Diversity through Coded Cooperation

Todd E. Hunter, *Member, IEEE* and Aria Nosratinia, *Senior Member, IEEE*

***Abstract*— Motivated by the recent works on the relay channel and cooperative diversity, this letter introduces *coded cooperation*, where cooperation is achieved through channel coding methods instead of a direct relay or repetition. Each codeword is partitioned into two subsets that are transmitted from the user's and partner's antennas, respectively. Coded cooperation achieves impressive gains compared to a non-cooperative system while maintaining the same information rate, transmit power, and bandwidth. We develop bounds on BER and FER and illustrate the advantage of coded cooperation under a number of different scenarios.**

***Index Terms*— User cooperation, diversity, transmit diversity, space-time coding.**

I. Introduction

COOPERATION between pairs of wireless communication agents [1], [2], [3], [4] achieves diversity by a signaling scheme that allows two single-antenna mobiles (users) to send their information using both of their antennas. The basic approach to the cooperation has been for a mobile to "listen" to a partner's transmission, and in a different time or frequency slot to retransmit either an amplified version of the received signal (amplify-and-forward) or a decoded version of the received signal (decode-and-forward). An overview of past work in this area is presented in [5].

This paper presents a user cooperation methodology called *coded cooperation*, where cooperative signaling is integrated with channel coding (first appeared in [6]). Due to the vagaries of the publication process, the appearance of this paper has been much delayed, to the extent that a derivative version of this work [7] as well as a tutorial on the subject of cooperation [5] have already appeared in print. For supplemental information and background, the interested reader is invited to consult [5], [7].

Our system model consists of two users both transmitting to a single destination. The channels between users (inter-user channels) and from each user to the destination (uplink channels) are mutually independent and subject to flat Rayleigh fading. The users transmit on orthogonal channels (e.g., TDMA, CDMA, or FDMA). The receivers have channel state information, but the transmitters do not. The instantaneous received SNR for the channel between users i and j is defined as $\gamma_{i,j}(n)$. In this paper, we examine the cases where the channel gain is Rayleigh distributed, thus $\gamma_{i,j}(n)$ has exponential distribution with mean $\Gamma_{i,j} = E\left[\gamma_{i,j}(n)\right]$. The quality of each channel in the sequel is represented by its average SNR $\Gamma_{i,j}$.

Manuscript received February 20, 2004; revised January 10, 2005; accepted February 6, 2005. The associate editor coordinating the review of this letter and approving it for publication was A. Yener. This work was supported in part by the NSF under grants CCR-9985171 and CNS-0435429.

A. Nosratinia is with the Multimedia Communications Laboratory, The University of Texas at Dallas, TX 75083-0688, USA (e-mail: aria@utdallas.edu).

T. Hunter is with Nortel Networks, Richardson, TX, USA, (e-mail: toddhu@nortel.com).

Digital Object Identifier 10.1109/TWC.2006.02006.

II. Coded Cooperation

In this section we briefly discuss the principles of coded cooperation. The interested reader is also referred to [5], [7] as well as [8], [9]. Assume that the baseline wireless system uses a rate-R channel code. The idea of coded cooperation is to use the *same* overall rate for coding and transmission (thus no more system resources are used), however, the coded symbols are re-arranged between the two users such that better diversity is attained.

Specifically, assume each user has K information bits per block, and N coded bits per block, so that $R = K/N$. We divide the transmission of the N coded bits into two successive time segments, which we call *frames*. Thus, each codeword of length N is divided into two segments of lengths N_1 and N_2, respectively, where $N_1 + N_2 = N$. In the first segment, a sub-codeword of rate $R_1 = K/N_1$ is broadcast by the user and is received, to varying degrees, by the base station as well as the partner. Each user will thus receive a noisy version of the coded message from its partner. If a user can correctly decode a partner's message, determined by the CRC code in each frame, the user will compute and transmit the N_2 bits for the partner.[1] If a user cannot correctly decode a partner, N_2 additional parity bits for the user's own data will be transmitted.

We should emphasize that each user always transmits a total of N bits per source block over the two frames, and the users only transmit in their own multiple access channels. Fig. 1 shows an implementation for a TDMA cooperative system. FDMA and CDMA implementations can be analogously constructed.[2]

The coded cooperation framework is very flexible and can be used with virtually any channel coding scenario. For example, the overall code may be a block or convolutional code, or a combination of both. The code bits for the two frames may be partitioned through puncturing, product codes, or other forms of concatenation. In this letter, we employ a simple but effective implementation using rate-compatible punctured convolutional (RCPC) codes [12]. See [7] for implementations with turbo codes and space-time transmission.

[1] We define the level of cooperation as N_2/N, the percentage of the total bits per each source block that the user transmits for his partner.

[2] Note that many current CDMA systems are actually hybrid CDMA/FDMA systems that use several uplink frequencies (see for example [10],[11]). Thus, we can avoid the difficulties of simultaneous transmission and reception on the same carrier, yet still preserve the advantages of CDMA, by having the partners use different carriers.

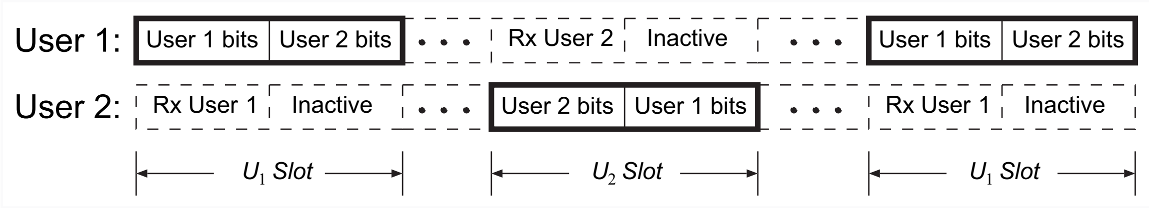

Fig. 1. Coded cooperation implementation for a system using TDMA.

The users act independently in the second frame, with no knowledge of whether their own first frame was correctly decoded. As a result, there are four possible cooperative cases for the transmission of the second frame, illustrated in Fig. 2. In Case 1, both users successfully decode each other, so that they each transmit for their partner in the second frame, resulting in the fully cooperative scenario. In Case 2, neither user successfully decodes their partner's first frame, and the system reverts to the non-cooperative case for that pair of source blocks. In Case 3, User 2 successfully decodes User 1, but User 1 does not successfully decode User 2. Consequently, neither user transmits the second set of code bits for User 2 in the second frame, but instead both transmit the second set for User 1. These two independent copies of User 1's bits are optimally combined. Case 4 is identical to Case 3 with the roles of User 1 and User 2 reversed.

Clearly the destination must know which of these four cases has occurred in order to correctly decode the received bits. Two methods have been proposed to address this issue [13]. In one method, the base station decodes according to the assumption of Case 1, 2, 3, and 4 successively until CRCs indicate successive decoding. Probabilistic analysis shows that this results in negligible *average* increase in computational complexity. In the second method, one additional bit is transmitted by each user to indicate its state to the base station.

III. Pairwise Error Probability

The pairwise error probability is written as [14, (12.13)]

$$P(\boldsymbol{c} \to \boldsymbol{e}|\boldsymbol{\gamma}) = Q\left(\sqrt{2\sum_{n\in\eta}\gamma(n)}\right) \tag{1}$$

where $Q(x)$ denotes the Gaussian Q-function [15, (2-1-97)]. The instantaneous received SNR values are denoted by vector $\boldsymbol{\gamma}$. The transmitted codeword is $\boldsymbol{c}$, the erroneously decoded codeword is $\boldsymbol{e}$, and the set η is the set of all n for which $c(n) \neq e(n)$, thus $|\eta| = d$ is the Hamming distance between $\boldsymbol{c}$ and $\boldsymbol{e}$. We assume a linear code where error probabilities are independent of transmitted codeword, thus the conditional PEP will be denoted simply by $P(d|\boldsymbol{\gamma})$.

A. Coded Cooperation in Slow Fading

In slow fading the SNR is constant over a block, i.e., $\gamma_{i,0}(n) = \gamma_{i,0}$ for $n = 1, \ldots, N$. For Case 1 (Fig. 2), when both users successfully decode each other's first frame, each user's coded bits are divided between the two user channels. Thus for User 1 we rewrite (1) as

$$P(d|\gamma_{1,0}, \gamma_{2,0}) = Q\left(\sqrt{2d_1\gamma_{1,0} + 2d_2\gamma_{2,0}}\right) \tag{2}$$

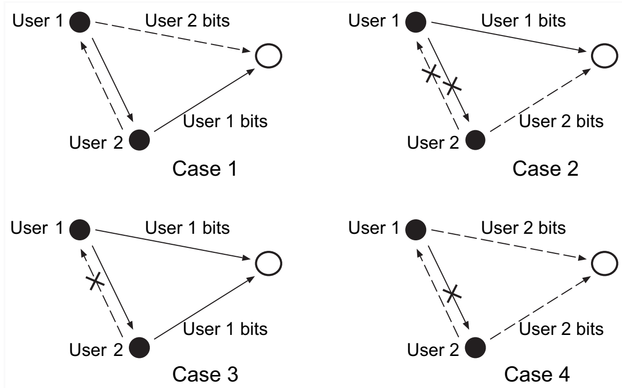

Fig. 2. Four cooperative cases for second frame transmission based on the first frame decoding results.

where d_1 and d_2 are the portions of the error event bits transmitted through User 1's and User 2's channel respectively, such that $d_1 + d_2 = d$.

To obtain the unconditional PEP we must average (2) over the fading distributions

$$P(d) = \int_0^\infty \int_0^\infty P(d|\gamma_{1,0}, \gamma_{2,0}) p(\gamma_{1,0}) p(\gamma_{2,0}) d\gamma_{1,0} d\gamma_{2,0} \tag{3}$$

where $p(\cdot)$ denotes a pdf. Using the alternate form of the Q-function [16] and the well-known MGF function method [14], we find, for Rayleigh fading

$$P(d) = \frac{1}{\pi}\int_0^{\pi/2} \left(1 + \frac{d_1\Gamma_{1,0}}{\sin^2\theta}\right)^{-1} \left(1 + \frac{d_2\Gamma_{2,0}}{\sin^2\theta}\right)^{-1} d\theta. \tag{4}$$

which can be bounded thus:

$$P(d) \leq \frac{1}{2}\left(\frac{1}{1 + d_1\Gamma_{1,0}}\right)\left(\frac{1}{1 + d_2\Gamma_{2,0}}\right). \tag{5}$$

For large SNR, the PEP is inversely proportional to the product of the average SNR of the uplink channels. Thus, if d_1 and d_2 are both non-zero, full diversity order of two is achieved when both partners successfully receive each other and cooperate.

For Case 3, where User 1 does not successfully decode User 2, but User 2 successfully decodes User 1, both users send the same additional parity bits for User 1 in the second frame. These bits are optimally combined at the destination, so that the conditional PEP (2) for User 1 becomes

$$\begin{aligned} P(d|\gamma_{1,0}, \gamma_{2,0}) &= Q\left(\sqrt{2d_1\gamma_{1,0} + 2d_2(\gamma_{1,0} + \gamma_{2,0})}\right) \\ &= Q\left(\sqrt{2d\gamma_{1,0} + 2d_2\gamma_{2,0}}\right) \end{aligned} \tag{6}$$

and the unconditional PEP becomes (7).

$$P(d) = \frac{1}{\pi} \int_0^{\pi/2} \left(1 + \frac{d\Gamma_{1,0}}{\sin^2\theta}\right)^{-1} \left(1 + \frac{d_2\Gamma_{2,0}}{\sin^2\theta}\right)^{-1} d\theta \leq \frac{1}{2} \left(\frac{1}{1+d\Gamma_{1,0}}\right) \left(\frac{1}{1+d_2\Gamma_{2,0}}\right) \quad (7)$$

Equation (7) illustrates that User 1 again achieves full diversity order two for Case 3 (for d_1 and d_2 non-zero).

B. Coded Cooperation in Fast Fading

For fast fading, the fading coefficients are no longer constant over the code word, but are i.i.d. across the coded bits. Thus, for Case 1, we generalize (1) as

$$P(d|\gamma_{1,0}, \gamma_{2,0}) = Q\left(\sqrt{2\sum_{n\in\eta_1} \gamma_{1,0}(n) + 2\sum_{n\in\eta_2} \gamma_{2,0}(n)}\right) \quad (8)$$

where the set η_i is the portion of the d error event bits transmitted through User i's channel. The cardinalities of η_1 and η_2 are d_1 and d_2 respectively, where again $d_1 + d_2 = d$.

Averaging over the fading to obtain the unconditional PEP now involves a d-fold integration, for which the techniques of [14] again provide a tractable solution. After some manipulation, we obtain for Rayleigh fading

$$P(d) = \frac{1}{\pi} \int_0^{\pi/2} \left[\prod_{n\in\eta_1} \left(1 + \frac{\Gamma_{1,0}}{\sin^2\theta}\right)^{-1}\right] \left[\prod_{n\in\eta_2} \left(1 + \frac{\Gamma_{2,0}}{\sin^2\theta}\right)^{-1}\right] d\theta. \quad (9)$$

Applying our assumption that $\Gamma_{1,0}$ and $\Gamma_{2,0}$ are constant over n results in (10).

Equation (10) shows that the diversity order for fast fading is equal to the total Hamming weight $d = d_1 + d_2$. This is also true for no cooperation. For statistically dissimilar uplink channels ($\Gamma_{1,0} \neq \Gamma_{2,0}$), (10) indicates definite improvement for the user with the lower uplink average SNR, which is an important practical result. Intuitively we see that coded cooperation does not provide additional diversity when the average uplink SNR are equal.

For Case 3 the conditional PEP (8) for User 1 becomes (11) and unconditional PEP becomes (12).

Equation (12) shows that User 1 does achieve improved diversity ($d + d_2$ vs. d) for Case 3.

IV. Bit and Block Error Rate Analysis

A. Cooperative Case Probabilities

The cooperative case probabilities are determined by the BLER of the first frame transmission. The BLER for a terminated convolutional code is bounded by [17, (12)], [18, (11)]

$$P_{block}(\boldsymbol{\gamma}) \leq 1 - (1 - P_E(\boldsymbol{\gamma}))^B \leq B \cdot P_E(\boldsymbol{\gamma}) \quad (13)$$

where B is the number of trellis branches in the code word, and $P_E(\boldsymbol{\gamma})$ is the error event probability conditioned on $\boldsymbol{\gamma}$, the vector state of the channel. P_E is bounded as [19, (4.3.51)]

$$P_E(\boldsymbol{\gamma}) \leq \sum_{d=d_f}^{\infty} a(d)\, P(d|\boldsymbol{\gamma}), \quad (14)$$

where d_f is the code free distance and $a(d)$ is the number of error events of Hamming weight d.

We parameterize the four cases by $\Theta \in \{1, 2, 3, 4\}$ and we can express the conditional probability for Case 1 ($\Theta = 1$) in (15) (bounds for the other cases are developed similarly).

To calculate end-to-end error probabilities, we need the unconditional probability $P(\Theta)$:

$$P(\Theta) = \int_{\boldsymbol{\gamma}_{1,2}} \int_{\boldsymbol{\gamma}_{2,1}} P(\Theta|\boldsymbol{\gamma}_{1,2}, \boldsymbol{\gamma}_{2,1}) p(\boldsymbol{\gamma}_{1,2}) p(\boldsymbol{\gamma}_{2,1})\, d\boldsymbol{\gamma}_{1,2} d\boldsymbol{\gamma}_{2,1}. \quad (16)$$

For slow fading, vectors $\boldsymbol{\gamma}_{1,2}$ and $\boldsymbol{\gamma}_{2,1}$ reduce to scalars $\gamma_{1,2}$ and $\gamma_{2,1}$. For reciprocal inter-user channels, $\gamma_{1,2} = \gamma_{2,1}$, and $P(\Theta|\gamma_{1,2})$ is conditioned on a single variable, reducing (16) to a single integral. For independent inter-user channels, the unconditional first-frame BLER of the two users are independent, and $P(\Theta)$ has form analogous to the first line of (15), e.g., for Case 1,

$$P(\Theta = 1) = (1 - P_{block,1}) \cdot (1 - P_{block,2}). \quad (17)$$

To obtain tight bounds in slow fading, we use the limit-before-average technique from [17] with the appropriate conditional (on fading) PEP to evaluate (16) and (17). For example, for Case 1 with reciprocal inter-user channels we have (18).

With independent inter-user channels, we compute the unconditional BLER for User i as (19) and apply the results to (17). The unconditional probabilities for the other cases are evaluated similarly. Note that (18) and (19) must be computed numerically due to the minimization.

For fast fading, tightness of the bounds does not present a difficulty and one can use the unconditional PEPs directly without the need for limit-before-averaging.

B. End-to-End Error Analysis

The overall end-to-end unconditional BER is equal to the average of the unconditional BER over the four possible transmission scenarios discussed in Section IV-A as

$$P_b = \sum_{i=1}^{4} P_b(\Theta) P(\Theta = i) \quad (20)$$

The end-to-end BLER has a similar expression.

The conditional BLER is given by (13)–(14), and the conditional BER is bounded by [19, (4.4.8)]

$$P_b(\boldsymbol{\gamma}, \Theta) \leq \frac{1}{k_c} \sum_{d=d_f}^{\infty} c(d) P(d|\boldsymbol{\gamma}, \Theta) \quad (21)$$

where $c(d)$ is the number of information bit errors for code words or error events with Hamming weight d, and k_c is the number of input bits for each branch of the code trellis.

We again use the limit-before-average technique [17] with the appropriate conditional PEP expressions to obtain tight bounds for slow fading. The unconditional BER and BLER are shown in (22).

$$P(d) = \frac{1}{\pi}\int_0^{\pi/2}\left(1+\frac{\Gamma_{1,0}}{\sin^2\theta}\right)^{-d_1}\left(1+\frac{\Gamma_{2,0}}{\sin^2\theta}\right)^{-d_2} d\theta \le \frac{1}{2}\left(\frac{1}{1+\Gamma_{1,0}}\right)^{d_1}\left(\frac{1}{1+\Gamma_{2,0}}\right)^{d_2} \tag{10}$$

$$\begin{aligned} P(d|\boldsymbol{\gamma}_{1,0},\boldsymbol{\gamma}_{2,0}) &= Q\left(\sqrt{2\sum_{n\in\eta_1}\gamma_{1,0}(n) + 2\sum_{n\in\eta_2}\gamma_{1,0}(n) + 2\sum_{n\in\eta_2}\gamma_{2,0}(n)}\right) \\ &= Q\left(\sqrt{2\sum_{n\in\eta}\gamma_{1,0}(n) + 2\sum_{n\in\eta_2}\gamma_{2,0}(n)}\right) \end{aligned} \tag{11}$$

$$P(d) = \frac{1}{\pi}\int_0^{\pi/2}\left(1+\frac{\Gamma_{1,0}}{\sin^2\theta}\right)^{-d}\left(1+\frac{\Gamma_{2,0}}{\sin^2\theta}\right)^{-d_2} d\theta \le \frac{1}{2}\left(\frac{1}{1+\Gamma_{1,0}}\right)^{d}\left(\frac{1}{1+\Gamma_{2,0}}\right)^{d_2} \tag{12}$$

$$\begin{aligned} P(\Theta=1|\boldsymbol{\gamma}_{1,2},\boldsymbol{\gamma}_{2,1}) &= (1-P_{block,1}(\boldsymbol{\gamma}_{1,2}))(1-P_{block,2}(\boldsymbol{\gamma}_{2,1})) \\ &\ge (1-P_{E,1}(\boldsymbol{\gamma}_{1,2}))^B(1-P_{E,2}(\boldsymbol{\gamma}_{2,1}))^B \\ &\ge (1-B\,P_{E,1}(\boldsymbol{\gamma}_{1,2}))(1-B\,P_{E,2}(\boldsymbol{\gamma}_{2,1})) \end{aligned} \tag{15}$$

For fast fading, tight bounds are obtained using the unconditional (on fading) $P(d|\Theta)$ expression directly in the summation (14) or (21), in lieu of computing (22) [17]. Applying these and the previous results to (20) gives tight approximations for the end-to-end bit and block error probabilities. (Although the case probabilities (Section IV-A) are not all strictly upper bounds, i.e. (15) and (18), we can say that, due to the tightness of the limit-before-average technique [17], we obtain a tight approximation for (20). This is demonstrated in the results shown in the following section.)

Whenever both users cooperate (Case 1) each user's message sees two independent fading paths and a diversity order of two is achieved. When a user's message does not benefit from cooperation the diversity is one. Therefore, the overall diversity order, interpreted as the slope of the error rate curve, is the average of the diversities in the four cases, weighted by the probabilities of the four cases. These probabilities are determined by the inter-user channel conditions. At high inter-user SNR, Case 1 is dominant and coded cooperation achieves full diversity order of two.[3] We note that in order to operate at realistic SNR's, some of our simulations are not in this dominant mode, and for that reason some of the simulations show diversity less than two.

V. Performance Evaluation

We implement coded cooperation using a family of RCPC codes with memory $M = 4$, puncturing period $P = 8$, and rate $1/4$ mother code given by Hagenauer [12]. For slow fading, we choose overall code rate $R = 1/4$. The source data block size is $K = 128$ bits. We computed via computer enumeration the distance spectra $a(d)$ and $c(d)$, including the partitioning of the Hamming weight d into d_1 and d_2. For the simulations, we use a 16-bit CRC code with generator polynomial given by coefficients 15935 (hexadecimal notation). For our analysis we assume perfect error detection. Since all comparisons are between systems with equal information rate K bits per source block, and equal code rate R, we plot the BER versus the channel SNR. Plotting BER versus the information bit SNR yields identical results, with the x-axis values shifted by $10\log R$ dB. Also, for brevity we omit BLER results, which can be found in [13].

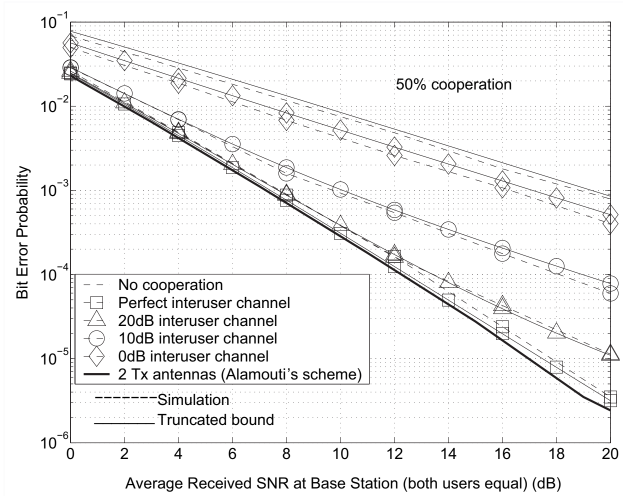

Fig. 3. Performance in slow Rayleigh fading with 50% cooperation, equal uplink SNR, and reciprocal inter-user channels.

Fig. 3 shows the BER for slow fading with reciprocal inter-user channels of various qualities. The users have statistically similar uplink channels ($\Gamma_{1,0} = \Gamma_{2,0}$), and the level of cooperation is 50%. Coded cooperation with a perfect inter-

[3] For any fixed set of probabilities, the errors of diversity order one will eventually dominate at high enough uplink SNR (even though such SNR's may be unrealistic in practice). Strictly speaking, to achieve diversity order of two, the ratio of the case probabilities in the asymptote must keep up with the increased uplink SNR. Therefore to make the above statement more precise, one more condition must be added. For example, one might say: "diversity of two is achieved if a fixed uplink to inter-user SNR ratio is maintained in the asymptote."

$$P(\Theta=1) \;\geq\; \int_0^\infty \left(1-\min\left[1, \sum_{d=d_f}^{\infty} a(d)P(d|\gamma_{1,2})\right]\right)^B \times \left(1-\min\left[1, \sum_{d=d_f}^{\infty} a(d)P(d|\gamma_{1,2})\right]\right)^B p(\gamma_{1,2})\, d\gamma_{1,2} \quad (18)$$

$$P_{block,i} \leq 1-\int_0^\infty \left(1-\min\left[1, \sum_{d=d_f}^{\infty} a(d)\, P(d|\gamma_{i,j})\right]\right)^B p(\gamma_{i,j})d\gamma_{i,j} \quad (19)$$

$$\begin{aligned} P_b(\Theta) &\leq \int_0^\infty\int_0^\infty \min\left[\frac{1}{2}, \frac{1}{k_c}\sum_{d=d_f}^{\infty} c(d)P(d|\gamma_{1,0},\gamma_{2,0},\Theta)\right] p(\gamma_{1,0})p(\gamma_{2,0})d\gamma_{1,0}d\gamma_{2,0} \\ P_{block}(\Theta) &\leq 1-\int_0^\infty\int_0^\infty \left(1-\min\left[1, \sum_{d=d_f}^{\infty} a(d)\, P(d|\gamma_{1,0},\gamma_{2,0},\Theta)\right]\right)^B \\ &\quad \cdot p(\gamma_{1,0})p(\gamma_{2,0})d\gamma_{1,0}d\gamma_{2,0} \end{aligned} \quad (22)$$

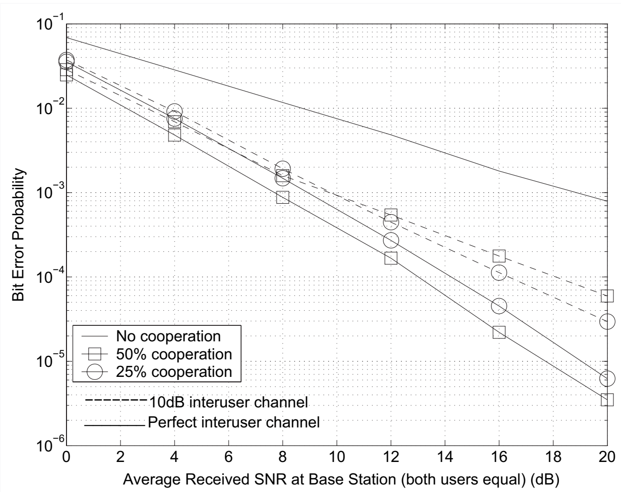

Fig. 4. Comparison of 50% and 25% cooperation in slow Rayleigh fading, equal uplink SNR.

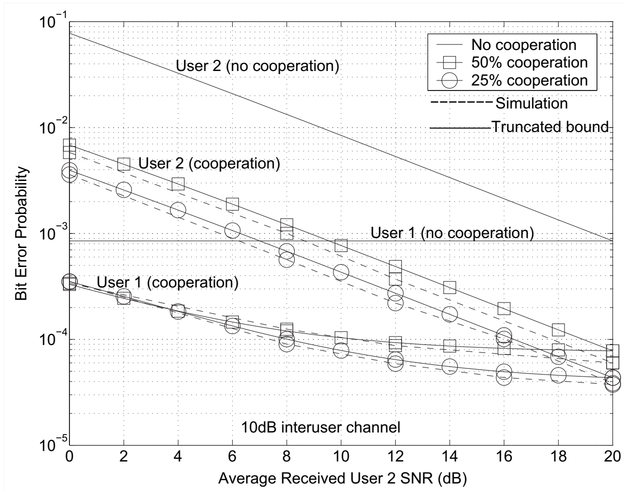

Fig. 5. Performance under asymmetric uplink conditions in slow Rayleigh fading.

user channel performs virtually identical to a two-antenna transmit diversity system that uses Alamouti signaling [20] and a rate-$1/4$ outer code. This confirms that coded cooperation does achieve full diversity. When the inter-user channel is not ideal, the improvements brought about by cooperation are still dramatic. For example, when the inter-user channel has 10dB average SNR, the gain is about 9dB at BER=10^{-3}. Coded cooperation still achieves significant gain even when the inter-user channel is much worse than the uplink channels, e.g., we see a 2–3dB gain for an inter-user channel with average SNR 0dB over the range of 0–20dB average uplink SNR.

Fig. 4 compares the performance of coded cooperation at 50% and 25%, for both a perfect inter-user channel and one with average SNR of 10dB. The user uplink channels again have equal average SNR. When the inter-user channel is perfect, both users always cooperate (we have Case 1 exclusively), and 50% cooperation yields better performance. This is predicted by the PEP of (5), since we expect the product $d_1 \cdot d_2$ to be maximized for 50% cooperation (d_1 and d_2 should be approximately equal). However, as the inter-user channel becomes worse, the situation changes. Fig. 4 shows that 25% cooperation becomes better than 50% cooperation for the 10dB inter-user channel, by as much as 2dB for higher uplink SNR. For poor inter-user channels, a stronger code in the first frame is more important to the overall performance than maximizing the product $d_1 \cdot d_2$. This is a result of averaging over the four cooperative cases.

In Fig. 5, we examine the performance of coded cooperation when the users have statistically dissimilar uplink channels. We fix the average uplink SNR for User 1 at 20dB, while

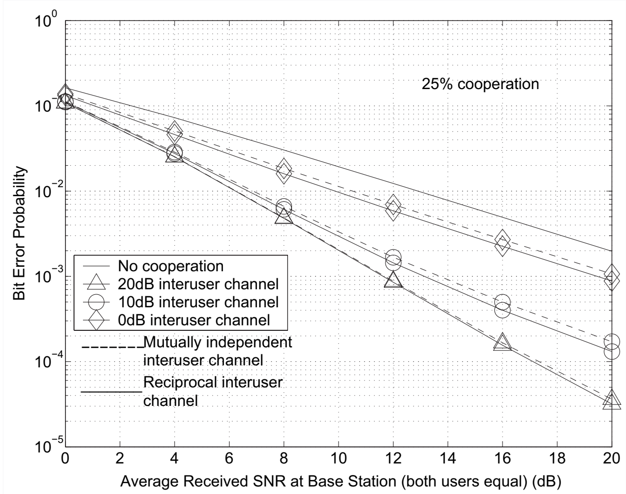

Fig. 6. Comparison (analytical bound) of coded cooperation for reciprocal and mutually independent inter-user channels of various qualities.

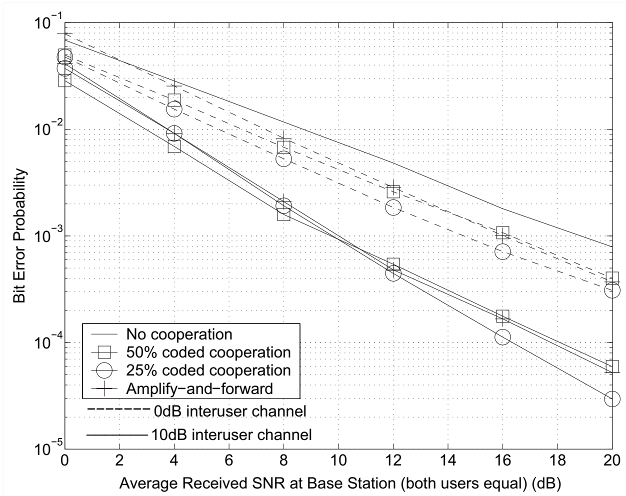

Fig. 7. Comparison of coded cooperation with amplify-and-forward under slow Rayleigh fading.

varying User 2's average uplink SNR from 0dB to 20dB. The inter-user channel has average SNR 10dB. Fig. 5 shows that User 2, with the worse uplink channel, improves dramatically with coded cooperation, exhibiting a gain of 11–13dB relative to no cooperation. More interestingly however, User 1, with the better uplink channel, also achieves a marked improvement in performance by cooperating, a result that is not necessarily intuitive. Thus, even a user with a very good uplink channel has a strong motivation to cooperation in a slow-fading environment.

In order to simplify the plots, we show in Figures 3 and 5 the analytical bounds truncated using only the first few terms of the distance spectrum, which is sufficient for our purposes. Because of this approximation, these bounds appear slightly tighter than that of [17]. Using all of the terms gives tight upper bounds with convergence behavior similar to [17].

Previous works on user cooperation ([1], [2], [3], [4]) generally assume that the channels between the users are reciprocal for slow fading. This is justifiable in TDD (Time-Division Duplex) channels, but not elsewhere. However, this is not a major factor because, even if we assume fully independent inter-user channels, the gains of coded cooperation can be maintained via a judicious choice of cooperation level. Fig. 6 shows that for 25% cooperation, the results for reciprocal and independent inter-user channels are well within 1dB of each other. For more comprehensive results the reader is referred to [13].

In i.i.d. fast fading, coded cooperation acts as a re-distributor of network resources. In other words, coded cooperation will improve the performance of the partner with the poor uplink SNR at the expense of the partner with the better uplink SNR. Since the network quality is often measured by the worst-case performance, even in i.i.d. fading coded cooperation has advantages to offer. In the interest of brevity we limit our discussion of coded cooperation in fast fading and refer the interested reader to [7].

Next, we wish to compare our method to the existing cooperative methods. The one method that has been shown to achieve full diversity is the Amplify-and-Forward protocol [3], [4]. We implement a coded version of the amplify-and-forward protocol for a fair comparison. We use a rate-$1/2$ convolutional code, resulting in an overall rate of $1/4$, since amplify-and-forward has repetition. The overall code rate for coded cooperation is also $R = 1/4$. Fig. 7 shows the comparison of simulated BER for slow Rayleigh fading and equal uplink average SNR. Coded cooperation maintains an edge of up to 1–2dB over amplify-and-forward, depending on the uplink SNR. The level of cooperation that achieves the best performance for coded cooperation varies between 50% and 25%, depending on the channel conditions. The level of cooperation for amplify-and-forward, of course, is inflexible (set at 50%) since repetition is a core part of that protocol.

VI. Conclusions and Related Work

We propose a method where cooperation is integrated with channel coding. Diversity is achieved by partitioning a user's code word into two parts. Each user receives the first codeword partition from the partner, and upon successful decoding (determined via a CRC code), transmits the *second* codeword partition. The two partitions are thus received at the destination through independent fading channels. This coded cooperation framework may be implemented using block or convolutional codes, and various methods of partitioning the code words (puncturing, product codes, parallel and serial concatenation, etc). We analyze the performance of this method, showing impressive gains in slow Rayleigh fading, even when the inter-user channel is much worse than the uplink channel.

In a subsequent work [7], we consider two extensions to the coded cooperation framework. The first uses space-time signaling concepts to improve the performance in fast fading. The second extension involves implementing coded cooperation using turbo codes. Although it has not been explicitly addressed, extension to multiple users can be achieved in a straight forward manner by dividing the second frame (cooperation frame) into multiple subframes. Also the code rate of the system is completely flexible and can be optimized by adjusting the size of the frames; this is an interesting subject for future research.

REFERENCES

[1] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity–Part I: System description," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1927–1938, November 2003.

[2] ——, "User cooperation diversity–Part II: Implementation aspects and performance analysis," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1939–1948, November 2003.

[3] J. N. Laneman and G. W. Wornell, "Energy-efficient antenna sharing and relaying for wireless networks," in *Proc. IEEE Wireless Communcations and Networking Conference (WCNC)*, Chicago, IL, September 2000, pp. 7–12.

[4] J. N. Laneman, G. W. Wornell, and D. N. C. Tse, "An efficient protocol for realizing cooperative diversity in wireless networks," in *Proc. IEEE International Symposium on Information Theory (ISIT)*, Washington, D. C., June 2001, p. 294.

[5] A. Nosratinia, T. Hunter, and A. Hedayat, "Cooperative communication in wireless networks," *IEEE Commun. Mag.*, vol. 42, no. 10, pp. 68–73, October 2004.

[6] T. E. Hunter and A. Nosratinia, "Cooperative diversity through coding," in *Proc. IEEE International Symposium on Information Theory (ISIT)*, Laussane, Switzerland, July 2002, p. 220.

[7] M. Janani, A. Hedayat, T. E. Hunter, and A. Nosratinia, "Coded cooperation in wireless communications: Space-time transmission and iterative decoding," *IEEE Trans. Signal Processing*, vol. 52, no. 2, pp. 362 – 371, Feb. 2004.

[8] T. E. Hunter and A. Nosratinia, "Coded cooperation under slow fading, fast fading, and power control," in *Proc. Asilomar Conference on Signals, Systems and Computers*, Pacific Grove, CA, November 2002.

[9] ——, "Performance analysis of coded cooperation diversity," in *Proc. IEEE International Conference on Communications (ICC)*, Anchorage, AK, May 2003.

[10] E. I. S.-. (IS-95), "Mobile station – base station compatibility standard for dual-mode wideband spread spectrum celluar system."

[11] 3rd Generation Partnership Project 3G TS 25.101, "UE radio transmission and reception (FDD)."

[12] J. Hagenauer, "Rate-compatible punctured convolutional codes (RCPC codes) and their applications," *IEEE Trans. Commun.*, vol. 36, no. 4, pp. 389–400, April 1988.

[13] T. E. Hunter, "Coded cooperation: a new framework for user cooperation in wireless systems," Ph.D. dissertation, University of Texas at Dallas, Richardson, Texas, USA, 2004.

[14] M. K. Simon and M.-S. Alouini, *Digital Communication over Fading Channels: A Unified Approach to Performance Analysis*. New York: John Wiley and Sons, 2000.

[15] J. G. Proakis, *Digital Communications*, 3rd ed. New York: McGraw-Hill, 1995.

[16] J. W. Craig, "A new, simple, and exact result for calculating the probability of error for two-dimensional signal constellations," in *Proc. IEEE MILCOM*, McLean, VA, October 1991, pp. 571–575.

[17] E. Malkamäki and H. Leib, "Evaluating the performance of convolutional codes over block fading channels," *IEEE Trans. Inform. Theory*, vol. 45, no. 5, pp. 1643–1646, July 1999.

[18] S. Kallel and C. Leung, "Efficient ARQ schemes with multiple copy decoding," *IEEE Trans. Commun.*, vol. 40, pp. 642–650, March 1992.

[19] A. Viterbi and J. K. Omura, *Principles of Digital Communcation and Coding*. New York: McGraw-Hill, 1979.

[20] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Select. Areas Commun.*, vol. 16, no. 8, pp. 1451–1458, October 1998.