (10), the pairwise error probability between C and $\tilde{C}$ can be upper-bounded as

$$P(C \to \tilde{C}) \le \binom{2rN-1}{rN}\left(\prod_{i=1}^{r}\lambda_i\right)^{-N}\left(\frac{\rho}{M}\right)^{-rN} \tag{16}$$

where r is the rank of $\Delta \circ R$, and $\lambda_1, \lambda_2, \ldots, \lambda_r$ are the nonzero eigenvalues of $\Delta \circ R$. As a consequence, we can formulate the design criteria for time-correlated fading channels as follows.

a) Design for diversity advantage: The minimum rank of $\Delta \circ R$ over all pairs of distinct signals C and $\tilde{C}$ should be as large as possible.

b) Design for coding advantage: The minimum value of the product $\prod_{i=1}^{r}\lambda_i$ over all pairs of distinct signals C and $\tilde{C}$ should be maximized.

In the sequel, we will discuss the ultimate limits on the maximum achievable diversity imposed by the time-correlated channel model. First, we will provide the general performance limits, and then we will describe some results obtained for two special cases: square space–time signals with an arbitrary time correlation matrix, and nonsquare space–time signals with a full rank time correlation matrix.

If the minimum rank of $\Delta \circ R$ is ν for any pair of distinct signals C and $\tilde{C}$, we say that the set of space–time signals achieves a diversity of νN. For fixed time duration T, the number of transmit antennas M, and time correlation matrix R, the maximum achievable diversity or *full diversity* is defined as the maximum diversity level that can be achieved by space–time signals of size $T \times M$. For example, for quasi-static fading channels

$$R = \begin{bmatrix} 1 & 1 & \ldots & 1 \\ 1 & 1 & \ldots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \ldots & 1 \end{bmatrix}_{T\times T}.$$

In this case, the maximum achievable diversity is $\min(M,T)N$. For rapid-fading channels, $R = I_T$, so the maximum achievable diversity is TN.

Assume that the time correlation matrix R is of rank $\Gamma(1 \le \Gamma \le T)$. Since the matrix $\Delta \circ R$ is of size $T \times T$, its rank is at most T. Furthermore, according to a rank inequality on Hadamard products ([27 p. 307]), the rank of matrix $\Delta \circ R$ can be upper-bounded as

$$\mathrm{rank}(\Delta \circ R) \le \mathrm{rank}(\Delta)\,\mathrm{rank}(R).$$

The rank of Δ cannot be greater than $\min(M,T)$. Therefore, for space–time signals of size $T \times M$ operating in time-correlated fading environment, the maximum achievable diversity is upper-bounded by $\min(M\Gamma, T)N$, where Γ is the rank of the time-correlation matrix R.

For space–time signals of square size, i.e., $T = M$, the maximum achievable diversity cannot be greater than MN. Now we will show that this upper bound can be achieved for any time-correlated fading channel. Note that both Δ and R are nonnegative definite, and all of the diagonal entries of R are nonzero. Therefore, we can apply Schur's theorem on Hadamard products ([27, p. 309]): if Δ is positive definite (i.e., of full rank), then $\Delta \circ R$ is also positive definite (i.e., of full rank). As a consequence, we arrive at the following theorem.

Theorem 1: If a set of space–time signals of size $M \times M$ achieves full diversity (MN) for quasi-static fading channels, then it also achieves full diversity (MN) for any time-correlated fading channel, independently of the time correlation matrix R.

[2] Suppose that $A = \{a_{i,j}\}$ and $B = \{b_{i,j}\}$ are two matrices of size $m \times n$. The *Hadamard product* of A and B is defined as

$$A \circ B = \begin{bmatrix} a_{1,1}b_{1,1} & \ldots & a_{1,n}b_{1,n} \\ \ldots & \ldots & \ldots \\ a_{m,1}b_{m,1} & \ldots & a_{m,n}b_{m,n} \end{bmatrix}.$$

$$\begin{aligned}(\boldsymbol{D}-\tilde{\boldsymbol{D}})\boldsymbol{R}(\boldsymbol{D}-\tilde{\boldsymbol{D}})^{\mathcal{H}} &= I_N \otimes \left[\sum_{i=1}^{M}(D_i-\tilde{D}_i)R(D_i-\tilde{D}_i)^{\mathcal{H}}\right] \\ &= I_N \otimes \begin{bmatrix} \sum_{i=1}^{M}|c_1^i-\tilde{c}_1^i|^2 r_{1,1} & \sum_{i=1}^{M}(c_1^i-\tilde{c}_1^i)(c_2^i-\tilde{c}_2^i)^* r_{1,2} & \ldots & \sum_{i=1}^{M}(c_1^i-\tilde{c}_1^i)(c_T^i-\tilde{c}_T^i)^* r_{1,T} \\ \sum_{i=1}^{M}(c_2^i-\tilde{c}_2^i)(c_1^i-\tilde{c}_1^i)^* r_{2,1} & \sum_{i=1}^{M}|c_2^i-\tilde{c}_2^i|^2 r_{2,2} & \ldots & \sum_{i=1}^{M}(c_2^i-\tilde{c}_2^i)(c_T^i-\tilde{c}_T^i)^* r_{2,T} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{i=1}^{M}(c_T^i-\tilde{c}_T^i)(c_1^i-\tilde{c}_1^i)^* r_{T,1} & \sum_{i=1}^{M}(c_T^i-\tilde{c}_T^i)(c_2^i-\tilde{c}_2^i)^* r_{T,2} & \ldots & \sum_{i=1}^{M}|c_T^i-\tilde{c}_T^i|^2 r_{T,T} \end{bmatrix} \\ &= I_N \otimes \left\{\left[(C-\tilde{C})(C-\tilde{C})^{\mathcal{H}}\right] \circ R\right\} \\ &= I_N \otimes \{\Delta \circ R\} \end{aligned} \tag{15}$$

The result in Theorem 1 is very interesting. It is well known that if a set of space–time signals of square size achieves full diversity for quasi-static fading channels, then it also achieves full diversity for rapid fading channels [3]. However, it is not obvious that it can also achieve full diversity for any time-correlated fading channels. It follows from Theorem 1 that all of the space–time signals of square size designed for quasi-static fading channels can also be used to achieve full diversity in any time-correlated fading environment.

Theorem 1 does not hold for nonsquare signals (i.e., $T \neq M$). However, if the time correlation matrix is of full rank, we can establish another result. We observe that the diagonal entries of Δ are $\|\boldsymbol{c}_t - \tilde{\boldsymbol{c}}_t\|_F^2$, $t = 1, 2, \ldots, T$, where $\boldsymbol{c}_t$ and $\tilde{\boldsymbol{c}}_t$ are the tth rows of C and $\tilde{C}$, respectively. Thus, we can apply Schur's theorem again: if all of $\|\boldsymbol{c}_t - \tilde{\boldsymbol{c}}_t\|_F^2$'s are nonzero for any pair of distinct signals C and $\tilde{C}$, then $\Delta \circ R$ is of full rank whenever the time correlation matrix R is of full rank. This implies that the maximum achievable diversity is TN for any full-rank time correlation matrix R. This result is summarized in the following theorem.

Theorem 2: If a set of space–time signals of size $T \times M$ achieves full diversity (TN) for rapid fading channels, then it also achieves full diversity (TN) for any time-correlated fading channel, provided that the time correlation matrix R is of full rank.

From Theorem 1, we observe that the space–time signals of square size achieving full diversity in quasi-static fading channels can also achieve full diversity in time-correlated fading channels, irrespective of the time correlation matrix. Thus, an efficient way to design robust space–time signals for time-correlated fading channels is to consider the construction of space–time signals of square size. In this case, the problem of designing robust space–time signals for time-correlated fading channels is reduced to that of designing space–time signals for quasi-static fading channels. It follows that the abundant classes of space–time signals of square size designed for quasi-static fading channels, for example, cyclic codes [10], codes from orthogonal designs [4]–[7], parametric codes [13], Cayley codes [12], and so on, may also be used for time-correlated fading channels.

From Theorem 2, we can also see that for channels exhibiting low temporal correlation, i.e., the time correlation matrix R is of full rank, the problem of designing space–time signals is equivalent to that of designing space–time signals for rapid fading channels. In this case, one may consider the construction of space–time signals of nonsquare size to achieve higher diversity order.

IV. Simulation Results

To illustrate the above analytical results, we performed some computer simulations. We considered downlink transmission from a base station with multiple transmit antennas to a mobile station. Assuming that the mobile station is moving at 55 mi/h, the carrier frequency is 900 MHz, and the bandwidth is 30 kHz, the normalized Doppler frequency is approximately $f_D = 0.0025$. The fading channels were modeled by the Jakes fading model [26], in which the temporal correlation is determined by the normalized Doppler frequency f_D.

The above channel model results in a very slowly time-varying channel. For space–time signals of small size, the channel will be approximately constant, so we used this channel model when simulating the quasi-static scenario. The time-correlated case was simulated by assuming that the temporal correlation was caused by imperfect interleaving. We used the slowly time-varying channel model with the Takeshita–Constello interleaver [25] given by

$$\Pi(i) = \mathrm{mod}\left(\frac{i(i+1)}{2}, I\right), \qquad i = 0, 1, 2, \ldots, I-1 \tag{17}$$

and the length of interleaving, I, was set to 128. In this case, we also calculated the entries of 2×2 and 4×4 temporal correlation matrices numerically. Averaging over 100 000 realizations of the fading channel with $f_D = 0.0025$ and using the Takeshita–Constello interleaver, the magnitudes of the entries of the 2×2 correlation matrix were obtained as

$$\begin{bmatrix} 0.9967 & 0.8384 \\ 0.8384 & 0.9967 \end{bmatrix} \tag{18}$$

and the magnitudes of the values in the 4×4 correlation matrix were obtained as

$$\begin{bmatrix} 0.9967 & 0.8384 & 0.8437 & 0.8310 \\ 0.8384 & 0.9967 & 0.8384 & 0.8437 \\ 0.8437 & 0.8384 & 0.9967 & 0.8384 \\ 0.8310 & 0.8437 & 0.8384 & 0.9967 \end{bmatrix}. \tag{19}$$

The fast fading channel model was simulated by generating independent channel coefficients for each discrete-time instant. We compared the performance of each space–time modulation scheme over quasi-static channels, shown as solid lines with stars ("$*$"), time-correlated channels, shown as dotted lines with dots ("$\cdot$"), and rapid fading channels, shown as dashed lines with circles ("$\circ$").

In case of two transmit antennas, the following space–time signals with $L = 4$ elements are optimal from the viewpoint of maximizing diversity product in quasi-static fading channels [12], [13]

$$V_0 = \sqrt{\frac{2}{3}} \begin{bmatrix} \boldsymbol{j} & 1-\boldsymbol{j} \\ -1-\boldsymbol{j} & -\boldsymbol{j} \end{bmatrix}, \quad V_1 = \sqrt{\frac{2}{3}} \begin{bmatrix} -\boldsymbol{j} & -1-\boldsymbol{j} \\ 1-\boldsymbol{j} & \boldsymbol{j} \end{bmatrix}$$

$$V_2 = \sqrt{\frac{2}{3}} \begin{bmatrix} -\boldsymbol{j} & 1+\boldsymbol{j} \\ -1+\boldsymbol{j} & \boldsymbol{j} \end{bmatrix}, \quad V_3 = \sqrt{\frac{2}{3}} \begin{bmatrix} \boldsymbol{j} & -1+\boldsymbol{j} \\ 1+\boldsymbol{j} & -\boldsymbol{j} \end{bmatrix}$$

where $\boldsymbol{j} = \sqrt{-1}$. The spectral efficiency of this scheme is 1 bit/s/Hz. Fig. 1 depicts the performance of this scheme with one receive antenna under the three channel conditions. We can see that all three curves have the same diversity order, consistent with the results of Theorem 1. The performance over the time-correlated channel (with the time correlation matrix in (18)) is close to the performance over the quasi-static fading channel.

Fig. 2 provides the simulation results for the Alamouti scheme [4] with quaternary phase-shift keying (QPSK) for two transmit and one receive antennas, giving a spectral efficiency of 2 bits/s/Hz. We observe that all the bit-error rate curves have approximately the same asymptotic slope under the three channel conditions. Moreover, the performance of the code over the quasi-static channel is the best, and the performance over the rapid fading channel is the worst. Fig. 3 shows the performance of the parametric code [13] with $L = 16$ for two transmit and one receive antennas. The spectral efficiency of this scheme is also 2 bits/s/Hz. We observe that the performance of this code is almost the same for the three different channel conditions.

We also simulated a signal set designed for four transmit antennas. We chose the full-diversity quasi-orthogonal space–time block code [9] given by

$$\begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ x_3 & x_4 & x_1 & x_2 \\ -x_4^* & x_3^* & -x_2^* & x_1^* \end{bmatrix} \tag{20}$$

where x_1 and x_2 were chosen from QPSK constellation $\mathcal{A} = \{\pm 1, \pm \boldsymbol{j}\}$, and x_3 and x_4 were chosen from the rotated QPSK constellation $e^{\boldsymbol{j}\frac{\pi}{4}}\mathcal{A} = \{\pm e^{\boldsymbol{j}\frac{\pi}{4}}, \pm e^{\boldsymbol{j}\frac{3\pi}{4}}\}$. The spectral efficiency of this scheme is 2 bits/s/Hz. Fig. 4 depicts the performance of this design with one receive antenna. The figure shows that all the bit-error rate curves have approximately the same asymptotic slope under the three channel conditions.

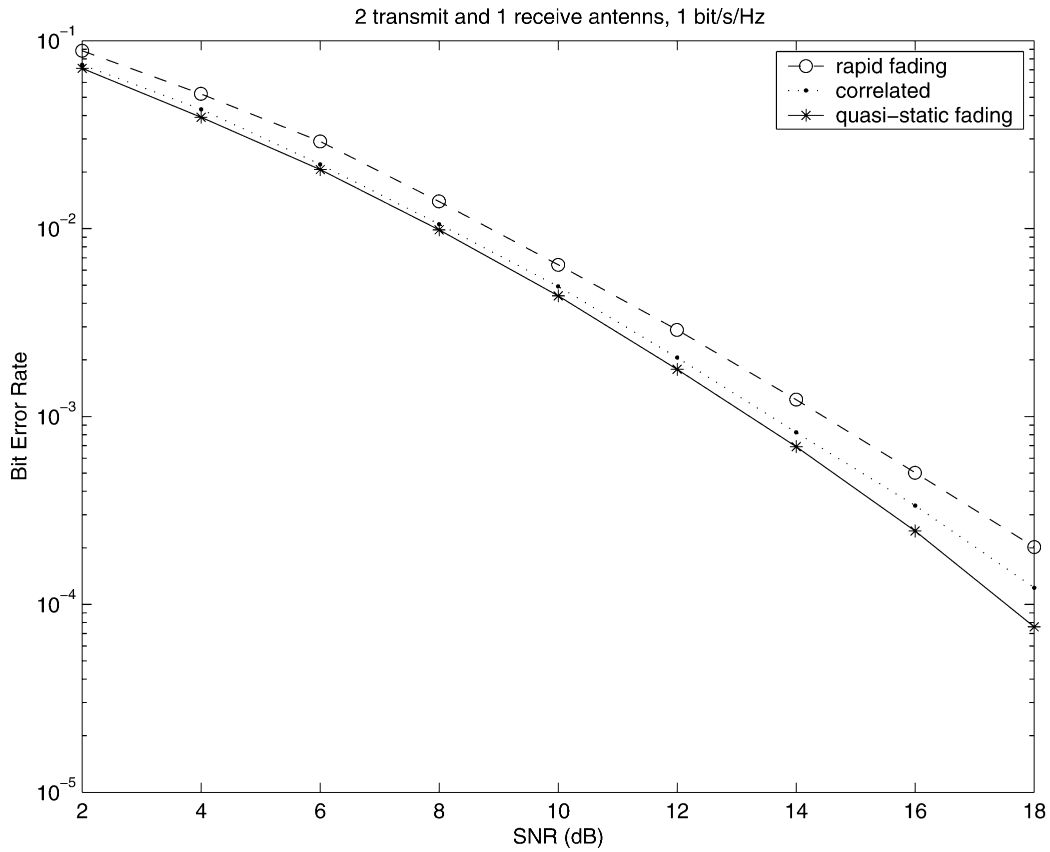

Fig. 1. Bit-error rate performance of the optimal space–time signals with $L = 4$ elements.

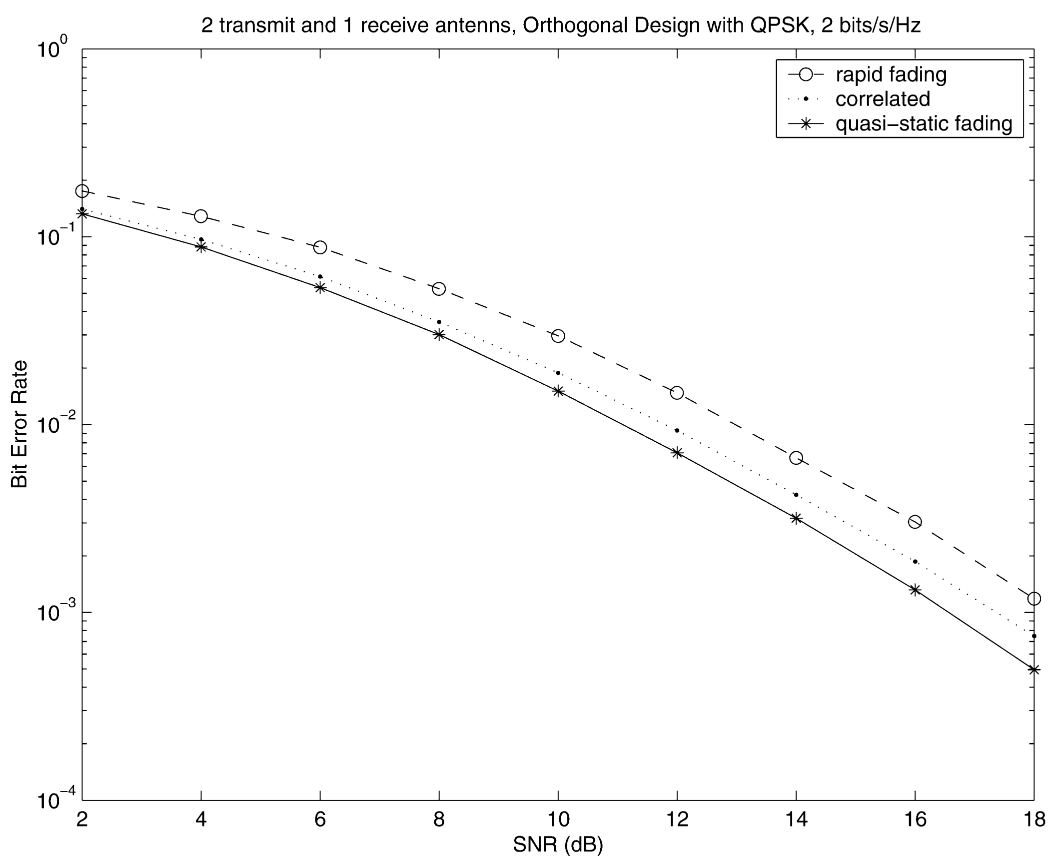

Fig. 2. Bit-error rate performance of the Alamouti scheme with QPSK.

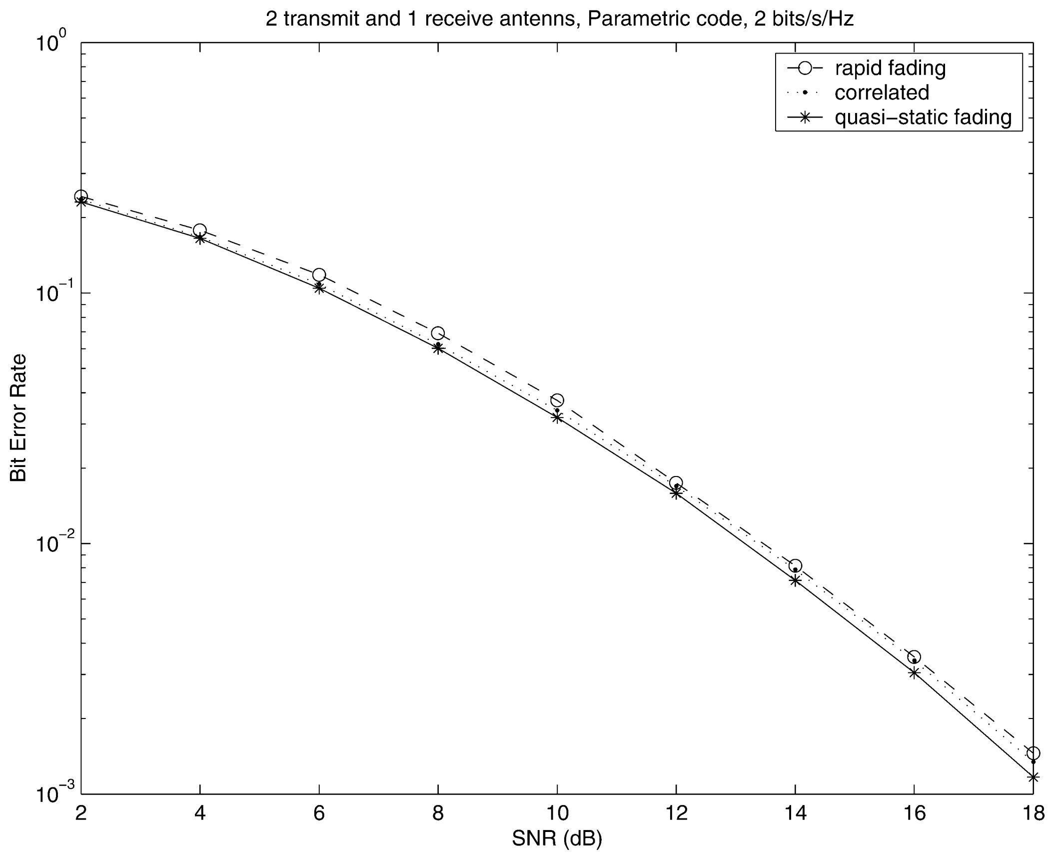

Fig. 3. Bit-error rate performance of the parametric code with $L = 16$.

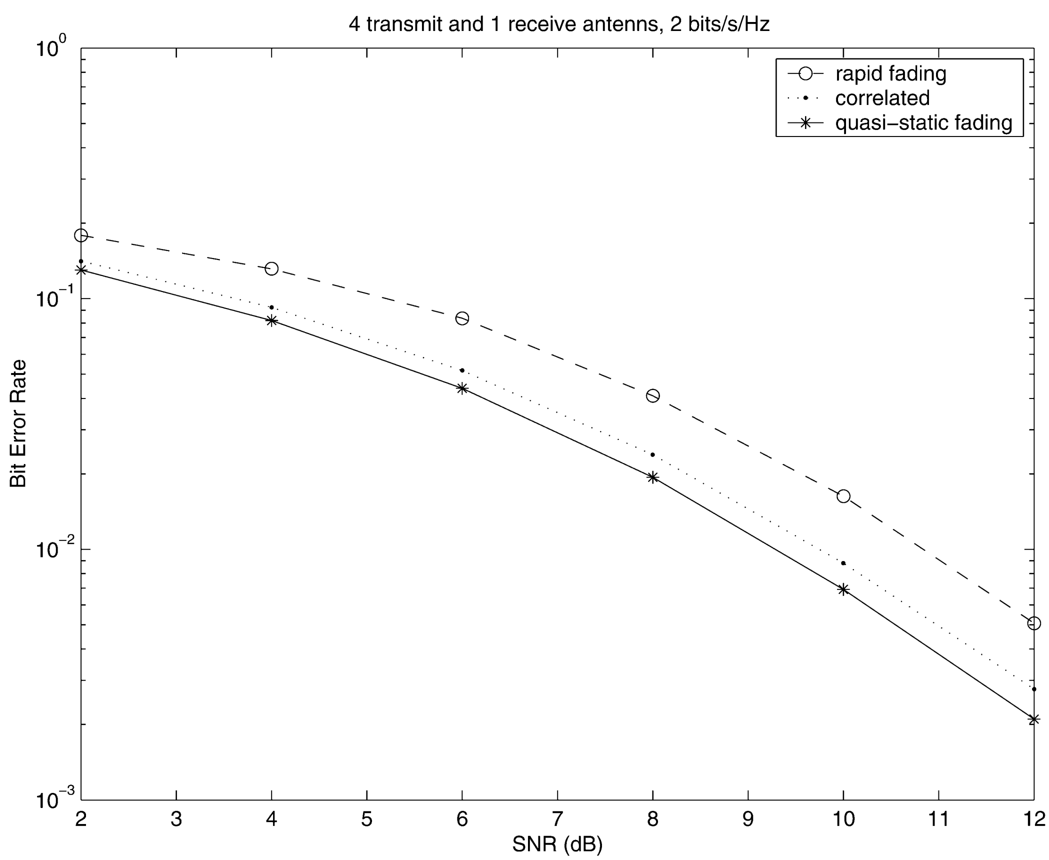

Fig. 4. Bit-error rate performance of the full-diversity quasi-orthogonal design with QPSK.

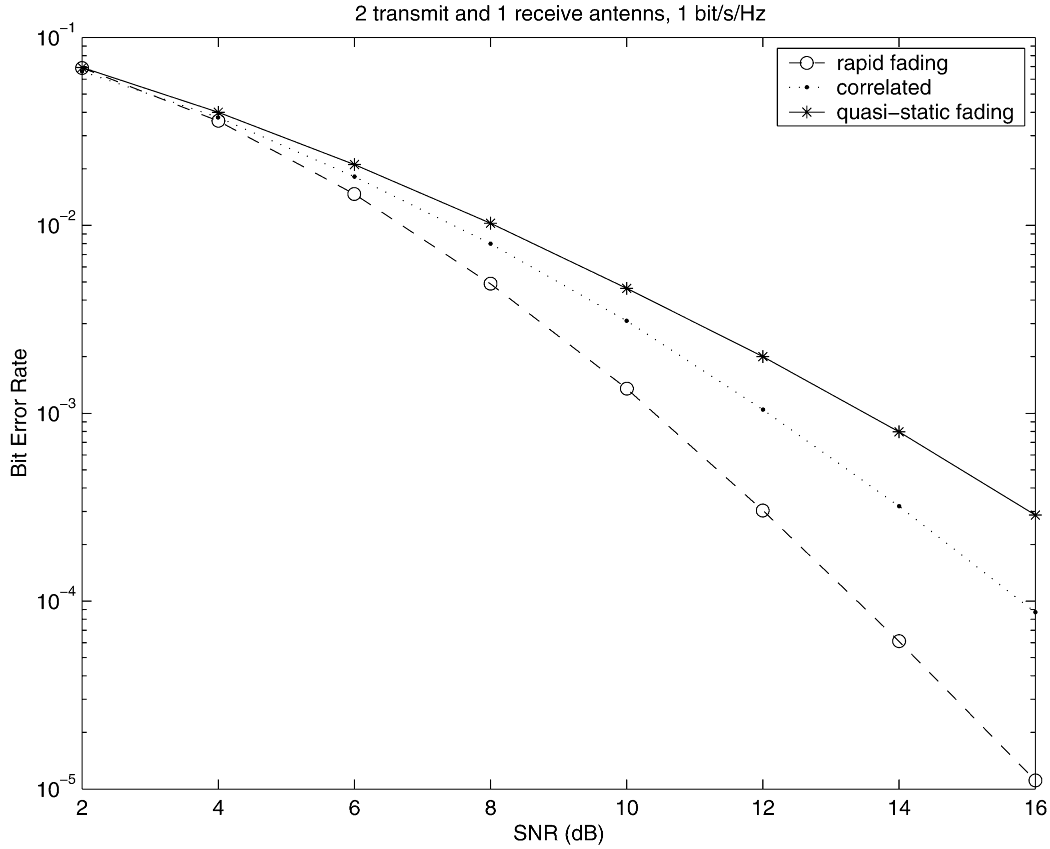

Fig. 5. Bit-error rate performance of the repetition of the Alamouti scheme with QPSK.

The simulations in Figs. 1–4 confirm the theoretical results of Theorem 1. The additional observation is that in Figs. 1–4, the performance of the simulated schemes over quasi-static fading channels is better than that over rapid-fading channels. It seems that for these modulation schemes, the worst channel is the rapid-fading channel, not the quasi-static fading channel, in contrast to the observations from space–time trellis codes [2], [20], [21].

From Theorem 2, we also know that in case of rapid-fading channels (low temporal correlation), one may consider the construction of nonsquare space–time signals to achieve higher diversity order. For example, we constructed a design for two transmit antennas by repeating the Alamouti scheme two times as follows:

$$\begin{bmatrix} x_1 & -x_2^* & x_1 & -x_2^* \\ x_2 & x_1^* & x_2 & x_1^* \end{bmatrix}^{\mathcal{T}} \tag{21}$$

where x_1 and x_2 are chosen from QPSK constellation. The spectral efficiency of this scheme is 1 bit/s/Hz. Fig. 5 depicts the performance of this design with one receive antenna. We can see that the bit-error rate curve of this design over the time-correlated fading channel (with the time correlation matrix in (19)) has a better diversity order than that over the quasi-static fading channel. We observe a 2-dB gain at a bit-error rate of 10^{-4}. Compared to the bit-error rate curve over the rapid-fading channel, there is an additional 2-dB gain that can be further obtained if we increase the interleaving size.

V. Conclusion

In this correspondence, we provided a new approach to the performance analysis of space–time modulation for time-correlated Rayleigh-fading channels. We showed that the space–time signals of square size achieving full diversity in quasi-static fading environment can also achieve full diversity in any time-correlated fading environment, independently of the time correlation matrix. This result implies that various classes of space–time signals of square size designed for quasi-static fading channels may also be used for time-correlated fading channels. The simulations confirmed our theoretical results. In addition, we observed that in case of transmitting space–time signals of square size over spatially independent channels, the quasi-static fading channel model seems to be the best scenario, so the use of interleavers does not seem beneficial in this case.

We also showed that if the time correlation matrix is of full rank, the design criterion for time-correlated fading channels is the same as that for rapid-fading channels. In this case, one may consider the construction of space–time signals of nonsquare size to achieve higher diversity order. The simulation results showed that the performance of the nonsquare design over the rapid fading channel had a better diversity order than that over the quasi-static fading situation. The interleaved nonsquare design also outperformed the noninterleaved (quasi-static) scenario, suggesting that the use of interleavers can improve the performance of nonsquare space–time signals.

References

[1] J.-C. Guey, M. P. Fitz, M. R. Bell, and W.-Y. Kuo, "Signal design for transmitter diversity wireless communication systems over Rayleigh fading channels," *IEEE Trans. Commun.*, vol. 47, pp. 527–537, Apr. 1999.

[2] V. Tarokh, N. Seshadri, and A. R. Calderbank, "Space-time codes for high data rate wireless communication: Performance criterion and code construction," *IEEE Trans. Inform. Theory*, vol. 44, pp. 744–765, Mar. 1998.

[3] V. Tarokh, A. Naguib, N. Seshadri, and A. R. Calderbank, "Space-time codes for high data rate wireless communications: Performance criteria in the presence of channel estimation errors, mobility, and multiple paths," *IEEE Trans. Commun.*, vol. 47, pp. 199–207, Feb. 1999.

[4] S. Alamouti, “A simple transmit diversity technique for wireless communications,” *IEEE J. Select. Aareas Commun.*, vol. 16, pp. 1451–1458, Oct. 1998.

[5] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, “Space-time block codes from orthogonal designs,” *IEEE Trans. Inform. Theory*, vol. 45, pp. 1456–1467, Sept. 1999.

[6] G. Ganesan and P. Stoica, “Space-time block codes: A maximum SNR approach,” *IEEE Trans. Inform. Theory*, vol. 47, pp. 1650–1656, May 2001.

[7] O. Tirkkonen and A. Hottinen, “Square-matrix embeddable space-time block codes for complex signal constellations,” *IEEE Trans. Inform. Theory*, vol. 48, pp. 384–395, Feb. 2002.

[8] W. Su and X.-G Xu, “On space-time block codes from complex orthogonal designs,” *Wireless Personal Commun.*, vol. 25, no. 1, pp. 1–26, Apr. 2003.

[9] W. Su and X.-G. Xia, “Quasiorthogonal space-time block codes with full diversity,” in *Proc. IEEE GLOBECOM’02*, vol. 2, 2002, pp. 1098–1102.

[10] B. M. Hochwald and W. Sweldens, “Differential unitary space-time modulation,” *IEEE Trans. Commun.*, vol. 48, pp. 2041–2052, Dec. 2000.

[11] A. Shokrollahi, B. Hassibi, B. M. Hochwald, and W. Sweldens, “Representation theory for high-rate multiple-antenna code design,” *IEEE Trans. Inform. Theory*, vol. 47, pp. 2335–2367, Sept. 2001.

[12] B. Hassibi and B. M. Hochwald, “Cayley differential unitary space-time codes,” *IEEE Trans. Inform. Theory*, vol. 48, pp. 1485–1503, June 2002.

[13] X.-B. Liang and X.-G. Xia, “Unitary signal constellations for differential space-time modulation with two transmit antennas: Parametric codes, optimal designs and bounds,” *IEEE Trans. Inform. Theory*, vol. 48, pp. 2291–2322, Aug. 2002.

[14] Q. Yan and R. Blum, “Robust space-time block coding for rapid fading channels,” in *Proc. IEEE GLOBECOM*, vol. 1, 2001, pp. 460–464.

[15] S. Zummo and S. Al-Semari, “Space-time coded QPSK for rapid fading channels,” *Proc. IEEE Int. Symp. Personal, Indoor and Mobile Radio Communications (PIMRC)*, vol. 1, pp. 504–508, 2000.

[16] W. Firmanto, B. Vucetic, and J. Yuan, “Space-time TCM with improved performance on fast fading channels,” *IEEE Commun. Letters*, vol. 5, pp. 154–156, Apr. 2001.

[17] H. Bölcskei and A. Paulraj, “Performance of space-time codes in the presence of spatial fading correlation,” in *Proc. Asilomar Conf. Signals, Systems and Computers*, vol. 1, 2000, pp. 687–693.

[18] M. P. Fitz, J. Grimm, and S. Siwamogsatham, “A new view of performance analysis techniques in correlated Rayleigh fading,” in *Proc. IEEE Wireless Communications and Networking Conf. (WCNC)*, Sept. 1999, pp. 139–144.

[19] S. Siwamogsatham, M. P. Fitz, and J. Grimm, “A new view of performance analysis of transmit diversity schemes in correlated Rayleigh fading,” *IEEE Trans. Inform. Theory*, vol. 48, pp. 950–956, Apr. 2002.

[20] S. Siwamogsatham and M. P. Fitz, “Robust space-time codes for correlated Rayleigh fading channels,” *IEEE Trans. Signal Processing*, vol. 50, pp. 2408–2416, Oct. 2002.

[21] Z. Safar and K. J. R. Liu, “Performance analysis of space-time codes over correlated Rayleigh fading channels,” in *Proc. IEEE Int. Conf. Communications*, vol. 5, Anchorage, AK, May 2003, pp. 3185–3189.

[22] H. El Gamal, “On the robustness of space-time coding,” *IEEE Trans. Signal Processing*, vol. 50, pp. 2417–2428, Oct. 2002.

[23] K. Leeuwin-Boulle and J. C. Belfiore, “The cutoff rate of time correlated fading channels,” *IEEE Trans. Inform. Theory*, vol. 39, pp. 612–617, Mar. 1993.

[24] E. Baccarelli, “Performance bounds and cutoff rates for data channels affected by correlated randomly time-variant multipath fading,” *IEEE Trans. Commun.*, vol. 46, pp. 1258–1261, Oct. 1998.

[25] O. Y. Takeshita and D. J. Constello Jr, “New classes of algebraic interleavers for turbo-codes,” in *Proc. IEEE Int. Symp. Information Theory*, MIT, Cambridge, MA, 1998, p. 419.

[26] W. C. Jakes, *Microwave Mobile Communications*. New York: Wiley, 1974.

[27] R. A. Horn and C. R. Johnson, *Topics in Matrix Analysis*. Cambridge, U.K.: Cambridge Univ. Press, 1991.

New Family of p-ary Sequences With Optimal Correlation Property and Large Linear Span

Ji-Woong Jang, Young-Sik Kim, Jong-Seon No, *Member, IEEE*, and Tor Helleseth, *Fellow, IEEE*

***Abstract*—For an odd prime p and integers n, m, and k such that $n = (2m+1)k$, a new family of p-ary sequences of period $p^n - 1$ with optimal correlation property is constructed using the p-ary Helleseth–Gong sequences with ideal autocorrelation, where the size of the sequence family is p^n. That is, the maximum nontrivial correlation value $R_{\max}$ of all pairs of distinct sequences in the family does not exceed $p^{\frac{n}{2}} + 1$, which means the family has optimal correlation in terms of Welch's lower bound. The symbol distribution of the sequences in the family is enumerated. It is also shown that the linear span of the sequences in the family is $(m+2)n$ except for the m-sequence in the family.**

***Index Terms*—Family of sequences, optimal correlation, p-ary sequences.**

I. Introduction

In the wireless communication systems employing code-division multiple-access (CDMA) scheme, a signature sequence is assigned to each user, which makes it possible to distinguish his signal from those of the other users. In design of sequences for CDMA system, the most important properties of the sequences are low periodic correlation between all pairs of distinct sequences and large family size. For an odd prime p, families of p-ary sequences of period $p^n - 1$ with optimal correlation property have been found, where the optimality of correlation values means that maximum magnitude of out-of-phase autocorrelation and cross-correlation values of any pairs of sequences of period $p^n - 1$ in the family is upper-bounded by $R_{\max} = p^{\frac{n}{2}} + 1$. Sidelnikov constructed a family of p-ary sequences with optimal correlation property and a family of prime-phase sequences with optimal correlation property was introduced by Kumar and Moreno [3]. By extending the alphabet size, Liu and Komo [7] constructed p-ary Kasami sequences. The family of p-ary bent sequences also has the optimal correlation property. Using the p-ary bent functions given by Kumar and Moreno, a family of balanced p-ary sequences with optimal correlation property was constructed by Moriuchi and Imamura [9]. The known families of p-ary sequences of period $p^n - 1$ with optimal correlation property are listed in Table I. The family size of the sequences due to Sidelnikov and to Kumar and Moreno are larger than that of the others in Table I. But the linear span of the sequences due to Sidelnikov and to Kumar and Moreno are much smaller than those of the others.

In this correspondence, for an odd prime p and integers n, m, and k such that $n = (2m+1)k$, a new family of p-ary sequences of period $p^n - 1$ with optimal correlation property is constructed using the p-ary Helleseth–Gong sequences with ideal autocorrelation, where the size of the sequence family is p^n. That is, the maximum nontrivial correlation value $R_{\max}$ of all pairs of distinct sequences in the family does not exceed $p^{\frac{n}{2}} + 1$, which means the family has optimal correlation with respect to Welch's lower bound. The symbol distribution of the

Manuscript received January 9, 2003; revised February 18, 2004. This work was supported in part by the Korean Ministry of Information and Communications and the Norwegian Research Council.

J.-W. Jang, Y.-S. Kim, and J.-S. No are with the School of Electrical Engineering and Computer Science, Seoul National University, Seoul 151-742, Korea (e-mail: jsno@snu.ac.kr).

T. Helleseth is with the Department of Informatics, University of Bergen, N-5020 Bergen, Norway (e-mail: Tor.Helleseth@ii.uib.no).

Communicated by K. G. Paterson, Associate Editor for Sequences.

Digital Object Identifier 10.1109/TIT.2004.831837