Do Realized Skewness and Kurtosis Predict the Cross-Section of Equity Returns?*

Diego Amaya
HEC Montreal and UQAM

Peter Christoffersen
Rotman, CBS and CREATES

Kris Jacobs
University of Houston and Tilburg University

Aurelio Vasquez
ITAM School of Business

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Abstract

Yes. We use intraday data to compute weekly realized variance, skewness and kurtosis for individual equities and assess whether this week's realized moments predict next week's stock returns in the cross-section. We sort stocks each week according to their realized moments, form decile portfolios, and analyze subsequent weekly returns. We find a very strong negative relationship between realized skewness and next week's stock returns, and a positive relationship between realized kurtosis and next week's stock returns. We do not find a strong relationship between realized volatility and stock returns. A trading strategy that buys stocks in the lowest realized skewness decile and sells stocks in the highest realized skewness decile generates an average weekly return of 24 basis points with a t-statistic of 3.65. A similar strategy that buys stocks with high realized kurtosis and sells stocks with low realized kurtosis produces a weekly return of 14 basis points with a t-statistic of 2.12. Our results are robust across sample periods, portfolio weightings, and firm characteristics, and they are not captured by the Fama-French and Carhart factors.

JEL Codes: G11, G12, G17

Keywords: Realized volatility; skewness; kurtosis; equity markets; return prediction.

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Correspondence to: Peter Christoffersen, Phone: 416-946-5511, Email: peter.christoffersen@rotman.utoronto.ca.

1 Introduction

We examine the relationship between higher moments computed from intraday returns and future stock returns. Extending the well-known concept of realized volatility (Hsieh (1991), Andersen, Bollerslev, Diebold, and Ebens (2001)), computed from intraday squared returns, we compute realized skewness and kurtosis from intraday cubed and quartic returns. We show that the realized moments have well-defined convergence limits under realistic assumptions and that they are measured reliably in finite samples.

The relationship between higher moments and stock returns has been a topic of study since Kraus and Litzenberger (1976), who show theoretically that coskewness is a determinant of the cross-section of stock returns. Going beyond comovements, three different types of theoretical arguments suggest that assets' skewness may explain asset returns. Barberis and Huang (2008) demonstrate that assets with greater skewness have lower returns under cumulative prospect theory. Mitton and Vorkink (2007) obtain a similar result for expected skewness using heterogeneous investor preference for skewness, and Brunnermeier, Gollier, and Parker (2007) also predict a negative relationship between skewness and returns using an optimal expectations framework. Theory therefore unambiguously predicts a negative relationship between an asset's skewness and its return. To the best of our knowledge no such theoretical results are available for kurtosis.

Recent papers confirm that higher moments of the underlying stock return distribution are related to future returns. Ang, Hodrick, Xing, and Zhang (2006) find that stocks with higher idiosyncratic volatility have lower subsequent returns. Boyer, Mitton, and Vorkink (2010) find that stocks with higher expected idiosyncratic skewness yield lower future returns. Grouping stocks by industry, Zhang (2006) documents a negative relation between skewness and stock returns. Kelly (2011) studies the relationship between tail estimates and returns. Skewness measures extracted from options yield contradictory results on the relation between option implied skewness and future returns in the cross-section. While Xing, Zhang and Zhao (2010) and Rehman and Vilkov (2010) document a positive relation, Conrad, Dittmar, and Ghysels (2008) find a negative one. As for kurtosis, Conrad, Dittmar, and Ghysels (2008) report that risk-neutral kurtosis and stock returns are positively related.

Our empirical strategy uses a very extensive sample of weekly data. We aggregate daily realized moments to obtain weekly realized volatility, skewness, and kurtosis measures for over two million firm-week observations. We sort stocks into deciles based on the current-week realized moment and compute the subsequent one-week return of the trading strategy that buys the portfolio of stocks with a high realized moment (volatility, skewness or kurtosis) and sells the portfolio of stocks with a low realized moment.

When sorting on realized volatility, the resulting portfolio return differences are not statistically significant. However, when sorting by realized skewness, the long-short value-weighted portfolio produces an average weekly return of -24 basis points with a t-statistic of -3.65. This exceeds the premiums reported in Boyer, Mitton, and Vorkink (2010) and in Zhang (2006), which are −67 and −36 basis points per month, respectively. The resulting four factor Carhart risk adjusted alpha for the long-short skewness portfolio is −23 basis points per week. We find a positive relation between realized kurtosis and subsequent stock returns. For realized kurtosis, the long-short value-weighted portfolio generates a weekly return of 14 basis points with a t-statistic of 2.12. The four factor Carhart alpha of 11 basis points per week further supports the value of realized kurtosis as a predictor of stock returns.

We confirm the negative relation between realized skewness and future returns, and the positive relation between realized kurtosis and future returns, using Fama-MacBeth regressions. We also investigate the robustness of these findings to controlling for a number of well-documented determinants of returns: lagged return (Jegadeesh (1990), Lehmann (1990) and Gutierrez and Kelley (2008)), realized volatility, firm size (Fama and French (1993)), the book-to-market ratio (Fama and French (1993)), market beta, historical skewness, idiosyncratic volatility (Ang, Hodrick, Xing, and Zhang (2006)), coskewness (Harvey and Siddique (2000)), maximum return (Bali, Cakici, and Whitelaw (2009)), the number of analysts that follow the firm (Arbel and Strebel (1982)), illiquidity (Amihud (2002)), and the number of intraday transactions. Two-way sorts on realized skewness and firm characteristics also confirm that the relationship between realized skewness and returns is significant. However, two-way sorts show that the relationship between realized kurtosis and returns is not always significant when controlling for other firm characteristics. Finally, results for realized skewness and realized kurtosis are robust to the January effect and are significant when considering only NYSE stocks.

To verify that our measures of higher moments are not contaminated by microstructure noise, and to make sure that we are effectively measuring asymmetry and fat tails, we investigate two additional measures of skewness and kurtosis using high frequency data. The first measure is an enhanced version of the realized moment that uses the subsampling methodology suggested by Zhang, Mykland, and Ait-Sahalia (2005) to compute realized volatility. This subsampling methodology ensures that useful data is not ignored and provides a more robust estimator of the realized moment. The second approach uses percentiles of the high-frequency return distribution as alternative measures to capture skewness and kurtosis. We find that the negative relation between realized skewness and future stock returns is robust to using different measures of skewness, but the resulting long-short returns are smaller. However, the relation between realized kurtosis and stock returns is not always positive for the alternative measures. In addition, we show that the cross-sectional results also obtain for monthly holding periods.

We also compare the long-short returns obtained by sorting on our skewness measure with long-short returns obtained by sorting on other available skewness measures. We find that sorting using the skewness measure proposed by Zhang (2006) and the expected skewness measure proposed by Boyer, Mitton, and Vorkink (2010) also yields negative long-short returns. However, in our sample these returns are economically smaller than the ones we obtain using realized skewness, and they are not statistically significant. Sorting on historical skewness also does not produce statistically significant results. Our findings therefore complement and strengthen the results obtained using alternative skewness estimates in Boyer, Mitton, and Vorkink (2010), Zhang (2006), Xing, Zhang and Zhao (2010), but also suggest that our realized skewness measure may provide a simpler and cleaner estimate of skewness, which therefore leads to economically and statistically stronger results.

Ang, Hodrick, Xing, and Zhang (2006) find that stocks with high idiosyncratic volatility earn low returns. Motivated by their findings, we further explore the relationship between realized skewness, idiosyncratic volatility and subsequent stock returns. We find that when idiosyncratic volatility increases, low skewness stocks are compensated with higher returns while high skewness stocks are compensated with lower returns. Therefore, skewness provides a partial explanation of the idiosyncratic volatility puzzle. We also show that similar findings obtain when using realized volatility instead of idiosyncratic volatility.

Finally, we verify the limiting properties of realized higher moments, based on a continuous-time specification of equity price dynamics that includes stochastic volatility and jumps. We show how the limits of the higher realized moments are determined by the jump parameters of the continuous-time price process. Using Monte Carlo techniques, we verify that the measurement of the realized higher moments is robust to the presence of market microstructure noise as well as to quote discontinuities in existence prior to decimalization. We also show that our cross-sectional results hold up when using jump-robust measures of realized volatility to compute higher moments.

The remainder of the paper is organized as follows. Section 2 estimates the weekly realized higher moments from intraday returns and constructs portfolios based on these moments. Section 3 computes raw and risk-adjusted returns on portfolios sorted on realized volatility, skewness and kurtosis, and estimates Fama-MacBeth regressions including various control variables. Section 3 also investigates the interaction of volatility, skewness and returns. Section 4 contains a series of robustness checks. Section 5 investigates the limiting properties of the realized higher moments as well as the significance of our results when using jump-robust realized volatility estimators. Section 6 concludes.

2 Constructing Moment-Based Portfolios

We first describe the data. We then show how the realized higher moments are computed. Finally, we form portfolios by sorting stocks into deciles based on the weekly realized moments, and then report on the characteristics of these portfolios.

2.1 Data

We analyze every listed stock in the Trade and Quote (TAQ) database from January 4, 1993 to September 30, 2008. TAQ provides historical tick by tick data for all stocks listed on the New York Stock Exchange, American Stock Exchange, Nasdaq National Market System, and SmallCap issues.

We record prices every five minutes starting at 9:30 EST and construct five-minute log-returns for the period 9:30 EST to 16:00 EST for a total of 78 daily returns. We construct the five minute grid by using the last recorded price within the preceding five-minute period. If there is no price in a period, the return for that period is set to zero.

To ensure sufficient liquidity, we require that a stock has at least 80 daily transactions to construct a daily measure of realized moments.[1] The average number of intraday transactions per day for a stock is over one thousand. The weekly realized moment estimator is the average of the available daily estimators (Wednesday through Tuesday). Only one valid day of the realized moment is required to have a weekly estimator. Stocks with prices below $5 are excluded from the analysis.

We use data from three additional databases. From the Center for Research and Security Prices (CRSP) database, we use daily returns of each firm to calculate weekly returns (from Tuesday close to Tuesday close), historical equity skewness, market beta, lagged return, idiosyncratic volatility, maximum return over the previous month, and illiquidity; we use monthly returns to compute coskewness as in Harvey and Siddique (2000);[2] we use daily volume to compute illiquidity; and we use outstanding shares and stock prices to compute market capitalization. COMPUSTAT is used to extract the Standard and Poor's issuer credit ratings and book values to calculate book-to-market ratios of individual firms. From Thomson Returns Institutional Brokers Estimate System (I/B/E/S), we obtain the number of analysts that follow each individual firm. These variables are discussed in more detail in Appendix A.

2.2 Computing Realized Higher Moments

We first define the intraday log returns for each firm. On day t, the ith intraday return is given by

$$r_{t,i} = p_{t-1+\frac{i}{N}} - p_{t-1+\frac{i-1}{N}}, \tag{1}$$

where p_l is the natural logarithm of the price observed at time l and N is the number of return observations in a trading day. We use five-minute returns so that in 6.5 trading hours we have $N = 78$.

The well-known daily realized variance (Andersen and Bollerslev (1998) and Andersen, Bollerslev, Diebold and Labys (2003)) is obtained by summing squares of intraday high-frequency returns

$$RDVar_t = \sum\nolimits_{i=1}^{N} r_{t,i}^2. \tag{2}$$

As is standard, we do not estimate the mean of the high-frequency return because it is dominated by the variance at this frequency.

[1] We repeated the analysis using a minimum of 100, 250 and 500 transactions instead. The results are similar.

[2] Computing co-moments with high-frequency data is not straightforward due to synchronicity problems between stock and index returns. We leave this problem for future research.

An appealing characteristic of this volatility measure compared to other estimation methods is its model-free nature (see Andersen, Bollerslev, Diebold, and Labys (2001) and Barndorff-Nielsen and Shephard (2002) for details). Moreover, as we will discuss below, realized variance converges to a well-defined quadratic variation limit as the sampling frequency N increases.

Given that we are interested in measuring the asymmetry of the daily return's distribution, we construct a measure of ex-post realized daily skewness based on intraday returns standardized by the realized variance as follows

$$RDSkew_t = \frac{\sqrt{N}\sum_{i=1}^{N} r_{t,i}^3}{RDVar_t^{3/2}}. \tag{3}$$

The interpretation of this measure is straightforward: negative values indicate that the stock's return distribution has a left tail that is fatter than the right tail, and positive values indicate the opposite.

We are interested in extremes of the return distribution more generally, and so we also construct a measure of realized daily kurtosis defined by

$$RDKurt_t = \frac{N\sum_{i=1}^{N} r_{t,i}^4}{RDVar_t^2}. \tag{4}$$

The limits of the third and fourth moment when the sampling frequency N increases will be analyzed below as well.

Our cross-sectional asset pricing analysis below is conducted at the weekly frequency. We therefore construct weekly realized moments from their daily counterparts as follows. If t is a Tuesday then we compute

$$RVol_t = \left(\frac{252}{5}\sum_{i=0}^{4} RDVar_{t-i}\right)^{1/2}, \tag{5}$$

$$RSkew_t = \frac{1}{5}\sum_{i=0}^{4} RDSkew_{t-i}, \tag{6}$$

$$RKurt_t = \frac{1}{5}\sum_{i=0}^{4} RDKurt_{t-i}. \tag{7}$$

Our cross-sectional analysis below is conducted at the weekly frequency and t will therefore denote a week from this point on. Note that, as is standard, we have annualized the realized volatility measure to facilitate the interpretation of results.

We compute the $RVol_t$, $RSkew_t$, and $RKurt_t$ for more than two million firm-week observations during our January 1993 to September 2008 sample period. Figure 1 summarizes the realized moments. The top-left panel of Figure 1 displays a histogram of the realized volatility measure pooled across firms and weeks. As often found in the realized volatility literature, the unconditional distribution of realized equity volatility appears to be roughly log-normally distributed. The top-right panel in Figure 1 shows the time-variation in the cross-sectional percentiles using three-month moving averages. The cross-sectional dispersion in realized equity volatility is clearly not constant over time and seems to have decreased through our sample period.

The middle-left panel of Figure 1 shows the histogram of realized equity skewness. The skewness distribution is very fat-tailed and strongly peaked around zero. The middle-right panel of Figure 1 shows the time-variation in the cross-sectional skewness percentiles. The cross-sectional dispersion in realized equity skewness has increased through our sample.

The bottom-left panel of Figure 1 shows the histogram of realized equity kurtosis. Similar to realized volatility, realized kurtosis appears to be approximately log-normally distributed. The vast majority of our sample has a kurtosis above 3, strongly suggesting fat-tailed returns. The bottom-right panel of Figure 1 shows that the cross-sectional distribution of realized equity kurtosis has become more disperse over time, matching the result found for realized skewness.

2.3 Portfolio Sort Characteristics

Each Tuesday, we form portfolios by sorting stocks into deciles based on the weekly realized moments. Table 1 reports the time-series sample averages for the moments and different firm characteristics, by decile. Panel A reports the time-series averages for realized volatility, Panel B for realized skewness, and Panel C for realized kurtosis. Column 1 represents the portfolio of stocks with the smallest average realized moment, and column 10 is for the portfolio of stocks with the highest realized moment. The characteristics include firm size, book-to-market ratio, realized volatility over the previous week, historical skewness using daily returns from the previous month, market beta from the market model regression, lagged return, illiquidity as in Amihud (2002), coskewness as in Harvey and Siddique (2000), idiosyncratic volatility as in Ang, Hodrick, Xing, and Zhang (2006), the number of analysts from I/B/E/S, credit rating, stock price, the number of intraday transactions, and the number of stocks per decile. On average there are 257 companies per decile each week.

Table 1, Panel A displays results for the ten decile portfolios based on realized volatility. Realized volatility increases from 18.8% for the first decile to 145.0% for the highest decile. Interestingly, realized skewness has a negative relation with realized volatility and realized kurtosis shows an increasing pattern through the volatility deciles. Furthermore, companies with high realized volatility tend to be small, followed by fewer analysts, less coskewed with the market, and they have a lower stock price. A positive relation exists between realized volatility and historical skewness, market beta, lagged return, idiosyncratic volatility and maximum return. Finally, no pattern is observed between realized volatility and book-to-market, number of intraday transactions, and credit rating.

Panel B of Table 1 shows that realized skewness equals −1.04 for the first decile portfolio and 1.02 for the tenth decile. Firms with a high degree of asymmetry, either positive or negative, are small, highly illiquid, followed by fewer analysts, and the number of intraday transactions for these firms is lower.

Panel C of Table 1 reports on the decile portfolios based on realized kurtosis. The average kurtosis ranges from 3.9 to 16.6 across the deciles. Firm characteristics that are positively related to realized kurtosis include realized volatility, historical skewness, lagged return, idiosyncratic skewness, illiquidity and maximum return. Variables that have a negative relation with realized kurtosis include size, market beta, coskewness, number of I/B/E/S analysts, stock price, and number of intraday transactions.

In summary, Table 1 strongly suggests that firm-specific realized volatility, skewness, and kurtosis all contain unique information about the cross-sectional and temporal distribution of equity returns. We now attempt to exploit this moment-based information for predicting the cross-section of equity returns.

3 Realized Moments and the Cross-Section of Stock Returns

In this section, we first analyze the relationship between the current week's returns and the previous week's realized volatility, realized skewness, and realized kurtosis. Second, we use the Fama and MacBeth (1973) methodology to conduct cross-sectional regressions and to determine the significance of each higher realized moment individually and simultaneously, and also when controlling for firm-specific factors. Third, we investigate the interaction of returns, realized volatility and skewness.

3.1 Sorting Stock Returns on Realized Volatility

Every Tuesday, stocks are ranked into deciles according to their realized volatility. Then, using returns over the following week, we construct value- and equal-weighted portfolios. Table 2, Panel A reports the time series average of weekly returns for decile portfolios based on the level of realized volatility.

The value-weighted returns show a decreasing pattern, from 20 basis points for decile 1 to 8 basis points for decile 10. On the other hand, equal-weighted returns increase from 22 basis points to 27 basis points. Thus, the returns of the long-short portfolio, namely one that buys stocks in decile 10 and sells stocks in decile 1, are negative for value-weighted portfolios and positive for equal-weighted ones. The negative relation between individual volatility and stock returns for value-weighted portfolios is consistent with Ang, Hodrick, Xing, and Zhang (2006). However, neither the value-weighted nor the equal-weighted long-short portfolios are statistically significant, and this is the case for raw returns as well as for alphas from the Carhart four factor model. The four factor model employs the three Fama and French (1993) factors (excess market-return, size and book-to-market factors) and the Carhart (1997) momentum factor.

We conclude that realized volatility and future stock returns are not robustly related when using our measure of realized volatility.

3.2 Sorting Stock Returns on Realized Skewness

Table 2, Panel B reports the time-series average of weekly returns for decile portfolios grouped by realized skewness.

The value-weighted and equal-weighted returns both show a monotonically decreasing pattern between realized skewness and the average stock returns over the subsequent week. The return for the portfolio of stocks with the lowest level of skewness is 40 basis points for value-weighted portfolios and 55 basis points for equal-weighted portfolios, while the returns for stocks with the highest level of realized skewness is 17 basis points for value-weighted and 12 for equal-weighted portfolios. The weekly return difference between portfolio 10 and 1 is −24 basis points for value-weighted returns and −43 for equal-weighted returns. Both differences are statistically significant at the one percent level. This result is consistent with recent theories stating that stocks with lower skewness command a risk premium. For prominent examples, see Barberis and Huang (2008), Brunnermeier, Gollier, and Parker (2007), and Mitton and Vorkink (2007). The equal-weighted return difference is larger than the value-weighted return difference, suggesting that the relationship between skewness and subsequent returns is larger for small firms.

We also assess the empirical relationship between realized skewness and stock returns by adjusting for standard measures of risk. Panel B of Table 2 presents, for each decile, alphas relative to the Carhart four factor model. Note that alphas are large and statistically significant for value- and equal-weighted portfolios across deciles. In addition, the difference between the alphas of the tenth and first deciles is −23 and −46 basis points for value- and equal-weighted portfolios, respectively. Note also that the magnitude of the alphas is very similar to that of raw returns, which shows that standard measures of risk do not account for the return provided by the realized skewness exposure.

The sign of the cross-sectional relationship between skewness and subsequent stock returns is consistent with the findings of other studies that use different measures of skewness, but the magnitude is larger. Boyer, Mitton, and Vorkink (2010) use a model that incorporates firm characteristics in order to measure the expected skewness over a given horizon. They report that a strategy that buys stocks with the highest one-month expected skewness and sells stocks with the smallest one-month expected skewness generates an average return of −67 basis points per month. Zhang (2006) measures expected skewness for a stock by allocating it into a peer group (e.g. industry) and uses recent returns from this group to compute its skewness measure. In this case the long-short strategy produces risk-adjusted returns of −36 basis points per month.

Our long-short skewness returns are also large when compared with the standard four factor returns. In our sample the weekly return on the market factor is 12 basis points per week, the size factor return is 2 basis points, the value factor return is 10 basis points and momentum yields 20 basis points per week on average.

In conclusion, we find strong evidence that realized skewness predicts the cross section of stock returns. Realized skewness is an important determinant of the cross-sectional variation in subsequent one-week returns, and its effect is not captured by standard measures of risk.

3.3 Sorting Stock Returns on Realized Kurtosis

Panel C of Table 2 documents the average next-week stock returns for decile portfolios based on realized kurtosis. Value-weighted and equal-weighted portfolio returns both increase with the level of realized kurtosis. For value weighted portfolios, decile 1 has an average weekly return of 17 basis points, compared to 31 basis points for decile 10. Thus, the long-short portfolio generates a return of 14 basis points with a t-statistic of 2.12. A similar result is found for the equal-weighted portfolio, where the long-short realized kurtosis premium equals 16 basis points with a t-statistic of 2.98.

The results for the Carhart four-factor alpha are of the same magnitude as those of the raw returns. The value-weighted alpha for the long-short portfolio is 11 basis points and the equal-weighted alpha is 13 basis points. The equal-weighted alpha is significant at the 1% level, the value-weighted alpha at the 5% level.

Comparing Panels A, B, and C of Table 2, we conclude that, while the results for kurtosis are fairly strong, realized skewness appears to be the most reliable moment-based predictor of subsequent one-week equity returns in the cross section.

3.4 Fama-MacBeth Regressions

To further assess the relationship between future returns and realized volatility, realized skewness, and realized kurtosis, we carry out various cross-sectional regressions using the method proposed in Fama and MacBeth (1973). Each week t, we compute the realized moments for firm i and estimate the following cross-sectional regression on the week $t+1$ returns

$$r_{i,t+1} = \gamma_{0,t} + \gamma_{1,t} RVol_{i,t} + \gamma_{2,t} RSkew_{i,t} + \gamma_{3,t} RKurt_{i,t} + \phi_t' Z_{i,t} + \varepsilon_{i,t+1}, \tag{8}$$

where $r_{i,t+1}$ is the weekly return (in bps) of the ith stock for week $t+1$, and where $Z_{i,t}$ represent a vector of characteristics and controls for the ith firm observed at the end of week t. The characteristics and controls included are the week t return (in bps), firm size, book-to-market, market beta, historical skewness, idiosyncratic volatility, coskewness, maximum monthly return (in bps), number of analysts, illiquidity, and number of intraday transactions.

Table 3 reports the time-series average of the γ and ϕ coefficients for six cross-sectional regressions. The first column presents the results of the regression of the stock return on lagged realized volatility. The coefficient associated with realized volatility is 3.5 with a Newey-West t-statistic of 0.26. This confirms that there does not seem to be a significant relationship between realized volatility and stock returns. The second and third columns confirm the relation between the stock return and lagged realized skewness and realized kurtosis respectively. In column 2, the coefficient associated with realized skewness is -22.4 with a Newey-West t-statistic of -7.90. Similarly, in column 3, the coefficient on realized kurtosis is 1.3 with a t-statistic of 3.49. In the fourth column, we report regression results using all higher moments simultaneously. The coefficients on lagged realized skewness and realized kurtosis remain statistically significant, and are again negative and positive respectively. The third and fourth realized moments appear to explain different aspects of stock returns.

In the fifth column, we include lagged returns in the regression, given the strong evidence of the return reversal effect in short run returns (Jegadeesh (1990), Gutierrez and Kelley (2008)). Even though the coefficient of realized skewness decreases to -3.9, it remains significant and negative. The coefficient of realized kurtosis increases to 1.6 with a Newey-West t-statistic of 3.83. The coefficient on lagged return is negative and statistically significant, as expected.

In the last column, we add all control variables to ensure that realized skewness and realized kurtosis are not a manifestation of previously documented relationships between firm characteristics and stock returns. We find that the coefficients of realized skewness and realized kurtosis are still significant, with Newey-West t-statistics of -2.73 and 2.15, and preserve their signs with coefficients of -4.7 and 0.7, respectively. The negative sign on the coefficient related to size and the positive sign of the coefficient related to book-to-market confirm existing results in the literature. We include control variables related to the illiquidity and visibility of individual stocks. This includes the number of intraday transactions, the measure of illiquidity proposed in Amihud (2002), and the number of analysts following a stock (see Arbel and Strebel (1982)). We also control for the previously documented relationships between stock returns and firm characteristics, such as idiosyncratic volatility (Ang, Hodrick, Xing, and Zhang (2006)), the maximum daily return over the previous month (Bali, Cakici, and Whitelaw (2009)), and the stock's coskewness, as measured by the variability of the stock's return with respect to changes in the level of volatility following Harvey and Siddique (2000). Finally, we control for the market beta computed with a regression using daily returns on the market over the previous 12 months.

The results in the last column of Table 3 show that the economic and statistical significance of realized skewness and realized kurtosis for the cross-section of weekly returns is robust to the inclusion of various control variables. Variables such as realized volatility, idiosyncratic volatility, and coskewness do not play a significant role in the cross-section of returns at a weekly level, while variables such as lagged return, maximum daily return, and size are relevant.

3.5 Realized Skewness and Realized Volatility

We now further examine the interaction between the effects of realized skewness and realized volatility on returns. We construct portfolios using a double sort on realized skewness and realized volatility and then examine subsequent stock returns. First, we form five quintile portfolios with different levels of realized skewness. Within each of these portfolios, we form five portfolios that have different levels of realized volatility.[3] Panel B of Table 4 reports the equal-weighted returns for the 25 portfolios as well as the difference between the high realized volatility quintiles and low realized volatility quintiles. We observe that in low skewness portfolios, higher realized volatility translates

[3] Sorting on realized volatility first, and subsequently on realized skewness, does not change the results.

into higher returns. The portfolio that buys quintile 5 (stocks with high realized volatility and low realized skewness) and sells quintile 1 (stocks with low realized volatility and low realized skewness) has a weekly return of 47 basis points with a t-statistic of 3.31. Hence, in the case of low skewness stocks, investors are compensated with higher returns when holding high volatility stocks. However, for stocks with high skewness, we find that portfolios containing stocks with low volatility have higher subsequent returns than portfolios containing stocks with high volatility. In this case, the long-short portfolio return premium is -28 basis points with a t-statistic of -2.22. Overall, the portfolio with the lowest return is the one with stocks that have high skewness and high volatility. Panel A of Table 4 demonstrates that similar results obtain for value-weighted portfolios, but in this case the results are not statistically significant.

Panel A of Figure 2 shows the value- and equal-weighted returns for the 25 portfolios double sorted on realized skewness and realized volatility. For equal-weighted returns in the right panel, realized skewness increases from -0.772 to $+0.762$ and realized volatility increases from 20% to about 120% for the 25 portfolios. Portfolios with low realized volatility of 20% have very similar returns for all five levels of realized skewness: between 20 and 30 basis points for equal-weighted portfolios. However, as realized volatility increases, the return of low and high realized skewness portfolios strongly diverges. Portfolios with high realized volatility of 120% report the highest and the lowest return of all 25 portfolios. Stocks with the lowest realized skewness earn the highest average equal-weighted return of 77 basis points; and stocks with the highest realized skewness earn the lowest average return of -9 basis points. Hence, it is important to account for skewness when analyzing the return/volatility relationship. Highly volatile stocks may earn low returns, which seems counterintuitive, but the reason is that their skewness is high.[4]

3.6 Realized Skewness and Idiosyncratic Volatility

Building on our findings regarding volatility and skewness, we now investigate whether realized skewness can explain the idiosyncratic volatility puzzle uncovered by Ang, Hodrick, Xing, and Zhang (2006). They find that stocks with high idiosyncratic volatility earn lower returns than stocks with low idiosyncratic volatility, contradicting the implications of mean-variance models. Table 5 replicates the idiosyncratic volatility puzzle in Ang, Hodrick, Xing, and Zhang (2006) for our sample. For value-weighted portfolios (Panel A of Table 5), we find a weekly premium of -0.24% with a t-statistic of -1.73 for the period 1993-2008. This result is comparable to that of Ang, Hodrick, Xing, and Zhang (2006) who find a monthly premium of -1.06% with a t-statistic of -3.10 for the period 1963-2000. Interestingly, Panel B of Table 5 indicates that the puzzle does not obtain for equal-weighted portfolios, where we do not observe significant differences across quintiles.

To study the interaction between realized skewness and idiosyncratic volatility on stock returns we employ double sorting. We first sort stocks by realized skewness and form quintile portfolios.

[4]This finding is supported by Golec and Tamarkin (1998), who show that gamblers at horse races accept bets with low returns and high volatility only because they enjoy the high positive skewness offered by these bets.

Quintile 1 has stocks with the lowest level of realized skewness and quintile 5 has stocks with the highest level of realized skewness. Then, within each quintile portfolio, we sort stocks by idiosyncratic volatility.[5] Table 6 reports the results for value-weighted and equal-weighted portfolios. The equal-weighted portfolio results (Panel B) are very similar to those for realized volatility reported in Panel B of Table 4. In particular, we observe that the premium of the high idiosyncratic volatility portfolio (quintile 5) minus the low idiosyncratic volatility portfolio (quintile 1) decreases as the level of skewness increases. The premium for low realized skewness is 35 basis points and decreases to −43 basis points for high realized skewness. The highest returns are observed for the portfolios with low skewness and the lowest return of −20 basis points is for the portfolio with high idiosyncratic volatility and high skewness. Panel B of Figure 2 shows the returns of the 25 equal-weighted portfolios for different levels of idiosyncratic volatility. Just as with realized volatility in Panel A of Figure 2, high idiosyncratic volatility is compensated with high returns only if skewness is low. Investors are willing to accept low returns and high idiosyncratic volatility in exchange for high positive skewness. Panel A of Table 6 and Panel B of Figure 2 indicate that value-weighted portfolios display a similar but less significant pattern.

Investors trade high idiosyncratic volatility and low returns for high skewness, because they prefer skewness. Preference for skewness seems to partly explain the idiosyncratic volatility puzzle.

4 Robustness Analysis

In this section, we further explore the relation between realized moments and stock returns. First, we investigate if the relation between current-week realized moments and next-week returns is present in different subsamples. Second, we check if our findings are robust to alternative measures of skewness and kurtosis. Third, we investigate if the relation between moments and subsequent returns exists regardless of firm characteristics. Fourth, we use monthly rather than weekly holding periods for returns.

4.1 Subsamples

Panel A of Table 7 reports value- and equal-weighted returns of portfolios sorted on realized skewness across different subsamples. Keim (1983) documents calendar-related anomalies for the month of January, in which stocks have higher returns than in the rest of the year. Panel A of Table 7 presents the average weekly returns for the month of January and for the rest of the year for both value- and equal-weighted portfolios. As expected, returns for the month of January are consistently higher than returns for the rest of the year.

The difference between the returns of portfolios with high-skewness stocks and portfolios with low-skewness stocks is negative and significant for both January and non-January periods. This is

[5] An unconditional two-way sort on realized volatility and realized skewness produces similar results than the conditional two-way sort.

the case for value-weighted as well as equal-weighted portfolios.

We previously documented that stocks with high and low levels of skewness tend to be small. Hence, we examine if the effect of skewness is exclusively driven by small NASDAQ stocks. By only including stocks from the New York Stock Exchange (NYSE), row 3 of Table 7 shows that the effect of realized skewness is present among NYSE stocks. Hence, small NASDAQ stocks are not driving our results.

In Table 7, Panel B, we analyze the value-weighted and equal-weighted returns of portfolios sorted on realized kurtosis for different subsamples. The long-short portfolio returns are positive for all subsamples. As expected, in January the long-short portfolio earns higher returns compared to the rest of the year. We also confirm that the effect of realized kurtosis is not driven by small NASDAQ stocks. For value-weighted portfolios the realized kurtosis premium is positive, but often not statistically significant.

4.2 Alternative Measures of Skewness and Kurtosis

The analysis of alternative measures of skewness and kurtosis serves two purposes. On the one hand we want to investigate if our findings depend on the implementation of the realized skewness and kurtosis measures. On the other hand the literature already contains analyses of skewness measures that are constructed in a radically different way. A natural question is if the long-short returns constructed using these alternative skewness measures are similar to the long-short returns we document in Table 2.

First, we investigate the robustness with respect to the implementation of realized skewness by analyzing two alternative estimators. The first estimator, $SubRSkew$, uses the subsampling methodology suggested by Zhang, Mykland, and Ait-Sahalia (2005), which provides measures robust to microstructure noise. This method consists of constructing subsamples that are spaced every minute. Instead of one realized measure based on a single five-minute return grid, we end up with five estimators of realized skewness using subsamples of 5-minute returns for the period 9:30 EST to 16:00 EST. Subsamples start every minute (at 9:00, 9:01, 9:02, 9:03 and 9:04), but we use 5-minute returns. Subsequently, the realized skewness estimator is computed as the average of the five (overlapping) estimators obtained from the subsamples.[6]

The second alternative estimator of intraday skewness depends solely on quartiles from the intraday return distribution. As proposed in Bowley (1920), a measure of skewness that is based on quartiles can be defined as

$$SK2_t = (Q_3 + Q_1 - 2Q_2)/(Q_3 - Q_2), \tag{9}$$

where Q_i is the i^{th} quartile of the five-minute return distribution F, that is $Q_1 = F^{-1}(0.25)$,

[6] Neuberger (2011) has recently suggested another interesting alternative measure of realized skewness which we have not pursued here.

$Q_2 = F^{-1}(0.5)$, and $Q_3 = F^{-1}(0.75)$.

The literature contains some radically different approaches to measuring (expected) skewness. Zhang (2006) measures the skewness for a given firm by the cross-sectional skewness of the firms in that industry. Boyer, Mitton, and Vorkink (2010) construct a measure of expected idiosyncratic skewness that controls for firm characteristics. These two measures are discussed in more detail in Appendix A.[7]

Table 8 includes our results for $RSkew$ from Table 2, for the $SK2$ measure based on quartiles in (9), and for the estimator $SubRSkew$ which is based on the subsampling methodology suggested by Zhang, Mykland, and Ait-Sahalia (2005). Furthermore, we include simple historical skewness computed using daily returns over different horizons. The problem with computing historical skewness from daily returns is well-known: one needs a sufficiently long window to capture outliers that identify skewness, but longer windows may lead to artificial smoothness in the resulting skewness series. We report results for one-year, two-year, and five-year historical skewness. We also investigated one-month and six-month skewness, but we did not obtain significant estimates using these windows. We also include the industry skewness implemented by Zhang (2006), $IndSkewRes$, and the expected idiosyncratic skewness as constructed in Boyer, Mitton, and Vorkink (2010), $ExpSkew$.

Table 8 reports on the long-short returns. For value-weighted returns, we obtain statistically significant negative long-short returns for the alternative measures $SubRSkew$ and $SK2$. However, the long-short return is smaller than that of the $RSkew$ measure. Out of the three measures of historical skewness, only the 60-month yields a negative long-short return, but it is not statistically significant. The same holds for the measure in Zhang (2006), $IndSkewRes$, and for $ExpSkew$, the measure from Boyer, Mitton, and Vorkink (2010). We performed an elaborate robustness analysis with respect to the implementation of these measures, and we obtained similar results. Conclusions for the equal-weighted returns largely confirm the value-weighted results. The most important difference is that the long-short return for the $RSkew$ measure is much larger, and as a result the difference with the long-short returns for the $SK2$ and $SubRSkew$ measures is larger.

In summary, we conclude that all measures of realized skewness yield statistically significant negative long-short returns, consistent with theory. The estimate of a long-short value-weighted return of -24 basis points obtained using $RSkew$ is larger than the estimates obtained using alternative measures of realized skewness, and alternative skewness measures mostly yield estimates that are not statistically significant. The equal-weighted estimate for $RSkew$ is much larger, suggesting that the relationship is stronger for small firms. Interestingly, point estimates obtained using the $SK2$ and $SubRSkew$ measures are not very different from the estimate obtained using $ExpSkew$

[7] Using the methodology in Bakshi, Kapadia, and Madan (2003), Conrad, Dittmar, and Ghysels (2008) extract higher risk neutral moments from equity options and find that subsequent stock returns are negatively related to risk neutral volatility, negatively related to risk neutral skewness, and positively related to risk neutral kurtosis. Using the same methodology, Rehman and Vilkov (2010) and Xing, Zhang and Zhao (2010) find a positive relation between risk neutral skewness and future stock returns. It is difficult to compare these measures with physical skewness because risk premia are large. Moreover, these risk neutral measures can only be reliably estimated for a relatively limited number of stocks.

in the equal-weighted case.

For kurtosis, we also implemented two alternative estimators. The first kurtosis estimator uses the subsampling methodology suggested by Zhang, Mykland, and Ait-Sahalia (2005). The second alternative measure for intraday kurtosis uses the octiles of the intraday return distribution as proposed by Moors (1988). In particular, the centered kurtosis measure is defined as

$$KR2_t = ((E_7 - E_5) + (E_3 - E_1))/(E_6 - E_2) - 1.23,$$

where E_i is the i^{th} octile of the five-minute return distribution F, that is $E_i = F^{-1}(i/8)$.

We do not report on the two alternative measures of realized kurtosis, because they produced mixed results for the long-short portfolio returns. While the $KR2$ measure confirms the positive long-short returns, the subsampling measure produces small negative returns. None of the long-short returns is statistically significant. This confirms that the results for realized kurtosis are less robust than those for realized skewness.

4.3 Realized Moments and other Firm Characteristics

This section further analyzes the interaction between realized moments and other firm characteristics. Consider size as an example. As pointed out by Fama and French (2008), to ensure the validity of an anomaly, small (microcaps), medium, and large firms ought to all exhibit the anomaly. We use a double sorting methodology to analyze the realized skewness premium and the realized kurtosis premium for five different size portfolios. We first sort stocks into quintiles by size and then, within each quintile, we sort stocks again by realized skewness (or realized kurtosis) into quintiles. Then we compute the value- and equal-weighted return for each portfolio and the difference between the highest and lowest realized skewness (or kurtosis) quintiles. This difference represents a realized skewness (or kurtosis) premium conditional on size. This double sorting methodology analyzes the value- and equal-weighted return of the long-short portfolio, quintile 5 minus quintile 1, for each size quintile. With this methodology, we can assess if the realized moment premium is economically significant for all size levels. We also provide a similar analysis for the following firm characteristics: lagged return, market beta, BE/ME, realized volatility, historical skewness, illiquidity, number of intraday transactions, maximum return over the previous month, number of I/B/E/S analysts, idiosyncratic volatility and coskewness. We perform all analyses for realized skewness and realized kurtosis. Realized volatility is not included, because its relation with future stock returns is not statistically significant.

Panel A of Table 9 reports the value- and equal-weighted results for skewness. In row 1, we double sort stock returns on realized skewness across different levels of firm size and, for each firm size quintile, we compute the return of the portfolio that buys the highest realized skewness stocks (within a given size quintile) and sells the lowest realized skewness stocks (within that same size quintile). For value-weighted returns, the realized skewness premium of -75 basis points for quintile 1 can be earned by buying small stocks (microcaps) with high realized skewness and selling small stocks with low realized skewness. For big firms in quintile 5, the corresponding premium is −26 basis points. All five size groups exhibit the realized skewness anomaly, but the premium is larger for small stocks. This finding explains why the effect of realized skewness is weaker for value-weighted portfolios when compared to equal-weighted portfolios, as evident in Table 2. The stronger negative effect of skewness for small firms is also consistent with Chan, Chen, and Hsieh (1985), who show that there are risk differences between small and large firms. The realized skewness premium and t-statistics are of similar magnitude for equal-weighted returns.

Panel A of Table 9 reports realized skewness premia for value- and equal-weighted portfolios conditional on the various firm characteristics. For value-weighted returns the realized skewness premia are negative and statistically significant in most cases. Only two quintiles yield positive returns when double sorting by lagged return and one quintile is not significant when double sorting by book-to-market. For equal weighted portfolios, the realized skewness premia are negative and statistically significant for all firm characteristics. The relationship between realized skewness and subsequent returns is robust to all firm characteristics and is not a proxy for any of them.

In the previous section, Fama-MacBeth regressions showed that lagged returns have explanatory power for next-week returns. The third row in Panel A of Table 9 shows that realized skewness is negatively associated with next week value-weighted returns for quintiles 1 to 3 of lagged return, and that the effect is stronger for past losers (stocks with the smallest lagged return). For equal-weighted returns, the realized skewness premia are negative for any level of lagged return. Furthermore, as the market beta increases, the long-short skewness premium becomes more negative. This pattern is also observed for idiosyncratic volatility or realized volatility, confirming the results in Tables 4 and 6.

Panel B of Table 9 reports value- and equal-weighted results from a double sort on realized kurtosis and firm characteristics. The results are more robust for equal- than for value-weighted portfolios and are not as strong as those for realized skewness since not all realized kurtosis premia are statistically significant. Overall, it is clear from Table 9 that realized skewness is a stronger predictor of future stock returns than realized kurtosis.

4.4 Monthly Returns

Thus far our empirics have been based on weekly returns and weekly realized moments. In this section we keep the weekly frequency when computing realized moments but we increase the return holding period from one week to one month.

Table 10 contains the results for overlapping monthly returns. As for the weekly returns in Table 2, we report the value-weighted and equal-weighted returns of decile portfolios formed from realized moments, and the return difference between portfolio 10 (highest realized moment) and portfolio 1 (lowest realized moment). Each panel reports on both value-weighted and equal-weighted portfolios and include t-statistics computed from robust standard errors. Alpha is again computed using the

Carhart four factor model.

Panel A reports the results for realized volatility. The insignificant relationship between volatility and weekly returns found in Panel A of Table 2 is evident for monthly returns in Table 10 as well.

Panel B in Table 10 reports the relationship between realized skewness and subsequent monthly returns. The strong negative relationship between realized skewness and returns in Table 2 is confirmed when using monthly returns. This relationship is significant for raw returns as well as alphas, and for value-weighted and equal-weighted portfolios.

Finally, Panel C in Table 10 reports the cross-sectional relationship between weekly realized kurtosis and monthly returns. The positive relationship found for weekly returns in Table 2 is also evident for monthly returns in Table 10. The relationship appears to be stronger for equal-weighted returns than for value-weighted returns when using a one-month holding period. This was also the case for weekly returns in Table 2.

5 Properties of Realized Moments

The limiting properties of realized variance have been studied in detail in the econometrics literature, however, much less is known about realized skewness and kurtosis. In this section we therefore investigate the realized moments when assuming that the underlying continuous time price process follows a jump-diffusion with stochastic volatility. First, we derive closed-form solutions for the limits of the realized moments. Second, we allow for market microstructure noise and discontinuities in quoted prices, and provide Monte Carlo evidence on the realized higher moments. Third, we assess the significance of our cross-sectional return results when using jump-robust measures of realized volatility.

5.1 The Equity Price Process

To illustrate the properties of the realized moments defined in (2), (3), and (4), we assume that the log-price p_t of a security evolves according to the stochastic differential equation

$$dp_t = \left(\mu - \tfrac{1}{2}V_t - \mu_J\lambda\right)dt + \sqrt{V_t}dW_t^{(1)} + JdN_t, \tag{10}$$

$$dV_t = \kappa\left(\theta - V_t\right)dt + \sigma\sqrt{V_t}dW_t^{(2)}, \tag{11}$$

where μ is the drift parameter, κ is the mean reversion speed to the long-term volatility mean θ, and σ is the diffusion coefficient of the volatility process V_t. $W_t^{(1)}$ and $W_t^{(2)}$ denote two standard Brownian motions with correlation ρ, and N_t is an independent Poisson process with arrival rate λ. The jump size J is distributed $N\left(\mu_J, \sigma_J^2\right)$.

5.2 The Limits of the Realized Moments

Suppose that in the time interval $[0, T]$, for example a day, $N+1$ observations are available on p, and the distance between these observations is $\tau = T/N$, that is, the observation times are $t_i = i\tau$, for $i = 0, \ldots, N$. Then we define the realized moments by

$$RM(j) = \sum_{i=0}^{N} \left(p_{t_{i+1}} - p_{t_i}\right)^j \tag{12}$$

for $j = 1, 2, 3, 4$. The limits of these realized moments are given in the following proposition:

Proposition 1 *The realized moments defined in* (12) *for* $j = 1, 2, 3,$ *and* 4 *converge in mean square to the integrated moments*

$$\begin{aligned}
IM(1) &= \left(\mu - \frac{\theta}{2}\right) T + (\theta - V_0) \frac{\left(1 - e^{-\kappa T}\right)}{\kappa}, &(13)\\
IM(2) &= \left(\theta + \lambda\left(\mu_J^2 + \sigma_J^2\right)\right) T - (\theta - V_0) \frac{\left(1 - e^{-\kappa T}\right)}{\kappa}, &(14)\\
IM(3) &= \lambda\left(\mu_J^3 + 3\mu_J \sigma_J^2\right) T, &(15)\\
IM(4) &= \lambda\left(\mu_J^4 + 6\mu_J^2 \sigma_J^2 + 3\sigma_J^4\right) T. &(16)
\end{aligned}$$

Proof. See Appendix B. ■

This result is quite revealing. Note that while the limit for $j = 2$ contains both jump and diffusion parameters, the limits for $j = 3$ and $j = 4$ depend exclusively on jump parameters. This means that realized skewness and realized kurtosis, as defined in equations (3) and (4), complement the information captured by realized volatility. $IM(3)$, which is the limit of the numerator in $RDSkew$, is the only realized moment that accounts for the jump direction, since its sign depends on that of the average jump size, μ_J. $IM(4)$, which is the limit of the numerator in $RDKurt$, captures the magnitude of the jump.

5.3 Allowing for Market Microstructure Noise

So far, we have studied the realized moments in the idealized case where observed prices correspond to their theoretical counterparts. In practice, microstructure noise is present in high frequency prices. To simulate market microstructure noise, we define the observed log price p_t^* as

$$p_t^* = p_t + u_t, \tag{17}$$

where u_t is i.i.d. Gaussian noise with mean zero and variance σ_u^2. Hence, the observed log price p_t^* is a noisy observation of the non-observable true price p_t.

Several studies use Monte Carlo simulations to investigate the properties of realized variance estimators when allowing for market microstructure noise, see for instance Andersen, Bollerslev, and Meddahi (2011), Gonçalves and Meddahi (2009), and Ait-Sahalia and Yu (2009). Following their work, we conduct the following Monte Carlo study. We simulate 100,000 paths of the log price process p_t using the Euler scheme at a time interval $\tau = 1$ second. The parameters for the continuous part of the process are set to $\mu = 0.05$, $\kappa = 5$, $\theta = 0.04$, $\sigma = 0.5$, $V_0 = 0.09$, and $\rho = -0.5$. The parameters for the jump component are set at $\lambda = 100$, $\mu_J = 0.01$ and $\sigma_J = 0.05$. The microstructure noise, u_t, is modeled with a normal distribution of mean zero and standard deviation of 0.05%. These parameter values are similar to those employed by Ait-Sahalia and Yu (2009).

To assess the impact of the microstructure noise at different sampling frequencies, we use signature plots as proposed in Andersen, Bollerslev, Diebold, and Labys (2000). The signature plots provide the sample mean of a daily realized moment based on returns sampled at different intraday frequencies. We take as an observation period $T = 1$ day, that is $T = 1/252$, and we assume a day has 6.5 trading hours.

Panel A of Figure 3 shows the signature plots of $RM(j)$ (as defined in (12)) for $j = 2$, 3, and 4. This figure includes 99% confidence bands around the Monte Carlo estimates. For the second moments in the first row of panels the confidence intervals are very tight around the Monte Carlo estimate making them barely noticeable in the plot. For the third and fourth moments (the second and third row of panels), the 99% confidence intervals contain the Monte Carlo estimate as well as the theoretical limit.

The signature plot for the second moment $RM(2)$ depicts the well-known effect that microstructure noise has on realized volatility: as the sampling frequency increases (moving from right to left in the figure), the variance of the noise dominates that of the price process; but for lower frequencies, this effect attenuates. In contrast, the microstructure noise does not affect the signature plots of $RM(3)$ and $RM(4)$ in the same way. There is a small and insignificant bias in $RM(3)$ and $RM(4)$ relative to $IM(3)$ and $IM(4)$ but the bias does not increase with the intraday frequency.

5.4 Allowing for Quoted Price Discontinuity

Chakravarty, Wood, and Van Ness (2004) document that bid-ask spreads declined significantly following the decimalization of NYSE-listed companies in 2001. This indicates that pre-decimalization prices exhibit an additional bid-ask spread generated by fractional minimum increments. To gauge the effect of this discontinuity on the realized moment measures, we conduct a Monte Carlo study similar to the one above, with the exception that observed prices are now measured in sixteenths of a dollar. To isolate the effect of fractional minimum increments, we assume here that observed prices are not affected by microstructure noise.

Panel B of Figure 3 shows the signature plots for the realized moments. The plots reveal that realized volatility is the only moment affected by fractional changes in observed prices. As the frequency increases, the discontinuity of observed prices creates noise that is picked up by the volatility measure. However, the noise does not affect the third and fourth moments as shown by the 99% confidence intervals, which contain the Monte Carlo estimate as well as the theoretical limit.

In summary, we find that the third and fourth moments used in this paper have well-defined limits. Moreover, when estimated with an adequate sampling frequency, realized moments are not contaminated by simple market microstructure noise or discontinuous quotes.

5.5 Alternative Realized Volatility Estimators

For the affine jump-diffusion model that we assumed in (10)-(11) the limit of the sum of intraday squared returns in (14) can be written as the sum of jump variation and integrated variance

$$IM(2) = JV + IV$$

where

$$\begin{aligned} JV &\equiv \lambda\left(\mu_J^2 + \sigma_J^2\right)T \\ IV &\equiv \theta T + (V_0 - \theta)\frac{\left(1 - e^{-\kappa T}\right)}{\kappa} \end{aligned}$$

The $RDVar_t$ estimator in (2) that we have used in the empirics so far will capture both jumps and diffusive volatility in the limit. This does not invalidate it as an ex-post measure for the total daily quadratic variation, but it does suggest the use of more refined procedures for separately estimating IV and JV.

Several volatility estimators that are robust to jumps have been developed in the literature. They are designed to only capture IV and not JV in the limit. The so-called bipower variation estimator of Barndorff-Nielsen and Shephard (2004) is defined by

$$BPV_t = \frac{\pi}{2}\frac{N}{N-1}\sum_{i=1}^{N-1}|r_{t,i+1}|\,|r_{t,i}|$$

which converges in the limit to integrated variance, IV_t, when N approaches infinity, even in the presence of jumps.

Motivated by the presence of large jumps that may bias upward the bipower variation measure in realistic settings when N is finite, Andersen, Dobrev and Schaumburg (2010) have recently developed two alternative jump-robust estimators, defined by

$$\begin{aligned} MinRV_t &= \frac{\pi}{\pi-2}\left(\frac{N}{N-1}\right)\sum_{i=1}^{N-1}\min\left\{|r_{t,i}|, |r_{t,i+1}|\right\}^2, \text{ and} \\ MedRV_t &= \frac{\pi}{6-4\sqrt{3}+\pi}\left(\frac{N}{N-2}\right)\sum_{i=2}^{N-1}\text{median}\left\{|r_{t,i-1}|, |r_{t,i}|, |r_{t,i+1}|\right\}^2 \end{aligned}$$

These estimators will also both converge to IV_t when N goes to infinity and in the presence of large jumps they typically have better finite sample properties than BPV_t.

To assess the robustness of our cross-sectional return results, we report in Table 11 the equal-weighted and value-weighted weekly returns of the difference between portfolio 10 (highest realized moment) and portfolio 1 (lowest realized moment) when using the three alternative realized volatility estimators, BPV_t, $MinRV_t$ and $MedRV_t$. To facilitate comparisons, the first column of Table 11 uses the standard $RVol_t$ from (5) and thus reproduces the last column of Table 2. Each panel in Table 11 reports the equal-weighted and the value-weighted long-short returns. Alpha is again computed using the Carhart four factor model.

Panel A in Table 11 reports the long-short results for the three alternative realized volatility estimators. We find that the insignificant relationship between return and volatility remains when alternative estimators of realized volatility are used.

Panel B in Table 11 shows the long-short results for realized skewness when scaling by the three alternative realized volatility estimators. We see that the strong negative relationship between realized skewness and return found in Table 2 is robust to changing the denominator in

$$RDSkew_t = \frac{\sqrt{N}\sum_{i=1}^{N} r_{t,i}^3}{RDVar_t^{3/2}}. \tag{18}$$

to be any of the three jump-robust volatility estimators defined in this section.

Panel C in Table 11 presents the realized kurtosis results when scaling by the three alternative realized volatility estimators. Panel C shows that the positive relationship between return and realized kurtosis is also robust to changing the denominator in $RDKurt_t$ to be any of the jump-robust volatility estimators.

We conclude that the strong negative relationship between skewness and subsequent returns in the cross-section, as well as the positive relationship between kurtosis and subsequent returns, are not artefacts of the particular measure of realized volatility that we used above. The results hold up when we use estimators of realized volatility that are jump-robust.

6 Conclusions

We document the cross-sectional relationship between realized higher moments of individual stocks and future stock returns. We first introduce model-free estimates of higher moments based on the methodology used by Hsieh (1991) and Andersen, Bollerslev, Diebold, and Ebens (2001) to estimate realized volatility. We use five-minute returns to obtain a daily measure of realized volatility, realized skewness, and realized kurtosis, and subsequently aggregate this measure up to the weekly frequency. We find that realized skewness and realized kurtosis predict next week's stock returns in the cross-section, but realized volatility does not.

Realized skewness is negatively related to future stock returns. Value-weighted portfolios with low skewness outperform portfolios with high skewness by 24 basis points per week. Realized kurtosis is positively related to future stock returns. A portfolio that buys stocks with high realized kurtosis and sells stocks with low realized kurtosis generates an average weekly return of 14 basis points.

Fama-MacBeth regressions and double sorting confirm that realized skewness is not a proxy for firm characteristics such as lagged return, size, book-to-market, realized volatility, market beta, historical skewness, idiosyncratic volatility, coskewness, maximum return over the previous month, analysts coverage, illiquidity or number of intraday transactions. The forecasting ability of realized kurtosis is also found to be robust to these firm characteristics when employing Fama-MacBeth regressions. However, when double sorting using firm characteristics, the predictive power of realized kurtosis weakens.

We analyze the relationship between realized skewness and realized volatility in more detail. When double sorting on realized skewness and volatility, we find that stocks with negative skewness are compensated with high future returns. However, as skewness increases and becomes positive, the positive relation between volatility and returns turns into a negative relation. We conclude that investors may accept low returns and high volatility because they are attracted to high positive skewness.

We perform a similar analysis for realized skewness and idiosyncratic volatility. We find that portfolios with high idiosyncratic volatility compensate investors with higher returns only for low levels of skewness. For high levels of skewness, high idiosyncratic volatility leads to lower returns. This finding may help explain the idiosyncratic volatility puzzle in Ang, Hodrick, Xing, and Zhang (2006), who document that stocks with high idiosyncratic volatility earn low returns.

References

Ait-Sahalia, Y., and J. Yu, 2009, High Frequency Market Microstructure Noise Estimates and Liquidity Measures, Annals of Applied Statistics 3, 422-457.

Amihud, Y., 2002, Illiquidity and Stock Returns: Cross-Section and Time-Series Effects, Journal of Financial Markets 5, 31-56.

Andersen, T.G., and T. Bollerslev, 1998, Answering the Skeptics: Yes, Standard Volatility Models Do Provide Accurate Forecasts, International Economic Review 39, 885-905.

Andersen, T.G., T. Bollerslev, F. Diebold, and H. Ebens, 2001, The Distribution of Realized Stock Return Volatility, Journal of Financial Economics 61, 43-76.

Andersen, T.G., T. Bollerslev, F.X. Diebold, and P. Labys, 2000, Great Realizations, Risk, 105-108.

Andersen, T.G., T. Bollerslev, F.X. Diebold, and P. Labys, 2001, The Distribution of Realized Exchange Rate Volatility, Journal of the American Statistical Association 96, 42-55.

Andersen, T.G., Bollerslev, T., Diebold, F.X. and P. Labys, 2003, Modeling and Forecasting Realized Volatility, Econometrica, 71, 529-626.

Andersen, T.G., T. Bollerslev, and N. Meddahi, 2011, Realized Volatility Forecasting and Market Microstructure Noise, Journal of Econometrics 160, 220-234.

Andersen, T.G., Dobrev, D. and E. Schaumburg, 2010, Jump-Robust Volatility Estimation Using Nearest Neighbor Truncation, Federal Reserve Bank of New York Staff Report No. 465.

Ang, A., R.J. Hodrick, Y. Xing, and X. Zhang, 2006, The Cross-Section of Volatility and Expected Returns, Journal of Finance 61, 259-299.

Arditti, F., 1967, Risk and the Required Return on Equity, Journal of Finance 22, 19–36.

Arbel, A., and P. Strebel, 1982, The Neglected and Small Firm Effects, Financial Review 17, 201-218.

Bakshi, G., N. Kapadia, and D. Madan, 2003, Stock Return Characteristics, Skew Laws, and the Differential Pricing of Individual Equity Options, Review of Financial Studies 16, 101-143.

Bali, T., N. Cakici, and R. Whitelaw, 2009, Maxing Out: Stocks as Lotteries and the Cross-Section of Expected Returns, Journal of Financial Economics 99, 427-446.

Barndorff-Nielsen, O.E., and N. Shephard, 2002, Econometric Analysis of Realised Volatility and its Use in Estimating Stochastic Volatility Models, Journal of the Royal Statistical Society 64, 253-280.

Barndorff-Nielsen, O.E., and N. Shephard, 2004, Power and Bipower Variation with Stochastic Volatility and Jumps, Journal of Financial Econometrics 2, 1-37.

Barberis, N., and M. Huang, 2008, Stocks as Lotteries: The Implications of Probability Weighting for Security Prices, American Economic Review 98, 2066–2100.

Bowley, A.L., 1920, *Elements of Statistics*, Scribner's, New York.

Boyer, B., T. Mitton, and K. Vorkink, 2010, Expected Idiosyncratic Skewness, Review of Financial Studies 23, 169-202.

Brunnermeier, M., C. Gollier , and J. Parker, 2007, Optimal Beliefs, Asset Prices and the Reference for Skewed Returns, American Economic Review 97, 159-165.

Campbell, R. H., and A. Siddique, 2000, Conditional Skewness in Asset Pricing Tests, Journal of Finance 55, 1263–1295.

Carhart, M., 1997, On Persistence in Mutual Fund Performance, Journal of Finance 52, 57-82.

Chan, K.C., N. Chen, and D.A. Hsieh, 1985, An Exploratory Investigation of the Firm Size Effect, Journal of Financial Economics 14, 451-471.

Chakravarty, S., R. A. Wood, and R. A. Van Ness, 2004, Decimals And Liquidity: A Study Of The NYSE, Journal of Financial Research 27, 75–94.

Conrad, J., R.F. Dittmar, and E. Ghysels, 2008, Ex Ante Skewness and Expected Stock Returns, University of North Carolina, Working Paper.

Dittmar, R.F., 2002, Nonlinear Pricing Kernels, Kurtosis Preference, and Evidence from the Cross Section of Equity, Journal of Finance 57, 369-403.

Duffie, D., J. Pan, and K. Singleton, 2000, Transform Analysis and Asset Pricing for Affine Jump-Diffusions, Econometrica 68, 1343–1376.

Fama, E., and K. French, 1993, Common Risk Factors in the Returns on Stocks and Bonds, Journal of Financial Economics 33, 3-56.

Fama, E., and K. French, 2008, Dissecting Anomalies, Journal of Finance 63, 1653-1678.

Fama, E., and M. J. MacBeth, 1973, Risk, Return, and Equilibrium: Empirical Tests, Journal of Political Economy 81, 607-636.

Golec, J., and M. Tamarkin, 1998, Bettors Love Skewness, Not Risk, at the Horse Track, Journal of Political Economy 106, 205-225.

Gonçalves, S., and N. Meddahi, 2009, Bootstrapping Realized Volatility, Econometrica 77, 283-306.

Gutierrez Jr, R., and E. Kelley, 2008, The Long-Lasting Momentum in Weekly Returns, Journal of Finance 63, 415-447.

Harvey, C., and A. Siddique, 2000, Conditional Skewness in Asset Pricing Tests, Journal of Finance 55, 1263-1295.

Hsieh, D., 1991, Chaos and Nonlinear Dynamics: Application to Financial Markets, Journal of Finance, 46, 1839-1877.

Jegadeesh, N., 1990, Evidence of Predictable Behavior of Security Returns, Journal of Finance 45, 881-898.

Keim, D.B., 1983, Size-Related Anomalies and Stock Return Seasonality, Journal of Financial Economics 12, 13-32.

Kelly, B., 2011, Tail Risk and Asset Prices, Chicago Booth Research Paper No. 11-17.

Kraus, A., and R. Litzenberger, 1976, Skewness Preference and the Valuation of Risk Assets, Journal of Finance 31, 1085-1100.

Lehmann, B.N., 1990, Fads, Martingales, and Market Efficiency, Quarterly Journal of Economics 105, 1-28.

Mitton, T., and K. Vorkink, 2007, Equilibrium Underdiversification and the Preference for Skewness, Review of Financial Studies 20, 1255-1288.

Moors, J.J.A., 1988, A Quantile Alternative for Kurtosis, The Statistician 37, 25–32.

Neuberger, A., 2011, Realized Skewness, Working Paper, Warwick Business School.

Rehman, Z., and G. Vilkov, 2010, Risk-Neutral Skewness: Return Predictability and Its Sources, Working Paper, BlackRock and Goethe University.

Scott, R., and P. Horvath, 1980, On the Direction of Preference for Higher Order Moments, Journal of Finance 35, 915-919.

Xing, Y., X. Zhang, and R. Zhao, 2010, What Does Individual Option Volatility Smirks Tell Us about Future Equity Returns?, Journal of Financial and Quantitative Analysis 45, 641-662.

Zhang, L., P. Mykland, and Y. Ait-Sahalia, 2005, A Tale of Two Time Scales: Determining Integrated Volatility with Noisy High-Frequency Data, Journal of the American Statistical Association 100, 1394-1411.

Zhang, Y., 2006, Individual Skewness and the Cross-Section of Average Stock Returns, Working Paper.

Appendix A: Data

- Following Fama and French (1993), size (in billions of dollars) is computed each June as the stock price times the number of outstanding shares. The market equity value is held constant for a year.

- Following Fama and French (1993), book-to-market is computed as the ratio of book common equity over market capitalization (size). Book common equity is defined using COMPUSTAT's book value of stockholders' equity plus balance-sheet deferred taxes and investment tax credit minus the book value of preferred stock. The ratio is then computed as the book common equity at the end of the fiscal year over size at the end of December.

- Historical skewness for stock i on day t is defined as

$$HSkew_{i,t} = \frac{1}{N} \sum_{s=0}^{N} \left(\frac{r_{i,t-s} - \mu_i}{\sigma_i} \right)^3, \tag{19}$$

  where N is the number of trading days, $r_{i,t-s}$ is the daily log-return of stock i on day $t-s$, μ_i is the mean over the last month for stock i and σ_i is the standard deviation of stock i for that month. We use 20 trading days to estimate historical skewness.

- Market beta is computed at the end of each month using a regression of daily returns over the past 12 months.

- Following Ang, Hodrick, Xing, and Zhang (2006), idiosyncratic volatility is defined as

$$idvol_{i,t} = \sqrt{var(\varepsilon_{i,t})}, \tag{20}$$

  where $\varepsilon_{i,t}$ is the error term of the three-factor Fama and French (1993) regression. The regression is estimated with daily returns over the previous 20 trading days.

- Following Harvey and Siddique (2000), coskewness is defined as

$$CoSkew_{i,t} = \frac{E\left[\varepsilon_{i,t}\varepsilon_{m,t}^2\right]}{\sqrt{E\left[\varepsilon_{i,t}^2\right]} E\left[\varepsilon_{m,t}^2\right]}, \tag{21}$$

  where $\varepsilon_{i,t}$ is obtained from $\varepsilon_{i,t} = r_{i,t} - \alpha_i - \beta_i r_{m,t}$, where $r_{i,t}$ is the monthly return of stock i on month t, $r_{m,t}$ is the market monthly return on month t. This regression is estimated at the end of each month using monthly returns for the past 24 months.[8]

[8] Harvey and Siddique (2000) use data for the past 60 months to estimate coskewness. This approach considerably reduces our sample of firms. We therefore estimate coskewness using only 24 months, which is also done in Harvey and Siddique (2000).

- Following Zhang (2006), industry skewness is the cross-sectional intra-group return skewness computed from daily raw returns over one month. Industry skewness is defined as

$$IndSkew_{i,t} = \frac{\sum_i (R_i - IndMean_{i,t})}{IndStd_{i,t}^3}$$

where R_i are the daily raw returns of all stocks on industry i over one month, and $IndMean_{i,t}$ and $IndStd_{i,t}$ are the mean and standard deviation of the daily raw returns of all stocks on industry i over one month. There are 49 industries according to the classification system available on Kenneth French's website.

The industry skewness residual, $IndSkewRes_{i,t}$, is the residual of the regression of $IndSkew_{i,t}$ on $IndMean_{i,t}$.

- Following Boyer, Mitton, and Vorkink (2010), expected idiosyncratic skewness is computed in three steps. In the first step, the residual $\varepsilon_{i,d}$ of the Fama and French (1993) three factor model is computed on day d for firm i over the number of days in the set $N(t)$. The regression coefficients that define the residual are estimated with daily returns over a five year period. Then, idiosyncratic volatility and idiosyncratic skewness are defined as

$$\begin{aligned} iv_{i,t} &= \left(\frac{1}{N(t)} \sum \varepsilon_{i,d}^2 \right)^{1/2}, \\ is_{i,t} &= \frac{1}{N(t)} \frac{\sum \varepsilon_{i,d}^3}{iv_{i,t}^3}. \end{aligned}$$

In the second stage, we estimate the following cross-sectional regressions at the end of each month t

$$\begin{aligned} is_{i,t} &= \beta_{0,t} + \beta_{1,t} is_{i,t-60} + \beta_{2,t} iv_{i,t-60} + \beta_{3,t} mom_{i,t-60} + \beta_{4,t} turn_{i,t-60} \\ &\quad + \beta_{5,t} small_{i,t-60} + \beta_{6,t} medium_{i,t-60} + \beta_{7,t} NASDAQ_{i,t-60} \\ &\quad + \beta_{industry,t} IndustryDummy_{i,t-60}. \end{aligned}$$

In this equation, $mom_{i,t-60}$ is the momentum defined as the cumulative return for firm i over months $t-72$ and $t-61$, $turn_{i,t-60}$ is the average daily turnover of the firm over month $t-60$, $small_{i,t-60}$ and $medium_{i,t-60}$ are dummy variables for the firms in first and second terciles according to their market capitalization, $NASDAQ_{i,t-60}$ is a dummy variable for the firms trading on NASDAQ, and $IndustryDummy_{i,t-60}$ is an industry dummy based on the two digit SIC code classification system available on Kenneth French's website. In the final stage, we use the regression coefficients of the previous regression to estimate expected idiosyncratic skewness for each firm,

$$\begin{aligned} ExpSkew_{i,t} &= \beta_{0,t} + \beta_{1,t} is_{i,t} + \beta_{2,t} iv_{i,t} + \beta_{3,t} mom_{i,t} + \beta_{4,t} turn_{i,t} \\ &\quad + \beta_{5,t} small_{i,t} + \beta_{6,t} medium_{i,t} + \beta_{7,t} NASDAQ_{i,t} \\ &\quad + \beta_{industry,t} IndustryDummy_{i,t}. \end{aligned}$$

- Maximum return is defined as the maximum daily return over the previous month.
- Following Amihud (2002), stock illiquidity on day t is measured as the average of the ratio of the absolute value of the return over the dollar value of the trading volume over the previous year

$$illiquidity_{i,t} = \frac{1}{N} \sum_{s=0}^{N} \left(\frac{|r_{i,t-s}|}{|volume_{i,t-s} * price_{i,t-s}|} \right), \tag{22}$$

  where N is the number of trading days, $r_{i,t-s}$ is the daily log-return of stock i on day $t-s$, $volume_{i,t-s}$ is the daily volume of stock i on day $t-s$ and $price_{i,t-s}$ is the price of stock i on day $t-s$. We use 252 trading days to estimate illiquidity.
- The credit rating is retrieved from COMPUSTAT and is then assigned a numerical value as follows: AAA=1, AA+=2, AA=3, AA-=4, A+=5, A=6, A-=7, BBB+=8, BBB=9, BBB-=10, BB+=11, BB=12, BB-=13, B+=14, B=15, B-=16, CCC+=17, CCC=18, CCC-=19, CC=20, C=21 and D=22. When no rating is available, the default credit rating value is 8.

Appendix B: Limits of Realized Moments

The process for the log-price p_t in equation (10) belongs to the class of models commonly known as affine jump diffusion models. The affine structure of this process yields closed-form solutions for the moment generating function, MGF. In this appendix, we find an explicit representation of the MGF of $p_{t_i} - p_{t_{i-1}}$, which is then used to prove the limits of the realized measures.

Corollary 1 *The MGF of p_t is given by*

$$\begin{aligned} \psi_t(u) &= E\left[\exp\left(\tilde{u} \cdot [p_T, V_T]^\intercal\right) \middle| \mathcal{F}_t\right] \\ &= e^{\alpha(u,T-t) + up_t + \beta_2(u,T-t)V_0}, \end{aligned}$$

where

$$\begin{aligned} \alpha(u,t) &= (\mu - \lambda\mu_J)\, ut + \frac{\kappa\theta}{\sigma^2}\left((\gamma + b)\, t + 2\log\left(1 - \frac{\gamma + b}{2\gamma}\left(1 - e^{-\gamma t}\right)\right)\right) + \lambda t\left(\phi(u) - 1\right), \\ \beta_2(u,t) &= -\frac{a\left(1 - e^{-\gamma t}\right)}{2\gamma - (\gamma + b)\left(1 - e^{-\gamma t}\right)}, \end{aligned}$$

where $a = u - u^2$, $b = \sigma\rho u - \kappa$, $\gamma = \sqrt{b^2 + a\sigma^2}$, *and* $\tilde{u} = [u, 0]^\intercal$. $\mathcal{F}_t$ *denotes the information set generated by the process* $[p_t, V_t]^\intercal$.

Proof. Using the transform analysis in Duffie, Pan, and Singleton (2000), we have that

$$E\left[\exp\left(\tilde{u} \cdot [p_T, V_T]^\intercal\right) \middle| \mathcal{F}_t\right] = \exp\left(\mathcal{A}\left(\tilde{u}, t\right) + \mathcal{B}\left(\tilde{u}, t\right) \cdot [p_t, V_t]^\intercal\right),$$

where $\mathcal{B}(t) = [\beta_1(t), \beta_2(t)]^\intercal$ and $\tilde{u} = [u, 0]^\intercal$. Solving the system of ODEs

$$\begin{aligned} \dot{\mathcal{B}}(t) &= \begin{bmatrix} 0 & 0 \\ -\frac{1}{2} & -\kappa \end{bmatrix} \begin{bmatrix} \beta_1(t) \\ \beta_2(t) \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} \beta_1(t) \\ \beta_2(t) \end{bmatrix}^\intercal \begin{bmatrix} 0 & 0 \\ -\frac{1}{2} & -\kappa \end{bmatrix} \begin{bmatrix} \beta_1(t) \\ \beta_2(t) \end{bmatrix}, \\ \dot{\mathcal{A}}(t) &= \begin{bmatrix} \mu - \lambda\mu_J \\ \kappa\theta \end{bmatrix} \cdot \begin{bmatrix} \beta_1(t) \\ \beta_2(t) \end{bmatrix} + \lambda\left(\phi(u) - 1\right), \end{aligned}$$

with $\mathcal{A}(T) = u$ and $\mathcal{B}(T) = [u, 0]^\intercal$ concludes the proof. ■

Armed with the MGF of p_t, we can now derive the MGF for $p_{t_{i+1}} - p_{t_i}$.

Corollary 2 *The MGF of* $p_{t_{i+1}} - p_{t_i}$ *is given by*

$$\begin{aligned} \varphi_t(u, \tau) &= E\left[\exp\left(u\left(p_{t+\tau} - p_t\right)\right) \middle| \mathcal{F}_0\right] \\ &= \exp\left(\alpha(u, \tau)\right) \exp\left(\frac{\beta(u, \tau) e^{-\kappa t}}{1 - \beta(u, \tau) \frac{\sigma^2}{2\kappa}\left(1 - e^{-\kappa t}\right)}\right) \left(1 - \beta(u, \tau) \frac{\sigma^2}{2\kappa}\left(1 - e^{-\kappa t}\right)\right)^{-\frac{2\kappa\theta}{\sigma^2}} \end{aligned}$$

Proof. From the definition of the MGF and using the law of iterated expectations, we have:

$$\begin{aligned} E\left[\exp\left(u\left(p_{t+\tau} - p_t\right)\right) \middle| \mathcal{F}_0\right] &= E\left[E\left[\exp\left(u\left(p_{t+\tau} - p_t\right)\right) \middle| \mathcal{F}_t\right] \middle| \mathcal{F}_0\right] \\ &= E\left[\exp\left(\alpha(u, \tau) + \beta(u, \tau) V_t\right) \middle| \mathcal{F}_0\right] \\ &= \exp\left(\alpha(u, \tau)\right) E\left[\exp\left(\beta(u, \tau) V_t\right) \middle| \mathcal{F}_0\right] \\ &= \exp\left(\alpha(u, \tau)\right) \exp\left(-A\left(-\beta(u, \tau), t\right) - B\left(-\beta(u, \tau), t\right) V_0\right), \end{aligned}$$

where

$$A(\varsigma, t) = \frac{\varsigma e^{-\kappa t}}{1 + \varsigma \frac{\sigma^2}{2\kappa}\left(1 - e^{-\kappa t}\right)}, \quad B(\varsigma, t) = \frac{2\kappa\theta}{\sigma^2} \log\left(1 + \varsigma \frac{\sigma^2}{2\kappa}\left(1 - e^{-\kappa t}\right)\right),$$

are the coefficients of the affine representation of the MGF for V_t. ■

To prove Proposition 1, we need to find the expected value of the limits of the realized moments defined in (12). The following proposition establishes these limits.

Corollary 3 *The limit*

$$\lim_{N \to \infty} E\left[\sum_{i=1}^{N} \left(p_{t_i} - p_{t_{i-1}}\right)^j\right] \tag{23}$$

converges to (13), (14),(15) *and* (16) *for* $j = 1$, 2, 3 *and* 4, *respectively.*

Proof. We start by rewriting equation (23) as

$$\begin{aligned}\lim_{N\to\infty} E\left[\sum_{i=1}^{N}\left(p_{t_i}-p_{t_{i-1}}\right)^j\right] &= \lim_{N\to\infty}\sum_{i=1}^{N} E\left[\left(p_{t_i}-p_{t_{i-1}}\right)^j\right] \\ &= \lim_{N\to\infty}\sum_{i=1}^{N}\frac{\partial^j\varphi_t}{\partial u^j}(u,\tau)\bigg|_{u=0}.\end{aligned}$$

The Taylor series expansion of $\frac{\partial^j\varphi_t}{\partial u^j}(u,\tau)\Big|_{u=0}$ around $\tau=0$ yields

$$\frac{\partial\varphi_t}{\partial u}(u,\tau)\bigg|_{u=0} = \left(\mu-\frac{\theta}{2}+\frac{1}{2}e^{-\kappa t}(\theta-V_0)\right)\tau+O\left(\tau^2\right), \tag{24}$$

$$\frac{\partial^2\varphi_t}{\partial u^2}(u,\tau)\bigg|_{u=0} = \left(\theta+\lambda\left(\mu_J^2+\sigma_J^2\right)-\frac{1}{2}e^{-\kappa t}(\theta-V_0)\right)\tau+O\left(\tau^2\right), \tag{25}$$

$$\frac{\partial^3\varphi_t}{\partial u^3}(u,\tau)\bigg|_{u=0} = \lambda\left(\mu_J^3+3\mu_J\sigma_J^2\right)\tau+O\left(\tau^2\right), \tag{26}$$

$$\frac{\partial^4\varphi_t}{\partial u^4}(u,\tau)\bigg|_{u=0} = \lambda\left(\mu_J^4+6\mu_J^2\sigma_J^2+3\sigma_J^4\right)\tau+O\left(\tau^2\right), \tag{27}$$

where $O(\tau)^2$ denotes all terms of order 2 and above. As N tends to infinity, τ converges to zero, so that the only remaining terms in equations (24), (25), (26), and (27) are those of order 1. The limit of the sum of these terms coincides with the definition of the Riemann-Stieltjes integral, so that integrating these terms with respect to t over the sampling interval $[0,T]$ gives (13), (14), (15), and (16). ■

Corollary 4 *The limit of the variance of the realized moments is*

$$\lim_{N\to\infty}\sum_{i=1}^{N} Var\left[\left(p_{t_i}-p_{t_{i-1}}\right)^j\right]=0,$$

for $j=1,\ 2,\ 3$ *and* 4.

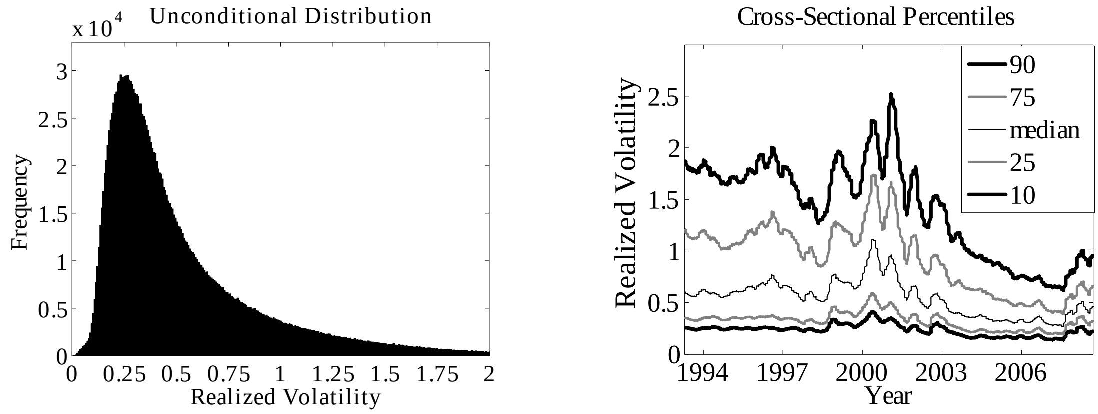
Proof. Follows by arguments similar to those made to prove (23). ■

The results in this appendix ensure mean-squared or L^2 convergence of the realized moments.

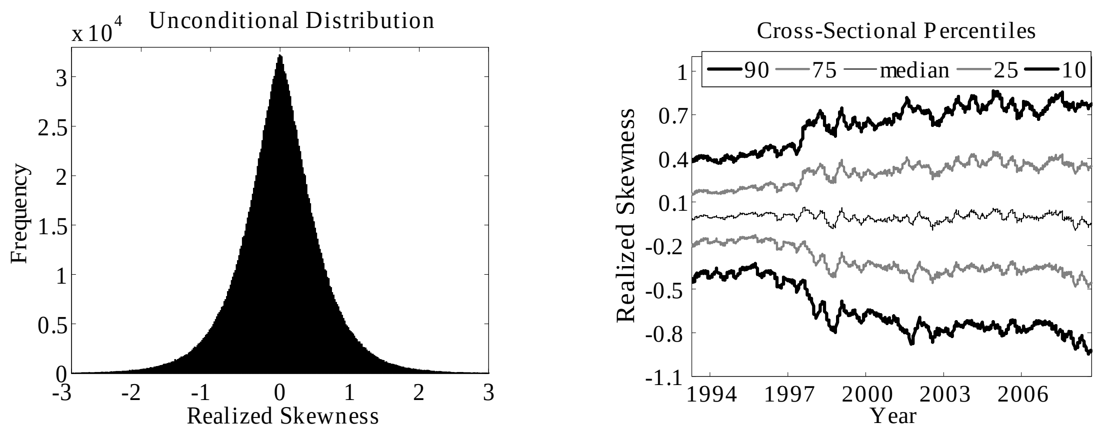
Figure 1
Histogram and Percentiles of Realized Moments

We display the histograms and various percentiles of realized moments for the cross-section of stocks over the period January 1993 to September 2008. Figures for realized volatility, realized skewness, and realized kurtosis are reported in Panel A, Panel B, and Panel C respectively. The sample contains 2,052,752 firm-week observations.

Panel A: Realized Volatility



Panel B: Realized Skewness



Panel C: Realized Kurtosis

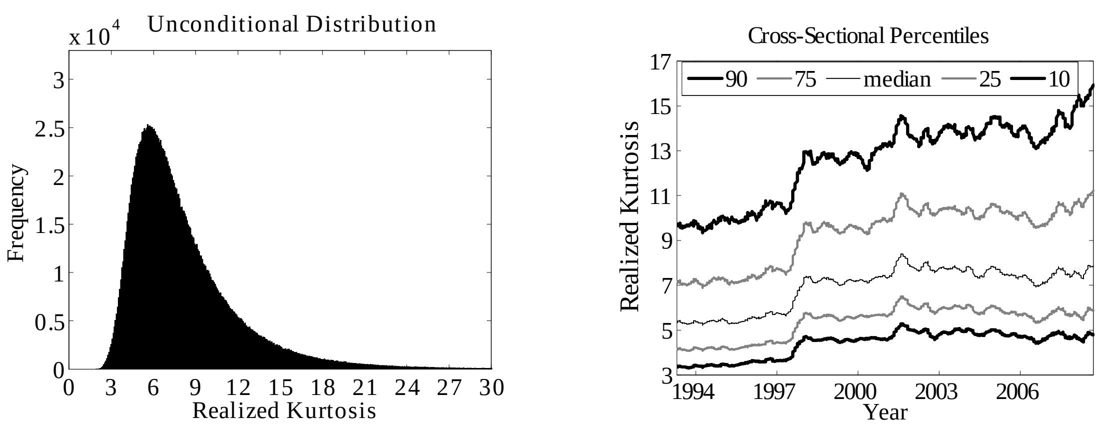


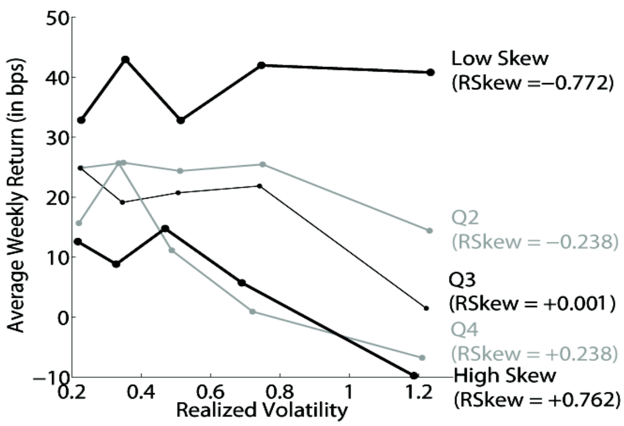
Figure 2

Interaction between Realized Volatility, Realized Skewness, and Stock Returns

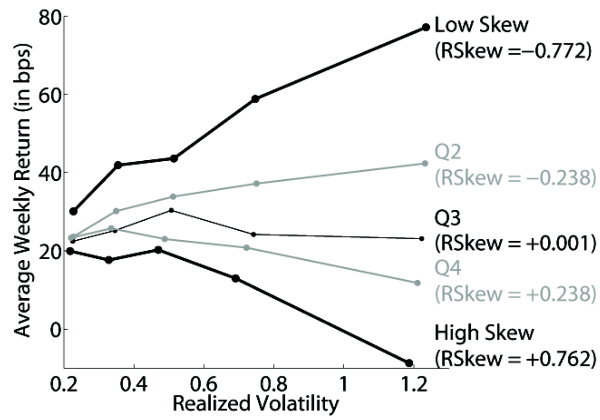
Each week, stocks are first ranked by realized skewness into five quintiles and then, within each quintile, stocks are sorted once again into five quintiles by realized volatility (Panel A) and idiosyncratic volatility (Panel B). We report value- and equal-weighted returns (in bps) for different levels of volatility and realized skewness. Each line represents different quintiles of realized skewness. The average realized skewness in each quintile is reported in parentheses.

Panel A: Interaction between Realized Volatility, Realized Skewness, and Stock Returns

Value-Weighted

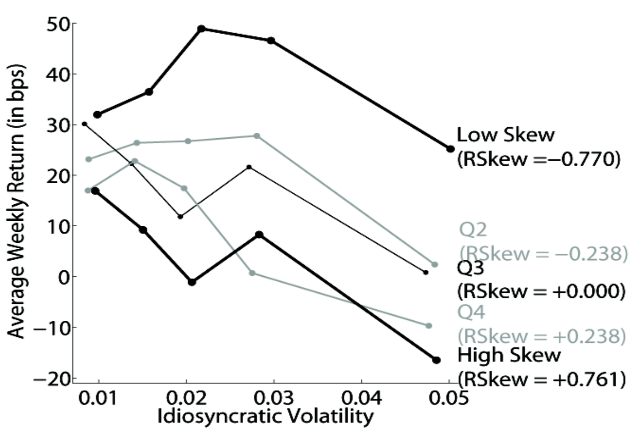


Equal-Weighted



Panel B: Interaction between Idiosyncratic Volatility, Realized Skewness, and Stock Returns

Value-Weighted



Equal-Weighted

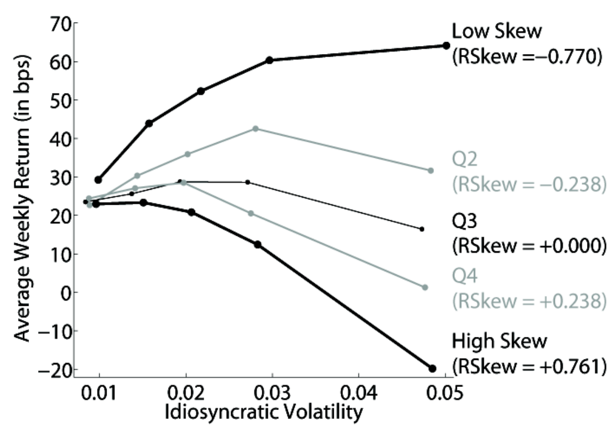


Figure 3
Signature Plots for Realized Moments

We show signature plots for the three daily realized moments, $RM(2)$, $RM(3)$, and $RM(4)$ computed using equation (12). The intraday sampling frequency on the horizontal axis is in seconds. The dotted lines represent the theoretical limit of realized moments as given by equations (14), (15), and (16). Monte Carlo estimates are plotted in continuous dark lines. Confidence intervals at 99% are shown in grey lines. In Panel A, observed prices have a microstructure noise component, which is simulated from a mean-zero normal distribution with a standard deviation of 0.05%. In Panel B, prices are only observed in sixteenths of a dollar.

Panel A: Market Microstructure Noise | Panel B: Discrete Price Changes

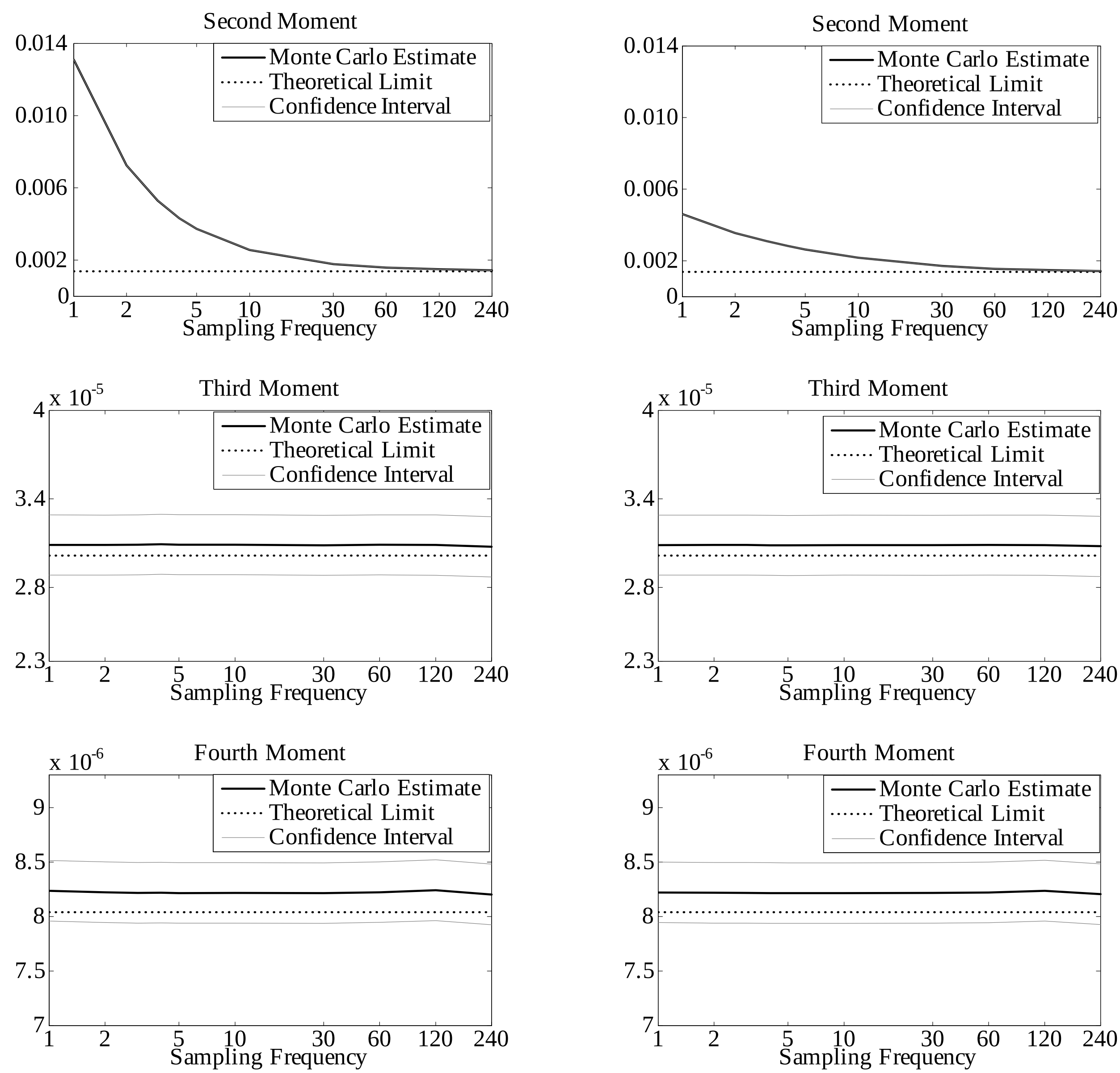


Table 1
Characteristics of Portfolios Sorted by Realized Moments

Each week, stocks are ranked by their realized moment and sorted into deciles. The equal-weighted characteristics of those deciles are computed over the same week. This procedure is repeated for every week from January 1993 through September 2008. Panel A displays the average results for realized volatility, Panel B for realized skewness, and Panel C for realized kurtosis. Average characteristics of the portfolios are reported for the realized moment, Size ($ market capitalization in $billions), BE/ME (book-to-market equity ratio), Realized volatility (weekly realized volatility computed with high-frequency data), Historical Skewness (one month historical skewness from daily returns), Market Beta, Lagged Return, Idiosyncratic Volatility (computed as in Ang, Hodrick, Xing, and Zhang (2006)), Coskewness (computed as in Harvey and Siddique (2000)), Maximum Return (of the previous month), Illiquidity (annual average of the absolute return over daily dollar trading volume times 10^6, as in Amihud (2002)), Number of Analysts (from I/B/E/S), Credit Rating (1= AAA, 8= BBB+, 17= CCC+, 22=D), Price (stock price), Intraday Transactions (intraday transactions per day) and Number of Stocks.

Panel A: Characteristics of Portfolios Sorted by Realized Volatility

Deciles	1	2	3	4	5	6	7	8	9	10
Realized Volatility	0.188	0.254	0.308	0.369	0.447	0.543	0.660	0.803	1.001	1.450
Realized Skewness	0.024	0.020	0.012	0.004	0.000	-0.004	-0.009	-0.014	-0.024	-0.026
Realized Kurtosis	7.0	6.9	7.0	7.2	7.5	7.8	8.1	8.4	8.8	9.4
Size	13.84	12.14	9.04	6.19	4.26	2.89	1.91	1.21	0.79	0.46
BE/ME	0.654	0.573	0.509	0.507	0.488	0.459	0.428	0.421	0.417	0.440
Historical Skewness	0.109	0.102	0.109	0.126	0.144	0.166	0.191	0.212	0.241	0.311
Market Beta	0.72	0.82	0.89	0.97	1.07	1.18	1.28	1.36	1.36	1.32
Lagged Return	0.003	0.004	0.004	0.005	0.005	0.006	0.007	0.008	0.010	0.028
Idiosyncratic Volatility	0.011	0.013	0.015	0.017	0.020	0.023	0.027	0.031	0.036	0.048
Coskewness	0.003	-0.007	-0.018	-0.028	-0.032	-0.035	-0.041	-0.045	-0.049	-0.050
Maximum Return	0.032	0.036	0.041	0.047	0.055	0.064	0.074	0.085	0.099	0.137
Illiquidity	0.005	0.005	0.006	0.009	0.013	0.020	0.035	0.080	0.126	0.562
Number of Analysts	10.0	10.6	10.2	9.5	8.8	7.9	6.9	5.8	4.7	3.2
Credit Rating	7.0	7.4	7.9	8.4	8.7	8.9	8.9	8.8	8.7	8.4
Price	62.2	53.6	38.9	33.5	29.6	27.7	24.7	21.4	17.8	15.0
Intraday Transactions	1,216	1,499	1,576	1,568	1,628	1,617	1,558	1,438	1,268	1,189
Number of Stocks	256	257	257	257	257	257	257	257	257	257

Table 1 (Continued)

Panel B: Characteristics of Portfolios Sorted by Realized Skewness

Deciles	1	2	3	4	5	6	7	8	9	10
Realized Skewness	-1.04	-0.50	-0.31	-0.17	-0.05	0.05	0.17	0.31	0.50	1.02
Realized Volatility	0.618	0.613	0.614	0.614	0.610	0.606	0.600	0.590	0.577	0.579
Realized Kurtosis	11.4	7.9	7.0	6.6	6.4	6.4	6.6	7.0	7.8	11.1
Size	2.96	4.41	5.22	5.75	6.22	6.22	6.09	5.69	4.91	3.49
BE/ME	0.476	0.471	0.480	0.483	0.486	0.488	0.500	0.495	0.488	0.490
Historical Skewness	0.140	0.160	0.163	0.167	0.162	0.167	0.171	0.173	0.184	0.209
Market Beta	1.05	1.14	1.12	1.11	1.09	1.10	1.11	1.11	1.10	1.02
Lagged Return	-0.041	-0.025	-0.014	-0.006	0.003	0.011	0.020	0.030	0.042	0.061
Idiosyncratic Volatility	0.026	0.025	0.024	0.024	0.023	0.023	0.023	0.024	0.024	0.025
Coskewness	-0.038	-0.031	-0.028	-0.029	-0.027	-0.026	-0.025	-0.024	-0.025	-0.027
Maximum Return	0.067	0.067	0.066	0.065	0.065	0.065	0.066	0.068	0.069	0.072
Illiquidity	0.116	0.071	0.082	0.066	0.076	0.059	0.058	0.067	0.102	0.158
Number of Analysts	6.5	7.6	7.9	8.1	8.1	8.1	8.2	8.2	8.0	6.9
Credit Rating	8.4	8.3	8.3	8.3	8.3	8.3	8.3	8.3	8.3	8.4
Price	29.8	32.6	33.2	32.2	31.5	31.4	33.0	33.7	33.9	33.1
Intraday Transactions	998	1,324	1,516	1,613	1,676	1,683	1,633	1,595	1,426	1,095
Number of Stocks	256	257	257	257	257	257	257	257	257	257

Panel C: Characteristics of Portfolios Sorted by Realized Kurtosis

Deciles	1	2	3	4	5	6	7	8	9	10
Realized Kurtosis	3.9	4.7	5.3	5.9	6.5	7.2	8.0	9.2	10.9	16.6
Realized Volatility	0.485	0.537	0.570	0.589	0.603	0.614	0.623	0.638	0.654	0.711
Realized Skewness	0.005	0.007	0.006	0.006	0.007	0.003	0.000	-0.006	-0.012	-0.034
Size	15.59	8.80	6.22	4.89	3.88	3.27	2.80	2.38	1.98	1.36
BE/ME	0.481	0.492	0.476	0.502	0.477	0.478	0.476	0.480	0.480	0.514
Hskew	0.129	0.149	0.156	0.165	0.170	0.166	0.180	0.177	0.186	0.220
Beta	1.09	1.15	1.16	1.15	1.14	1.13	1.12	1.08	1.03	0.90
Lagged Return	0.004	0.006	0.007	0.007	0.008	0.008	0.008	0.008	0.009	0.015
Idiosyncratic Volatility	0.019	0.022	0.023	0.024	0.024	0.025	0.025	0.026	0.026	0.027
Coskewness	-0.012	-0.017	-0.020	-0.023	-0.027	-0.030	-0.031	-0.036	-0.040	-0.046
Maximum Return	0.053	0.061	0.065	0.067	0.068	0.069	0.070	0.070	0.071	0.076
Illiquidity	0.014	0.029	0.035	0.046	0.061	0.070	0.074	0.109	0.112	0.306
Number of Analysts	11.4	10.1	9.1	8.4	7.8	7.4	6.9	6.4	5.8	4.4
Credit Rating	7.8	8.1	8.3	8.3	8.4	8.4	8.4	8.5	8.5	8.5
Price	36.4	37.9	36.2	34.0	33.6	32.4	30.2	29.5	28.6	25.6
Intraday Transactions	3,250	2,195	1,795	1,553	1,349	1,217	1,067	917	736	483
Number of Stocks	256	257	257	257	257	257	257	257	257	257

Table 2
Realized Moments and the Cross-Section of Stock Returns

We report value- and equal-weighted weekly returns (in bps) of decile portfolios formed from realized moments, the corresponding t-statistics (in parentheses), and the return difference between portfolio 10 (highest realized moment) and portfolio 1 (lowest realized moment) over the period January 1993 to September 2008. Panel A displays the results for realized volatility, Panel B for realized skewness, and Panel C for realized kurtosis. Each panel reports the value-weighted portfolios and the equal-weighted portfolios. Raw returns (in bps) are obtained from decile portfolios sorted solely from ranking stocks based on the realized moment measure. Alpha is the intercept from time-series regressions of the returns of the portfolio using the Carhart four factor model.

Panel A: Realized Volatility and the Cross-Section of Stock Returns

	Low	2	3	4	5	6	7	8	9	High	High-Low
					Value weighted						
Raw Returns	19.98	21.66	26.28	20.59	18.41	18.10	19.51	14.09	17.01	8.07	-11.92
	(3.24)	(3.06)	(3.27)	(2.19)	(1.69)	(1.39)	(1.32)	(0.81)	(0.93)	(0.41)	(-0.65)
Alpha, C4	20.34	22.40	28.11	22.93	20.87	20.48	21.56	17.61	21.40	12.58	-9.42
	(3.25)	(3.13)	(3.48)	(2.42)	(1.90)	(1.56)	(1.44)	(1.01)	(1.16)	(0.63)	(-0.51)
					Equal weighted						
Raw Returns	21.96	24.10	25.82	30.19	28.56	31.92	29.15	32.04	34.04	27.04	5.08
	(3.93)	(3.62)	(3.55)	(3.75)	(3.13)	(2.97)	(2.35)	(2.29)	(2.16)	(1.55)	(0.33)
Alpha, C4	21.17	23.83	26.29	30.33	29.51	33.36	30.61	34.57	37.08	29.83	7.00
	(3.73)	(3.53)	(3.57)	(3.71)	(3.19)	(3.07)	(2.44)	(2.44)	(2.32)	(1.70)	(0.45)

Panel B: Realized Skewness and the Cross-Section of Stock Returns

	Low	2	3	4	5	6	7	8	9	High	High-Low
					Value weighted						
Raw Returns	40.49	33.84	27.24	19.93	26.64	15.61	20.58	9.00	5.25	16.69	-23.80
	(4.12)	(3.49)	(2.77)	(1.96)	(2.83)	(1.76)	(2.29)	(1.00)	(0.61)	(1.98)	(-3.65)
Alpha, C4	39.56	35.70	28.49	21.19	29.81	16.80	22.34	10.45	8.06	18.67	-22.55
	(3.97)	(3.63)	(2.87)	(2.06)	(3.14)	(1.88)	(2.46)	(1.15)	(0.93)	(2.19)	(-3.42)
					Equal weighted						
Raw Returns	54.99	45.73	36.08	30.78	29.43	21.13	23.78	18.22	13.29	11.54	-43.44
	(5.06)	(4.12)	(3.31)	(2.91)	(2.87)	(2.11)	(2.41)	(1.87)	(1.41)	(1.29)	(-8.91)
Alpha, C4	55.92	47.21	37.47	32.50	30.40	22.08	24.96	19.81	14.34	12.06	-45.52
	(5.07)	(4.20)	(3.39)	(3.03)	(2.93)	(2.18)	(2.50)	(2.01)	(1.50)	(1.33)	(-9.26)

Panel C: Realized Kurtosis and the Cross-Section of Stock Returns

	Low	2	3	4	5	6	7	8	9	High	High-Low
					Value weighted						
Raw Returns	17.17	20.83	18.30	27.24	17.55	15.04	21.81	26.27	18.91	30.74	13.57
	(1.80)	(2.32)	(2.06)	(3.06)	(1.93)	(1.61)	(2.47)	(2.81)	(2.16)	(3.58)	(2.12)
Alpha, C4	18.67	23.12	20.76	29.29	18.68	15.23	22.61	27.25	20.98	31.18	10.85
	(1.94)	(2.55)	(2.33)	(3.26)	(2.04)	(1.62)	(2.53)	(2.88)	(2.37)	(3.57)	(1.67)
					Equal weighted						
Raw Returns	22.68	25.66	25.81	25.25	27.30	27.00	27.86	30.47	34.39	38.45	15.77
	(2.28)	(2.52)	(2.48)	(2.37)	(2.58)	(2.59)	(2.70)	(2.96)	(3.45)	(4.21)	(2.98)
Alpha, C4	23.70	27.90	27.32	26.94	28.85	28.39	29.17	31.22	35.03	38.12	12.75
	(2.35)	(2.71)	(2.60)	(2.50)	(2.69)	(2.69)	(2.79)	(2.99)	(3.47)	(4.12)	(2.41)

Table 3
Fama-MacBeth Cross-Sectional Regressions

We report results from Fama-MacBeth cross-sectional regressions of weekly stock returns (in bps) on firm characteristics for the period January 1993 to September 2008. Firm characteristics are Realized Volatility, Realized Skewness, Realized Kurtosis, Lagged Return (in bps), Size (market capitalization in $billions), BE/ME (book-to-market equity ratio), Market Beta, Historical Skewness (one month historical skewness from daily returns), Idiosyncratic Volatility (computed as in Ang, Hodrick, Xing, and Zhang (2006)), Coskewness (computed as in Harvey and Siddique (2000) with 24 months of data), Maximum Return (of previous month in bps), Number of Analysts (from I/B/E/S), Illiquidity (annual average of the absolute return over daily dollar trading volume times 10^6, as in Amihud (2002)), and Number of Intraday Transactions. We report the average of the coefficient estimates for the weekly regressions along with the Newey-West t-statistic (in parentheses).

	(1)	(2)	(3)	(4)	(5)	(6)
Intercept	27.7	31.5	21.0	20.7	16.8	187.6
	(3.80)	(3.23)	(1.99)	(2.54)	(2.03)	(6.76)
Realized Volatility	3.5			0.1	6.7	-3.1
	(0.26)			(0.01)	(0.46)	(-0.40)
Realized Skewness		-22.4		-24.3	-3.9	-4.7
		(-7.90)		(-9.21)	(-2.07)	(-2.73)
Realized Kurtosis			1.3	1.3	1.6	0.7
			(3.49)	(2.99)	(3.83)	(2.15)
Lagged Return					-0.032	-0.041
					(-10.55)	(-14.27)
log (Size)						-13.2
						(-5.48)
log (BE/ME)						3.2
						(1.80)
Market Beta						-8.1
						(-1.20)
Historical Skewness						17.4
						(9.98)
Idiosyncratic Volatility						-284.0
						(-1.17)
Coskewness						15.4
						(0.99)
Maximum Monthly Return						-0.014
						(-2.09)
log (Number of Analysts+1)						0.28
						(0.12)
Illiquidity						-8.2
						(-0.48)
log (Number of Intraday transactions)						13.6
						(5.31)
Rsquare	0.025	0.003	0.003	0.031	0.039	0.096

Table 4
Double Sorting on Realized Skewness and Realized Volatility

Each week, stocks are first sorted by realized skewness into five quintiles and then, within each quintile, stocks are sorted once again into five quintiles by realized volatility. Panel A reports value-weighted returns and Panel B reports equal-weighted returns. The value- and equal-weighted average weekly returns (in bps) are reported for all double sorted portfolios as well as for the difference between portfolio five (High volatility) and one (Low volatility) for each level of realized skewness. T-statistics are in parentheses.

	Panel A: Value-Weighted					
	Low	2	3	4	High	High-Low RVol
1 (Low Realized Skewness)	32.7	43.1	32.7	42.0	40.8	8.1
	(4.32)	(4.47)	(2.45)	(2.47)	(2.07)	(0.46)
2	25.0	25.6	24.1	25.6	14.8	-10.2
	(3.40)	(2.70)	(1.82)	(1.50)	(0.70)	(-0.55)
3	24.9	19.1	20.8	21.5	1.1	-23.7
	(3.59)	(2.11)	(1.68)	(1.35)	(0.06)	(-1.35)
4	15.4	25.5	10.6	0.7	-6.9	-22.2
	(2.27)	(2.96)	(0.88)	(0.04)	(-0.35)	(-1.25)
5 (High Realized Skewness)	12.6	8.7	14.6	6.1	-9.6	-22.2
	(1.89)	(0.95)	(1.34)	(0.44)	(-0.53)	(-1.34)

	Panel B: Equal-Weighted					
	Low	2	3	4	High	High-Low RVol
1 (Low Realized Skewness)	30.0	42.0	43.2	59.0	77.0	47.2
	(4.50)	(4.92)	(3.70)	(4.16)	(4.47)	(3.31)
2	23.8	30.2	33.5	37.2	43.0	18.9
	(3.73)	(3.74)	(3.09)	(2.61)	(2.37)	(1.23)
3	22.5	25.3	30.0	24.2	23.0	0.6
	(3.70)	(3.28)	(3.02)	(1.77)	(1.36)	(0.04)
4	23.4	25.3	23.1	20.8	12.0	-11.7
	(3.88)	(3.33)	(2.40)	(1.60)	(0.70)	(-0.80)
5 (High Realized Skewness)	19.9	17.6	20.2	13.0	-9.0	-28.4
	(3.32)	(2.33)	(2.20)	(1.09)	(-0.56)	(-2.22)

Table 5
Idiosyncratic Volatility and the Cross-Section of Stock Returns

We report value- and equal-weighted weekly returns (in bps) of quintile portfolios ranked by idiosyncratic volatility, their t-statistics (in parentheses), and the difference between portfolio 5 (highest idiosyncratic volatility) and portfolio 1 (lowest idiosyncratic volatility) over the period January 1993 to September 2008. Panel A reports on value-weighted portfolios and Panel B reports on equal-weighted portfolios. Raw returns (in bps) are obtained for quintile portfolios sorted solely on the idiosyncratic volatility measure. Alpha is the intercept from the time-series regressions for the return on the portfolio that buys portfolio 5 and sells portfolio 1, using the Carhart four-factor model.

	Panel A: Value-Weighted					
	Low	2	3	4	High	High-Low
Raw Returns	24.61	22.76	15.05	18.40	0.48	-24.14
	(3.46)	(2.79)	(1.46)	(1.39)	(0.03)	(-1.73)
Alpha, C4	25.95	24.54	16.36	21.71	5.09	-22.53
	(3.63)	(2.99)	(1.57)	(1.62)	(0.30)	(-1.59)

	Panel B: Equal-Weighted					
	Low	2	3	4	High	High-Low
Raw Returns	23.61	30.38	32.69	32.23	20.96	-2.64
	(3.96)	(3.96)	(3.30)	(2.50)	(1.30)	(-0.20)
Alpha, C4	23.59	30.91	33.50	34.64	23.11	-2.14
	(3.91)	(3.98)	(3.34)	(2.65)	(1.42)	(-0.16)

Table 6
Double Sorting on Realized Skewness and Idiosyncratic Volatility

Each week, stocks are first ranked by realized skewness into five quintiles and then, within each quintile, stocks are sorted once again into five quintiles based on idiosyncratic volatility. Panel A reports on value-weighted returns and Panel B reports on equal-weighted returns. The value- and equal-weighted average weekly returns (in bps) are reported for all double sorted portfolios as well as for the difference between portfolio five (high idiosyncratic volatility) and one (low idiosyncratic volatility) for each level of realized skewness. T-statistics are in parentheses.

	Panel A: Value-Weighted					
	Low	2	3	4	High	High-Low
1 (Low Realized Skewness)	32.1	36.1	48.9	46.6	25.1	-6.9
	(3.98)	(3.81)	(3.89)	(2.99)	(1.35)	(-0.43)
2	23.2	26.4	26.5	28.1	2.6	-20.6
	(2.91)	(2.95)	(2.19)	(1.83)	(0.13)	(-1.22)
3	30.2	22.3	12.1	21.1	0.9	-29.4
	(3.98)	(2.54)	(1.09)	(1.46)	(0.05)	(-1.90)
4	16.8	22.8	17.5	0.1	-9.5	-26.3
	(2.25)	(2.67)	(1.56)	(0.01)	(-0.53)	(-1.67)
5 (High Realized Skewness)	17.0	9.0	-1.1	8.3	-16.4	-33.4
	(2.29)	(1.01)	(-0.11)	(0.65)	(-1.03)	(-2.44)

	Panel B: Equal-Weighted					
	Low	2	3	4	High	High-Low
1 (Low Realized Skewness)	29.3	44.0	51.9	60.5	64.0	34.8
	(4.44)	(5.05)	(4.53)	(4.27)	(3.76)	(2.48)
2	22.8	30.2	36.0	42.4	32.0	9.0
	(3.64)	(3.74)	(3.26)	(2.97)	(1.83)	(0.61)
3	23.5	26.1	28.4	28.3	17.0	-6.8
	(3.99)	(3.37)	(2.79)	(2.14)	(0.98)	(-0.47)
4	24.5	26.4	28.6	20.3	2.0	-22.9
	(4.07)	(3.47)	(2.93)	(1.56)	(0.09)	(-1.64)
5 (High Realized Skewness)	23.0	23.3	21.0	12.4	-20.0	-42.6
	(3.79)	(3.10)	(2.31)	(1.06)	(-1.32)	(-3.40)

Table 7
Realized Moments and Returns for Different Subsamples

We report weekly returns (in bps) and t-statistics of quintile portfolios formed from ranking stocks by their realized moments. We also report the difference between portfolio 5 (highest realized moment) and portfolio 1 (lowest realized moment). Panel A displays results for realized skewness and Panel B for realized kurtosis. Each panel reports the value- and equal-weighted quintile portfolio returns for the month of January, for all months excluding January, and only for NYSE stocks.

Panel A: Realized Skewness Effects for Different Subsamples

	Value weighted					
	Low	2	3	4	High	High-Low
Raw Returns, January	54.76	43.49	36.75	18.08	24.20	-30.57
	(2.11)	(1.45)	(1.43)	(0.68)	(0.90)	(-1.72)
Raw Returns, Non-January	34.31	20.93	19.47	13.99	8.09	-26.22
	(3.41)	(2.03)	(2.08)	(1.52)	(0.93)	(-4.80)
Raw Returns, NYSE	44.61	31.23	26.48	13.90	13.06	-31.55
	(3.08)	(2.04)	(1.82)	(1.01)	(1.02)	(-3.51)

	Equal weighted					
	Low	2	3	4	High	High-Low
Raw Returns, January	93.27	55.86	59.54	54.28	37.21	-56.05
	(2.72)	(1.74)	(1.98)	(1.76)	(1.31)	(-4.13)
Raw Returns, Non-January	46.36	31.37	22.15	17.90	10.11	-36.25
	(4.05)	(2.78)	(2.08)	(1.75)	(1.06)	(-8.36)
Raw Returns, NYSE	66.84	44.06	28.57	18.80	2.48	-64.36
	(4.81)	(3.08)	(2.06)	(1.40)	(0.21)	(-12.27)

Panel B: Realized Kurtosis Effects for Different Subsamples

	Value weighted					
	Low	2	3	4	High	High-Low
Raw Returns, January	31.25	38.43	37.81	30.27	58.70	27.44
	(1.20)	(1.44)	(1.38)	(1.26)	(2.10)	(1.46)
Raw Returns, Non-January	17.49	21.28	13.61	23.45	19.44	1.95
	(1.82)	(2.35)	(1.45)	(2.51)	(2.24)	(0.39)
Raw Returns, NYSE	25.41	22.47	16.71	33.42	32.15	6.75
	(1.75)	(1.59)	(1.24)	(2.71)	(2.89)	(0.75)

	Equal weighted					
	Low	2	3	4	High	High-Low
Raw Returns, January	47.14	58.03	63.79	60.40	70.76	23.62
	(1.54)	(1.82)	(1.97)	(1.93)	(2.34)	(1.59)
Raw Returns, Non-January	22.05	22.56	23.74	26.26	33.23	11.18
	(2.10)	(2.05)	(2.16)	(2.44)	(3.35)	(2.66)
Raw Returns, NYSE	25.85	31.95	28.70	33.81	40.34	14.49
	(1.63)	(2.16)	(2.11)	(2.67)	(3.72)	(1.88)

Table 8
Long-Short Returns for Alternative Skewness Measures

We report the long-short weekly returns computed as in Table 2 for the following skewness measures: realized skewness ($RSkew$), interquartile skewness ($SK2$) defined as $(Q_3 + Q_1 - 2Q_2)/(Q_3 - Q_2)$ where Q_i is the i^{th} quartile of the five-minute return distribution, $SubRSkew$ is the average realized skewness over different subsamples as suggested by Zhang, Mykland, and Ait-Sahalia (2005), historical skewness ($HSkew_t$) computed with daily returns across different horizons (12 months, 24 months and 60 months), industry skewness residual ($IndSkewRes$) computed with one-month daily returns as in Zhang (2006), and expected idiosyncratic skewness ($ExpSkew$) computed as in Boyer, Mitton, and Vorkink (2010). The data sample is from January 1993 to September 2008.

	RSkew	SK2	SubRSkew	HSkew$_{12M}$	HSkew$_{24M}$	HSkew$_{60M}$	IndSkewRes	ExpSkew
				Value weighted				
Raw Returns	-23.80	-7.84	-9.34	3.27	2.26	-7.44	-1.43	-16.85
	(-3.65)	(-1.63)	(-1.61)	(0.51)	(0.34)	(-1.00)	(-0.17)	(-1.71)
Alpha, C4	-22.55	-9.38	-11.28	1.91	1.15	-8.51	-3.56	-20.43
	(-3.42)	(-1.92)	(-1.92)	(0.29)	(0.17)	(-1.12)	(-0.42)	(-2.05)
				Equal weighted				
Raw Returns	-43.44	-11.06	-13.72	-0.78	-3.74	-7.42	-3.98	-11.24
	(-8.91)	(-2.43)	(-3.33)	(-0.11)	(-0.48)	(-0.90)	(-0.58)	(-1.06)
Alpha, C4	-45.52	-13.61	-17.15	-2.88	-5.32	-8.83	-5.72	-13.44
	(-9.26)	(-2.98)	(-4.10)	(-0.40)	(-0.68)	(-1.06)	(-0.82)	(-1.29)

Table 9
Double Sorting on Firm Characteristics, then on Realized Moments

Based on a given firm characteristic, stocks are sorted into five quintiles each week. Within each quintile, stocks are sorted once again into five quintiles based on the realized moment measure. Then, for each realized moment, we compute value- and equal-weighted average weekly returns of the difference between portfolios five and one, along with the t-statistic (in parentheses). Panel A displays the results for realized skewness and Panel B for realized kurtosis. Firm characteristics are Size ($ market capitalization in $billions), BE/ME (book-to-market equity ratio), Lagged Return, Realized Volatility (weekly realized volatility computed with high-frequency data), Historical Skewness (one month historical skewness from daily returns), Illiquidity (annual average of the absolute return over daily dollar trading volume times 10^6, as in Amihud (2002)), Number of Intraday Transactions, Maximum Return (of previous month), Number of Analysts (from I/B/E/S), Market Beta, Idiosyncratic Volatility (computed as in Ang, Hodrick, Xing, and Zhang (2006)) and Coskewness (computed as in Harvey and Siddique (2000)).

Panel A: Double Sorting on Firm Characteristics, then on Realized Skewness

	Value Weighted					Equal Weighted				
Quintiles	Low	2	3	4	High	Low	2	3	4	High
Size	-75.5	-34.8	-31.5	-10.6	-25.8	-77.0	-36.9	-33.7	-9.5	-16.9
RSkew 5-1	(-10.42)	(-5.82)	(-6.13)	(-2.09)	(-4.29)	(-10.23)	(-6.17)	(-6.49)	(-1.91)	(-3.66)
BE/ME	-60.8	-28.1	-11.9	-0.6	-12.8	-58.7	-39.7	-36.0	-29.3	-24.6
RSkew 5-1	(-6.13)	(-3.85)	(-1.80)	(-0.09)	(-1.93)	(-7.91)	(-6.97)	(-6.70)	(-5.59)	(-4.33)
Lagged Return	-20.5	-17.8	-11.5	5.1	4.2	-30.5	-9.9	-9.9	-3.3	-14.3
RSkew 5-1	(-2.18)	(-2.53)	(-1.73)	(0.72)	(0.46)	(-5.47)	(-2.28)	(-2.50)	(-0.68)	(-2.32)
Realized Volatility	-17.0	-29.0	-33.5	-33.5	-44.4	-10.1	-26.3	-31.7	-46.6	-88.9
RSkew 5-1	(-3.43)	(-4.24)	(-3.88)	(-3.06)	(-3.53)	(-3.70)	(-7.45)	(-6.45)	(-8.17)	(-11.50)
Realized Kurtosis	-31.4	-20.5	-23.7	-30.1	-26.8	-24.9	-29.5	-38.5	-46.0	-48.1
RSkew 5-1	(-4.57)	(-2.81)	(-3.49)	(-3.92)	(-3.67)	(-3.84)	(-4.77)	(-6.98)	(-8.33)	(-8.68)
Historical Skewness	-18.2	-29.1	-23.5	-18.3	-26.6	-31.9	-37.8	-37.4	-38.1	-43.6
RSkew 5-1	(-2.36)	(-3.95)	(-2.75)	(-2.20)	(-3.02)	(-5.88)	(-7.14)	(-7.20)	(-6.87)	(-7.22)
Illiquidity	-26.0	-20.5	-16.3	-38.6	-80.7	-15.3	-18.7	-27.6	-40.3	-77.6
RSkew 5-1	(-4.30)	(-4.29)	(-2.93)	(-6.55)	(-9.80)	(-3.01)	(-3.64)	(-5.52)	(-7.41)	(-11.55)
Intra. Transactions	-18.8	-20.9	-13.7	-19.9	-30.6	-37.0	-37.6	-37.8	-39.4	-37.7
RSkew 5-1	(-3.64)	(-4.29)	(-2.93)	(-4.05)	(-4.42)	(-8.02)	(-7.53)	(-6.99)	(-6.13)	(-5.01)
Maximum Return	-20.0	-21.0	-35.6	-21.1	-52.1	-17.5	-25.9	-5.3	-39.7	-57.6
RSkew 5-1	(-3.77)	(-3.03)	(-4.22)	(-2.18)	(-4.48)	(-2.20)	(-1.56)	(-0.39)	(-2.40)	(-5.76)
Nb. of Analysts	-27.9	-34.9	-30.0	-17.0	-30.2	-55.8	-50.1	-39.2	-18.6	-19.4
RSkew 5-1	(-3.76)	(-5.62)	(-5.47)	(-3.05)	(-5.02)	(-8.45)	(-8.72)	(-7.38)	(-3.67)	(-3.72)
Market Beta	-16.5	-23.2	-27.8	-39.9	-24.4	-29.6	-31.9	-32.8	-44.2	-62.0
RSkew 5-1	(-3.00)	(-3.99)	(-3.99)	(-4.74)	(-2.37)	(-6.34)	(-7.55)	(-6.93)	(-8.06)	(-9.01)
Idio. Volatility	-14.3	-19.1	-46.3	-52.3	-52.3	-6.3	-20.8	-32.3	-54.0	-82.4
RSkew 5-1	(-2.87)	(-2.67)	(-5.37)	(-4.95)	(-4.35)	(-2.45)	(-6.02)	(-6.62)	(-8.68)	(-11.04)
Coskewness	-13.2	-41.3	-19.5	-16.3	-37.2	-37.9	-16.9	-27.7	-32.9	-24.3
RSkew 5-1	(-1.56)	(-4.60)	(-2.62)	(-2.21)	(-5.08)	(-6.75)	(-3.03)	(-5.42)	(-6.42)	(-5.09)

Table 9 (Continued)

Panel B: Double Sorting on Firm Characteristics, then on Realized Kurtosis

	Characteristics - Value Weighted					Characteristics - Equal Weighted				
Quintiles	Low	2	3	4	High	Low	2	3	4	High
Size	-0.8	10.7	1.9	-1.1	4.0	4.3	12.1	1.8	-1.0	-0.6
RKurt 5-1	(-0.08)	(1.36)	(0.32)	(-0.22)	(0.78)	(0.42)	(1.53)	(0.31)	(-0.19)	(-0.13)
BE/ME	4.5	-0.7	0.4	14.5	0.4	14.3	-3.6	9.5	11.6	21.4
RKurt 5-1	(0.49)	(-0.09)	(0.07)	(2.47)	(0.07)	(2.05)	(-0.61)	(1.83)	(2.21)	(3.54)
Lagged Return	-11.7	-2.0	3.9	18.5	14.5	16.7	8.9	9.6	21.2	15.8
RKurt 5-1	(-1.16)	(-0.28)	(0.58)	(2.68)	(1.71)	(2.10)	(1.76)	(2.28)	(4.16)	(2.24)
Realized Volatility	3.1	2.6	15.0	8.2	36.2	3.8	1.7	15.3	16.3	20.3
RKurt 5-1	(0.68)	(0.39)	(1.55)	(0.67)	(2.50)	(1.22)	(0.45)	(2.82)	(1.91)	(1.88)
Realized Skewness	9.5	-2.5	2.7	4.6	10.7	14.1	7.2	13.1	9.5	8.5
RKurt 5-1	(1.14)	(-0.33)	(0.36)	(0.62)	(1.64)	(2.28)	(1.24)	(2.27)	(1.72)	(1.65)
Historical Skewness	5.2	9.2	7.7	1.6	19.9	12.3	17.5	11.7	13.0	2.7
RKurt 5-1	(0.66)	(1.43)	(1.04)	(0.19)	(2.53)	(2.34)	(3.45)	(2.20)	(2.19)	(0.41)
Illiquidity	6.1	-3.7	-4.8	4.0	4.6	3.7	0.6	2.5	2.6	12.6
RKurt 5-1	(1.13)	(-0.84)	(-0.84)	(0.56)	(0.44)	(0.77)	(0.12)	(0.51)	(0.40)	(1.37)
Intra. Transactions	5.5	7.4	-2.3	7.3	5.1	25.1	17.7	13.1	11.4	-2.4
RKurt 5-1	(1.00)	(1.41)	(-0.43)	(1.46)	(0.78)	(5.23)	(3.48)	(2.41)	(1.84)	(-0.35)
Maximum Return	-8.3	15.1	9.0	23.9	5.1	14.3	11.5	30.0	28.1	-0.8
RKurt 5-1	(-1.51)	(2.28)	(1.18)	(2.46)	(0.41)	(1.87)	(0.45)	(2.17)	(1.61)	(-0.08)
Nb. of Analysts	9.8	4.3	-1.9	3.0	2.3	17.9	5.9	3.6	-1.9	4.3
RKurt 5-1	(1.02)	(0.63)	(-0.34)	(0.59)	(0.41)	(2.08)	(0.92)	(0.69)	(-0.41)	(0.82)
Market Beta	7.4	6.8	7.4	-0.3	-1.6	27.7	14.1	11.2	-0.8	-4.9
RKurt 5-1	(1.35)	(1.12)	(1.20)	(-0.04)	(-0.16)	(5.32)	(3.29)	(2.34)	(-0.15)	(-0.68)
Idio. Volatility	-4.1	9.3	16.8	5.9	18.4	3.5	9.0	10.9	19.4	19.7
RKurt 5-1	(-0.80)	(1.42)	(1.96)	(0.57)	(1.45)	(1.17)	(2.33)	(2.08)	(2.68)	(2.04)
Coskewness	5.0	11.2	2.0	-0.6	4.0	13.5	17.1	4.1	10.8	5.2
RKurt 5-1	(0.62)	(1.44)	(0.27)	(-0.09)	(0.60)	(2.17)	(2.81)	(0.80)	(2.12)	(1.08)

Table 10

Weekly Realized Moments and the Cross-Section of Monthly Stock Returns

We report the value- and equal-weighted monthly returns (in bps) of decile portfolios formed from realized moments, their Newey-West t-statistics (in parentheses) and the return difference between portfolio 10 (highest realized moment) and portfolio 1 (lowest realized moment) over the period January 1993 to September 2008. Panel A displays the results for realized volatility, Panel B for realized skewness, and Panel C for realized kurtosis. Raw returns (in bps) are obtained from decile portfolios sorted solely from ranking stocks based on the realized moment measure. Alpha is the intercept from time-series regressions of the returns of the portfolio that buys portfolio 10 and sells portfolio 1 using the Carhart four factor model.

Panel A: Realized Volatility and Monthly Stock Returns

	Low	2	3	4	5	6	7	8	9	High	High-Low
						Value weighted					
Raw Returns	78.20	78.93	88.84	91.56	79.02	65.72	71.95	49.58	10.37	-10.54	-88.74
	(4.04)	(3.66)	(3.66)	(3.12)	(2.30)	(1.64)	(1.52)	(0.92)	(0.18)	(-0.15)	(-1.42)
Alpha, C4	80.87	82.55	94.28	96.90	88.86	73.98	81.11	65.02	20.66	3.35	-79.18
	(6.70)	(5.54)	(5.49)	(3.90)	(3.17)	(2.12)	(1.89)	(1.31)	(0.38)	(0.06)	(-1.39)
						Equal weighted					
Raw Returns	89.54	94.83	98.43	107.19	102.54	111.95	103.15	102.50	93.36	81.74	-7.80
	(4.58)	(4.29)	(4.14)	(4.01)	(3.32)	(3.00)	(2.32)	(1.99)	(1.63)	(1.25)	(-0.13)
Alpha, C4	90.74	96.73	101.45	110.41	107.55	119.58	111.45	113.46	105.05	95.69	3.29
	(7.68)	(6.50)	(5.99)	(5.10)	(4.23)	(3.85)	(2.84)	(2.48)	(2.02)	(1.62)	(0.06)

Panel B: Realized Skewness and Monthly Stock Returns

	Low	2	3	4	5	6	7	8	9	High	High-Low
						Value weighted					
Raw Returns	112.31	84.25	81.82	83.82	89.20	66.91	71.81	64.06	64.49	77.01	-35.30
	(3.77)	(2.80)	(2.78)	(2.86)	(3.09)	(2.47)	(2.52)	(2.33)	(2.54)	(3.10)	(-2.63)
Alpha, C4	113.21	90.22	87.41	88.15	95.03	72.04	76.23	68.93	71.94	83.01	-31.86
	(5.83)	(4.21)	(4.16)	(3.59)	(3.93)	(3.06)	(2.90)	(2.70)	(2.95)	(3.56)	(-2.43)
						Equal weighted					
Raw Returns	128.61	114.74	102.22	96.81	102.05	86.29	97.79	84.10	84.18	88.65	-39.96
	(3.37)	(2.97)	(2.72)	(2.66)	(2.82)	(2.49)	(2.86)	(2.47)	(2.54)	(2.80)	(-3.29)
Alpha, C4	135.21	122.44	109.63	103.28	108.85	92.72	104.18	91.94	90.10	94.06	-42.81
	(5.80)	(4.71)	(4.39)	(3.65)	(3.73)	(3.21)	(3.34)	(2.98)	(2.90)	(3.18)	(-3.73)

Panel C: Realized Kurtosis and Monthly Stock Returns

	Low	2	3	4	5	6	7	8	9	High	High-Low
						Value weighted					
Raw Returns	62.01	78.22	72.57	99.75	76.19	79.11	80.72	93.86	94.12	92.40	30.39
	(2.04)	(2.81)	(2.64)	(3.69)	(2.66)	(2.84)	(2.89)	(3.39)	(3.52)	(3.32)	(1.78)
Alpha, C4	67.54	84.49	79.56	104.51	80.99	83.10	85.44	97.57	98.17	94.97	25.77
	(3.54)	(4.37)	(4.12)	(4.64)	(3.35)	(3.45)	(3.35)	(3.80)	(3.88)	(3.64)	(1.64)
						Equal weighted					
Raw Returns	78.77	87.09	89.19	95.85	95.81	97.60	93.39	109.74	116.13	121.75	42.98
	(2.48)	(2.53)	(2.54)	(2.63)	(2.62)	(2.62)	(2.56)	(2.96)	(3.22)	(3.44)	(2.54)
Alpha, C4	84.93	95.24	96.24	103.37	102.92	104.74	100.63	116.01	122.31	125.89	39.30
	(4.24)	(4.06)	(4.12)	(3.61)	(3.47)	(3.36)	(3.11)	(3.50)	(3.65)	(3.93)	(2.75)

Table 11
Alternative Realized Volatility Estimators, Higher Moments, and the Cross-Section of Stock Returns

We report the value-weighted and equal-weighted weekly returns (in bps) of the difference between portfolio 10 (highest realized moment) and portfolio 1 (lowest realized moment) during January 1993 to September 2008. Stocks are ranked based on their realized moments using different realized volatility estimators. Panel A displays the results for alternative realized volatility estimators, Panel B for realized skewness scaled by alternative realized volatility estimators, and Panel C for realized kurtosis scaled by alternative realized volatility estimators. Each panel reports the value-weighted and the equal-weighted long-short returns. Alpha is the intercept from time-series regressions of the returns of the portfolio that buys portfolio 10 and sells portfolio 1 using the Carhart four factor model.

Panel A: Long-Short Returns for Alternative Realized Volatility Estimators

	RV	BPV	minRV	medRV
	Value weighted			
Raw Returns	-11.92	-21.75	-20.21	-21.66
	(-0.65)	(-1.15)	(-1.10)	(-1.18)
Alpha, C4	-9.42	-17.72	-17.33	-18.77
	(-0.51)	(-0.92)	(-0.93)	(-1.01)
	Equal weighted			
Raw Returns	5.08	0.39	-0.26	0.14
	(0.33)	(0.02)	(-0.02)	(0.01)
Alpha, C4	7.00	2.85	1.54	2.01
	(0.45)	(0.17)	(0.10)	(0.13)

Panel B: Long-Short Returns for Realized Skewness Scaled by Alternative Realized Volatility Estimators

Realized Skewness scaled by	RV	BPV	minRV	medRV
	Value weighted			
Raw Returns	-23.80	-16.64	-19.96	-21.75
	(-3.65)	(-3.23)	(-3.86)	(-3.72)
Alpha, C4	-22.55	-18.35	-22.64	-24.50
	(-3.42)	(-3.51)	(-4.31)	(-4.14)
	Equal weighted			
Raw Returns	-43.44	-24.31	-27.29	-31.22
	(-8.91)	(-7.14)	(-7.95)	(-8.58)
Alpha, C4	-45.52	-26.58	-29.38	-33.45
	(-9.26)	(-7.74)	(-8.46)	(-9.09)

Panel C: Long-Short Returns for Realized Kurtosis Scaled by Alternative Realized Volatility Estimators

Realized Kurtosis scaled by	RV	BPV	minRV	medRV
	Value weighted			
Raw Returns	13.57	11.69	13.14	12.44
	(2.12)	(1.88)	(2.21)	(2.06)
Alpha, C4	10.85	8.32	9.79	8.98
	(1.67)	(1.33)	(1.64)	(1.48)
	Equal weighted			
Raw Returns	15.77	14.69	18.13	18.54
	(2.98)	(2.45)	(3.32)	(3.59)
Alpha, C4	12.75	10.68	14.77	15.54
	(2.41)	(1.81)	(2.72)	(3.02)