LETTERS TO THE EDITOR

PHYSICAL SCIENCES

Do Faster-than-Light Group Velocities imply Violation of Causality ?

DISCUSSION of the properties of hypothetical systems[1-6] in which group velocities exceed the speed of light has considered chiefly whether such systems would violate the axioms of special relativity. In particular they appear at first sight to violate causality.

Feinberg[1], discussing Klein–Gordon particles of imaginary mass (tachyons), has an argument purporting to show that no causality violation is observable. His argument is incomplete, because the "events" which he considers are the emission or absorption of single tachyons. It seems possible to construct a more general Gedankenexperiment than Feinberg's, in which a well defined pattern of correlated tachyons is used as a signal, and causality violation is observable.

The second class of "superluminal" systems considered is the propagation of sound in ultradense matter. Bludman and Ruderman[6], in a model calculation, find a spectrum with two branches. For long waves ($k \to 0$) one "optical" branch has $\omega = \text{constant}$, and hence a group velocity $d\omega/dk = 0$, while the second branch has $\omega = c_s k$, where c_s can exceed unity. (We use units in which the velocity of light in a vacuum is 1.) Although this group velocity is k-dependent, and eventually approaches unity as $k \to 0$, it seems possible to make a wave packet out of long waves alone, and to use this packet to transmit superluminal signals which violate causality.

The situation is reminiscent of the controversy which raged following the publication[7] of the theory of special relativity. It was already known that the group velocity of electromagnetic waves in a dispersive medium could exceed unity for frequencies close to a resonance, so that causality could apparently be violated. This violation was obviously spurious, because the macroscopic refractive index is a consequence of the scattering of electromagnetic waves by electrons and nuclei, while between scattering processes the propagation velocity is 1. The paradox was resolved by Sommerfeld and Brillouin[8-10], who showed that one must distinguish between the signal velocity and the group velocity, and that for electromagnetic waves the signal velocity always satisfies $V_s \leq 1$.

It is easy to extend the method of Sommerfeld and Brillouin to other wave equations to show that

$$V_s \leq V_\infty \qquad (1)$$

where V_∞ is the limiting phase velocity at infinite frequency. For both tachyons and the acoustic branch of the Bludman–Ruderman spectrum, $V_\infty = 1$ and the signal velocity is ≤ 1, vindicating Feinberg's argument *a posteriori*. (Paradoxically, we can make no such statement about the Bludman–Ruderman optical branch, because here $V_\infty = C_s > 1$.)

In the case of tachyons, the proof is trivial, for the characteristic curves of the Klein–Gordon equation

$$(\Box - m^2)\, \varphi(\mathbf{X}, t) = \left(\nabla^2 - \frac{\partial^2}{\partial t^2} - m^2\right) \varphi(\mathbf{X}, t) = 0 \qquad (2)$$

are the straight line generators of the light cone

$$|\dot{\mathbf{X}}| = t \qquad (3)$$

whatever the sign of m^2. Hence it follows[11] that if initially for a wave propagating in the Z direction

$$\varphi(\mathbf{X}, 0) = 0 \qquad (Z > 0) \qquad (4)$$

then at time t,

$$\varphi(\mathbf{X}, t) = 0 \qquad (Z > t) \qquad (5)$$

All signal transmission experiments can be regarded as particular cases of a general Gedankenexperiment, in which a shutter is opened at some time $t = 0$, so that an initial wave packet with the property (4) is allowed to propagate.

We should, however, remark that a tachyon wave packet with the property (4) cannot be built out of plane waves with real frequencies alone. A wave packet localized on one side of a shutter has decaying components and cannot be normalized in a time-independent manner. This is the way in which the non-localizability of tachyons manifests itself in classical wave theory, according to a private communication from A. Peres. A fuller account will be published elsewhere.

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Neutron Starquakes and Pulsar Periods

THE outer layers of neutron stars form a solid crust with a calculable rigidity (shear modulus) very soon after the stars are born. Subsequent changes in stellar shape from oblate toward spherical, as the neutron star angular velocity decreases, will induce stresses in the crust until the maximum shear strain which the solid can support is reached. Beyond this yield point there will be a sudden relaxation of the stress, and a very slight change in stellar shape and moment of inertia. The calculated accompanying jump in angular velocity is close to that which has been observed in a pulsar.

When matter is squeezed to densities up to about 10^{14} g cm^{-3}, all of the protons present are contained in nuclei. (Above this density they form a quantum superfluid together with the much more abundant neutrons.) At densities above 10^8 g cm^{-3} the nuclei are embedded in a relativistic degenerate electron Fermi sea. The momenta of these relativistic electrons are high enough for them to be perturbed only slightly by the coulomb fields of the embedded nuclei, and they therefore act qualitatively like