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Hyoduk Shin, Tunay I. Tunca

Institutions: Northwestern University, Stanford University

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Do Firms Invest in Forecasting Efficiently? The Effect of Competition on Demand Forecast Investments and Supply Chain Coordination

Hyoduk Shin
Kellogg School of Management, Northwestern University, Evanston, Illinois 60208,
hyoduk-shin@kellogg.northwestern.edu

Tunay I. Tunca
Graduate School of Business, Stanford University, Stanford, California 94305, tunca_tunay@gsb.stanford.edu

We study the effect of downstream competition on incentives for demand forecast investments in supply chains. We show that with common pricing schemes, such as wholesale price or two-part tariffs, downstream firms under Cournot competition overinvest in demand forecasting. Analyzing the determinants of overinvestment, we demonstrate that under wholesale price contracts and two-part tariffs, total demand forecast investment can be very significant, and as a result, the supply chain can suffer substantial losses. We show that an increased number of competing retailers and uncertainty in consumer demand tend to increase inefficiency, whereas increased consumer market size and demand forecast costs reduce the loss in supply chain surplus. We identify the causes of inefficiency, and to coordinate the channel with forecast investments, we explore contracts in the general class of market-based contracts used in practice. When retailers' forecast investments are not observable, such a contract that employs an index-price can fully coordinate the supply chain. When forecast investments are observable to others, however, the retailers engage in an "arms race" for forecast investment, which can result in a significant increase in overinvestment and reduction in supply chain surplus. Furthermore, in that case, simple market-based contracts cannot coordinate the supply chain. To solve this problem, we propose a uniform-price divisible-good auction-based contracting scheme, which can achieve full coordination when forecast investments are observable. We also demonstrate the desirable properties for implementability of our proposed coordinating contracting schemes, including incentive-compatible and reliable demand forecast information revelation by the retailers, and being regret-free.

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1. Introduction

Fueled by an increasingly dynamic business environment and growing availability of advanced software and tools, demand forecasting has gained an elevated importance among practitioners in recent years. Today, companies spend billions of dollars annually on software, personnel, and consulting fees to achieve accurate demand forecasts (Aiyer and Ledesma 2004). From a broader supply chain perspective, accuracy of a firm's demand forecast is important not only for itself but also for its partners because the quality of forecasts often affects the performance of the entire supply chain, including vertical partners as well as horizontal competitors (cf. Chen 2003). Therefore, it becomes an important question whether the large amounts of money and resources directed toward demand forecasting are spent efficiently.

An important yet understudied factor in analyzing demand forecast investments is horizontal competition. Many industry observers point out that competition increases companies' incentives to obtain more accurate forecasts (see, e.g., Schreibfeder 2002, Rishi 2006, Demery 2007). Indeed, under increased pressure from competition, margins fall and companies are forced to obtain and utilize sharper information to assess consumer demand, thus making better use of their shrinking slice of the industry profits. Consequently, focused on their own profits, companies might overinvest in demand forecasting at levels that are substantially inefficient for the supply chain as a whole. In integrated channels, such destructive behavior can be controlled by centralized decision making. However, in decentralized supply chains, it is harder to prevent the losses caused by downstream companies' self-interested behavior in demand forecast investment.

In many cases, carefully designed vertical contracts can be useful to remedy the misalignment and coordinate the supply chain (cf. Cachon 2003). However, designing and implementing efficient contracts between a supplier and competing downstream partners (e.g., manufacturers or retailers) with dispersed private information such as demand forecasts, pose important challenges. First, the contract mechanism employed must ensure that each downstream partner shares his private demand forecast. This is a difficult task because firms would not want the information they shared with the supplier to be available to their competitors. For instance, normally, if a supplier provides a crucial component to two competing manufacturers, the manufacturers would not share information with the supplier unless it is guaranteed that the information will not be shared with the competitor (Lee and Whang 2000). Second, even if they agree to share information, competing retailers might have incentives to distort their information when sharing it, i.e., truthful information sharing might be challenging. Third, with or without explicit information sharing, a contract agreed between a supplier and downstream partner might "leak" information to a firm about other firms' information (cf. Li 2002). Thus, after contracting, a downstream partner can update his information based on what he learns from the contract. With his updated information, he might then "regret" his contracted quantity and look for ways to alter it. This problem can undermine both the implementation of a given contracting scheme and the realization of its intended outcome. Combining these factors, when competing downstream partners have correlated private signals, coordination becomes challenging, and under simple, common contract structures, firms might have distorted incentives both in production and in demand forecast investments.

In this paper, we have four main goals. First, we demonstrate that under common contracting schemes, such as wholesale price contracts and two-part tariffs, downstream competition indeed causes overinvestment in demand forecasting, reducing the efficiency of the entire supply chain. Second, we show that the extent of overinvestment and resulting supply chain losses can be very severe, and we study the factors that affect the severity of these losses. Third, we explore the effect of observability of forecast investments by other firms, and show that investment transparency increases overinvestment and reduces supply chain surplus. Thus, our results suggest that it is preferable to promote secrecy of demand forecast investments in a supply chain. Finally, we propose *market-based contract* schemes as a solution to the coordination problem. Market-based contracts utilize base unit prices determined by market mechanisms and are commonly used in various forms in many industries ranging from energy and steel to electronics (see, e.g., Priddle 1998, Faruqui and Eakin 2000, Hoyt et al. 2007, Nagali et al. 2008). We show that when the forecast investments are unobservable, a market-based contract can fully coordinate the supply chain, including investment in demand forecasting by downstream parties. When there is investment observability, coordination becomes more complex. We demonstrate that in this case a uniform-price (divisible good) auction that gives the retailers extended flexibility in their orders can fully coordinate the supply chain. Furthermore, the coordinating contracts we propose achieve full and reliable revelation of retailers' private information. In addition, even though information leaks through contracting, the mechanisms we propose are regret-free, i.e., no downstream firm wants to change his order quantity *even after* observing the contracting outcome. Therefore our proposed contracts also satisfy important but hard-to-achieve implementability properties.

The remainder of this paper is organized as follows. Section 2 reviews the relevant literature. Section 3 presents the model. Section 4 demonstrates the emergence of overinvestment in demand forecasting with commonly employed contracting schemes and the determinants of overinvestment and supply chain inefficiency when forecast investments are unobservable. Section 5 presents the market-based contracting scheme that fully coordinates the supply chain for the unobservable forecast investment case and analyzes its properties. Section 6 discusses the effect of demand forecast investment observability and studies the uniform-price auction-based contracts that coordinate the supply chain for that case. Section 7 offers our concluding remarks. Proofs for propositions that are not provided in the paper and supplemental technical analysis are given in the electronic companion to this paper, which is available as part of the online version at http://or.journal.informs.org/.

2. Literature Review

Vertical disintegration and inefficiencies due to decentralized decision making in supply chains have been explored in many studies. The extensive literature on supply chain coordination examines mechanisms that can resolve the misalignment of incentives by making different parties act according to the way a centralized decision maker would behave in various settings. (See Cachon 2003, Chen 2003 for comprehensive surveys.) Some examples of contracting schemes are revenue sharing (Cachon and Lariviere 2005), channel rebates (Taylor 2002), and quantity flexibility contracts (Tsay 1999). There is also a large literature in economics on double-marginalization (Spengler 1950) and vertical restraints (cf. Tirole 1990). From the perspective of these two main branches of literature, we explore an important type of incentive misalignment, namely distortions resulting from private information and the incentives to invest in demand forecasting under downstream competition.

Various issues related to demand forecasting in supply chain management have been studied in the literature (see, e.g., Fisher and Raman 1996, Cachon and Lariviere 2001, Terwiesch et al. 2005, among others). Aviv (2001) explores the benefits of vertical sharing of demand forecast information by comparing scenarios with and without demand information sharing in a collaboratively managed supply chain with a single supplier and a single retailer. Lariviere (2002) studies a noncooperative setting in which the retailer's cost-effectiveness in forecasting is private information. He explores buy-back and quantity flexibility contracts to simultaneously induce truthful revelation and optimal investment in forecasts, showing that the latter type of contract can coordinate the supply chain and screen retailers who are efficient forecasters. Taylor and Xiao (2009) show that under rebate contracts in a single supplier/single retailer setting, the retailer might overinvest in demand forecasting, and the system can benefit from the retailer having an inferior forecasting technology. Furthermore, a return contract instead of a rebate contract can coordinate the supply chain in this model. Özer et al. (2011) show that under standard game-theoretic assumptions, demand forecast communication between a supplier and a retailer are uninformative in equilibrium. They experimentally demonstrate that this conclusion does not necessarily hold with human subject interactions. They further introduce a trust-embedded model and show its explanatory power. In our paper, we consider a decentralized supply chain with multiple competing retailers and costly information acquisition. Our results point to a type of inefficiency that has not yet been explored in the literature, namely, the overinvestment in demand forecasting due to downstream competition.

Our paper is also a part of the literature on information sharing in oligopoly. The classic literature in this area demonstrates the difficulties of inducing competing oligopolists to share private information (cf. Novshek and Sonnenschein 1982; Vives 1984; Gal-Or 1985, 1986; Li 1985; Shapiro 1986; Raith 1996; Jin 2000), and potential ways to address this issue (e.g., Ziv 1993, Jain et al. 2010). One of the primary conclusions derived from this literature is that when competing firms have private information on a common uncertain variable, they do not want to share this information with their competitors. Li (2002) analyzes information sharing in a one-to-many supply chain and concludes that competing downstream firms refuse to share demand information not only with other downstream firms, but also with the supplier. Zhu (2004) shows that information transparency in an online procurement market under oligopoly can hurt the participating firms. Li and Zhang (2008) show that when supplier-retailer information sharing confidentiality can be achieved, under certain parameter regions, retailers truthfully report their information, and supply chain profit can be maximized. Ha and Tong (2008) investigate the value of vertical information sharing in supply chains that compete with each other. They explore menu contracts and linear price contracts between the manufacturer and the retailer in each supply chain, finding that the value of vertical information sharing is positive for the menu contracts and negative for the linear contracts. Considering two competing supply chains under production diseconomies, Ha et al. (2011) identify the conditions under which vertical information sharing benefits a supply chain. We suggest a mechanism that effectively yields demand information revelation as equilibrium behavior with endogenous demand forecasting. Furthermore, our proposed contracting scheme achieves coordination of investment in demand forecasting.

Li et al. (1987) examine the welfare consequences of investment in demand forecasting under a single-layer Cournot oligopoly with linear investment costs. The investment level of each competitor is observable to others. They consider social welfare and show that when demand forecasting cost is high, there is underinvestment in forecasting compared to the welfare-maximizing level; whereas for low forecasting cost levels, there is overinvestment in demand forecasting. In our model, we consider a disintegrated vertical channel, which introduces incentive alignment issues. We study supply chain surplus, demonstrate that there is overinvestment with common contracting schemes no matter what the magnitude of forecasting costs is, and explore the determinants of overinvestment and supply chain efficiency.

When competing downstream retailers have private demand forecasts, unless they are compelled by a mechanism that ties incentives to truthful reporting, they have strong incentives to distort information, especially when sharing it with competitors. Studying the impact of strategic spot trading in supply chains, Mendelson and Tunca (2007) demonstrate that partial truthful information sharing in the supply chain can be achieved in a decentralized spot market. Our proposed contracting scheme in this paper shows that the upstream firm can offer a contract to implement the supply chain surplus-maximizing contract by inducing full truthful information revelation and aggregation in an incentive-compatible way as well as achieving coordination in demand forecast investments.

A number of researchers in economics literature have studied efficient mechanism design with information acquisition. The papers in this area regard information acquisition as hidden action, implying that the level of effort (investment) is unobservable to other players. Bergemann and Välimäki (2002) show that when agents have private valuations of a good and acquire costly independent information, a Vickrey-Clarke-Groves mechanism (VCG; see Clarke 1971, Groves 1973) achieves regret-free (or ex-post) efficient allocation and an ex-ante efficient level of information acquisition. Under common valuations and independent signals, efficient mechanism design is also examined in a group of studies, including Dasgupta and Maskin (2000), Jehiel and Moldovanu (2001), and Perry and Reny (2002). With uncorrelated signals and under the assumption that deviations from equilibrium can be detected with positive probability, Mezzetti (2002) demonstrates that a mechanism that achieves ex-post efficient allocation with efficient ex-ante information acquisition exists. When the signals are correlated, Cremer and McLean (1985, 1988) establish the existence of regret-free efficient mechanisms with full surplus extraction, but with no information acquisition and interdependence among the payoffs of the agents. Following Cremer and McLean, Obara (2008) studies efficient allocation in perfect Bayesian equilibrium with information acquisition for correlated signals and shows that there is no mechanism that guarantees full efficiency in a Bayesian equilibrium, even if the objective of a regret-free implementation is relaxed. In our paper, in an environment of common values and correlated signals, in which the quantity decision of each retailer affects the payoffs of the other retailers through downstream competition, we present a contracting scheme that achieves efficient production quantities and investment in information acquisition with full surplus extraction in a regret-free way.

3. The Model

A supplier sells a good to n retailers who compete as a Cournot oligopoly in the consumer market.[1] The (inverse) consumer demand curve is given by $p_c = K - \sum_{i=1}^{n} q_i$, where p_c is the clearing price in the consumer market and q_i is the quantity that retailer i, $1 \leqslant i \leqslant n$, orders and sells in the consumer market.[2]

For simplicity in exposition, we normalize each retailer's reservation value to zero.[3] In the demand curve, K is uncertain with mean K_0 and variance σ_0^2. The supplier's unit production cost is c_0. There are four time periods indexed by $t = 1$ to 4. Figure 1 illustrates the model timeline. At time $t = 1$, there is no information asymmetry among the participants. The supplier offers retailers a contract, which specifies a payment function $P(\mathbf{q})$, where $\mathbf{q} = (q_1, \ldots, q_n)$.

At $t = 2$, each retailer invests in demand forecasting in order to obtain a private signal about the state of the demand (i.e., K). Demand forecasting is costly; therefore, before making the forecast, each retailer decides how much to invest. The more a retailer invests, the more accurate the signal he obtains about the state of the demand. At $t = 3$, each retailer i receives his private demand forecast, s_i, and places his order q_i to the supplier.[4] At $t = 4$, supplier delivers the good, consumer market demand is realized, and competing as a Cournot oligopoly, retailers sell the good in the consumer market.

We assume unbiased signals and affine conditional expectations for the information structure of the signal (see, e.g., Ericson 1969). That is, $\mathrm{E}[s_i \mid K] = K$, and $\mathrm{E}[K \mid s]$ is affine in s, for all $s = (s_1, \ldots, s_n)$. There are many conjugate pair distributions for the demand intercept and signals that satisfy these assumptions, such as normal/multivariate normal, beta/binomial, and gamma/poisson, respectively (see Ericson 1969 for a detailed discussion). Define $\nu_i \triangleq \sigma_0^2/\sigma_i^2$, where $\sigma_i^2 \triangleq \mathrm{E}[\mathrm{Var}[s_i \mid K]]$, $i = 1, \ldots, n$. ν_i is the expected precision of retailer i's demand signal, s_i, relative to the precision of K. As the expected precision of the demand signal ν_i increases, σ_i^2 decreases. Denote the cost function for demand forecasting by C. That is, to have an expected forecast precision ν_i, retailer i must invest $C(\nu_i)$. Clearly, to achieve higher precision, one needs to invest more, i.e., C is nondecreasing. We also assume that C is convex, nonidentically zero, and twice differentiable with $C(0) = 0$.[5] We first start in §§4 and 5 with the case where the investment level of each retailer is unobservable by other parties. We then explore the impact of observability of forecast investments in §6.

Given this structure, retailer i's profit is

$$\Pi_i(q_i, \mathbf{q}_{-i}, \nu_i) \triangleq q_i \cdot (K - Q) - P(q_i, \mathbf{q}_{-i}) - C(\nu_i), \tag{1}$$

the supplier's profit is

$$\Pi_S(\mathbf{q}) \triangleq \sum_{i=1}^{n} P(q_i, \mathbf{q}_{-i}) - c_0 Q, \tag{2}$$

and the total supply chain profit is

$$\Pi_{SC}(\mathbf{q}, \nu) \triangleq \Pi_S(\mathbf{q}) + \sum_{i=1}^{n} \Pi_i(q_i, \mathbf{q}_{-i}, \nu_i) = Q(K - Q - c_0) - \sum_{i=1}^{n} C(\nu_i), \tag{3}$$

where $\mathbf{q}_{-i} = (q_1, \ldots, q_{i-1}, q_{i+1}, \ldots, q_n)$ for $i = 1, \ldots, n$, $Q = \sum_{j=1}^{n} q_j$, the total quantity ordered, and $\nu = (\nu_1, \ldots, \nu_n)$.

3.1. Definition of Equilibrium

For all contracting schemes examined in this paper, we explore the Bayesian Nash equilibrium of the game. In equilibrium, retailer i, $1 \leqslant i \leqslant n$, selects his profit-maximizing order quantity q_i and signal precision ν_i for $i = 1, \ldots, n$, given the contracting scheme offered by the supplier. That is,

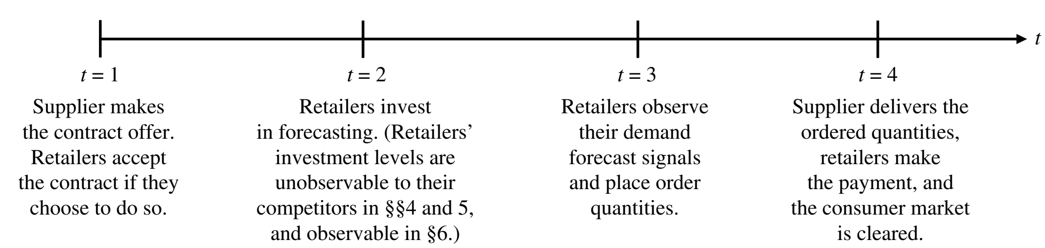
$$\mathrm{E}[\Pi_i(q_i, \mathbf{q}_{-i}, \nu_i) \mid s_i] \geqslant \mathrm{E}[\Pi_i(q_i', \mathbf{q}_{-i}, \nu_i') \mid s_i], \tag{4}$$

for any alternative order quantities q_i' and investment levels ν_i', given the other retailers' equilibrium order strategies and investment levels, for $1 \leqslant i \leqslant n$.

3.2. The First-Best Benchmark

To understand the effect of competition on investment in demand forecasting and supply chain efficiency, we need to derive the centralized first-best benchmark outcome for

Figure 1. The model timeline.



the supply chain. The first-best benchmark assumes that the supply chain is fully coordinated, i.e., all decisions are made in a centralized manner, and all information in the supply chain is available to the decision maker. Given this, the first-best problem can be formulated as

$$\max_{Q(s),\,\nu}\left\{\mathrm{E}[Q(K-Q-c_0)]-\sum_{i=1}^{n}C(\nu_i)\right\}. \tag{5}$$

We denote this first-best case with superscript ^{FB}. The following lemma provides the solution to this problem.

Lemma 1. *The first-best total production quantities and investment level in demand forecasting are given by*

$$Q^{FB}(s)=\frac{K_0-c_0}{2}+\frac{\sum_{i=1}^{n}\nu_i^{FB}(s_i-K_0)}{2(1+\sum_{i=1}^{n}\nu_i^{FB})}, \tag{6}$$

$$\nu_i^{FB}=\nu^{FB}=\nu^*\cdot 1_{\{C'(0)<\sigma_0^2/4\}}, \tag{7}$$

for $i=1,\ldots,n$, *where* $1_{\{\cdot\}}$ *is the indicator function, and* ν^* *is the unique solution to the equation*

$$\frac{\sigma_0^2}{4(1+n\nu)^2}-C'(\nu)=0. \tag{8}$$

One observation from Lemma 1 is that it is optimal to invest in demand forecasting only if $C'(0)<\sigma_0^2/4$. This condition reflects the trade-off between the benefit of investing in demand forecasting and the costs. $C'(0)$ is the marginal cost of acquiring information at zero information level, and $\sigma_0^2/4$ is the expected marginal benefit of that information. The condition states that if the expected benefit of information is lower than the cost at the zero information level, it is optimal not to acquire any information at all because at higher information levels the benefits are not higher than when one has no information, and the marginal costs stay the same or go up as one acquires more information.

If $C(\cdot)$ is linear, the expected total supply chain profit is the same as long as the total investment level is the same, given the optimal total production function. Hence, there is a continuum of optimal solutions, among which the symmetric optimal solution is one. This continuum includes the solution where the entire supply chain invests in only one signal because the central planner is indifferent between investing in one signal or multiple ones.[6] However, when $C(\cdot)$ is strictly convex, although investing in a single cost function to get a single signal is also in the feasible set of the optimization problem, the planner would not choose that option because there are decreasing returns to investment, and it is optimal to "spread" the cost equally among as many cost functions as available.[7]

The first-best benchmark is the idealized best and achieves the fully coordinated outcome by utilizing normally nonexistent advantages, such as centralized decision making (no incentive issues) and the pooling of the dispersed private information of multiple agents (no informational asymmetry). Throughout the paper, we compare the outcomes of the contracting schemes we examine to this fully coordinated first-best benchmark.

3.3. The Challenges in Achieving Full Coordination

Before we analyze the outcome in the decentralized supply chain, let us first discuss the sources of inefficiency in that setting and lay out the framework for the challenges for coordination. There are two main sources of inefficiency.

The first main source of inefficiency, as with all models that deal with decentralized decision making in supply chains and coordination, is vertical disintegration. The supplier and the retailers make decisions to optimize their own disparate profit functions, which could result in misalignment of the production quantities in the supply chain.

The second main source of inefficiency in our setting is downstream competition. It has three consequences on misalignment.

The first consequence is the misalignment of expected production quantity decisions among competing downstream retailers. When a retailer faces competition, the value of the expected marginal unit he sells is different for him compared to the value of that unit for the supply chain because the retailer does not internalize the negative effect of that unit on other retailers' revenues due to reduction of consumer price. As a result, in a decentralized setting the expected order quantity of a given retailer differs from that in the centralized solution. Here we are specifically separating the misalignment in expected order quantities from the misalignment in the random component of the order quantities (i.e., the way the retailer's respond to their forecast signals in their orders, which we discuss next). This misalignment in expected order quantities would exist even with no uncertainty or private information.

The second misalignment of incentives caused by downstream competition stems from decision making with private demand forecast signals. Facing competition from the other retailers in the market, who also act by utilizing their own signals, each retailer uses his signal not only to predict demand but also to predict the other retailers' forecasts, and conjectures how they will respond to their forecasts in their order quantities. Furthermore, in decentralized equilibrium, when making his quantity decision based on his signal, a retailer does not take into account the impact of his order quantity on other retailers' revenues. This differs from the first-best solution, where the retailers' signals are used only to forecast the demand and the order quantities are determined centrally, taking the impact of the solution's reaction to each signal on the entire supply chain. A coordinating contracting scheme should align each retailer's equilibrium reaction to his signal with the centralized solution for each realization of demand forecast signals, i.e., achieve *state-wise* quantity coordination.

Finally, a third kind of misalignment that downstream competition creates is misalignment in incentives in demand forecast investment. As we mentioned, there is a divergence between how each retailer uses his demand signal in equilibrium and how that signal is used in the centralized solution. Consequently, there is a divergence between how the accuracy of a retailer's signal affects his profits in equilibrium and how that accuracy affects the centralized supply chain surplus. In conjunction with this divergence, each retailer fails to internalize the impact of a marginal increase in the accuracy of his signal on the other retailers' revenues, while the centralized solution takes the full impact of such an increase on the entire supply chain. Therefore, a distortion in the retailers' incentives in investing to increase the accuracy of their signals emerges relative to the centralized supply chain optimum. In the remainder of the paper, we show how these sources of misalignment can create inefficiencies and how supply chain coordination can be achieved in the face of these challenges.[8]

4. Overinvestment in Forecasting Under Unobservable Investments

We start our analysis by exploring the case where retailers' demand forecast investments are not observable to one another. In §6, we introduce investment observability to study its effect on incentives for demand forecast investments and coordination.

4.1. Overinvestment Under Common Pricing Schemes

Our first goal is to show that under common contracting schemes, such as simple wholesale pricing and two-part tariff, competing retailers tend to overinvest in demand forecasting. For the wholesale pricing scheme, which we denote by the superscript ^{ws}, the supplier sets a constant unit price w^{ws} to maximize her expected profit, i.e., the pricing scheme she offers is $P(\mathbf{q}) = w^{ws} q_i$. For two-part tariff scheme, denoted by ^{tpt}, the supplier announces the pricing scheme as $P(\mathbf{q}) = w_0^{tpt} + w_1^{tpt} q_i$ and chooses w_0^{tpt} and w_1^{tpt}. For ease of exposition, we start by giving the equilibrium solution for the two-part tariff scheme. The common wholesale price contract corresponds to the case where $w_0 = 0$.

Lemma 2. *Given the pricing scheme $P(\mathbf{q}) = w_0 + w_1 q_i$, there exists a unique equilibrium. In equilibrium $q_i(s_i) = \alpha_0^{tpt} + \alpha_s^{tpt}(s_i - K_0)$ and $\nu_i^{tpt} = \nu^{tpt}$, for all i, where $\alpha_0^{tpt} = (K_0 - w_1)/(n+1)$, $\alpha_s^{tpt} = \nu^{tpt}/(2+(n+1)\nu^{tpt})$, $\nu^{tpt} = \nu^* \cdot 1_{\{C'(0)<\sigma_0^2/4\}}$, and ν^* is the unique solution to the equation*

$$\frac{\sigma_0^2}{(2+(n+1)\nu)^2} - C'(\nu) = 0. \tag{9}$$

Notice that, similar to the first-best case, the retailers invest in demand forecasting only when the marginal cost of investment is less than the marginal benefits at the zero investment level: if $C'(0) \geqslant \sigma_0^2/4$, then the retailers' equilibrium investment in the decentralized setting is the same with the first-best investment level, which is zero. When this condition is not satisfied, however, the retailer's investment level can diverge from the first-best, as we explore next.

Proposition 1. (i) *The optimal wholesale price contract for the supplier is specified by the wholesale price $w^{ws} = (K_0 + c_0)/2$, and the optimal two-part tariff contract for the supplier is specified by the two parameters*

$$w_0^{tpt} = \frac{1}{4n}\left(\frac{(K_0 - c_0)^2}{n} + \frac{4n\nu^{tpt}(1+\nu^{tpt})\sigma_0^2}{(2+(n+1)\nu^{tpt})^2}\right) - C(\nu^{tpt}), \tag{10}$$

$$w_1^{tpt} = \frac{(n-1)K_0 + (n+1)c_0}{2n}, \tag{11}$$

where $\nu^{tpt} = \nu^{ws}$, as given in Lemma 2.

(ii) *For $n \geqslant 2$, under simple wholesale pricing and two-part tariff schemes, each retailer overinvests in demand forecasting in equilibrium, i.e., $\nu^{ws} = \nu^{tpt} \geqslant \nu^{FB}$. The inequality is strict when $C'(0) < \sigma_0^2/4$.*

Proposition 1 states the first main result of our paper: it is indeed the case that under common contracting schemes, such as wholesale price contracts and two-part tariffs, downstream competition causes overinvestment in demand forecasting. To explore the reason for this, and referring back to the challenges for coordination we discussed in §3.3, first consider the effect of vertical disintegration. Vertical misalignment due to double-marginalization would cease to exist if the wholesale price is set to equal the supplier's marginal production cost c_0. This corresponds to the case with $w_0 = 0$ and $w_1 = c_0$. But by Equation (9), we can see that this would not solve the overinvestment problem, because by Lemma 2 the fixed unit price affects only the expected quantity produced by the retailers and does not influence the retailers' information usage and value. On the other hand, to test the effect of downstream competition, we can check the outcome in the absence of competition, i.e., when $n = 1$. As can be seen by substituting $n = 1$ into (9) and comparing to (8), when there is only one downstream retailer, demand forecast investment level under wholesale price and two-part tariff contracts is the same as the first-best level. That is, without downstream competition, even under vertical disintegration, there is no overinvestment in demand forecasting for any wholesale price and two-part tariff contract (whether $w_1 = c_0$ or $w_1 > c_0$). This is because when there is only one retailer, the information available to the supply chain in the first-best case is identical to the information available to the retailer (namely s_1). Thus, the retailer's estimate for the demand realization is identical to that of the centralized supply chain. Furthermore, because there is no competition, the retailer does not have to estimate competitors' quantities, and hence his signal affects his profits only through his demand estimate, which is exactly the case for the centralized supply chain as well. Consequently, with only one downstream retailer, the retailer's incentive to invest in demand forecasting is identical to that of supply chain first-best, and there is no misalignment in incentives to invest in forecasting. Hence, the cause of overinvestment in this setting is downstream competition.

Wholesale price contracts, even in the absence of private information, would not be able to coordinate the supply chain, but two-part tariff contracts can fully coordinate the supply chain in the presence of downstream competition (i.e., with $n \geqslant 2$) if there is no private demand information (and hence forecast investment is also not an issue).[9] However, once retailers have private demand forecasts, two-part tariff contracts can no longer coordinate the supply chain even without demand forecast investments. To see this, consider the (commonly employed) case with private demand forecasts and exogenously given forecast precisions, $\nu_i = \nu$ for all i. In that case, by (6) the first-best total quantity would be $(K_0 - c_0)/2 + \nu/(2(1+n\nu))\sum_{i=1}^{n}(s_i - K_0)$, while the equilibrium total quantity under the supplier's profit maximizing two-part tariff contract would be $(K_0 - c_0)/2 + \nu/(2+(n+1)\nu)\sum_{i=1}^{n}(s_i - K_0)$. That is, even though the expected quantities ($(K_0 - c_0)/2$) are aligned, there would be a mismatch in the effect of the forecast signals on the total production quantity, as we discussed in §3.3, and statewise supply chain production quantity coordination could not be achieved.[10] In particular, comparing the expressions above, the multiplier of s_i for the decentralized case is $\nu/(2+(n+1)\nu)$, which is greater than the multiplier of s_i in the centralized case, $\nu/2(1+n\nu)$. In other words, in the decentralized case each retailer reacts to his signal more strongly than a central planner would.

In the presence of demand forecast investments, the misalignment in the way retailers' forecast signals factor in their payoffs in equilibrium also results in an emergence of overinvestment in forecasting, as stated in part (ii) of Proposition 1. A marginal increase in the forecast accuracy of a retailer has a bigger positive impact on that retailer's profit than it has for the centralized solution. This is not only because the retailer reacts to his signal more strongly in the decentralized equilibrium than a central planner would, but also because when deciding on the level of investment to increase the accuracy of his own demand forecast, a retailer does not take into account the negative impacts of his increased demand forecast accuracy on his competitors. Consequently, each retailer has an amplified incentive to invest in demand forecasting relative to the supply chain first-best, and in equilibrium, each ends up overinvesting in demand forecasting.[11]

Finally, note that there are certain cross-effects among the two types of the information-related misalignment caused by downstream competition. Specifically, the misalignment in the way the retailers react to their signals is a cause for overinvestment in demand forecasting, and when this misalignment (i.e., the retailers' overreaction to their signals) becomes stronger, the misalignment in forecast investments also becomes larger. Conversely, the misalignment in forecast investments feeds back and amplifies the misalignment in the retailers' usage of their signals. In particular, under overinvestment, a retailer's signal accuracy is higher, which results in the retailer's increased confidence in his signal and an even stronger overreaction to it, i.e., $\alpha_s = \nu/(2+(n+1)\nu)$, which is increasing in ν. The marginal benefit of increased precision in this feedback loop is balanced by the marginal cost of acquiring increased accuracy to yield the resulting forecast precision in equilibrium.

4.2. Determinants of Overinvestment and Efficiency

The emergence of overinvestment in demand forecasting raises important questions about supply chain efficiency: How severe can overinvestment and the resulting supply chain surplus loss become?[12] What are the determinants of the efficiency loss in demand forecast investments and supply chain surplus? We explore the answers to these questions next.[13]

Proposition 2. *For $n \geqslant 2$, considering all possible increasing and convex forecast investment cost functions,*

(i)

$$\frac{\nu^{ws}}{\nu^{FB}} = \frac{\nu^{tpt}}{\nu^{FB}} \leqslant \frac{2n}{n+1}. \tag{12}$$

The upper bound in (12) *can be achieved for linear demand forecast cost functions, i.e., when $C(\nu) = c_f \cdot \nu$ for $c_f > 0$.*

(ii) $\mathrm{E}[\Pi_{SC}^{ws}]/\mathrm{E}[\Pi^{FB}]$ *and* $\mathrm{E}[\Pi_{SC}^{tpt}]/\mathrm{E}[\Pi^{FB}]$ *can attain any value in* $(0, 1)$. *Specifically, near-full-inefficiency for both contracting schemes can occur in the limit for $C(\nu) = c_f \cdot \nu$ and as $\eta \to \infty$ and $\sigma_0^2 \to \infty$; and near-full-efficiency for both contracting schemes can occur in the limit for all convex increasing cost functions as $\sigma_0^2/4 \to (C'(0))^+$, with the additional condition $\eta \to \infty$ for the case of wholesale price contracting.*

Proposition 2 states that the equilibrium investment level under the wholesale and two-part tariff contracts can be highly inefficient. Specifically, the demand forecast investment can reach up to $2n/(n+1)$ times the first-best level, converging to twice the first-best level as $n \to \infty$. That is, downstream competition can result in substantial waste in the channel in terms of money and resources spent on demand forecasting under commonly used contract structures. But the adverse effect of demand forecast overinvestment is not limited to the cost of investment. In fact, considering the ripple effects of overinvestment in demand forecasting on contracted quantities, in equilibrium, the losses in supply chain profits can be substantial, as part (ii) of Proposition 2 also states, especially when downstream competition is intense and uncertainty is high.[14]

Given the detrimental effects of demand forecast overinvestment, we next explore the factors that determine the severity of this overinvestment and the resulting supply chain loss. To illustrate the effects of these factors, we analyze the comparative statics for the linear demand forecast investment cost case, which is commonly used in the literature (see, e.g., Li et al. 1987).

PROPOSITION 3. *Consider the case $C(\nu) = c_f \cdot \nu$ and $n \geqslant 2$. $\mathrm{E}[\Pi_{SC}^{ws}]/\mathrm{E}[\Pi^{FB}]$ and $\mathrm{E}[\Pi_{SC}^{tpt}]/\mathrm{E}[\Pi^{FB}]$ are increasing in c_f and K_0, and decreasing in σ_0^2 and c_0. Furthermore, $\mathrm{E}[\Pi_{SC}^{ws}]/\mathrm{E}[\Pi^{FB}]$ is increasing in n if*

$$2 \leqslant n \leqslant 1 + \frac{(K_0 - c_0)^2}{2(\sigma_0^2 - 4\sqrt{c_f \sigma_0^2} + 4c_f)}$$

and decreasing in n otherwise; and $\mathrm{E}[\Pi_{SC}^{tpt}]/\mathrm{E}[\Pi^{FB}]$ is monotonically decreasing in n. In addition

$$\lim_{n\to\infty} \frac{\mathrm{E}[\Pi_{SC}^{tpt}]}{\mathrm{E}[\Pi^{FB}]} = \lim_{n\to\infty} \frac{\mathrm{E}[\Pi_{SC}^{ws}]}{\mathrm{E}[\Pi^{FB}]} = \frac{(K_0 - c_0)^2}{(K_0 - c_0)^2 + \sigma_0^2 + 4c_f - 4\sqrt{c_f \sigma_0^2}}.$$

When demand uncertainty (σ_0^2) increases, the marginal value of demand forecast investment increases. Consequently, a rise in market demand uncertainty increases the relative overinvestment in demand forecasting. Hence, supply chain efficiency under common contracting schemes decreases with increased σ_0^2. On the other hand, an increase in the market's profitability potential, by either an increase in K_0 or a decrease in c_0, reduces the relative effect of the inefficiency resulting from overinvestment. When the marginal cost of investment in demand forecast precision (c_f) increases, forecast investment decreases in equilibrium for the common contracting schemes as well as the first-best. As a result, profit losses due to demand forecasting become less significant, and the surplus efficiency ratios increase.

The effect of the number of competing retailers on profits is subtle. For wholesale price contracting, the increase in number of retailers reduces the efficiency loss due to double-marginalization. This reduction facilitates increased coordination in quantities and increases supply chain profits. On the other hand, as we have seen in Proposition 2, an increase in the number of retailers increases overinvestment in demand forecasting. As a result, under wholesale pricing, there can be a threshold level of downstream competition below which the supply chain surplus efficiency is increasing in n, but above which the negative effect of overinvestment becomes dominant to decrease the supply chain profits relative to the first-best level. The two-part tariff contract is more effective in coordinating quantities, and hence the overinvestment effect is dominant, decreasing the surplus efficiency ratios as the number of competing retailers increases.[15]

5. A Market-Based Contracting Scheme for Full Coordination

So far, we have shown that downstream competition causes overinvestment in demand forecasting under common contracting schemes, and the resulting supply chain losses can be substantial. These results call for the question of how one can fully coordinate the supply chain through contracting, specifically in both statewise production quantities and demand forecast investments.

The value of market information for pricing procurement contracts has been recognized by practitioners for a long time (see, e.g., Faruqui and Eakin 2000). Many companies and industries have developed ways to utilize market information in pricing, employing a class of contracts, generally called *index-based* or *market-based* contracts. Such contracts utilize a *base unit price* determined by incorporating market demand and supply information. The specific way the base unit price is determined varies greatly among industries, companies, and contracts, and it can take various names such as "index-price," "market-price," "benchmark price," or "base price." After this price is set, individual contracts use it as the unit price and add certain augmentations, which often include volume discounts.

Companies and industries have adopted various ways to determine the base unit price. In certain cases, a centralized marketplace exists for the product. A good example is the natural gas market in the United States. Most procurement contracts between natural gas suppliers and retailers in the United States today are based on a particular price called "Henry Hub Price."[16] Located in Erath, Louisiana, Henry Hub is at the junction of several major natural gas pipelines in the southwest United States and brings major buyers and sellers of natural gas together. The Henry Hub Price is set by equating supply and demand at this junction. Suppliers and retailers of natural gas then complete the contracts by taking the Henry Hub price as the base unit price and modifying the contracts, in many cases by applying certain volume discounts.[17] In other cases, a small set of (usually large) buyers and sellers get together to set the "benchmark price." For example, in the steel industry, a benchmark price for iron ore is set between a large supplier and a large buyer incorporating market supply and demand conditions at a given time. Contracts in the industry are then written taking this benchmark price as the base unit price (see, e.g., Hoyt et al. 2007).[18] Yet, in other cases, especially when there is no centralized marketplace for the traded good, an industrial buyer like Hewlett-Packard (HP) can determine a central market-price with its suppliers utilizing information about market conditions. This price then serves as a base unit price for the contracts that are written between the suppliers and HP, and volume discounts off this market price are often employed (Nagali et al. 2008). In short, although specific mechanisms employed to set the base unit price vary, all market-based contracts work with the same core idea of utilizing the market's aggregated information in pricing.

In this section we propose a contracting scheme that belongs to this general market-based contract structure. Specifically, we study contracts that utilize an endogenously formed index-price, which effectively aggregates the market's demand information. The index-price is used as a unit price, and the contract is augmented with volume discounts.

Consider the pricing scheme offered by the supplier to retailer i,

$$P(\mathbf{q}) = w_0 + \bar{p}(\mathbf{q})q_i - w_d q_i^2, \tag{13}$$

where $\bar{p}(\mathbf{q}) = w_1 + w_2 \sum_{j=1}^n q_j$ is the *index-price*, and $w_d > 0$. The timing is as follows: The supplier offers the contract to the retailers, and the retailers decide whether to participate, comparing their expected payoffs from the contract to their reservation value (normalized to zero). Then each participating retailer makes his demand forecast investment. After obtaining their forecasts, retailers simultaneously submit their orders to the supplier. Based on these orders, the index-price $\bar{p}$ is determined. Each retailer then pays the total price as given in (13), the supplier delivers the order quantities, and the consumer market is cleared.

When $w_2 > 0$, the price index increases in total production quantity. In this sense, this contracting scheme incorporates the market's opinion into pricing. If the retailers receive high demand signals, their order quantities will be higher, which will in turn push the index-price up. That is, higher demand signals mean higher contract prices and vice-versa. We call this contracting scheme the *market-based contracting* and denote it with superscript ^{m}. The following proposition states that this contracting scheme can fully coordinate the supply chain.

Proposition 4. *For any given contract in the class defined in* (13), *there exists a unique equilibrium linear in retailers' order quantities. In equilibrium,* $q_i(s_i) = \alpha_0^m + \alpha_s^m(s_i - K_0)$, *and* $\nu_i = \nu^m$ *for all i, where*

$$\alpha_0^m = \frac{K_0 - w_1}{(n+1)(1+w_2) - 2w_d},$$

$$\alpha_s^m = \frac{\nu^m}{2(1+w_2-w_d) + ((n+1)(1+w_2) - 2w_d)\nu^m},$$

$$\nu^m = \nu^* \cdot 1_{\{C'(0) < \sigma_0^2/4\}},$$

and ν^* *is the unique solution to the equation*

$$\frac{(1+w_2-w_d)\sigma_0^2}{(2(1+w_2-w_d)+((n+1)(1+w_2)-2w_d)\nu)^2} - C'(\nu) = 0. \tag{14}$$

Furthermore, there is a unique contract in the class defined by (13) *that in equilibrium achieves the full supply chain coordination in both statewise production quantities and demand forecast investments, and in which the supplier extracts the entire supply chain surplus. In this contract* $w_1^m = c_0$, $w_2^m = w_d{}^m = 1$, *and*

$$w_0^m = \frac{(K_0 - c_0)^2}{4n^2} + \frac{\sigma_0^2 \nu^{FB}}{4(1+\nu^{FB})} \cdot \left(\frac{2 + (n+1)\nu^{FB}}{2(1+n\nu^{FB})}\right)^2 - C(\nu^{FB}). \tag{15}$$

Proof. Let $q_j(s_j) = \alpha_{0j} + \alpha_{sj}(s_j - K_0)$, for $\alpha_{0j}, \alpha_{sj} \in \mathbb{R}$ and $\nu_j \in \mathbb{R}_+$, for all $j \neq i$. Expected profit for retailer i after observing s_i under the given pricing scheme is

$$\mathrm{E}[\Pi_i^m \mid s_i] = q_i\Bigg(K_0 - w_1 + \frac{\nu_i(s_i - K_0)}{1+\nu_i} - (1+w_2-w_d)q_i - (1+w_2)\sum_{j\neq i}(\alpha_{0j} + \alpha_{sj}\mathrm{E}[s_j - K_0 \mid s_i])\Bigg) - C(\nu_i) - w_0. \tag{16}$$

Note that (16) is strictly concave in q_i if and only if $1 + w_2 - w_d > 0$, and an equilibrium cannot exist otherwise. Using the fact that $\mathrm{E}[s_j - K_0 \mid s_i] = (\nu_i/(1+\nu_i)) \cdot (s_i - K_0)$, the first-order condition for q_i from (16) is written as

$$q_i = \frac{1}{2(1+w_2-w_d)}\Bigg(K_0 - w_1 - (1+w_2)\sum_{j\neq i}\alpha_{0j} + \frac{\nu_i}{1+\nu_i}\bigg(1 - (1+w_2)\sum_{j\neq i}\alpha_{sj}\bigg)(s_i - K_0)\Bigg). \tag{17}$$

Observe that q_i is linear in $s_i - K_0$. Substituting (17) into (16) and again plugging in $\mathrm{E}[s_j - K_0 \mid s_i] = (\nu_i/(1+\nu_i)) \cdot (s_i - K_0)$, we have

$$\mathrm{E}[\Pi_i^m] = \frac{1}{4(1+w_2-w_d)}\Bigg(K_0 - w_1 - (1+w_2)\sum_{j\neq i}\alpha_{0j}\Bigg)^2 + \frac{\sigma_0^2\nu_i}{4(1+w_2-w_d)(1+\nu_i)} \cdot \Bigg(1 - (1+w_2)\sum_{j\neq i}\alpha_{s+j}\Bigg)^2 - C(\nu_i) - w_0. \tag{18}$$

The first-order condition for ν_i from (18) is

$$\frac{\sigma_0^2}{4(1+w_2-w_d)(1+\nu_i)^2}\Bigg(1 - (1+w_2)\sum_{j\neq i}\alpha_{sj}\Bigg)^2 - C'(\nu_i) = 0;$$

and the second-order condition,

$$\frac{-\sigma_0^2}{2(1+w_2-w_d)(1+\nu_i)^3}\Bigg(1 - (1+w_2)\sum_{j\neq i}\alpha_{sj}\Bigg)^2 - C''(\nu_i) < 0$$

is satisfied for all $\nu_i \geqslant 0$ provided $1 + w_2 - w_d > 0$. From (17) and the first-order condition, we have

$$\alpha_{0i}=\frac{1}{2(1+w_2-w_d)}\left(K_0-w_1-(1+w_2)\sum_{j\neq i}\alpha_{0j}\right),$$
$$\alpha_{si}=\frac{\nu_i}{2(1+w_2-w_d)(1+\nu_i)}\cdot\left(1-(1+w_2)\sum_{j\neq i}\alpha_{sj}\right), \tag{19}$$

and

$$C'(\nu_i)=\frac{\sigma_0^2}{4(1+w_2-w_d)}\cdot\left(\frac{1-(1+w_2)\sum_{j\neq i}\alpha_{sj}}{1+\nu_i}\right)^2. \tag{20}$$

Summing α_{0i} over all i, we obtain

$$\sum_{i=1}^{n}\alpha_{0i}=\frac{n(K_0-w_1)}{(n+1)(1+w_2)-2w_d}.$$

Plugging this back into (19) and simplifying, we have α_0^m. Solving α_{si} in (19), we obtain

$$\alpha_{si}=\frac{\nu_i(1-(1+w_2)\sum_{j=1}^{n}\alpha_{sj})}{2(1+w_2-w_d)+(1+w_2-2w_d)\nu_i}. \tag{21}$$

Substituting (21) into (20), we have

$$C'(\nu_i)=(1+w_2-w_d)\sigma_0^2\cdot\left(\frac{1-(1+w_2)\sum_{j=1}^{n}\alpha_{sj}}{2(1+w_2-w_d)+(1+w_2-2w_d)\nu_i}\right)^2, \tag{22}$$

for all i. Because (22) holds for all i, ν_i satisfies the same equation for all i. Furthermore, (22) cannot have multiple solutions for ν_i because the left-hand side is increasing whereas the right-hand side is strictly decreasing in ν_i. Hence $\nu_i=\nu$, for $\nu\geqslant 0$. Aggregating (19) over all i and substituting $\nu_i=\nu$, we obtain

$$\sum_{i=1}^{n}\alpha_{si}=\frac{n\nu}{2(1+w_2-w_d)+((n+1)(1+w_2)-2w_d)\nu}. \tag{23}$$

Plugging (23) back into (19) and simplifying, we have α_s^m. Substituting $\nu_i=\nu$, for all i, and simplifying (20), we obtain (14) for each retailer i, $1\leqslant i\leqslant n$. The existence and uniqueness of the solution to (14) and the validity of ν^m can be shown as in the proof of Lemma 2. Now, (14) is identical to (7) if and only if $w_2^m={w_d}^m=1$. This satisfies the condition $1+w_2-w_d>0$. Furthermore, given $w_2^m={w_d}^m=1$ and $\nu_i^m=\nu_i^{FB}$ and by (6), α_0^m and α_s^m, the total supply chain production quantity is the same as the first-best total supply chain production quantity for any s, i.e., the statewise production quantity coordination is achieved if and only if $w_1^m=c_0$. □

Note that when two parties engage in contracting, the final division of the surplus depends on the outside alternatives or reservation values for the parties. Therefore, when one considers the surplus split between the supplier and the retailers, one should keep in mind the implicit and potentially positive reservation value for the retailers. In the contracts with fixed transfer payment w_0, such as the coordinating market-based contract above (and all other contracts we study other than the wholesale price contract), as the sum of the retailers' total reservation values moves between zero and the maximum attainable surplus under the given contract structure, any surplus split between the supplier and the retailers can be achieved by adjusting w_0 appropriately.

As we have seen in §4, an important challenge in coordinating the supply chain with private demand forecasts is achieving statewise quantity coordination. Proposition 4 states that the market-based contracting scheme we propose can achieve not only statewise quantity coordination but also coordination in demand forecast investments. The critical element for the full coordination is the *index-price* that adjusts to reflect the market information. The index-price helps with coordination in two aspects. First, for a given retailer, it adjusts with the total quantity submitted by the other retailers, which in turn reflects those retailers' collective demand forecasts. In the coordinating contract, the price movements as a function of retailers' quantities are set in such a way that, in equilibrium, when a given retailer conjectures how his competitors will use their forecasts in their order quantities and how that will impact the index-price, his expected profit from a positive (negative) demand shock is balanced by the increase (decrease) in the price. As a result, a retailer knows that the other retailers' forecasts will have no expected net effect on his profits, and thus he can rely only on his own signal to make the optimal decision. Second, notice that the quantity ordered by a retailer impacts the other retailers' revenues negatively as it reduces the consumer price. Therefore, each unit ordered by a retailer reduces the competing retailers' revenues proportional to the total quantity ordered by them. Because the index-price increases with competitors' quantities, in the coordinating contract, for each unit a retailer orders, he pays for the negative impact of that unit on other retailers' revenues. That is, the coordinating contract adjusts the retailers' payments so that the retailers internalize the full impact of their order quantities on the supply chain surplus in their own profits for all state realizations. Consequently, in equilibrium, the retailers' profits are functionally aligned with their share in supply chain surplus, and hence quantity coordination is achieved for all state realizations, and the retailers' incentives to invest in demand forecasting are aligned with the centralized optimum.

Another important issue here is implicit dissemination of information from the implementation of this contract. Given that a retailer's payment depends on the competing retailers' order quantities that contain competitors' private forecasting information, each retailer can infer other retailers' signals from the contract outcome. This, in many cases, can create implementation problems. For a contracting scheme to be implementable, it is important that the outcome of the contracting scheme be regret-free in the sense that given the mechanism and *after observing the* *outcome*, no retailer should want to change his equilibrium order quantity based on what he learns *from* the outcome.[19] Formally, the equilibrium order quantities are *regret-free* if

$$\mathrm{E}\big[\Pi_i(q_i, \mathbf{q}_{-i}, \nu_i) \mid s_i, P(\mathbf{q})\big] \geqslant \mathrm{E}\big[\Pi_i(q_i', \mathbf{q}_{-i}, \nu_i) \mid s_i, P(\mathbf{q})\big], \tag{24}$$

for any alternative order quantities q_i', for $1 \leqslant i \leqslant n$. Note that on the left-hand side of Equation (24), the equilibrium q_i derived in Proposition 4 is only a function of s_i. If the equilibrium strategy profile satisfies (24), then each retailer can submit his order quantity based solely on his own demand forecast, knowing that he will not regret this decision even after learning others' forecasts, as it will be optimal for each realization of the contracting outcome.[20] We next show that the market-based contracting scheme satisfies this property.

Proposition 5. *The equilibrium under the market-based pricing scheme is regret-free, i.e., equilibrium* $\mathbf{q}$ *as given in Proposition* 4 *satisfies* (24).

Proof. When retailer i observes the price the supplier asks him to pay, i.e., $P_i(q_i, \mathbf{q}_{-i}) = w_0^m + c_0 q_i + (\sum_{j\neq i} q_j) q_i$, he can refer to $\sum_{j\neq i} q_j$ as

$$\sum_{j\neq i} q_j = \frac{P_i(q_i, \mathbf{q}_{-i}) - w_0^m}{q_i} - c_0. \tag{25}$$

By (25) and Proposition 4, it then follows that

$$\sum_{j\neq i} s_j = (n-1)K_0 + \frac{1}{\alpha_s^m}\left(\frac{P_i(q_i, \mathbf{q}_{-i}) - w_0^m}{q_i} - c_0 - (n-1)\alpha_0^m\right). \tag{26}$$

That is, in equilibrium, each retailer i can infer the sum of the remaining retailers' demand signals after observing the price the supplier asks him to pay, $P_i(\mathbf{q})$. Now, because by Proposition 4 $q_j(s_j) = (K_0 - c_0)/2n + \nu^m/2(1+n\nu^m) \cdot (s_j - K_0)$, and because

$$\mathrm{E}\left[K \,\middle|\, s_i, \sum_{j\neq i} s_j\right] = K_0 + \frac{\nu^m}{1+n\nu^m}\sum_{j=1}^{n}(s_j - K_0),$$

we have

$$\begin{aligned}\mathrm{E}\left[\Pi_i^m \,\middle|\, s_i, \sum_{j\neq i} s_j\right] &= q_i \mathrm{E}\left[K - \sum_{j\neq i} q_j - q_i - c_0 - \sum_{j\neq i} q_j \,\middle|\, s_i, \sum_{j\neq i} s_j\right] - w_0^m - C(\nu_i)\\ &= q_i\left(\frac{K_0 - c_0}{n} + \frac{\nu^m(s_i - K_0)}{1+n\nu^m} - q_i\right) - w_0^m - C(\nu_i).\end{aligned} \tag{27}$$

Note that (27) is concave in q_i, and taking the first-order condition we obtain the optimal order quantity function identical to the one derived in Proposition 4. Therefore, (24) is satisfied. □

Proposition 5 states that in the market-based pricing scheme, all retailers are satisfied with the quantities they ordered based only on their own signals, even after they revise their demand forecasts with the information leaked through the contracting mechanism. This information is again provided by the market index-price that adjusts to reflect the realization of forecast signals in equilibrium. In particular, each retailer i, $i = 1, \ldots, n$, estimates that under any realization of s_j, $j \neq i$, what he could learn about the demand realization from those signals would already be reflected in price. If the competitors' signals indicate a higher demand, then the increase in the index-price would cancel the expected additional upside implied by that signal state realization. Just as well, the negative implications of a low realization of competitors' signals would be canceled by their impact on the price. As a result, for any retailer, for *any realization of his competitors' signals*, choosing his order quantity based only on his own signal becomes optimal, and the contract is regret-free.

There are two additional desirable features of the market-based contracting scheme to note here. First, in most of the previous studies of information sharing with oligopolistic competition, information sharing is through direct revelation of signals (see, e.g., Gal-Or 1985 and the follow-up studies, as cited in the literature review). However, when sharing their signals with their competitors, unless the forecast information is verifiable the retailers would have incentives to distort their signals and not reveal them truthfully. By utilizing the quantities ordered by the retailers for information sharing, our proposed mechanism eliminates the need for both direct signal revelation and the presence of verifiability to ensure truth telling. In equilibrium, the retailers' true signals are revealed in an incentive-compatible way. Second, market-based contracting also addresses an important issue in the retailers' actual incentives to share private demand forecasts. As the literature on information sharing in oligopoly demonstrates, given the opportunity to share their demand signals, competing oligopolists choose not to share them in equilibrium (see, e.g., Li 2002 and the references therein). With the market-based contracting scheme we present, each retailer ends up willingly and endogenously revealing his demand signal through his order quantity in equilibrium, and this information gets incorporated in the outcome.

In short, the market-based contracting scheme coordinates the supply chain with a regret-free implementation and achieves efficient demand forecast investments, while facilitating horizontal information revelation among the competing retailers and effectively enabling information sharing between the retailers and the supplier. As a result, full and efficient utilization of the dispersed information in the supply chain is achieved, while preventing the information leakage from destroying the alignment of incentives in decentralized decision making in the process.

6. The Effect of Investment Observability

So far, we have examined the case where demand forecast investments are unobservable. However, in certain cases, information about firms' forecast investments can be available in certain forms to the industry and the firms' competitors. Such information shows itself in news stories and financial statements, as well as through diffusion and transparency of information in the industry about companies' projects, trade agreements, campaigns, and practices. Therefore, a relevant and interesting issue is the effect of demand forecast investment observability in supply chains, which we explore in this section.

The sequence of events again proceeds in the same way as described in §3, with the only difference that at $t=2$, each retailer, i, can observe his competitors' forecast investment levels, ν_j, for $j \neq i$. Therefore, at time $t=3$, when a retailer places his order he not only has received his demand forecast signal but also is aware of the level of forecast investment by his competitors, and he places his order based on these pieces of information.

6.1. Overinvestment and Efficiency Loss

We start by studying the question of whether the overinvestment behavior persists when retailers' demand forecast investments are observable.

Proposition 6. *For* $n \geqslant 2$,

(i) *There is overinvestment in demand forecasting under wholesale price and two-part tariff contracts with demand observability. Furthermore,*

$$1 \leqslant \frac{\nu_{obs}^{ws}}{\nu_{unobs}^{ws}} = \frac{\nu_{obs}^{tpt}}{\nu_{unobs}^{tpt}} \leqslant \sqrt{\frac{3n-1}{n+1}}. \tag{28}$$

That is, overinvestment with observable forecast investments is higher compared to the case with unobservable forecast investments. All bounds in (28) *are tight.*

(ii) *Investment observability reduces the supply chain profit for wholesale price and two-part tariff contracts. That is,* $\mathrm{E}[\Pi_{SC,\,obs}^{ws}] \leqslant \mathrm{E}[\Pi_{SC,\,unobs}^{ws}]$, *and* $\mathrm{E}[\Pi_{SC,\,obs}^{tpt}] \leqslant \mathrm{E}[\Pi_{SC,\,unobs}^{tpt}]$.

(iii) *The retailers overinvest in demand forecasting in equilibrium with observable investments under the market-based pricing scheme. The equilibrium overinvestment* (ν_{obs}^{m}/ν^{FB}) *can be very severe, and the resulting supply chain surplus loss can reach at least as high as* 50% *of the first-best level.*

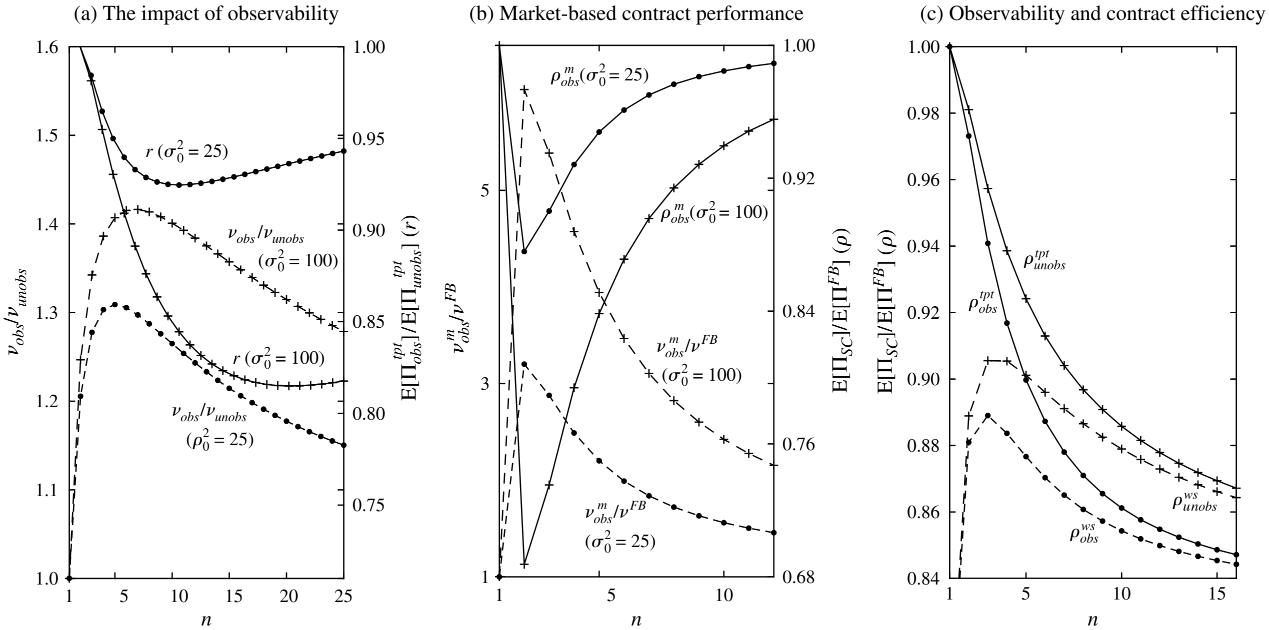
Proposition 6 attests that forecast observability amplifies both the overinvestment in forecasting and the ensuing supply chain inefficiency. In fact, the worst-case amplification of overinvestment increases with the number of retailers and can reach to $\sqrt{3}$.

To see why investment observability creates incentives for increasing forecast investments, notice that if a retailer (say Retailer 1) invests more, he will rely on the accuracy of his signal more. This means that if he receives a higher-than-expected signal, he will increase his order quantity compared to the case when he invested less and had a signal with lower accuracy. Suppose a competitor (Retailer 2) observes that Retailer 1 invested more in demand forecasting. Because the signals of the retailers are correlated, Retailer 2 is also likely to receive a high signal. Furthermore, seeing his high signal, Retailer 2 conjectures that Retailer 1's signal is likely to be high as well, and observing that Retailer 1's signal accuracy is high, he knows that Retailer 1 ordered a high quantity, which will depress the consumer price. As a result, Retailer 2 has reduced incentives to order and curbs his order quantity compared to the case when he does not observe the forecast investments. This reduction in Retailer 2's order quantity boosts Retailer 1's profits under a high signal realization. With a mirror-image argument, when Retailer 1 observes a low signal, Retailer 2 is likely to increase his order quantity, decreasing Retailer 1's profits. However, the expected profit increase for Retailer 1 under the high signal realization is, on average, greater than the decrease under the low signal realization.[21] Consequently, Retailer 1 benefits from observability of his demand forecast investment and thus has additional incentives to boost his investment. However, this added incentive for forecast investment translates into an arms race, yielding industry-wide overinvestment and reduced supply chain surplus, as stated in part (ii) of Proposition 6. Under both wholesale price and two-part tariff contracts, the supply chain is worse off with investment observability.

Part (iii) of Proposition 6 states that the market-based contracting scheme described in §5, which could coordinate the supply chain under unobservable demand forecast investments results in overinvestment under forecast investment observability. This is because when forecast investment levels are observable, in addition to the four factors of misalignment as given in §3.3, a retailer's incentive to make his competitors know that he has high forecast accuracy creates yet another source of misalignment, as we discussed in detail above, and that issue also needs to be addressed. Furthermore, this factor can cause a substantial amount of overinvestment as well as significant supply chain inefficiency, up to 50% of the efficient supply chain surplus.

Figure 2 demonstrates the effect of forecast investment observability and number of retailers on overinvestment and supply chain surplus. As shown in panel (a), observability significantly increases overinvestment in demand forecasting and reduces supply chain surplus. Yet the effect of increased number of retailers on both measures is nonmonotonic.[22] This is because there are two factors that play when the number of retailers increases. First, increased downstream competition increases retailers' aggregate incentives to signal the accuracy of their forecasts, leading to amplified overinvestment and reduced supply chain efficiency. On the other hand, increased number of retailers also makes the downstream market more

Figure 2. The effect of forecast investment observability on overinvestment and contract efficiency.



Notes. Panel (a) plots the relative overinvestment (ν_{obs}/ν_{ubobs}) and supply chain efficiency ($E[\Pi_{SC,obs}]/E[\Pi_{SC,unobs}]$) with observable forecast investments under the two-part tariff scheme. Panel (b) plots overinvestment (ν_{obs}^m/ν^{FB}) and supply chain efficiency ($E[\Pi_{SC,obs}^m]/E[\Pi^{FB}]$) with the market-based contracting scheme under investment observability. Panel (c) plots the supply chain efficiency ($E[\Pi_{SC}]/E[\Pi^{FB}]$) under wholesale pricing and two-part tariff schemes for the observable and unobservable investment cases. For panel (c), $\sigma_0^2 = 10$. For all panels $c_f = 0.2$, $K_0 = 5$, $c_0 = 0$.

competitive, which by reducing margins cools off retailers' incentives to invest in forecasting and improves supply chain surplus. As can be seen from the figure, the first effect is dominant for low n, i.e., an increasing number of retailers worsens overinvestment and supply chain surplus, and the adverse effect of investment observability is more pronounced for higher demand uncertainty. However, as n increases the second effect catches up, and for large n, an increased number of retailers reduces overinvestment and improves supply chain efficiency. As shown in panel (b), with observability of investment, overinvestment can reach even higher levels with market-based contracting. In addition, the supply chain efficiency can also be very low, especially for a small number of downstream retailers; however, with an increased number of retailers, inefficiency with market-based contracting dramatically decreases. On the other hand, as shown in panel (c) of Figure 2, the effect of an increased number of downstream retailers on supply chain efficiency can be the opposite for the wholesale price and two-part tariff contracts because the efficiency loss from overinvestment is a dominant factor, especially for large n values; and supply chain efficiency decreases with an increased number of downstream retailers.

An important strategic implication for supply chains emerges from these observations. Specifically, decreased observability of retailer demand forecast investments can bring substantial benefits to the supply chain. Therefore, it is recommended that the supply chain enforce the retailers conceal their demand forecast investments. Such enforcement might not always be possible because in certain cases, mandatory factors such as financial disclosure requirements could make these investments visible to outside parties. Furthermore, as we discussed above, the retailers have an intrinsic strategic inclination toward intimidating their competitors to reduce aggressive production by investing more in demand forecasting and disclosing their investments to the rest of the industry. However, our analysis suggests that by suppressing such behavior as a contract condition or by maintaining it as a supply chain practice, channel efficiency can be improved significantly. Another direction is studying contracting schemes that align incentives better in the supply chain, which we explore next.

6.2. Coordination Under Demand Forecast Investment Observability

Given that under forecast investment observability, simpler contracting forms and even market-based pricing can result in significant overinvestment and loss of surplus, the need for a solution that can utilize market information more efficiently under observability becomes evident. To this end, in this section we study a solution approach that employs a powerful market tool, namely uniform-price (divisible good) auctions.

Uniform-price auctions for divisible goods are implemented in practice in many markets, e.g., for federal treasury bill sales (Malvey et al. 1997), and in electricity procurement markets (Green 1999, Wilson 2002). Specifically, in this mechanism the seller submits a supply curve and each buyer submits a demand curve. The demand curves are aggregated and intersected with the supply curve to determine the market clearing price (cf. Wilson 1979). Our proposed contracting scheme utilizes a uniform-price auction to determine a base unit price, and it uses this price together with contractual augmentations to determine the final contract price to be charged to each retailer.

To derive the optimal contracts we first need to derive the equilibrium outcome in the uniform-price auction stage. We start by defining the equilibrium conditions and deriving the retailers' equilibrium bid behavior. Formally, the supplier announces the price function $\mathcal{Y}: \mathbb{R} \to \mathbb{R}$. Her supply curve $\mathcal{Y}(\cdot)$ indicates that at price p, she will supply $\mathcal{Y}(p)$ units in equilibrium. Similarly, each retailer i, $i = 1, \ldots, n$ submits a demand function $Q_i: \mathbb{R}^2 \to \mathbb{R}$, where the curve $Q_i(s_i; \cdot)$ specifies his demand at price p by $Q_i(s_i; p)$. The equilibrium price is set at the level that equates the supply with the aggregate demand. Define the vector of equilibrium demand curves for the retailers by $\mathbf{Q}$, the vector of equilibrium demand curves of the retailers other than i by $\mathbf{Q}_{-i}$, and the equilibrium price by $p_e(\mathbf{Q}, \mathbf{Y})$. Denote the equilibrium allocation for retailer i by $q_i(\mathbf{Q}, \mathcal{Y})$, the total equilibrium production level by $y(\mathbf{Q}, \mathcal{Y})$, and the vector of the quantities that the retailers other than i get in equilibrium by $\mathbf{q}_{-i}(\mathbf{Q}, \mathcal{Y})$, for $i = 1, \ldots, n$. Then, p_e solves

$$\sum_{i=1}^{n} Q_i(s_i; p_e) - \mathcal{Y}(p_e) = 0, \tag{29}$$

and

$$q_i(\mathbf{Q}, \mathcal{Y}) = Q_i(s_i; p_e) \quad \text{and} \quad y(\mathbf{Q}, \mathcal{Y}) = \mathcal{Y}(p_e). \tag{30}$$

A bidding equilibrium among retailers in this uniform-price auction mechanism satisfies the following condition: given the remaining players' trading strategies, each retailer's strategy maximizes his expected profit. That is, for any demand schedule Q_i',

$$\begin{aligned} &\mathrm{E}\big[\Pi_{mi}(q_i(Q_i, \mathbf{Q}_{-i}, \mathcal{Y}), \mathbf{q}_{-i}(Q_i, \mathbf{Q}_{-i}, \mathcal{Y}), p_e(Q_i, \mathbf{Q}_{-i}, \mathcal{Y})) \mid s_i\big] \\ &\quad \geqslant \mathrm{E}\big[\Pi_{mi}(q_i(Q_i', \mathbf{Q}_{-i}, \mathcal{Y}), \mathbf{q}_{-i}(Q_i', \mathbf{Q}_{-i}, \mathcal{Y}), \\ &\qquad\qquad p_e(Q_i', \mathbf{Q}_{-i}, \mathcal{Y})) \mid s_i\big], \end{aligned} \tag{31}$$

for $1 \leqslant i \leqslant n$.

We propose the following contract offer by the supplier. For constants w_0, w_d, β_0, and β_p, the payment charged to retailer i is $P_i(\mathbf{Q}) = w_0 + p_e \cdot q_i - w_d q_i^2$, where p_e and $\mathbf{q}$ are determined from the outcome of the uniform-price auction as described above with the posted supply curve $\mathcal{Y} = b_0 + \beta_p p$. Given the equilibrium as described above, we derive a linear equilibrium of the bidding game under the supplier's posted supply curve. In a linear equilibrium, the retailers' strategies are given as $Q_i(s_i, p) = \alpha_{0i} + \alpha_{si} \cdot (s_i - K_0) + \alpha_{pi} p$. Note that linearity arises endogenously in equilibrium. That is, we do not limit the retailers' strategies to only linear strategies. Rather, we conjecture the existence of a linear Bayesian Nash equilibrium and verify it by deriving it. When verifying this equilibrium, given all the other retailers' conjectured strategies, each retailer's strategy space is unrestricted, and he still optimally chooses to employ a linear strategy. We also focus on equilibria symmetric in α_p, i.e., $\alpha_{pi} = \alpha_p$ for all i. Once again, we do not impose symmetry as a constraint in a retailer's optimization problem, either. We conjecture the existence of a symmetric equilibrium and verify that conjecture as we described above. The equilibrium we find satisfies all requirements of a Bayesian Nash equilibrium.[23]

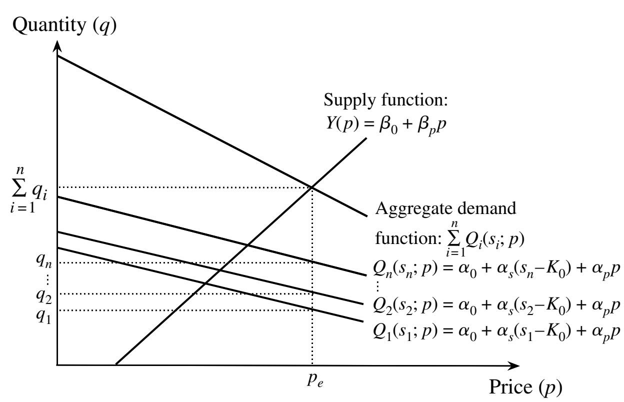
Our proposed mechanism proceeds as follows.

1. Supplier announces her pricing $P_i = w_0 + p_e \cdot q_i - w_d q_i^2$, including the posted supply curve $\mathcal{Y} = \beta_0 + \beta_p p$.
2. Retailers simultaneously invest in demand forecasting.
3. Retailers observe each others' investment levels. Each retailer submits his demand curve, Q_i, without observing his competitors' orders.
4. Retailers' orders are aggregated. The aggregate demand curve, $\sum_{i=1}^{n} Q_i(s_i; p)$, is intersected with the supply curve $\mathcal{Y} = \beta_0 + \beta_p p$. The intersection price gives the auction clearing price, p_e, and the retailers' allocated quantities, $q_1, \ldots, q_n$ are determined. Figure 3 provides a depiction of the auction outcome.
5. Using the clearing price, p_e, and retailers' allocated quantities from the auction outcome, final contract price paid by retailer i is determined, i.e., $P_i = w_0 + p_e \cdot q_i - w_d q_i^2$.

We next derive the equilibrium outcome for this mechanism.

Lemma 3. *Consider a pricing scheme that utilizes the uniform-price auction-based mechanism described above, and retailer i's payment is $P_i(\mathbf{Q}) = w_0 + p_e(\mathbf{Q}, \beta_0 + \beta_p p) q_i(\mathbf{Q}) - w_d q_i^2(\mathbf{Q})$, where the supplier's posted supply curve is $\mathcal{Y} = \beta_0 + \beta_p p$. Then given v, there exists*

Figure 3. Determination of the clearing price, p_e, as well as the retailer quantities $q_1, \ldots, q_n$ through the uniform-price auction.



Note. For simplicity in exposition, the figure illustrates the equilibrium with $s_n > \cdots > s_1$.

a Bayesian equilibrium with strategies $Q_i(s_i, p) = \alpha_{0i} + \alpha_{si}(s_i - K_0) + \alpha_p p$, *where for* $i = 1, \ldots, n$,

$$\alpha_{0i} = \frac{\beta_0}{n} + \frac{K_0 - \beta_0}{2nw_d}, \quad \alpha_{si} = \frac{\nu_i}{2w_d(1 + \sum_j \nu_j)}, \qquad \alpha_p = \frac{\beta_p}{n} - \frac{1 + \beta_p}{2nw_d}. \tag{32}$$

Furthermore, there exists a unique symmetric Nash equilibrium in forecast investment level, where $\nu_i = \nu$ *for all i, and* $\nu \geqslant 0$ *either equals to* 0, *or satisfies*

$$\frac{\sigma_0^2}{4n^2(1+n\nu)^3}\left(\frac{4\beta_p(n-1)(1+n\nu)^2}{1+\beta_p} + \frac{(n-1)(n-1+(n-3)n\nu)}{w_d} + \frac{4w_d\beta_p^2(1+n\nu)}{(1+\beta_p)^2}\right) - C'(\nu) = 0. \tag{33}$$

Utilizing the result of Lemma 3, we can now present the supplier's optimal contract fully coordinating the supply chain both in statewise quantities and forecast investments.

Proposition 7. (i) *With the uniform-price auction-based contract mechanism as described above, the supplier can coordinate both production quantities statewise and the investment in demand forecast by offering the contract with parameters,* $\beta_0 = -c_0$, $\beta_p = 1$, *and*

$$w_d = \frac{n^2 - 2(n-1)(1 + n\nu^{FB})}{2} + \sqrt{\frac{(2(n-1)(1+n\nu^{FB}) - n^2)^2}{4} - \frac{(n-1)(n-1+(n-3)n\nu^{FB})}{1+n\nu^{FB}}}, \tag{34}$$

where ν^{FB} *satisfies* (7).

(ii) *The equilibrium under the contracting scheme given in part* (*i*) *is regret-free.*

Similar to the market-based contracting we presented in §5, the key element of the mechanism for achieving full coordination here is the unit clearing price (governed by the supply curve) that, in equilibrium, aggregates market information and adjusts to it.[24] The mechanism again utilizes the price that dynamically adjusts with the order quantities to keep the retailers' incentives aligned with the supply chain objective. However, the reason the uniform-price auction-based mechanism can achieve coordination under observability while the simpler market-based mechanism cannot is that in the uniform-price auction, the retailers can submit flexible quantity schedules that allow their order quantity to change as the price changes instead of submitting a fixed order quantity. With demand forecast observability, as we discussed in §6.1, each retailer has added incentives to invest. To curb this additional incentive to invest, the parameter w_d in the market-based contract should be adjusted. However, this adjustment can cause a shift in incentives in coordinating the production quantities statewise. With the uniform-price auction, the retailers can submit demand schedules that make the demanded quantity depend on the realization of the price. For any given retailer, the realization of the price, in turn, reflects the aggregated private demand forecast information of his competitors. That is, with the possibility of submitting a continuum of price-quantity pairs instead of simple quantities only, each retailer can make the demand he is submitting *contingent* on the realization of the other retailers' signals. As a result, the supplier now does not have to tie the price movements precisely to eliminate the changes in retailers' expectations in demand realization in order to make them ignore their competitors' forecasts when deciding their quantities as in the market-based contract, i.e., she now has added flexibility in setting w_d. By adjusting that parameter in the uniform-price auction-based contract, she can still coordinate the order quantities statewise while simultaneously curbing the additional retailer incentives to overinvest caused by the observability of demand forecast investments.

Furthermore, this mechanism is also regret-free, i.e., no retailer would want to change his order strategy or allocation after observing the contracting outcome and learning about other retailers' forecasts through diffusion. This property emerges in this case from the retailers' ability to make their order quantities contingent on the price. Because the price aggregates the retailers' demand forecast information and adjusts to it, each realization of price communicates to a retailer the aggregated information in his competitors' demand forecasts. Essentially, a given retailer chooses each quantity point on the demand schedule he submits already conditional on the realization of his competitors' signals, and therefore his submitted strategy is optimal for each signal state realization.

Finally, as in the case for the market-based contracting scheme, the uniform-price auction implementation also achieves truthful revelation of demand forecasts by the retailers with voluntary participation. In summary, the uniform-price auction implementation we suggest is a desirable mechanism that manages to achieve information aggregation and supply chain coordination effectively, including cases when demand forecast investments are observable.

7. Concluding Remarks

In this paper we analyzed a phenomenon that can significantly undermine supply chain performance. Specifically, we showed that in a decentralized supply chain under common contracting schemes, such as wholesale price contracting and two-part tariffs, competition among downstream parties causes overinvestment in demand forecasting. We also studied the extent of resulting inefficiencies, demonstrating that overinvestment in demand forecasting and the consequent supply chain surplus loss can be significant. We showed that certain practically implementable contracts in the class of market-based contracts can coordinate the channel, eliminating overinvestment in demand forecasting. Finally, with forecast investment observability, overinvestment is amplified and simple market-index contracting schemes are no longer sufficient to coordinate the channel. We found that, in this case, a contracting mechanism that employs a uniform-price auction can coordinate the supply chain utilizing clearing price to disseminate the market information.

The sequence of contract offer and forecast investment we use in this paper is commonly used in models of supply chain contracting with demand forecast investments (Lariviere 2002, Taylor and Xiao 2009). Note that retailers' demand forecasting might also come earlier than the supplier's contract offer. In our case, the results are essentially robust to this change in sequence. Suppose the contract offer was made after the forecast investment. For coordinating market-based contracts in equilibrium, the retailers' investment strategies should be optimal given the contract offer, no matter what the order of actions is. Given that market-based contracts achieve the maximum possible expected supply chain profit, taking the retailers' strategies as given, the supplier's contract offer is optimal for her as well. Therefore, the equilibrium strategy profiles are preserved even if the contract offer is made after forecast investments. When the contracts are not coordinating (for the wholesale price or two-part tariff contracts), the results would still hold under the mild assumption that the contract structure is common knowledge, which is essentially true for virtually all procurement cases: the standard contract structure for the agreements is almost always known by both buyers and sellers even if the parameters within the contract structures change. If a seller decides to change the standards of the contract structure significantly (e.g., by introducing franchise fees, etc.), she normally informs her trading partners, and the parties come to an agreement about this important change in the long term.

In our model we analyzed the case of perfectly substitutable retailer products. An extension of our model could study the case where products are differentiated. In such a case, the types of misalignment caused by downstream competition would continue to exist, but their intensity would change depending on the degree of substitutability. If the product substitutability is very low, there can actually be underinvestment in retailers' demand forecasting rather than overinvestment. Furthermore, the market-based and uniform-price auction-based schemes can no longer coordinate the supply chain. Rather, with differentiated products, a modified, more complex version of a uniform-price auction might help achieve coordination. Extending our analysis to the case of imperfectly substitutable products and exploring supply chain coordination in such a setting would be an interesting future research direction.

We also focused on quantity competition among the retailers. Alternatively, one could explore downstream price competition, possibly with imperfectly substitutable products. In that case (as we also mention in endnote 8) all the misaligning effects of downstream competition would again still be present, albeit they would be adapted for price competition. Specifically, the overproduction effect would be replaced by an analogous "under-pricing" effect, where each retailer would price lower than the first-best optimal price for his product. The misalignment caused by decision making under private information would still be present because each retailer would use his demand signal to estimate the demand intercept as well as his competitors' equilibrium prices and would still not internalize the impact of his reaction to his signal on other retailers. Finally, the misalignment in investments would again be present because the marginal value of an increase in the precision of a retailer's signal would be different than that for the centralized solution. However, as we also discussed above, for the differentiated quantity competition, whether this effect leads to overinvestment or underinvestment could depend on the degree of substitutability of the retailers' products. The effect of forecast investment observability in boosting the incentives to forecast would also still be present in, again, an analog manner: observed higher forecast precision by a retailer would induce other retailers to adjust their pricing. Similar to the effect of observability for quantity competition, as we discussed in §6.1, the gains of showing the competitors a higher demand forecast investment would benefit a retailer more in the high-demand states than it would hurt in the low-demand states. In the balance there would be net extra benefits of investing in forecasting for a retailer, on average, when competitors can observe his demand forecast investments. Overall, exploring the effect of downstream price competition on demand forecast investments, determinants of inefficiency, observability, and coordination could also be an interesting future research avenue.

Another potential extension direction could be exploring the effect of forecast investment decisions in a supply chain setting with direct information sharing. One may expand the direct information sharing setting (see, e.g., Li 2002) with retailer demand forecast investments preceding information sharing and determination of the wholesale price by the supplier. Exploring the retailer incentives in forecast investments and supply chain efficiency in that case could be an interesting future research topic as well.

Every year, companies invest millions of dollars in software, personnel, and resources to accurately predict demand. Our analysis demonstrates that significant overinvestment in demand forecasting can often occur, resulting in substantial supply chain inefficiency. In addition, our results suggest that contracting schemes designed to be more sensitive to the market's pulse by utilizing explicit market mechanisms can yield significant gains. Consideration of such schemes by supply chain partners can ultimately help reduce inefficiencies in the supply chain and contribute to improving savings and performance.

8. Electronic Companion

An electronic companion to this paper is available as part of the online version that can be found at http://or.journal.informs.org/.

Endnotes

1. The downstream firms can be retailers or manufacturers, and the upstream firm can be a supplier or a manufacturer. Throughout the paper, for ease of exposition we refer to the downstream firms as retailers and the upstream firm as a supplier. We also refer to the upstream firm as female and the downstream firms as male.
2. This is the standard downward sloping Cournot inverse demand function with the slope coefficient normalized to 1 (see, e.g., Shapiro 1986, Li 2002). That is, the retailers' products are substitutes, and the increase in quantity for one of them decreases the consumer price for all retailers.
3. This is a common normalization in the literature (see, e.g., Schmalensee and Willig 1989). Our main results would be preserved if the retailers' reservation values were positive and identical.
4. For notational convenience, if retailer i decides not to contract at all, we set $q_i = 0$ by default. Note that for the wholesale price contract a retailer's ordering a zero quantity at this stage essentially means he is refusing the contract. However, for the two-part tariff scheme we study in §4, a retailer's participating in the contract requires a fixed payment. Also note that in equilibrium, order quantities are nonzero almost surely.
5. Note that this cost structure covers cases where there is a maximum level that the retailer's signal precision cannot exceed, no matter how much is invested in improving forecast accuracy (i.e., cases where the cost function vertically asymptotes to infinity at a certain $\nu > 0$). Furthermore, it also covers cases where the retailers are uncertain whether the forecast systems they invest will pay off, or more generally if they are uncertain about the precision of the information they will obtain from their systems. Specifically, in that case the ex-ante signal distribution could be viewed as the marginal distribution, i.e., the signal p.d.f. is the expectation of the probability density conditional on the signal precision realization.
6. By having many separate (and not perfectly correlated) signals with low accuracy, one can achieve the accuracy level of a single, more accurate signal because each separate signal would bring a new observation and a component of new observation of the underlying random variable. See, e.g., Ericson (1969) for the precise mathematical expressions for increase in forecast accuracy of an information set with multiple signals as the number of signals increases.
7. Note that although when cost functions are convex it is optimal to spread the investments equally among the available cost functions as much as possible, this spreading cannot be done indefinitely, considering that the number of available entities or teams that are set and readily equipped to make forecasts is normally finite. Even if one tried to split the forecasting tasks that one team could perform, at some point, the cost of further splitting forecasting tasks would exceed the benefits such a split could bring. In that sense, the convex cost functions we employ represent the best forecasting capabilities of the firms after all such possibilities are taken into consideration.
8. It should be noted that in our discussion for challenges here and in the following sections we provide the perspective of *quantity* competition among the retailers, which we study in this paper. Alternatively, one can explore the implications of downstream *price* competition (see, e.g., Gal-Or 1986 and Li and Zhang 2008 for modeling of downstream price competition with private demand information). In that setting, all the misaligning effects of downstream competition we discussed above would still be present with adjustments to the framework of price competition. See the concluding remarks for further discussion on this point.
9. The contract coefficients in that case would be

$$w_0^{tpt} = \left(\frac{K_0 - c_0}{2n}\right)^2 \quad \text{and} \quad w_1^{tpt} = \frac{(n-1)K_0 + (n+1)c_0}{2n}.$$

10. Note that quadratic contracts in the form of $P_i(\mathbf{q}) = w_0 + w_1 q_i + w_2 q_i^2$ can coordinate the supply chain in this setting with private demand information when there is no investment in demand forecasting. However, such contracts cannot coordinate demand forecast investments even when they eliminate the misalignment due to information usage. That is, in general, solving information misalignment in the supply chain does not automatically solve the misalignment in forecast investments. We provide a brief analysis and discussion of this contract structure in Section C of the online supplement.
11. It should be noted that the more substitutable the competitors' products are, the higher the negative cross-impact of their order quantities on each other, and hence the bigger the difference between their reaction to their signals and that of the centralized solution. When the products are imperfectly substitutable, under quantity or price competition, the overinvestment effect can soften; and for sufficiently differentiated products, even underinvestment can emerge. Please see the concluding remarks for further discussion on these alternative model settings.
12. Note that because our model is an abstraction of reality, the precise expressions for bounds on overinvestment and channel efficiency we derive in this section (as well as elsewhere in the paper) are dependent on the model assumptions we employ such as the linearity of consumer demand, additive demand uncertainty and the nature of the retailer competition. The reader should keep in mind that

obtaining conceptual insights from this analysis is our main intent here rather than the exact mathematical expressions we present for these bounds.

13. To avoid trivialities, throughout this section we focus on the case where forecast investment coordination is relevant, i.e., $C'(0) < \sigma_0^2/4$, and $n \geqslant 2$.

14. For certain conjugate signal-noise distribution pairs that satisfy the affine conditional expectations property, the variance of the demand intercept, σ_0^2, needs to be small enough so that the demand and the resulting consumer price remain positive in almost all state realizations (e.g., "normal-normal" conjugate distribution pair). In such cases, the limit $\sigma_0^2 \to \infty$ does not fit well with the model. However, for other such conjugate pairs (e.g., "gamma-Poisson"), taking σ_0^2 to infinity in the limit can be achieved without disturbing the positivity of the demand and price outcomes in our model.

15. These effects are also illustrated in panel (c) of Figure 2 in §6.1.

16. See Golombek et al. (1987), Priddle (1998), MacAvoy (2000) for details on the use of the Henry Hub Price in natural gas contracts.

17. We thank Dennis Rohan of Kimball Resources Inc. for valuable information on natural gas procurement contracts in the United States.

18. We thank Hau Lee and Jin Whang for providing this example.

19. See, e.g., Cremer and McLean (1985) and Klemperer and Meyer (1989) for extensive discussions of this issue.

20. Note that in certain cases the retailers can regret having entered the contracting agreement after they pay the fixed fee if they get a low demand signal. What we mean by being regret-free here is retailers still finding their quantities optimal after observing the realization of the price and hence inferring other retailers' private information, i.e., not regretting their quantity decisions. Also note that even though a retailer might regret having committed to the contract after receiving a particularly low demand signal, it is still optimal for him to order the same quantity in equilibrium at that time point because the fixed cost is then a sunk cost and his order quantity maximizes his overall expected payoff. That is, the retailers' actions are still dynamically optimal at each stage of the game.

21. For full details of this argument, please see Section B of the online supplement, which includes a numerical demonstration.

22. Panel (a) of Figure 2 presents the plots for the two-part tariff contracts. The plots are similar for the wholesale price contracts and are thus omitted.

23. Note that there could be Nash equilibria with asymmetric α_p values as well. However, because as in many models that study equilibrium in environments with private correlated information, when price coefficients, α_p, are not symmetric, tractability of the analysis is lost, and it is not possible to know the existence of or derive equilibria asymmetric in α_p. Therefore, as in most such studies, we focus on the symmetric equilibrium in the analysis (cf. O'Hara 1997).

24. The market-based contracting scheme we presented in §5 can also be implemented through uniform-price auction. In that case, each retailer submits a constant order quantity as his demand curve. That is, Retailer i's submitted order quantity is of the form, $Q_i(s_i) = \alpha_0 + \alpha_s(s_i - \mathring{a}_0)$, and the retailer demand curves as well as the aggregate demand curve in Figure 3 become flat for the market-based contract implementation.

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Online Supplement for
Do Firms Invest in Forecasting Efficiently?
The Effect of Competition on Demand Forecast Investments
and Supply Chain Coordination

Hyoduk Shin and Tunay I. Tunca

A Proofs of Propositions

Proof of Lemma 1: First, given $\mathbf{s} = (s_1, \ldots, s_n)$ and $\mathbf{v} = (\nu_1, \ldots, \nu_n)$, by (3),

$$\mathrm{E}\left[\Pi^{FB}|\mathbf{s},\mathbf{v}\right] = \left(\mathrm{E}\left[K|\mathbf{s},\mathbf{v}\right] - Q - c_0\right) Q - \sum_{i=1}^{n} C(\nu_i). \tag{A.1}$$

Since K and $\mathbf{s}$ satisfy affine conditional expectations property, by Ericson (1969),

$$\mathrm{E}\left[K|\mathbf{s},\mathbf{v}\right] = K_0 + \frac{\sum_{i=1}^{n} \nu_i (s_i - K_0)}{1 + \sum_{i=1}^{n} \nu_i}. \tag{A.2}$$

Taking the first derivative with respect to Q in (A.1), we have $Q = \left(\mathrm{E}\left[K|\mathbf{s},\mathbf{v}\right] - c_0\right)/2$. Plugging in (A.2) we obtain (6). Second, substituting (6) into (A.1), we have

$$\mathrm{E}\left[\Pi^{FB}\right] = \mathrm{E}\left[\mathrm{E}\left[\Pi^{FB}|\mathbf{s}\right]\right] = \frac{1}{4}\left((K_0 - c_0)^2 + \frac{\sigma_0^2 \sum_{i=1}^{n} \nu_i}{1 + \sum_{i=1}^{n} \nu_i}\right) - \sum_{i=1}^{n} C(\nu_i)\,. \tag{A.3}$$

Now, observe that the first term in (A.3) only depends on the sum of ν_i's. Suppose that there exists $i \neq j$, such that in the optimum $\nu_i \neq \nu_j$. Then replacing ν_i and ν_j with ν_i' and ν_j', respectively, where $\nu_i' = \nu_j' = \nu' = (\nu_i + \nu_j)/2$ and keeping ν_k constant for all $k \neq i, j$, $\sum_{i=1}^{n} \nu_i$ remains unchanged. If $C(\cdot)$ is strictly convex, then $C(\nu_i') + C(\nu_j') < C(\nu_i) + C(\nu_j)$. Therefore in the optimum, $\nu_i = \nu_j$ for all i, j. If $C(\cdot)$ is linear, then there is a continuum of optima and the symmetric solution is one of them. Thus, taking the first derivative of (A.3) and plugging in the symmetry, we obtain (8). Now, since C is convex, non-decreasing and non-identically zero, the left hand side of (8) is decreasing in ν, and will be strictly negative as $\nu \to \infty$. Hence if $C'(0) < \sigma_0^2/4$, there exists a unique $\nu > 0$ that satisfies (8). If $C'(0) \geq \sigma_0^2/4$ on the other hand, $\nu = 0$ will be the optimal ν. Thus (7) holds and the concavity of the objective function guarantees the optimality. □

Proof of Lemma 2: Let $q_j(s_j) = \alpha_{0j} + \alpha_{sj}\,(s_j - K_0)$, $\alpha_{0j}, \alpha_{sj} \in \mathbb{R}$ and $\nu_j \in \mathbb{R}_+$, for all $j \neq i$. Taking expectation given s_i, we obtain expected profit for retailer i after observing s_i

$$\mathrm{E}\left[\Pi_i^{tpt}|s_i\right] = q_i\Big(K_0 - w_1 + \frac{\nu_i\,(s_i - K_0)}{1+\nu_i} - q_i - \sum_{j\neq i}(\alpha_{0j} + \alpha_{sj}\mathrm{E}\,[s_j - K_0|s_i])\Big) - C(\nu_i) - w_0\,. \tag{A.4}$$

Note that the second order condition for q_i is satisfied. By $\mathrm{E}\,[s_j - K_0|s_i] = \nu_i/(1+\nu_i)\,(s_i - K_0)$, the first order condition for q_i is written as

$$q_i = \frac{1}{2}(K_0 - w_1 - \sum_{j\neq i}\alpha_{0j} + \frac{\nu_i}{1+\nu_i}(1 - \sum_{j\neq i}\alpha_{sj})\,(s_i - K_0)). \tag{A.5}$$

Observe that q_i is linear in $s_i - K_0$. By Substituting (A.5) into (A.4) and taking expectation, it follows that

$$\mathrm{E}\left[\Pi_i^{tpt}\right] = \frac{1}{4}(K_0 - w_1 - \sum_{j\neq i}\alpha_{0j})^2 + \frac{\sigma_0^2\,\nu_i}{4(1+\nu_i)}(1 - \sum_{j\neq i}\alpha_{sj})^2 - C(\nu_i) - w_0. \tag{A.6}$$

The first order condition for ν_i from (A.6) is $\frac{\sigma_0^2}{4(1+\nu_i)^2}\left(1 - \sum_{j\neq i}\alpha_{sj}\right)^2 - C'(\nu_i) = 0$, and the second order condition, $-\frac{\sigma_0^2}{2\,(1+\nu_i)^3}\left(1 - \sum_{j\neq i}\alpha_{sj}\right)^2 - C''(\nu_i) < 0$ is satisfied. From (A.5) and the first order condition of ν_i, we have

$$\alpha_{0i} = \frac{1}{2}(K_0 - w_1 - \sum_{j\neq i}\alpha_{0j})\,, \quad \alpha_{si} = \frac{1}{2}(\frac{\nu_i}{1+\nu_i}\,(1 - \sum_{j\neq i}\alpha_{sj}))\,, \tag{A.7}$$

and

$$C'(\nu_i) = \frac{\sigma_0^2}{4\,(1+\nu_i)^2}\,(1 - \sum_{j\neq i}\alpha_{sj})^2. \tag{A.8}$$

By summing over α_{0i} for all i, we have $\sum_{i=1}^{n}\alpha_{0i} = n(K_0 - w_1)/(n+1)$. Substituting this into (A.7) and simplifying, we obtain α_0^q. From (A.7), it follows

$$\alpha_{si} = \frac{\nu_i(1 - \sum_{j=1}^{n}\alpha_{sj})}{2+\nu_i}\,. \tag{A.9}$$

Plugging (A.9) into (A.8), we have

$$C'(\nu_i) = \frac{\sigma_0^2\left(1 - \sum_{j=1}^{n}\alpha_{sj}\right)^2}{(2+\nu_i)^2}\,, \tag{A.10}$$

for all i. Observe that since (A.10) holds for all i, the first order condition for all ν_i is identical. Further, since the difference between the two sides of (A.10) is strictly monotonic for $\nu_i > 0$, it can have at most one solution in ν_i. Thus, $\nu_i = \nu$, for some $\nu \geq 0$, for all i. Adding up (A.9) for all i, and plugging in $\nu_i = \nu$, we then have

$$\sum_{i=1}^{n}\alpha_{si} = \frac{n\nu}{2 + (n+1)\nu}\,. \tag{A.11}$$

Substituting (A.11) into (A.7) and simplifying, we obtain α_s^{tpt}. Finally, substituting (A.11) into (A.10), we obtain (9). Since C is convex, non-decreasing and non-identically zero, the left hand side of (9) is decreasing in ν, and becomes strictly negative as $\nu \to \infty$. Consequently, there exists a unique $\nu \geq 0$ that satisfies (9) if and only if $C'(0) < \sigma_0^2/4$, with $\nu^{tpt} = 0$ otherwise. This confirms ν^{tpt} and completes the proof. □

Proof of Proposition 1: Under a wholesale price contract, $w_0 = 0$ and $w_1 = w^{ws}$. Then, by Lemma 2, plugging in α_0^{tpt} and α_s^{tpt}, and summing up over all i, the equilibrium total order quantity is

$$Q^{ws}(\mathbf{s}) = \frac{n(K_0 - w^{ws})}{n+1} + \frac{\nu^{ws}}{2 + (n+1)\nu^{ws}} \sum_{i=1}^{n} (s_i - K_0) \,. \tag{A.12}$$

Plugging (A.12) in, the supplier's expected profit is

$$\mathrm{E}\left[\Pi_S^{ws}\right] = \mathrm{E}\left[(w^{ws} - c_0)\, Q^{ws}\right] = \frac{n(w^{ws} - c_0)\,(K_0 - w^{ws})}{n+1} \,. \tag{A.13}$$

It follows that $w^{ws} = (K_0 + c_0)/2$ maximizes the supplier's expected profit. For a two-part tariff contract, noticing that the participation constraint for each retailer must be binding, supplier's optimal w_0 has to equal the expected profit for each retailer. Then by Lemma 2, calculating the expected retailer profit and plugging back in $\mathrm{E}[\Pi_S^{tpt}]$, we obtain expected supplier profit as a function of w_1^{tpt} as

$$\mathrm{E}\left[\Pi_S^{tpt}\right] = \frac{n\,(K_0 - w_1^{tpt})}{n+1} \left((K_0 - c_0) - \frac{n\,(K_0 - w_1^{tpt})}{n+1} \right) + \frac{n\nu^{tpt}(1 + \nu^{tpt})\,\sigma_0^2}{(2 + (n+1)\nu^{tpt})^2} - n\, C(\nu^{tpt}) \,. \tag{A.14}$$

The first order condition for w_1^{tpt} gives (11) and is sufficient for optimality since (A.14) is concave in w_1^{tpt}. Further, (10) follows by plugging (11) back in α_0^{tpt} and α_s^{tpt} and equating expected retailer profit to the reservation value zero. This proves part (*i*).

For part (*ii*), from (9), notice that since

$$\left(\frac{1}{1 + \frac{n+1}{2}\nu} \right)^2 > \left(\frac{1}{1 + n\nu} \right)^2 , \tag{A.15}$$

for all $\nu > 0$, the left hand side of (9) is always greater than the left hand side of (8). Further, the left hand side of both equations are strictly decreasing in ν. It follows that if $\nu^{FB} > 0$, i.e., when $C'(0) < \sigma_0^2/4$, then $\nu^{ws} = \nu^{tpt} > \nu^{FB}$. In addition, by (7) and ν^{tpt}, $\nu^{tpt} = 0$ if and only if $\nu^{FB} = 0$. This completes the proof. ■

Proof of Proposition 2: For part (*i*), since, by Lemma 2, $\nu^{ws} = \nu^{tpt}$, we will only give the proof for ν^{ws}. Let $G(\nu^{ws}) = \sqrt{C'(\nu^{FB})/C'(\nu^{ws})}$. From (8) and (9), we have

$$\frac{\nu^{ws}}{\nu^{FB}} = \frac{2}{n+1} \left(n\, G(\nu^{ws}) + \frac{G(\nu^{ws}) - 1}{\nu^{FB}} \right) . \tag{A.16}$$

Since (A.16) is increasing in $G(\nu^{ws})$, and since by Proposition 1, $0 \le G(\nu^{ws}) \le 1$, ν^{ws}/ν^{FB} is maximized at $G(\nu^{ws}) = 1$, which is attainable for $C(\nu) = c_f \nu$, $c_f > 0$. Plugging in (A.16), we obtain the upper bound given in (12).

For part (ii), let $c_f > 0$ be given and let $C(\nu) = c_f \cdot \nu$. Then by (9), we obtain

$$\nu^{ws} = \nu^{tpt} = \frac{2}{n+1}\left(\sqrt{\frac{\sigma_0^2}{4c_f}} - 1\right). \tag{A.17}$$

By (5), (8), (A.17), Lemma 2 and Proposition 1, and taking expectations we obtain

$$\frac{\mathrm{E}\left[\Pi_{SC}^{ws}\right]}{\mathrm{E}\left[\Pi^{FB}\right]} = \frac{n\left((n+2)(K_0-c_0)^2 + 4\left(\sigma_0^2 + 4c_f - 4\sqrt{c_f\sigma_0^2}\right)\right)}{(n+1)^2\left((K_0-c_0)^2 + \sigma_0^2 + 4c_f - 4\sqrt{c_f\sigma_0^2}\right)}, \tag{A.18}$$

and

$$\frac{\mathrm{E}[\Pi_{SC}^{tpt}]}{\mathrm{E}\left[\Pi^{FB}\right]} = \frac{(n+1)^2(K_0-c_0)^2 + 4n\left(\sigma_0^2 + 4c_f - 4\sqrt{c_f\sigma_0^2}\right)}{(n+1)^2\left((K_0-c_0)^2 + \sigma_0^2 + 4c_f - 4\sqrt{c_f\sigma_0^2}\right)}. \tag{A.19}$$

Let $\{\mathbf{v}_k\} \subset \mathbb{N}_+ \times \mathbb{R}_+^3$ be a sequence of parameter vectors such that $\lim_{k\to\infty} n(k) = \infty$, and

$$\lim_{k\to\infty} \frac{K_0(k) - c_0(k)}{\sqrt{\sigma_0^2(k)} - 2\sqrt{c_f}} = 0. \tag{A.20}$$

Then by (A.18) and (A.19), we have

$$\lim_{k\to\infty} \frac{\mathrm{E}\left[\Pi_{SC}^{ws}\right]}{\mathrm{E}\left[\Pi^{FB}\right]} = \lim_{k\to\infty} \frac{\mathrm{E}[\Pi_{SC}^{tpt}]}{\mathrm{E}\left[\Pi^{FB}\right]} = 0. \tag{A.21}$$

For the upper bound of profit ratio under two-part tariff scheme, note that as $\sigma_0^2/4 \to (C'(0))^+$, $\nu^{FB} \to 0$ from (7) and (8). Further, plugging in (11), we can see that the optimal two-part tariff scheme achieves the first-best supply chain profit in the limit as $\nu^{FB} \to 0$. For the upper bound for the wholesale price scheme, observe that as $n \to \infty$, $w_1^{tpt} = \left((n-1)K_0 + (n+1)c_0\right)/2n \to (K_0 + c_0)/2 = w^{ws}$. Hence as $n \to \infty$, $\mathrm{E}[\Pi_{SC}^{ws}] \to \mathrm{E}[\Pi_{SC}^{tpt}]$, and the upper bound result for the wholesale price scheme follows similar to that for the two-part tariff scheme. This completes the proof. ■

Proof of Proposition 3: For the wholesale price contracts, taking partial derivative of (A.18) with respect to $(K_0 - c_0)^2$, we obtain

$$\frac{\partial}{\partial (K_0-c_0)^2}\left(\frac{\mathrm{E}\left[\Pi_{SC}^{ws}\right]}{\mathrm{E}\left[\Pi^{FB}\right]}\right) = \frac{n(n-2)\left(\sigma_0^2 + 4c_f - 4\sqrt{c_f\sigma_0^2}\right)}{(n+1)^2\left((K_0-c_0)^2 + \sigma_0^2 + 4c_f - 4\sqrt{c_f\sigma_0^2}\right)^2} \ge 0. \tag{A.22}$$

Hence $\mathrm{E}[\Pi_{SC}^{ws}]/\mathrm{E}\left[\Pi^{FB}\right]$ is increasing in K_0 and decreasing in c_0. Note that $\sigma_0^2+4c_f-4\sqrt{c_f\sigma_0^2}=\left(\sqrt{\sigma_0^2}-2\sqrt{c_f}\right)^2$, which is increasing in σ_0^2 and decreasing in c_f. By taking partial derivative of (A.18) with respect to $\left(\sqrt{\sigma_0^2}-2\sqrt{c_f}\right)^2$, we have

$$\frac{\partial}{\partial\left(\sqrt{\sigma_0^2}-2\sqrt{c_f}\right)^2}\left(\frac{\mathrm{E}[\Pi_{SC}^{ws}]}{\mathrm{E}[\Pi^{FB}]}\right)=-\frac{n(n-2)(K_0-c_0)^2}{(n+1)^2\left((K_0-c_0)^2+\sigma_0^2+4c_f-4\sqrt{c_f\sigma_0^2}\right)^2}\leq 0\,. \tag{A.23}$$

Therefore $\mathrm{E}[\Pi_{SC}^{ws}]/\mathrm{E}\left[\Pi^{FB}\right]$ is increasing in c_f and decreasing in σ_0^2. Similarly, it follows that

$$\frac{\partial}{\partial n}\left(\frac{\mathrm{E}[\Pi_{SC}^{ws}]}{\mathrm{E}[\Pi^{FB}]}\right)=\frac{2\left((K_0-c_0)^2-2(n-1)\left(\sigma_0^2+4c_f-4\sqrt{c_f\sigma_0^2}\right)\right)}{(n+1)^3\left((K_0-c_0)^2+\sigma_0^2+4c_f-4\sqrt{c_f\sigma_0^2}\right)}, \tag{A.24}$$

which is positive if $n\leq 1+(K_0-c_0)^2/2\left(\sigma_0^2+4c_f-4\sqrt{c_f\sigma_0^2}\right)$ and is negative otherwise. The proof for the two-part tariff scheme is similar. Lastly, taking limits for n to infinity ($n\to\infty$) in (A.18) and (A.19), we obtain

$$\lim_{n\to\infty}\frac{\mathrm{E}[\Pi_{SC}^{tpt}]}{\mathrm{E}[\Pi^{FB}]}=\lim_{n\to\infty}\frac{\mathrm{E}[\Pi_{SC}^{ws}]}{\mathrm{E}[\Pi^{FB}]}=\frac{(K_0-c_0)^2}{(K_0-c_0)^2+\sigma_0^2+4c_f-4\sqrt{c_f\sigma_0^2}}\,. \tag{A.25}$$

This completes the proof. ■

Proof of Proposition 6: To prove Proposition 6, we first present two lemmas:

Lemma A.1 *When demand forecast investment levels are observable, given the class of contracts $P_i(\mathbf{q})=w_0+w_1\,q_i$, there exists a unique subgame perfect equilibrium in retailer order quantities and demand forecast investment levels. In equilibrium $q_i(s_i,\mathbf{v})=\alpha_0^{tpt}+\alpha_{si}^{tpt}(\nu)\,(s_i-K_0)$ for all i, where*

$$\alpha_0^{tpt}=\frac{K_0-w_1}{n+1}\,,\quad \alpha_{si}^{tpt}(\mathbf{v})=\frac{\delta_i}{1+\sum_j\delta_j}\,, \tag{A.26}$$

and $\nu_i^{tpt}=\nu^*\cdot 1_{\{C'(0)<\sigma_0^2/4\}}$ where $\delta_i=\lambda_i/(2-\lambda_i)$, $\lambda_i=\nu_i/(1+\nu_i)$ and $\nu^*>0$ is the unique solution to the equation

$$\frac{2(1+\nu)\,(2+\nu)+(n-1)\nu\,(2+3\nu)}{(2+(n+1)\nu)^3\,(2+\nu)}-\frac{C'(\nu)}{\sigma_0^2}=0\,. \tag{A.27}$$

Proof: This proof follows the same steps in Li et al. (1987). First, given w_0, w_1, and $\mathbf{v}$, we have

$$\mathrm{E}\left[\Pi_i^{tpt}|s_i,\mathbf{v}\right]=q_i(K_0-w_1+\lambda_i(s_i-K_0)-q_i-\sum_{j\neq i}\mathrm{E}[q_j|s_i,\mathbf{v}])-C(\nu_i)-w_0\,. \tag{A.28}$$

Note that (A.28) is concave in q_i. The first order condition for q_i from (A.28) is written as

$$q_i = \frac{1}{2}\left(K_0 - w_1 + \lambda_i(s_i - K_0)\right) - \frac{1}{2}\sum_{j\neq i} \mathrm{E}[q_j|s_i, \mathbf{v}]\,, \tag{A.29}$$

which can be rewritten as

$$2\left(q_i - (\alpha_{0i} + \alpha_{si}(s_i - K_0))\right) = K_0 - w_1 + \lambda_i(s_i - K_0) - \sum_{j\neq i} \mathrm{E}[q_j|s_i, \mathbf{v}] - 2\left(\alpha_{0i} + \alpha_{si}(s_i - K_0)\right)\,. \tag{A.30}$$

From (A.26), we obtain

$$\alpha_{0i} = \frac{1}{2}(K_0 - w_1 - \sum_{j\neq i}\alpha_{0j})\,, \quad \text{and} \quad \alpha_{si} = \frac{\lambda_i}{2}(1 - \sum_{j\neq i}\alpha_{sj})\,. \tag{A.31}$$

Substituting (A.31) into (A.30) and using $\mathrm{E}[s_j - K_0|s_i] = \lambda_i(s_i - K_0)$, it follows that

$$2(q_i - (\alpha_{0i} + \alpha_{si}(s_i - K_0))) = -\sum_{j\neq i} \mathrm{E}[q_j - (\alpha_{0j} + \alpha_{sj}(s_j - K_0))|s_i, \mathbf{v}]\,. \tag{A.32}$$

Let $V_i(s_i) = q_i - (\alpha_{0i} + \alpha_{si}(s_i - K_0))$. Multiplying $V_i(s_i)$ to both sides of (A.32), taking expectation, and summing over all i, we get

$$\sum_{i=1}^{n} \mathrm{E}[V_i(s_i)^2] = -\sum_{i=1}^{n}\sum_{j=1}^{n} \mathrm{E}[V_i(s_i)V_j(s_j)]\,. \tag{A.33}$$

Note that $\mathrm{E}[V_i(s_i)V_j(s_j)]$ is an element in the variance covariance matrix of the random vector $V(\mathbf{s}) = (V_1(s_1), \ldots, V_n(s_n))$, which is positive semi-definite. Hence, (A.33) implies $\mathrm{E}[V_i(s_i)^2] = 0$, i.e. $q_i = \alpha_{0i} + \alpha_{si}(s_i - K_0)$ almost surely. Substituting $q_i = \alpha_{0i} + \alpha_{si}(s_i - K_0)$ into (A.28) and taking expectation, we have

$$\mathrm{E}\left[\Pi_i^{tpt}\right] = \frac{(K_0 - w_1)^2}{(n+1)^2} + \frac{\lambda_i \sigma_0^2}{\left(2 + (2 - \lambda_i)\sum_{j\neq i}\delta_j\right)^2} - C(\nu_i) - w_0\,, \tag{A.34}$$

where $\delta_i = \lambda_i/(2 - \lambda_i)$. Let $\Delta_{-i} = \sum_{j\neq i}\delta_j$. We find that

$$\frac{\partial \mathrm{E}\left[\Pi_i^{tpt}\right]}{\partial \nu_i} = \frac{\sigma_0^2\left(2 + (2 + \lambda_i)\Delta_{-i}\right)}{\left(2 + (2 - \lambda_i)\Delta_{-i}\right)^3(1 + \nu_i)^2} - C'(\nu_i)\,, \tag{A.35}$$

$$\frac{\partial^2 \mathrm{E}\left[\Pi_i^{tpt}\right]}{\partial \nu_i^2} = -\frac{2\sigma_0^2\left(4(1 + \Delta_{-i})^2(1 + \nu_i) - 4\Delta_{-i}(1 + \Delta_{-i}) - \Delta_{-i}^2\nu_i\right)}{\left(\Delta_{-i}\nu_i - 2(1 + \Delta_{-i})(1 + \nu_i)\right)^4} - C''(\nu_i) < 0\,. \tag{A.36}$$

Notice that (A.35) and (A.36) show that the game of investment in demand forecasting is concave, continuous and symmetric. Note that for $k \neq i$,

$$\frac{\partial^2 \mathrm{E}\left[\Pi_i^{tpt}\right]}{\partial \nu_i \partial \nu_k} = -\frac{4\sigma_0^2\left(4(1+\nu_i)+(4+8\nu_i+3\nu_i^2)\Delta_{-i}\right)}{(2+\nu_k)^2\left(\nu_i\Delta_{-i}-2(1+\nu_i)(1+\Delta_{-i})\right)^4} < 0\,, \tag{A.37}$$

and hence this game has a unique symmetric Nash equilibrium. Let $\Delta = \sum_j \delta_j$ and define

$$G(\tau,\Delta) = \frac{\sigma_0^2\left(4(1+\Delta)(1+\tau)^2 - 4\tau(1+\tau) - (1+\Delta)\tau^2\right)}{(1+\Delta)^3\left(2(1+\tau)-\tau\right)^4} - C'(\tau)\,. \tag{A.38}$$

Differentiating with respect to τ, we have

$$\frac{\partial G}{\partial \tau} = \frac{-2\sigma_0^2\left(\left(3\Delta-1\right)\tau^2 + 2\left(3\Delta+4\right)\tau + 4\right)}{(1+\Delta)^3\left(2+\tau\right)^5} - C''(\tau)\,. \tag{A.39}$$

Note that if $\Delta \geq 1/3$, $\partial G/\partial\tau < 0$. Further if $\Delta < 1/3$, $(3\Delta-1)\tau^2+2(3\Delta+4)\tau+4$ in the numerator, which is the second order polynomial with respect to τ, has only one positive root. In this case, by substituting $\tau = \nu_i$ and using $\Delta = \sum_j \delta_j$, we obtain $(3\Delta-1)\nu_i^2 + 2(3\Delta+4)\nu_i + 4 \geq 0$ for all i. Therefore $\partial G/\partial\tau < 0$ for $\tau \in [0, \max(\nu_i)]$. Note that $G(\nu_i,\Delta) = \partial\Pi_i^{tpt}/\partial\nu_i$. Suppose that $\mathbf{v}^0 = (\nu_1^0,\, \nu_2^0,\, \cdots,\, \nu_n^0)$ with some $\nu_i^0 < \nu_j^0$ is a Nash equilibrium. Then since a Nash equilibrium should satisfy the Kuhn-Tucker condition, $G(\nu_j^0,\, \Delta^0) = 0 \geq G(\nu_i^0,\, \Delta^0)$, which contradicts with $\partial G/\partial\tau < 0$. Thus, there are no asymmetric equilibria in the game of investment in demand forecasting, and hence we obtain the uniqueness of the equilibrium. □

Lemma A.2 *Given the pricing scheme $P(\mathbf{q}) = w_0 + \bar{p}(\mathbf{q})q_i - w_d\, q_i^2$, where $\bar{p}(\mathbf{q}) = w_1 + w_2\sum_{j=1}^n q_j$ with $1 + w_2 \geq 2w_d$, there exists an equilibrium in order quantities of the retailers. In equilibrium $q_i(s_i) = \alpha_{0i}^m + \alpha_{si}^m\,(s_i - K_0)$ for all i, where*

$$\alpha_{0i}^m = \frac{K_0 - w_1}{(n+1)(1+w_2) - 2w_d}\,, \quad \alpha_{si}^m = \frac{\eta_i}{(1+w_2-w_d) + (1+w_2)\sum_j \eta_j}\,, \tag{A.40}$$

where $\eta_i = (1+w_2-w_d)\lambda_i/(2(1+w_2-w_d) - \lambda_i(1+w_2))$. In any symmetric equilibrium, retailer's investment level is $\nu_i = \nu^m$ for all i, where $\nu^m = \nu^* \cdot 1_{\{C'(0) < \sigma_0^2/4\}}$ and ν^* is the unique solution to the equation

$$\frac{\left((3n-1)\nu^2 + 2(n+2)\nu + 4\right)(1+w_2)^2 - 2(\nu+1)((n+2)\nu+4)(1+w_2)w_d + 4(\nu+1)^2{w_d}^2}{((\nu+2)(1+w_2) - 2(\nu+1)w_d)\left(((n+1)\nu+2)(1+w_2) - 2(\nu+1)w_d\right)^3} - \frac{C'(\nu)}{\sigma_0^2(1+w_2-w_d)} = 0\,. \tag{A.41}$$

Proof: First, given w_0, w_1, w_2, w_d and $\mathbf{v}$, we have

$$\mathrm{E}\left[\Pi_i^m | s_i, \mathbf{v}\right] = q_i(K_0 - w_1 + \lambda_i(s_i - K_0) - (1 + w_2 - w_d)\, q_i - (1 + w_2) \sum_{j \neq i} \mathrm{E}[q_j | s_i, \mathbf{v}]) - C(\nu_i) - w_0 \,. \quad \text{(A.42)}$$

Note that (A.42) is concave in q_i if $w_d \leq 1 + w_2$. The first order condition for q_i from (A.42) is

$$q_i = \frac{1}{2(1 + w_2 - w_d)} \left(K_0 - w_1 + \lambda_i(s_i - K_0)\right) - \frac{1 + w_2}{2(1 + w_2 - w_d)} \sum_{j \neq i} \mathrm{E}[q_j | s_i, \mathbf{v}] \,. \quad \text{(A.43)}$$

From $\mathrm{E}[s_j - K_0 | s_i] = \lambda_i(s_i - K_0)$ and (A.40) for $j \neq i$, we obtain $q_i = \alpha_{0i} + \alpha_{si}(s_i - K_0)$, where

$$\alpha_{0i} = \frac{1}{2(1 + w_2 - w_d)} \left(K_0 - w_1 - (1 + w_2) \sum_{j \neq i} \alpha_{0j} \right) , \quad \alpha_{si} = \frac{\lambda_i}{2(1 + w_2 - w_d)} \left(1 - (1 + w_2) \sum_{j \neq i} \alpha_{sj} \right) . \quad \text{(A.44)}$$

By substituting α_{0j} and α_{sj} for $j \neq i$, we obtain α_{0i} and α_{si} as given in (A.40). Substituting $q_i = \alpha_{0i} + \alpha_{si}(s_i - K_0)$ into (A.42) and taking expectation, we have

$$\begin{aligned} \mathrm{E}\left[\Pi_i^m | \mathbf{v}\right] &= \frac{(1 + w_2 - w_d)^3 \lambda_i \sigma_0^2}{\left(2(1 + w_2 - w_d)^2 + (1 + w_2)\left(2(1 + w_2 - w_d) - \lambda_i(1 + w_2)\right) \sum_{j \neq i} \eta_j\right)^2} \\ &\quad + \frac{(1 + w_2 - w_d)(K_0 - w_1)^2}{\left(2(1 + w_2 - w_d) + (n - 1)(1 + w_2)\right)^2} - C(\nu_i) - w_0. \end{aligned} \quad \text{(A.45)}$$

Let $\Xi_{-i} = \sum_{j \neq i} \eta_j$. Taking derivative with respect to ν_i,

$$\frac{\partial \mathrm{E}\left[\Pi_i^m\right]}{\partial \nu_i} = \frac{\sigma_0^2 (1 + w_2 - w_d)^3 \left(2(1 + w_2 - w_d)^2 + (1 + w_2)\left(2(1 + w_2 - w_d) + \lambda_i(1 + w_2)\right) \Xi_{-i}\right)}{\left(2(1 + w_2 - w_d)^2 + (1 + w_2)\left(2(1 + w_2 - w_d) - \lambda_i(1 + w_2)\right) \Xi_{-i}\right)^3 (1 + \nu_i)^2} - C'(\nu_i) \,. \quad \text{(A.46)}$$

We focus on symmetric Nash equilibrium for the investment level in demand forecasting. Plugging $\nu_i = \nu_j = \nu$ into (A.46) and simplifying, we obtain (A.41). Using

$$\frac{\partial}{\partial \nu} \left(\frac{\left((3n - 1)\nu^2 + 2(n + 2)\nu + 4\right)(1 + w_2)^2 - 2(\nu + 1)((n + 2)\nu + 4)(1 + w_2) w_d + 4(\nu + 1)^2 {w_d}^2}{\left(((n + 1)\nu + 2)(1 + w_2) - 2(\nu + 1) w_d\right)^3} \right) \leq 0 \,, \quad \text{(A.47)}$$

for $1 + w_2 \geq 2w_d$, it follows that the left-hand side of (A.41) is strictly decreasing in ν for $1 + w_2 \geq 2w_d$. In addition, note that if $\sigma_0^2 / 4(1 + w_2 - w_d) < C'(0)$, the left-hand side of (A.41) is negative for all ν. Therefore any symmetric equilibrium of investment level can be written as $\nu^m = \nu^* \cdot 1_{C'(0) \leq \sigma_0^2/4(1 + w_2 - w_d)}$ where ν^* is the unique solution of (A.41). To prove the impossibility of coordination, first note that $w_d = w_2$ to achieve investment coordination of no investment case ($\nu^m = \nu^{FB} = 0$). By (6), to achieve statewise production quantity, we need

$$\frac{\nu^m}{2 + (n + 1 + (n - 1) w_2) \nu^m} = \frac{\nu^{FB}}{2(1 + n \nu^{FB})} \,, \quad \text{(A.48)}$$

$$\frac{K_0 - w_1}{2 + (n-1)(1+w_2)} = \frac{K_0 - c_0}{2n} \,. \tag{A.49}$$

For $\nu^{FB} > 0$ and $n \geq 2$, from (A.48), we obtain

$$\nu^m = \frac{2\nu^{FB}}{2 + (n-1)(1-w_2)\nu^{FB}} \,. \tag{A.50}$$

Substituting (A.50) into (A.41), using (8) and simplifying, we then have

$$\frac{4}{C'(\nu^{FB})} C'\left(\frac{2\nu^{FB}}{2 + (n-1)(1-w_2)\nu^{FB}}\right) - \frac{\left(2 + (n-1)(1-w_2)\nu^{FB}\right)^2}{1 + n\nu^{FB}} \\ \times \frac{2 + 3n\nu^{FB} + (n^2+n-1)(\nu^{FB})^2 - w_2\nu^{FB}\left(n + (n^2-2n+2)\nu^{FB}\right) + (n-1)w_2^2(\nu^{FB})^2}{2 + n(1-w_2)\nu^{FB}} = 0 \,. \tag{A.51}$$

Using the convexity of C and when $w_2 < 1$, it follows that both terms on the left hand side of (A.51) is increasing in w_2. Therefore the left hand side of (A.51) is monotonically increasing in w_2 for $w_2 < 1$. When $w_2 = 1$, the left hand side of (A.51) is $-8(n-1)\nu^{FB}/(1+n\nu^{FB}) < 0$. Thus there does not exist $w_2 < 1$ that satisfies (A.51). This contract scheme can not coordinate statewise production quantity in a supply chain. This completes the proof. □

For part (i), since $\nu^{ws}_{obs} = \nu^{tpt}_{obs}$ and $\nu^{ws}_{unobs} = \nu^{tpt}_{unobs}$, we only give the proof for $\nu^{ws}_{ob}/\nu^{ws}_{unobs}$. Note that

$$\sigma_0^2 \cdot \frac{(3n-1)\nu^2 + 2(n+2)\nu + 4}{(2+\nu)((n+1)\nu+2)^3} \geq \frac{\sigma_0^2}{4}\left(\frac{2}{2+(n+1)\nu}\right)^2 \,. \tag{A.52}$$

From (9) and (A.27) under $w_2 = 0$ with the convexity of C and (A.52), we obtain that $\nu^{ws}_{obs} \geq \nu^{ws}_{unobs}$, hence the lower bound of (28) is obtained. Given that $\nu^{ws}_{unobs} \geq \nu^{FB}$, overinvestment result for observable investment also follows. Further, using the similar function employed in the proof of part (i) of Proposition 2, the lower bound can be achieved. For the upper bound in part (i), let $y = \nu^{ws}_{obs}/\nu^{ws}_{unobs}$, $z = 2/\nu^{ws}_{unobs}$, and $G = C'(\nu^{ws}_{obs})/C'(\nu^{ws}_{unobs})$. From (9) and (A.27) under $w_2 = 0$, we have

$$\frac{\left((3n-1)y^2 + (n+2)zy + z^2\right)(z+n+1)^2}{(z+y)\left((n+1)y+z\right)^2} = G \,. \tag{A.53}$$

Note that left hand side of (A.53) is decreasing in y for all $n \geq 1$ and $z > 0$. Thus the maximum of y is achieved at the minimum of G, which is equal to 1 from the convexity of C and the previous lower bound result. When $G = 1$, by simplifying (A.53), we obtain

$$(n+1)^3y^4 + (n+4)(n+1)^2zy^3 + \left(3(n+1)(n+2)z^2 - (3n-1)(z+n+1)^2\right)y^2 \\ + \left((3n+4)z^3 - (n+2)z(z+n+1)^2\right)y + z^4 - z^2(z+n+1) = 0 \,. \tag{A.54}$$

Note that $(n+1)^3 > 0$, $(n+4)(n+1)^2 z > 0$, and $z^4 - z^2(z+n+1) < 0$. Further if $3(n+1)(n+2)z^2 - (3n-1)(z+n+1)^2 < 0$, then $(3n+4)z^3 - (n+2)z(z+n+1)^2 < 0$. By Descartes' sign rule, this guarantees that (A.54) has only one positive root. Since $\nu_{obs}^{ws} \geq \nu_{unobs}^{ws}$, (A.54) has the unique positive solution, which is greater than 1. Denote left hand side of (A.54) as $f(y,z)$. Note that for $z_1 < z_2$ and $y > 1$, if $f(y,z_2) < 0$, then $f(y,z_1) < 0$. This guarantees that the unique positive solution of (A.54) increases as z decreases. Thus maximum of y is achieved at $z = 0$, which is the upper bound in (28).

For part (ii), note that under wholesale pricing contract, the supplier's profit is $\mathrm{E}[\Pi_S^{ws}] = n(w^{ws} - c_0)(K_0 - w^{ws})/(n+1)$, from which the optimal wholesale price is given as $w^{ws} = (K_0 + c_0)/2$. Under this optimal wholesale price, the supply chain profit is

$$\mathrm{E}\left[\Pi_{SC}^{ws}\right] = \frac{n(n+2)(K_0 - c_0)^2}{4(n+1)^2} + \frac{n\nu_{obs}^{ws}(1+\nu_{obs}^{ws})\sigma_0^2}{(2+(n+1)\nu_{obs}^{ws})^2} - nC(\nu_{obs}^{ws})\,. \tag{A.55}$$

Note that the supply chain profit is the same as that under unobservable investment if the investment in forecasting is the same. Further, taking derivative of (A.55) with respect to ν_{obs}^{ws}, we obtain

$$\frac{\mathrm{dE}\left[\Pi_{SC}^{ws}\right]}{\mathrm{d}\nu_{obs}^{ws}} = \frac{n}{(2+(n+1)\nu_{obs}^{ws})^3}\left(\sigma_0^2(2-(n-3)\nu_{obs}^{ws}) - (2+(n+1)\nu_{obs}^{ws})^3 C'(\nu_{obs}^{ws})\right)\,. \tag{A.56}$$

Note that if $n \geq 3$, the numerator of (A.56) is strictly decreasing in ν_{obs}^{ws} and hence it is quasi-concave in ν_{obs}^{ws}. Further if $n = 2$, (A.56) is simplified to $n\sigma_0^2(2+\nu_{obs}^{ws})/(2+3\nu_{obs}^{ws})^3 - nC'(\nu_{obs}^{ws})$, which is strictly decreasing in ν_{obs}^{ws}. Thus, if $n = 2$, (A.55) is strictly concave in ν_{obs}^{ws}. Therefore, (A.55) is quasi-concave in ν_{obs}^{ws}. Denote the optimal ν that maximizes (A.55) as $\hat{\nu}$. Then we obtain $\hat{\nu} \leq \nu_{unobs}^{ws} \leq \nu_{obs}^{ws}$ by comparing corresponding first order conditions and using

$$\frac{\sigma_0^2(2-(n-3)\nu)}{(2+(n+1)\nu)^3} \leq \frac{\sigma_0^2}{(2+(n+1)\nu)^2} \leq \frac{\sigma_0^2((3n-1)\nu^2 + 2(n+2)\nu + 4)}{(\nu+2)(2+(n+1)\nu)^3}\,. \tag{A.57}$$

Therefore $\mathrm{E}[\Pi_{SC,\,obs}^{ws}] \leq \mathrm{E}[\Pi_{SC,\,unobs}^{ws}]$ follows. In addition, the equality can be achieved when $\nu_{unobs}^{ws} = \nu_{obs}^{ws}$ and the corresponding cost function is given in part (i). For two-part tariff scheme, the proof is similar.

To see part (iii), plugging $w_2 = 1$ and $w_d = 1$ in (A.41), we obtain

$$\frac{1+n\nu+2(n-1)\nu^2}{(1+n\nu)^3}\cdot\frac{\sigma_0^2}{4} - C'(\nu) = 0\,. \tag{A.58}$$

Notice that since

$$\frac{1+n\nu+2(n-1)\nu^2}{(1+n\nu)^3} > \left(\frac{1}{1+n\nu}\right)^2, \tag{A.59}$$

for all $\nu > 0$, the left hand side of (A.58) is greater than that of (8). Given that the left hand side of both equations are strictly decreasing in ν, we obtain $\nu^m > \nu^{FB}$, when $C'(0) < \sigma_0^2/4$ and $n > 1$. Now consider $n = 2$, and let $C(\nu) = c_f \cdot \nu$ and $c_f < \sigma_0^2/4$. Then the equilibrium investment level ν satisfies

$$8\nu^3 + 2(6-z)\nu^2 + 2(3-z)\nu + 1 - z = 0\,, \tag{A.60}$$

where $z = \sigma_0^2/(4c_f) > 1$. Note that (A.60) has a unique positive root when $z > 1$ by the Descartes' sign rule. Further,

$$r_\nu^m = \frac{\nu_{obs}^m}{\nu_{unobs}^m} = \frac{1}{2\sqrt{2}}\sqrt{z} + \frac{1}{2\sqrt{2}} + O\left(\frac{1}{\sqrt{z}}\right). \tag{A.61}$$

Hence when z approaches infinity, r_ν^m becomes unbounded. Similarly, we obtain

$$r_\pi^m = \frac{E[\Pi_{obs}^m]}{E[\Pi_{unobs}^m]} = \frac{1}{2} + \frac{3\sqrt{c_f}}{\sqrt{2\sigma_0^2}} + O(\sigma_0^2)\,. \tag{A.62}$$

Thus, the profit ratio approaches 50% when σ_0^2 approaches infinity. This completes the proof. ■

Proof of Lemma 3: First, given w_0, β_0, β_p, w_d, and $\mathbf{v}$, using $p_c = K - \sum_j q_j$ and $\sum_j q_j = \beta_0 + \beta_p p_e$, we have

$$\mathrm{E}[\pi_i^m|s_i,\mathbf{v}] = \mathrm{E}[\mathrm{E}[\pi_i^m|s_i,p_e,\mathbf{v}]|s_i,\mathbf{v}] = \mathrm{E}[q_i\left(\mathrm{E}[K|s_i,p_e,\mathbf{v}] - \beta_0 - (1+\beta_p)p_e + w_d q_i\right) - C(\nu_i) - w_0|s_i,\mathbf{v}]\,. \tag{A.63}$$

Let $q_j = \alpha_{0j} + \alpha_{sj}(s_j - K_0) + \alpha_p p_e$ for all $j \neq i$, where α_{0j}, α_{sj}, α_p are given in (32). Denote $\xi = 1/2w_d(1 + \sum_k \nu_k)$, then $\alpha_{sj} = \xi\nu_j$. Note that

$$\beta_0 + \beta_p p_e = q_i + \sum_{j\neq i}\alpha_{0j} + \xi\sum_{j\neq i}\nu_j(s_j - K_0) + \sum_{j\neq i}\alpha_p p_e\,. \tag{A.64}$$

From (A.64), we obtain

$$\sum_{j\neq i}\nu_j(s_j - K_0) = (\beta_0 - \sum_{j\neq i}\alpha_{0j} + (\beta_p - (n-1)\alpha_p)p_e - q_i)\xi^{-1}\,. \tag{A.65}$$

Using (A.65) and $\mathrm{E}[K|s_i,p_e,\mathbf{v}] = K_0 + \sum_j \nu_j(s_j - K_0)/(1+\sum_k \nu_k)$, we have

$$\mathrm{E}[K|s_i,p_e,\mathbf{v}] = K_0 + \frac{1}{1+\sum_j \nu_j}((\beta_0 - \sum_{j\neq i}\alpha_{0j} + (\beta_p - (n-1)\alpha_p)p_e - q_i)\xi^{-1} + \nu_i(s_i - K_0))\,. \tag{A.66}$$

Plugging (A.66) into (A.63), we obtain

$$\mathrm{E}[\pi_i^m|s_i,p_e,\mathbf{v}] = \frac{q_i}{(1+\sum_j \nu_j)\xi}\Bigg((K_0 - \beta_0)(1 + \sum_j \nu_j)\xi + \beta_0 - \sum_{j\neq i}\alpha_{0j} + \xi\nu_i(s_i - K_0)$$
$$+ (\beta_p - (n-1)\alpha_p - (1+\beta_p)(1+\sum_j \nu_j)\xi)p_e - (1 - w_d(1+\sum_j \nu_j)\xi)q_i\Bigg) - C(\nu_i) - w_0\,. \tag{A.67}$$

Since $\xi = 1/2w_d(1+\sum_k \nu_k)$, $1 - w_d(1+\sum_j \nu_j)\xi = 1/2 > 0$. Hence the objective function is concave in q_i. Then the equilibrium order quantity for retailer i is equal to $q_i(s_i, p_e) = \alpha_{0i} + \alpha_{si}(s_i - K_0) + \alpha_p p_e$, where α_{0i}, α_{si}, α_p are given in (32). Using those equations, it follows

$$p_e = \frac{2w_d}{1+\beta_p}\left(\frac{K_0-\beta_0}{2w_d} + \xi\sum_j \nu_j(s_j - K_0)\right), \tag{A.68}$$

$$q_i = \frac{1}{n}\left(\beta_0 + \frac{\beta_p(K_0-\beta_0)}{1+\beta_p} + \left(n-1+\frac{2\beta_p w_d}{1+\beta_p}\right)\xi\nu_i(s_i-K_0) - \left(1-\frac{2\beta_p w_d}{1+\beta_p}\right)\xi\sum_{j\neq i}\nu_j(s_j-K_0)\right). \tag{A.69}$$

Plugging (A.68) and (A.69), and taking expectation, we obtain

$$\begin{aligned}\mathrm{E}[\pi_i^m] = \frac{w_d}{n^2}\Bigg(&\left(\beta_0 + \frac{\beta_p(K_0-\beta_0)}{1+\beta_p}\right)^2 + \left(n-1+\frac{2\beta_p w_d}{1+\beta_p}\right)^2\xi^2\nu_i(1+\nu_i)\sigma_0^2 \\ &+\left(1-\frac{2\beta_p w_d}{1+\beta_p}\right)^2\xi^2\sum_{j\neq i}\nu_j(1+\nu_j)\sigma_0^2 - 2\left(n-1+\frac{2\beta_p w_d}{1+\beta_p}\right)\left(1-\frac{2\beta_p w_d}{1+\beta_p}\right)\xi^2\nu_i\sum_{j\neq i}\nu_j\sigma_0^2 \\ &+\left(1-\frac{2\beta_p w_d}{1+\beta_p}\right)^2\xi^2\sum_{j\neq i}\sum_{k\neq i,j}\nu_j\nu_k\sigma_0^2\Bigg) - C(\nu_i) - w_0.\end{aligned} \tag{A.70}$$

Note that $\mathrm{E}[\pi_i^m]$ is continuous and differentiable in ν_i and approaches $-\infty$ as ν_i goes to ∞. Thus in an equilibrium, ν_i either equals to 0 or satisfies first order condition. We focus on a symmetric equilibrium. In a symmetric equilibrium, the equilibrium investment level can be written as $\mathbf{v} = \nu \cdot \mathbf{1}$, where $\nu \geq 0$ either equals to 0, or satisfies the first order condition. We obtain (33) by differentiating (A.70) with respect to ν_i and setting $\nu_j = \nu_i$ for all $j \neq i$. □

Proof of Proposition 7: From (A.69), we obtain

$$Q = \sum_{j=1}^n q_j = \beta_0 + \frac{\beta_p(K_0-\beta_0)}{1+\beta_p} + \frac{\beta_p}{1+\beta_p}\sum_{j=1}^n \frac{\nu_j(s_j-K_0)}{1+\sum_k \nu_k}. \tag{A.71}$$

Note that the equilibrium total production quantity in (A.71) is equal to the first-best total production quantity in (6) when $\beta_0 = -c_0$ and $\beta_p = 1$. For those β_0 and β_p, (33) is simplified to

$$\frac{\sigma_0^2}{4n^2(1+n\nu)^3}\left(2(n-1)(1+n\nu)^2 + \frac{(n-1)(n-1+(n-3)n\nu)}{w_d} + w_d(1+n\nu)\right) - C'(\nu) = 0. \tag{A.72}$$

Further, if w_d satisfies

$$(1+n\nu)w_d^2 + (1+n\nu)\left(2(n-1)(1+n\nu) - n^2\right)w_d + (n-1)(n-1+(n-3)n\nu) = 0, \tag{A.73}$$

then (A.72) is equal to (8), first order condition for first-best investment level. We obtain (34) by solving (A.73). This proves part (i).

To see part (ii), note that we can obtain the expected profit given (s_i, $\mathbf{v}$) in (A.67) by considering the expected profit given (s_i, p_e, $\mathbf{v}$) first, and then taking expectation given (s_i, $\mathbf{v}$) from the law of iterated expectation. Further, we can optimize the expected profit given (s_i, p_e, $\mathbf{v}$) pointwise since the order quantity is the function of p_e as well. This guarantees that the equilibrium is regret-free. ■

B An Illustration of the Effect of Observability on Increasing Incentives for Demand Forecast Investments

B.1 An Example with Two Retailers

To demonstrate this net benefit from the increase in observable investment through competitors' order responses, consider the following simplified example with two competing retailers, which is also illustrated in Figure 4. Suppose that there are only two states of the world, $K = H$ and $K = L$, which are equally likely, and where $H > L$. Correspondingly, there are two possible forecast signals H and L that the retailers can receive. Also suppose there are two forecast precision levels available to the retailers, $\nu = h$ and $\nu = l$, where $h > l$. For simplicity, in this example, let the unit price each retailer pays be constant and equal to w.

Let us fix Retailer 2's signal precision level ν_2, and consider Retailer 1's additional incentives to acquire higher precision information due to observability. If Retailer 1 has the higher precision signal, $\nu_1 = h$, he has higher confidence for his demand signal s_1, and consequently, when $s_1 = H$, q_1 will be higher, compared to the case when $s_1 = H$ and $\nu_1 = l$. That is, for any q_2, $q_1(s_1 = H, \nu_1 = h) > q_1(s_1 = H, \nu_1 = l)$. Similarly, $q_1(s_1 = L, \nu_1 = h) < q_1(s_1 = L, \nu_1 = l)$. Denote Retailer 2's quantity under signal s_2 when he assumes that Retailer 1 has precision ν_1 by $q_2(s_2, \hat{\nu}_1 = \nu_1)$. Notice that

$$q_2(s_2|\hat{\nu}_1 = \nu_1) = \arg\max_q \mathrm{E}[q(K - q_1(s_1, \nu_1) - q) - w \cdot q|s_2]\,, \tag{B.1}$$

which is strictly decreasing in $E[q_1(s_1, \nu_1)|s_2]$. When $s_2 = H$, Retailer 2 conjectures that it is more likely that $s_1 = H$. But, as we discussed above, when $s_1 = H$, q_1 is higher for $\hat{\nu}_1 = h$ than for $\hat{\nu}_1 = l$. Thus, $E[q_1(s_1, h)|s_2 = H] > E[q_1(s_1, l)|s_2 = H]$, and consequently $q_2(s_2 = H|\hat{\nu}_1 = h) < q_2(s_2 = H|\hat{\nu}_1 = l)$. With a symmetric argument, $q_2(s_2 = L|\hat{\nu}_1 = h) > q_2(s_2 = L|\hat{\nu}_1 = l)$. That is, when Retailer 2 assumes that Retailer 1's forecast precision increases, q_2 gets "squeezed to the middle", as also demonstrated in panel (a) of Figure 4.[25] But, Retailer 1's profit is higher when Retailer 2's quantity gets "squeezed to the middle". This profit increase is illustrated in panel (b) of Figure 4. To see this, first notice that for all s_1,

$$\mathrm{E}[\Pi_1(q_1, q_2(s_2|\hat{\nu}_1), K)|s_1, \nu_1] = \mathrm{E}[q_1(K - q_2(s_2|\hat{\nu}_1) - q_1) - w \cdot q_1|s_1, \nu_1]\,. \tag{B.2}$$

For a given q_2 and a realization of K, define Retailer 1's *residual demand curve* as $R_1(s_1, \hat{\nu}_1) = E[K - q_2(s_2|\hat{\nu}_1)|s_1, \nu_1] - q_1$. Because $q_2(s_2|\hat{\nu}_1)$ is "squeezed to the middle" with an increase in Retailer 1's observable investment in demand forecasting as we described above, Retailer 1's residual demand curves

[25] We also provide a numerical illustration of this argument in Section B.2.

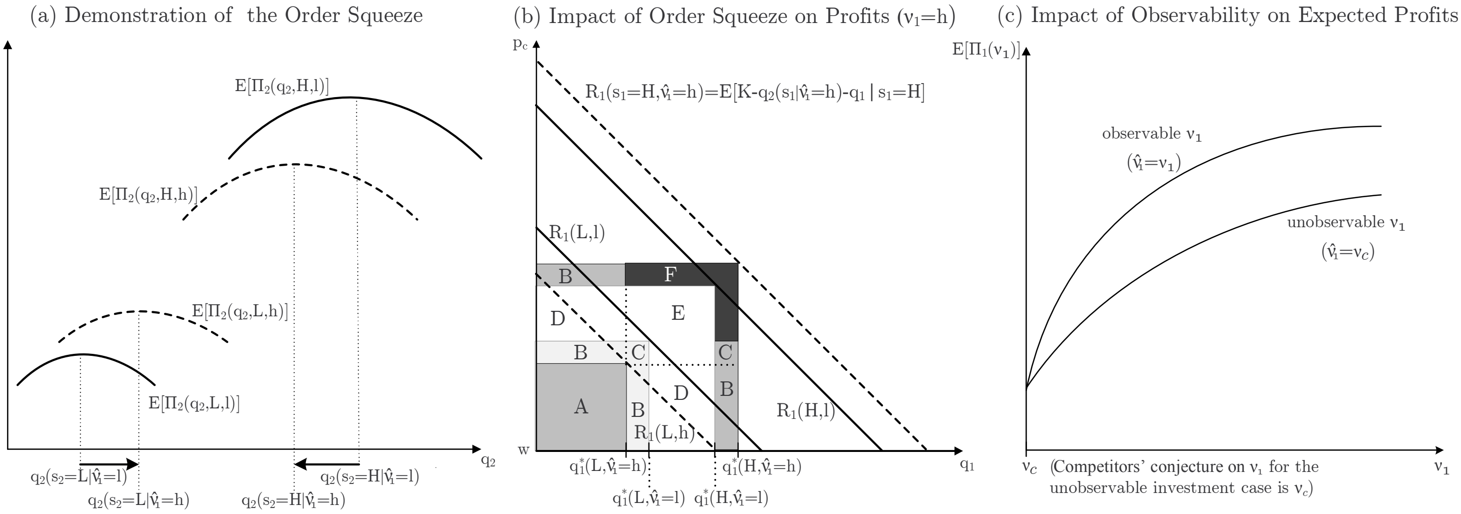

Figure 4: Illustration of the effect of forecast investment observability on creating extra incentives to invest. Panel (a) demonstrates the Order Squeeze, which refers to a retailer's reducing order quantities for high forecast levels and increasing those for low forecast levels when he conjectures (or sees) a high forecast investment for a competitor. Panel (b) demonstrates the profit increase for a retailer as a result of a competitor's order squeeze. Panel (c) illustrates the gap between the expected profits for a retailer when his investment level is observable and when it is unobservable to his competitors.

show the opposite effect, i.e., they "spread away from the middle". This is observable in panel (b) of Figure 4, where the pair of solid lines correspond to the residual demand curves for $s_1 = H$ and $s_1 = L$ for $\hat{\nu}_1 = l$, and the pair of dashed lines correspond to those for $\hat{\nu}_1 = h$. Consequently, Retailer 1's expected profit for a given residual demand curve is the rectangular area delineated by his optimal order quantity, q_1^*, the corresponding expected price, $E[K - q_2(s_2|\hat{v}_1)|s_1, \nu_1] - q_1^*$, and the two axes. That is, following the labeling in the figure, for $s_1 = L$ and $\hat{\nu}_1 = h$, $\Pi_1 = A$, and so forth. Then, Retailer 1's gain under the high demand signal from having an observable high demand forecast precision compared to an observable low demand forecast precision is given by $A+4B+2C+2D+E+F-(A+2B+C+2D+E) = 2B + C + F$. His corresponding loss because of the order squeeze when he receives the low demand signal is $A + 2B + C - A = 2B + C$. Therefore his net expected gain from observable high demand forecast investment is $(2B + C + F)/2 - (2B + C)/2 = F/2 > 0$.

Now, we have shown that Retailer 1 is better off if Retailer 2 assumes that Retailer 1 has a high forecast precision. Therefore, Retailer 1 is better off by showing to Retailer 2 that his investment level is higher. In particular, consider the forecast precision level that Retailer 2 assumes that Retailer 1 has when the investment level is unobservable. When his forecast investment is observable, Retailer 1 has extra benefits from investing, because his higher forecast precision level is observed by Retailer 2, who responds by an order squeeze as we discussed above, which makes Retailer 1 better off compared to the case if Retailer 1 did not observe his additional investment. This profit increase with observability is depicted in panel (c) of Figure 4 for the general model.[26]

[26]This effect of observability on increasing retailer incentives to invest in demand forecasting also holds if the retailers compete in prices rather than quantities. Also, if the retailers' knowledge of each others' forecast cost structures were

B.2 A Numerical Example for Order Quantity Squeeze

We next present an example to illustrate the "squeeze to the middle" effect for order quantities as defined and discussed in Section B.1.

Consider the demand curve, $p_c = K - q_1 - q_2$, and for simplicity, normalize the unit purchase and production costs for both firms to zero. The prior distribution of K is given by

$$K = \begin{cases} 14 \text{ (state H)}, & \text{with probability } 1/2\,, \\ 6 \text{ (state L)}, & \text{with probability } 1/2\,. \end{cases} \tag{B.1}$$

There are two potential forecast investment levels, $\nu = l$, and $\nu = h$, with low and high signal precisions, respectively, i.e., $h > l$. After choosing one of these investment levels, each retailer obtains a noisy signal $s_i \in \{H, L\}$. Specifically, if the investment level is l, then the probability distribution for the forecast signal is

$$P(s_i = H) = \begin{cases} 0.6 & \text{if} \quad K = 14\,, \\ 0.4 & \text{if} \quad K = 6\,; \end{cases} \tag{B.2}$$

and if the investment level is h, then

$$P(s_i = H) = \begin{cases} 0.9 & \text{if} \quad K = 14\,, \\ 0.1 & \text{if} \quad K = 6\,. \end{cases} \tag{B.3}$$

Both firms' signals are independent conditional on the value of K, i.e., $s_1 \perp s_2 | K$. Note that Retailer 1's expected profit is

$$E[\pi_1(q_1)|s_1] = E[q_1(K - q_1 - q_2)|s_1] = q_1(E[K|s_1] - q_1 - E[q_2|s_1])\,. \tag{B.4}$$

Fix Retailer 2's investment level at l. First consider the case where $\nu_1 = l$. Then the joint probability distribution for $(K,\, s_1,\, s_2)$ is given as:

$Pr(K,\, s_1,\, s_2)$	$K = 14$		$K = 6$	
$\nu_1 = l$	$s_2 = H$	$s_2 = L$	$s_2 = H$	$s_2 = L$
$s_1 = H$	$0.5 \cdot 0.6 \cdot 0.6 = \mathbf{0.18}$	$0.5 \cdot 0.6 \cdot 0.4 = \mathbf{0.12}$	$0.5 \cdot 0.4 \cdot 0.4 = \mathbf{0.08}$	$0.5 \cdot 0.4 \cdot 0.6 = \mathbf{0.12}$
$s_1 = L$	$0.5 \cdot 0.4 \cdot 0.6 = \mathbf{0.12}$	$0.5 \cdot 0.4 \cdot 0.4 = \mathbf{0.08}$	$0.5 \cdot 0.6 \cdot 0.4 = \mathbf{0.12}$	$0.5 \cdot 0.6 \cdot 0.6 = \mathbf{0.18}$

Table B.1: Joint probability distribution of K, s_1 and s_2 for investment level l.

Using Table B.1, for investment level l, by Bayes' rule, we have $P(K = 14|s_1 = H) = 0.6$, and consequently, $E[K|s_1 = H] = 0.6 \cdot 14 + 0.4 \cdot 6 = 10.8$. Further, again by Bayes' rule, $P(s_2 = H|s_1 = H) = (0.18 + 0.08)/(0.18 + 0.12 + 0.08 + 0.12) = 0.52$. It then follows that $E[q_2|s_1 = H] = 0.52 \cdot q_2(s_2 = H) + 0.48 \cdot q_2(s_2 = L)$. Therefore, by (B.4), when $s_1 = H$, Retailer 1's expected profit is

$$E[\pi_1(q_1)|s_1 = H] = q_1(H)(10.8 - q_1(H) - (0.52 \cdot q_2(s_2 = H) + 0.48 \cdot q_2(s_2 = L)))\,. \tag{B.5}$$

imperfect, the difference between the retailer incentives for investment under observable and unobservable investment cases would become smaller, but the intuition and the effect would still be valid.

The first order condition for (B.5) is

$$10.8 - 2q_1(H) - (0.52 \cdot q_2(H) + 0.48 \cdot q_2(L)) = 0\,, \tag{B.6}$$

and the second order condition is satisfied. Following similar steps, again by using Table B.1, Retailer 1's expected profit given $s_1 = L$ can be obtained as

$$E[\pi_1|s_1 = L] = q_1(L)(9.2 - q_1(L) - (0.48 \cdot q_2(H) + 0.52 \cdot q_2(L)))\,, \tag{B.7}$$

with the corresponding first order condition

$$9.2 - 2q_1(L) - (0.48 \cdot q_2(H) + 0.52 \cdot q_2(L)) = 0\,. \tag{B.8}$$

Following the same steps for Retailer 2, we also obtain the first order conditions for $q_2(H)$ and $q_2(L)$ as

$$10.8 - (0.52 \cdot q_1(H) + 0.48 \cdot q_1(L)) - 2q_2(H) = 0\,, \tag{B.9}$$

and

$$9.2 - (0.48 \cdot q_1(H) + 0.52 \cdot q_1(L)) - 2q_2(L) = 0\,. \tag{B.10}$$

Simultaneously solving (B.6), and (B.8)-(B.10), we obtain

$$q_i(s_i = H, \nu_1 = l) = 3.7255, \quad \text{and} \quad q_i(s_i = L, \nu_1 = l) = 2.9412, \quad \text{for} \quad i = 1,\, 2. \tag{B.11}$$

Second, consider the case where Retailer 1 invests at the high level, h. In this case, similar to the case for investment level l, the joint probability distribution for $(K,\, s_1,\, s_2)$ is given as:

$Pr(K,\, s_1,\, s_2)$	$K = 14$		$K = 6$	
$\nu_1 = h$	$s_2 = H$	$s_2 = L$	$s_2 = H$	$s_2 = L$
$s_1 = H$	$0.5 \cdot 0.9 \cdot 0.6 = \mathbf{0.27}$	$0.5 \cdot 0.9 \cdot 0.4 = \mathbf{0.18}$	$0.5 \cdot 0.1 \cdot 0.4 = \mathbf{0.02}$	$0.5 \cdot 0.1 \cdot 0.6 = \mathbf{0.03}$
$s_1 = L$	$0.5 \cdot 0.1 \cdot 0.6 = \mathbf{0.03}$	$0.5 \cdot 0.1 \cdot 0.4 = \mathbf{0.02}$	$0.5 \cdot 0.9 \cdot 0.4 = \mathbf{0.18}$	$0.5 \cdot 0.9 \cdot 0.6 = \mathbf{0.27}$

Table B.2: Joint probability distribution of K, s_1 and s_2 for investment level h.

Following the similar steps as above, and using Table B.2 and the Bayes' rule, we obtain Retailer 1's expected profit when $s_1 = H$ as

$$E[\pi_1|s_1 = H] = q_1(H)(13.2 - q_1(H) - (0.58 \cdot q_2(H) + 0.42 \cdot q_2(L)))\,. \tag{B.12}$$

The corresponding first order condition is

$$13.2 - 2q_1(H) - (0.58 \cdot q_2(H) + 0.42 \cdot q_2(L)) = 0\,. \tag{B.13}$$

When $s_1 = L$, Retailer 1's conditional expected profit is

$$E[\pi_1|s_1 = L] = q_1(L)(6.8 - q_1(L) - (0.42 \cdot q_2(H) + 0.58 \cdot q_2(L))), \tag{B.14}$$

from which we can again obtain the corresponding first order condition as

$$6.8 - 2q_1(L) - (0.42 \cdot q_2(H) + 0.58 \cdot q_2(L)) = 0. \tag{B.15}$$

Next, similarly, when $s_2 = H$, again employing the Bayes' rule, Retailer 2's expected profit can be written as

$$E[\pi_2|s_2 = H] = q_2(H)(10.8 - (0.58 \cdot q_1(H) + 0.42 \cdot q_1(L)) - q_2(H)), \tag{B.16}$$

and the corresponding first order condition is

$$10.8 - (0.58 \cdot q_1(H) + 0.42 \cdot q_1(L)) - 2q_2(H) = 0. \tag{B.17}$$

Lastly, when $s_2 = L$, conditional expectation of Retailer 2's profit is

$$E[\pi_2|s_2 = L] = q_2(L)(9.2 - (0.42 \cdot q_1(H) + 0.58 \cdot q_1(L)) - q_2(L)), \tag{B.18}$$

and taking derivative of (B.18) with respect to $q_2(L)$, we obtain the first order condition of $q_2(L)$ as

$$9.2 - (0.42 \cdot q_1(H) + 0.58 \cdot q_1(L)) - 2q_2(L) = 0. \tag{B.19}$$

Solving (B.13), (B.15), (B.17) and (B.19), simultaneously, we then have

$$q_1(s_1 = H, \nu_1 = h) = 4.9114, \quad \text{and} \quad q_1(s_1 = L, \nu_1 = h) = 1.7552, \tag{B.20}$$

and,

$$q_2(s_2 = H, \nu_1 = h) = 3.6071, \quad \text{and} \quad q_2(s_2 = L, \nu_1 = h) = 3.0596. \tag{B.21}$$

Therefore, by (B.11) and (B.21), we have

$$\begin{aligned} & q_2(s_2 = H, \nu_1 = l) = \ 3.7255 \to q_2(s_2 = H, \nu_1 = h) = 3.6071, \\ \text{and} \quad & q_2(s_2 = L, \nu_1 = l) = \ 2.9412 \to q_2(s_2 = L, \nu_1 = h) = 3.0596. \end{aligned} \tag{B.22}$$

That is Retailer 2 "squeezes her orders to the middle" when he observes that Retailer 1 increases his investment.

C An Analysis of Equilibrium under General Standard Quadratic Contracting Schemes

In this section, we explore general quadratic contracting schemes. In this contracting structure, the price that a retailer pays when ordering a quantity q_i is given by $P(\mathbf{q}) = w_0 + w_1\, q_i + w_2\, q_i^2$, for constants w_0, w_1, and w_2. Specifically, we will demonstrate two things: First, such contracts can coordinate the supply chain with private retailer demand forecasts when there is no investment in forecasting, and the retailer forecast precisions are fixed and equal. Second, full coordination is impossible with such contracts under downstream competition, when retailers invest in demand forecasting.

We start by deriving the retailers' equilibrium order quantities and investment levels under a general quadratic contracting scheme.

Lemma C.1 *Given the pricing scheme $P(\mathbf{q}) = w_0 + w_1\, q_i + w_2\, q_i^2$, provided that the second order condition $1 + w_2 > 0$ is satisfied, there exists a unique equilibrium. In equilibrium $q_i(s_i) = \alpha_0^q + \alpha_s^q\,(s_i - K_0)$ and $\nu_i^q = \nu^q$, for all i, where $\alpha_0^q = (K_0 - w_1)/(n + 1 + 2w_2)$, $\alpha_s^q = \nu^q/(2(1 + w_2) + (2w_2 + n + 1)\nu^q)$, $\nu^q = \nu^* \cdot 1_{\{C'(0) < \sigma_0^2/4(1+w_2)\}}$, and ν^* is the unique solution to the equation*

$$\frac{(1 + w_2)\sigma_0^2}{(2(1 + w_2) + (2w_2 + n + 1)\nu)^2} - C'(\nu) = 0\,. \tag{C.1}$$

Proof: Let $q_j(s_j) = \alpha_{0j} + \alpha_{sj}\,(s_j - K_0)$, α_{0j}, $\alpha_{sj} \in \mathbb{R}$ and $\nu_j \in \mathbb{R}_+$, for all $j \neq i$. Expected profit for retailer i after observing s_i is

$$\mathrm{E}\,[\Pi_i^q|s_i] = q_i\Big(K_0 - w_1 + \frac{\nu_i\,(s_i - K_0)}{1 + \nu_i} - (1 + w_2)q_i - \sum_{j\neq i}(\alpha_{0j} + \alpha_{sj}\mathrm{E}\,[s_j - K_0|s_i])\Big) - C(\nu_i) - w_0\,. \tag{C.2}$$

Note that (C.2) is concave in q_i if and only if $w_2 \geq -1$. Since $\mathrm{E}\,[s_j - K_0|s_i] = \nu_i/(1 + \nu_i)\,(s_i - K_0)$, the first order condition for q_i from (C.2) is written as

$$q_i = \frac{1}{2(1 + w_2)}(K_0 - w_1 - \sum_{j\neq i}\alpha_{0j} + \frac{\nu_i}{1 + \nu_i}(1 - \sum_{j\neq i}\alpha_{sj})\,(s_i - K_0)). \tag{C.3}$$

Observe that q_i is linear in $s_i - K_0$. Substituting (C.3) into (C.2) and taking expectation, we have

$$\mathrm{E}\,[\Pi_i^q] = \frac{1}{4(1 + w_2)}(K_0 - w_1 - \sum_{j\neq i}\alpha_{0j})^2 + \frac{\sigma_0^2\,\nu_i}{4(1 + w_2)\,(1 + \nu_i)}(1 - \sum_{j\neq i}\alpha_{sj})^2 - C(\nu_i) - w_0. \tag{C.4}$$

The first order condition for ν_i from (C.4) is $\frac{\sigma_0^2}{4(1+w_2)\,(1+\nu_i)^2}\,\Big(1 - \sum_{j\neq i}\alpha_{sj}\Big)^2 - C'(\nu_i) = 0$, and the second order condition, $-\frac{\sigma_0^2}{2(1+w_2)\,(1+\nu_i)^3}\,\Big(1 - \sum_{j\neq i}\alpha_{sj}\Big)^2 - C''(\nu_i) < 0$ is satisfied if $w_2 > -1$, i.e., if $w_2 > -1$, the objective function, (C.4), is strictly concave in ν_i and there exists unique maximizer. From (C.3) and using the first order condition, we have

$$\alpha_{0i} = \frac{1}{2(1+w_2)}(K_0 - w_1 - \sum_{j\neq i}\alpha_{0j})\,, \quad \alpha_{si} = \frac{1}{2(1+w_2)}(\frac{\nu_i}{1+\nu_i}\,(1-\sum_{j\neq i}\alpha_{sj}))\,, \tag{C.5}$$

and

$$C'(\nu_i) = \frac{\sigma_0^2}{4(1+w_2)\,(1+\nu_i)^2}\,(1-\sum_{j\neq i}\alpha_{sj})^2. \tag{C.6}$$

Summing over α_{0i} for all i, we obtain $\sum_{i=1}^n \alpha_{0i} = n(K_0 - w_1)/(n+1+2w_2)$. Substituting this into (C.5) and simplifying, we obtain α_0^q. Note that by (C.5)

$$\alpha_{si} = \frac{\nu_i(1-\sum_{j=1}^n \alpha_{sj})}{(1+2w_2)\nu_i + 2(1+w_2)}\,. \tag{C.7}$$

Plugging (C.7) into (C.6), we have

$$C'(\nu_i) = \frac{\sigma_0^2(1+w_2)\left(1-\sum_{j=1}^n \alpha_{sj}\right)^2}{\left((1+2w_2)\nu_i + 2(1+w_2)\right)^2}\,, \tag{C.8}$$

for all i. Observe that since (C.8) holds for all i, the first order condition for all ν_i is identical. Further, since the difference between the two sides of (C.8) is strictly monotonic for $\nu_i > 0$, it can have at most one solution in ν_i. It follows that $\nu_i = \nu$, for some $\nu \geq 0$, for all i. Adding up (C.7) for all i, and plugging in $\nu_i = \nu$, we then have

$$\sum_{i=1}^n \alpha_{si} = \frac{n\nu}{2(1+w_2) + (2w_2+n+1)\nu}\,. \tag{C.9}$$

Substituting (C.9) into (C.5) and simplifying, we obtain α_s^q. Finally, substituting (C.9) into (C.8), we obtain (C.1). Since C is convex, non-decreasing and non-identically zero, the left hand side of (C.1) is decreasing in ν, and will be strictly negative as $\nu \to \infty$. Consequently there exists a unique $\nu \geq 0$ that satisfies (C.1) if and only if $C'(0) < \sigma_0^2/4(1+w_2)$, with $\nu^q = 0$ otherwise. This confirms ν^q and completes the proof. □

Utilizing Lemma C.1, we can now proceed with the main result of our analysis in this section.

Proposition C.1 *If there is no demand forecasting and the retailers' forecast accuracies are fixed and equal, a standard quadratic pricing scheme can coordinate the supply chain. However, when $n \geq 2$, and there is positive investment in demand forecasting in the first-best solution, i.e., when $C'(0) < \sigma_0^2/4$, it is impossible to achieve full coordination simultaneously in statewise total production quantities and forecast investment levels using a standard quadratic pricing scheme.*

Proof: Consider any quadratic pricing scheme that coordinates the production quantities statewise. By (6), to achieve statewise production quantity, we need

$$\frac{\nu^q}{2(1+w_2)+(2w_2+n+1)\nu^q}=\frac{\nu^{FB}}{2(1+n\nu^{FB})}\,, \tag{C.10}$$

$$\frac{K_0-w_1}{n+1+2w_2}=\frac{K_0-c_0}{2n}\,. \tag{C.11}$$

First, suppose that forecast accuracies are fixed and equal for all retailers, for both the centralized and decentralized cases, and for conciseness in notation, denote this level by ν^{FB}. Then, when $w_1 = K_0-(n+1-(n-1)\nu^{FB}/(1+\nu^{FB}))(K_0-c_0)/(2n)$ and $w_2=(n-1)\nu^{FB}/(2(1+\nu^{FB}))$, (C.10) and (C.11) are satisfied. That is, with no investment in demand forecasting and symmetric forecast accuracy levels, statewise total production quantity coordination, and hence full supply chain coordination can be achieved by employing a general quadratic pricing scheme.

Now consider the full model, where the retailers invest in demand forecasting, and let us explore if full coordination can be achieved. For $\nu^{FB}>0$ and $n\geq 2$, from (C.10), we obtain

$$\nu^q=\frac{2(1+w_2)\nu^{FB}}{2+(n-1-2w_2)\,\nu^{FB}}\,. \tag{C.12}$$

Substituting (C.12) into (9), using (8) and simplifying, we then have

$$\frac{4(w_2+1)}{C'(\nu^{FB})}C'\left(\frac{2(1+w_2)\nu^{FB}}{2+(n-1-2w_2)\,\nu^{FB}}\right) - \left(4(\nu^{FB})^2w_2^2-4\nu^{FB}(2+(n-1)\nu^{FB})w_2+(2+(n-1)\nu^{FB})^2\right)=0\,. \tag{C.13}$$

Since C is convex, the first term on the left hand side of (C.13) is increasing in w_2. Now for $w_2 > 1/\nu^{FB}+(n-1)/2$, (C.12) can not be satisfied due to the non-negativity of ν^q and therefore we can restrict our attention to the region where $w_2\leq 1/\nu^{FB}+(n-1)/2$. By taking the first derivative of the second term in the left hand side of (C.13) and plugging in this upper bound for w_2 it then follows that this term is non-decreasing in w_2. Therefore the left hand side of (C.13) is monotonically increasing in w_2 for $w_2\leq 1/\nu^{FB}+(n-1)/2$. When $w_2=0$, left hand side of (C.13) is negative. Let

$$\overline{w}_2\triangleq\frac{1}{2(\nu^{FB})^2}\left(\nu^{FB}(2+(n-1)\nu^{FB})+1-\sqrt{2\nu^{FB}(2+(n+1)\nu^{FB})+1}\right)\in\left(0,\frac{1}{\nu^{FB}}+\frac{n-1}{2}\right)\,. \tag{C.14}$$

We then have

$$\left.\frac{2(1+w_2)\nu^{FB}}{2+(n-1-2w_2)\,\nu^{FB}}\right|_{w_2=\overline{w}_2}=\frac{1}{2}\left(\sqrt{2\nu^{FB}(2+(n+1)\nu^{FB})+1}-1\right)>\nu^{FB}\,, \tag{C.15}$$

and hence, again by convexity of C,

$$\frac{4(w_2+1)}{C'(\nu^{FB})} C' \left(\frac{2(1+w_2)\nu^{FB}}{2+(n-1-2w_2)\,\nu^{FB}} \right) \Bigg|_{w_2=\overline{w}_2} > 4(\overline{w}_2+1)\,. \tag{C.16}$$

Further,

$$4(\nu^{FB})^2 w_2^2 - 4\nu^{FB}(2+(n-1)\nu^{FB})w_2 + (2+(n-1)\nu^{FB})^2 \big|_{w_2=\overline{w}_2} = 4(\overline{w}_2+1)\,. \tag{C.17}$$

Plugging (C.16) and (C.17) in, it then follows that at $w_2 = \overline{w}_2$, the left hand side of (C.13) is positive. Hence there exists a unique $w_2^q \in (0, 1/\nu^{FB} + (n-1)/2)$ that satisfies (C.10). Given this w_2^q, w_1^q is uniquely determined from (C.11). Further, setting $w_0^q = \left(\mathrm{E}\left[\Pi_{SC}^q\right] - \mathrm{E}\left[\sum_i (w_1^q q_i + w_2^q q_i^2)\right]\right)/n > 0$ assures that the supplier extracts all expected supply chain surplus. Finally, note that since $w_2^q > 0$, it follows that $w_2 \geq -1$ holds, and hence the second order conditions for the optimality of q_i and ν_i are satisfied. Now,

$$\frac{2(1+w_2)\nu^{FB}}{2+(n-1-2w_2)\,\nu^{FB}} \Bigg|_{w_2=\frac{(n-1)\nu^{FB}}{2(1+\nu^{FB})}} = \nu^{FB}\,. \tag{C.18}$$

By (C.18) and plugging in and simplifying, it follows that the left hand side of (C.13) is negative at $w_2 = (n-1)\nu^{FB}/2(1+\nu^{FB})$. Then the monotonicity of (C.13) implies $w_2^q > (n-1)\nu^{FB}/2(1+\nu^{FB})$. Combining this with the monotonicity of (C.12) yields $\nu^q > \nu^{FB}$. Therefore the unique quadratic pricing scheme that achieves statewise supply chain production coordination results in overinvestment in demand forecasting. That is, simultaneous coordination of quantities in each state and investments by the full quadratic contracting scheme is impossible. This completes the proof. ■