DOA Estimation and Achievable Rate Analysis for 3D Massive MIMO in Aeronautical Communication Systems

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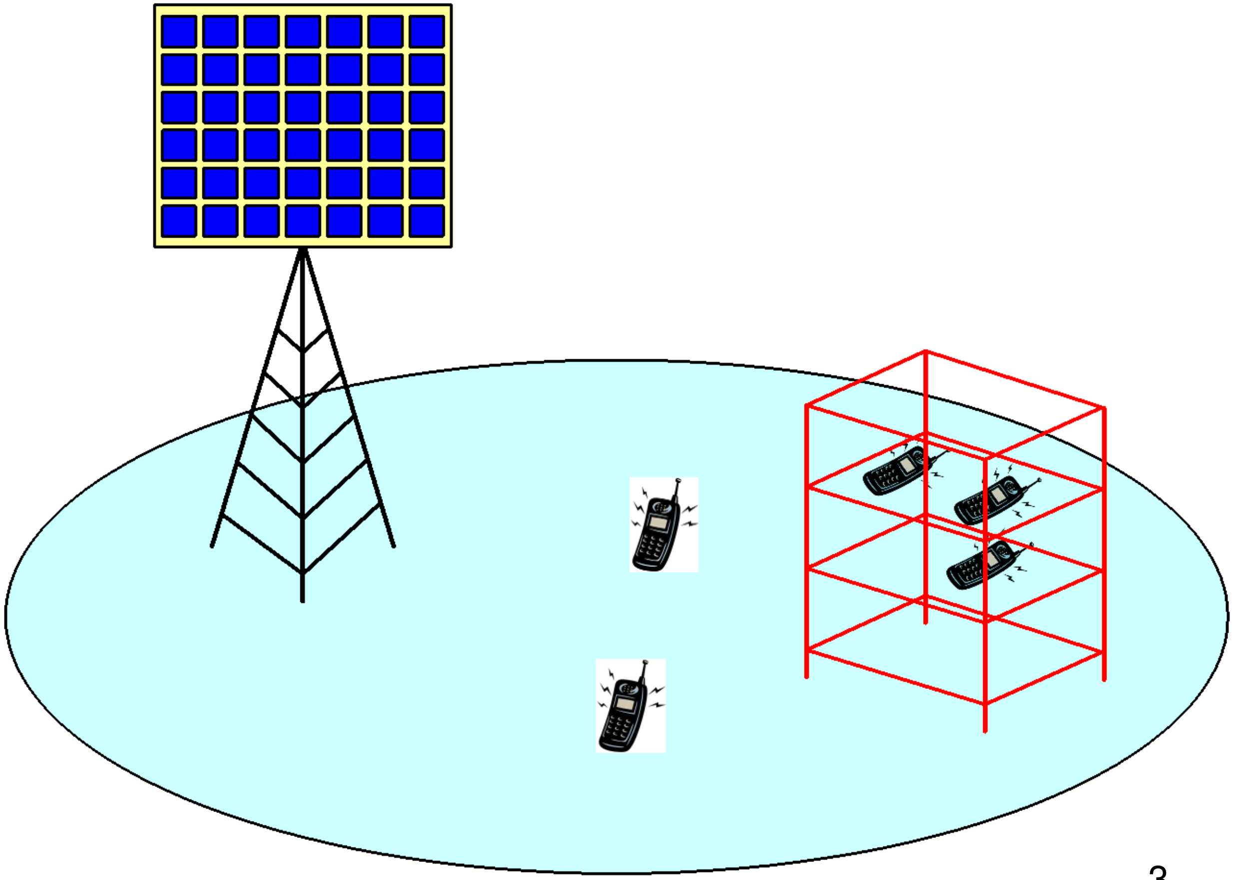
Outline

- Motivation
- DOA estimation algorithm
- Channel model
- Contribution:

  - MSE characterization.

  - Rate analysis.

3D massive MIMO systems



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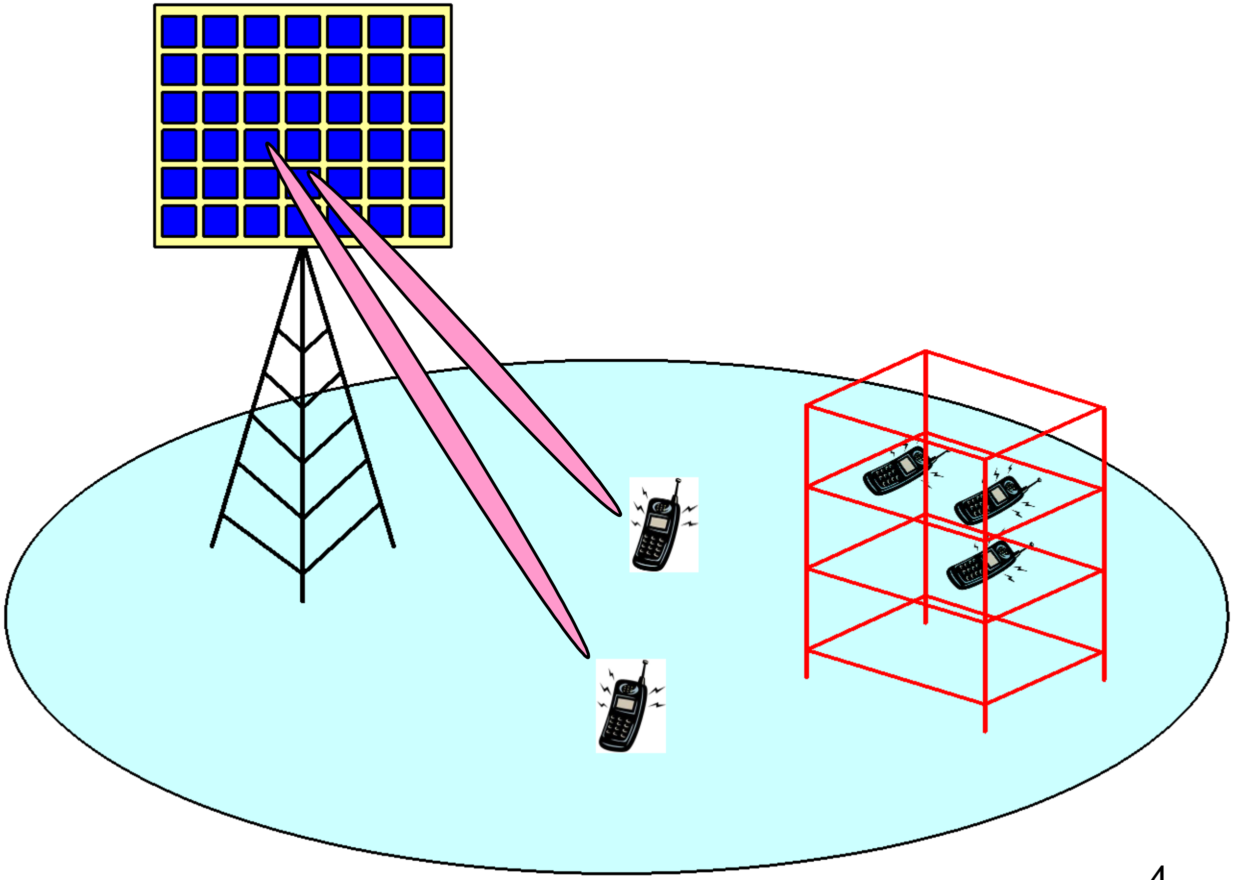

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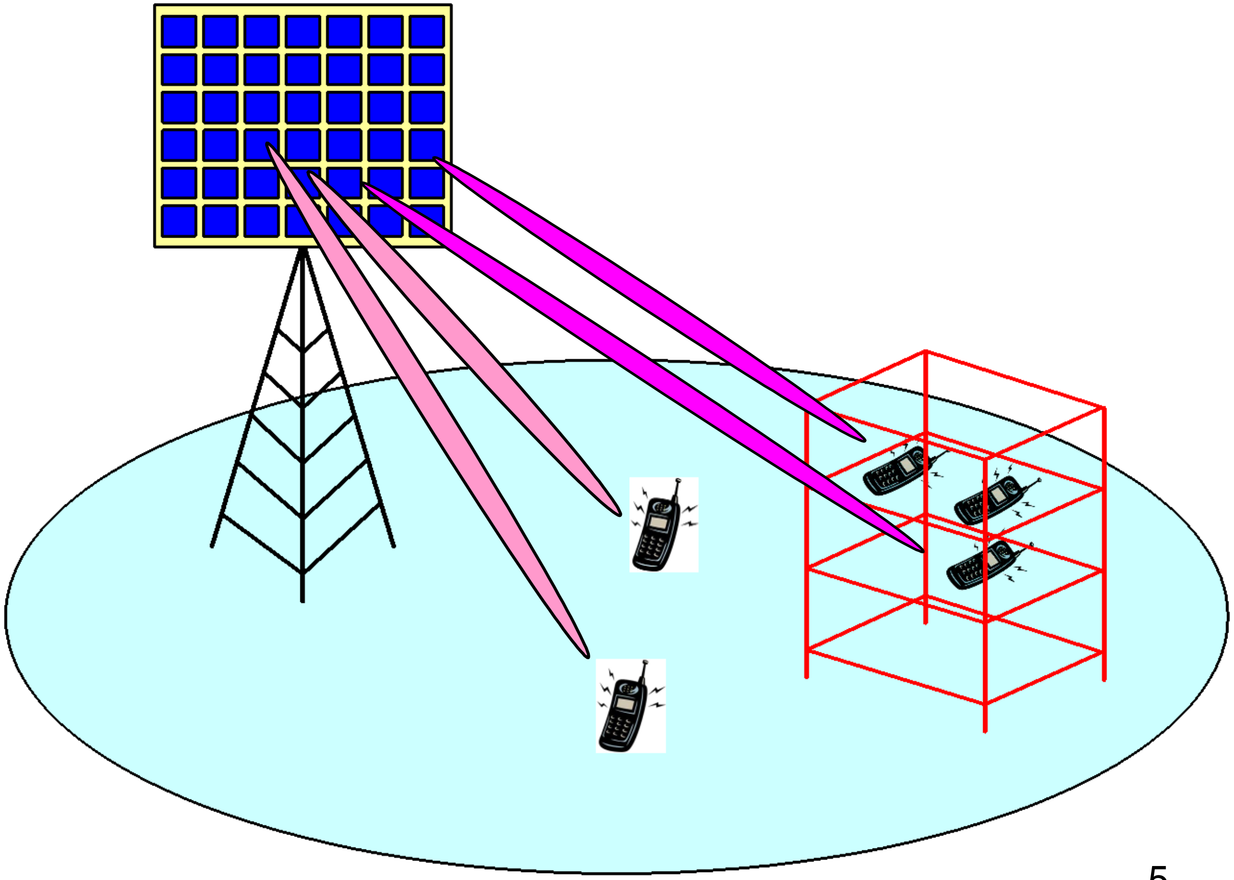
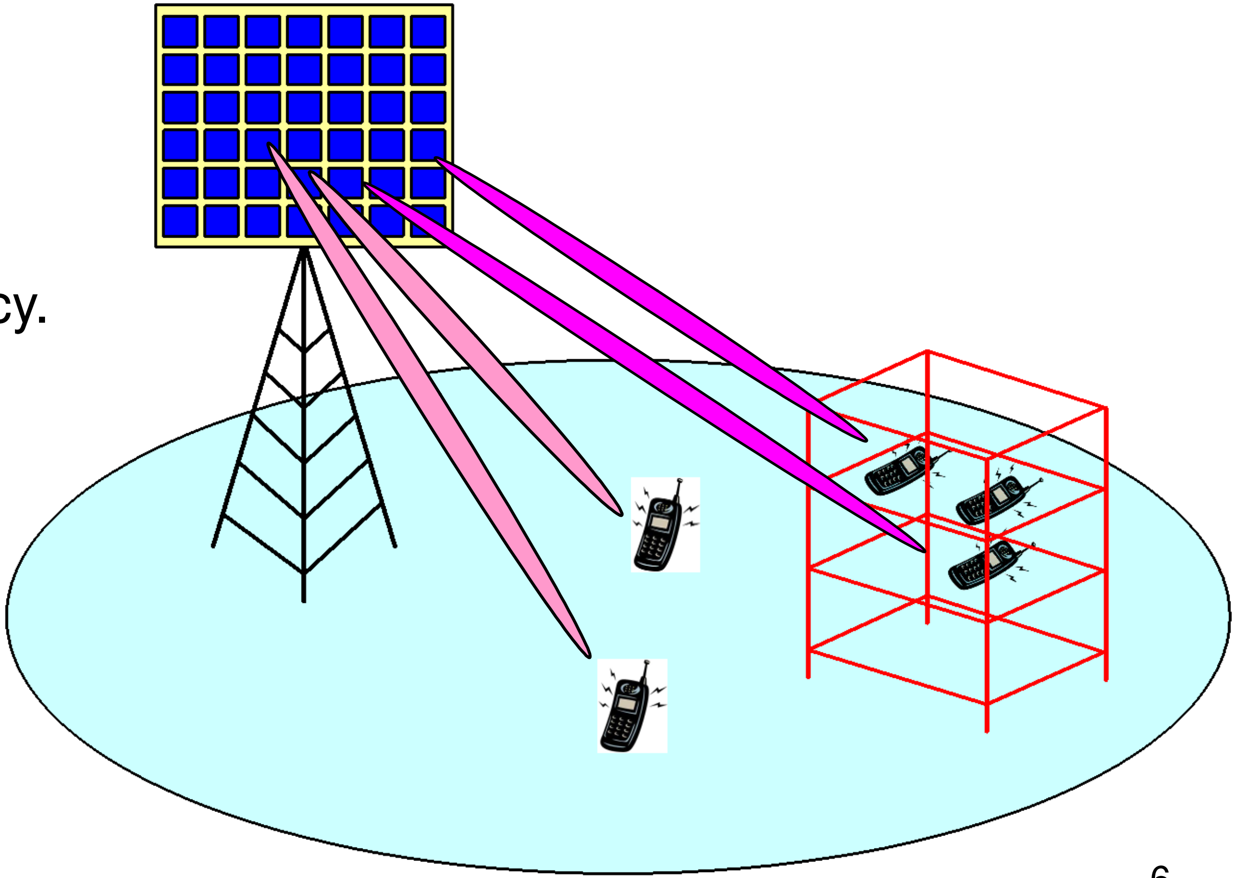


3D massive MIMO systems

- Spectral efficiency.
- Spatial diversity.



Why DOA estimation is important in massive MIMO systems?

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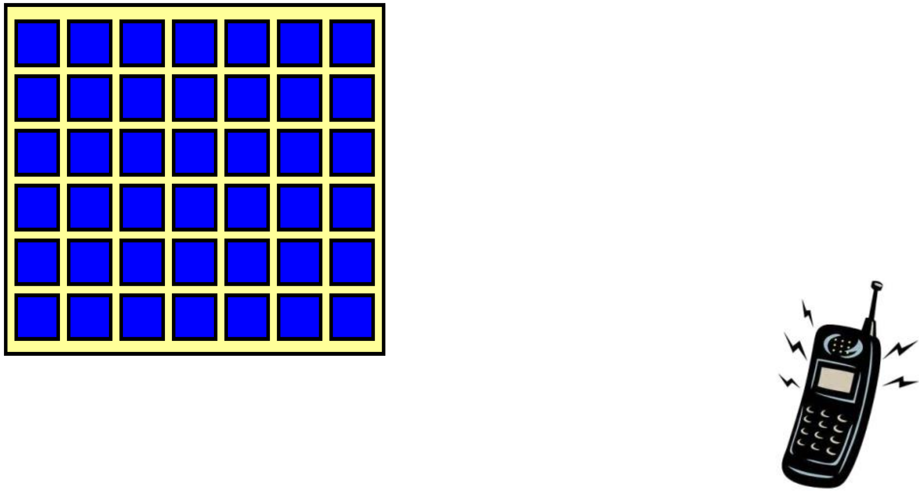

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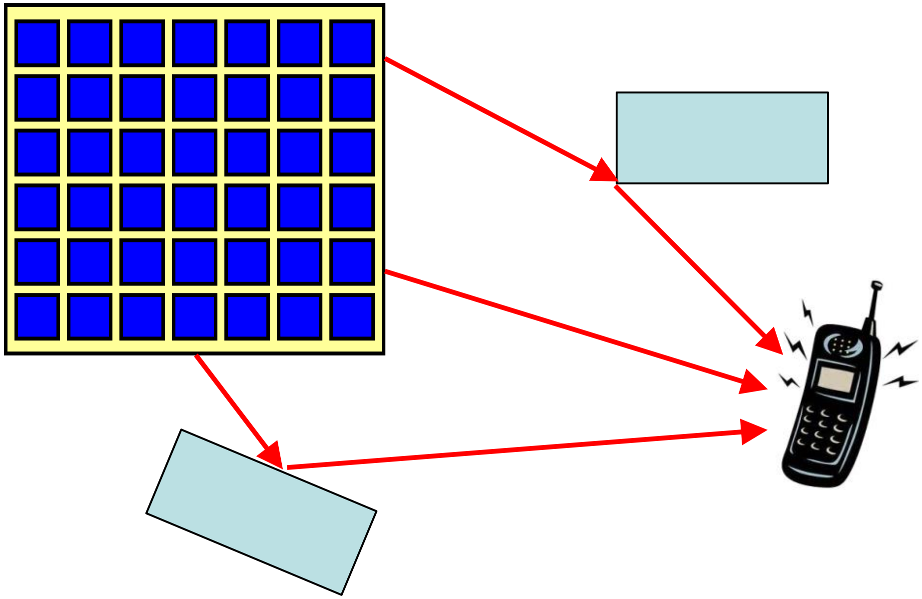

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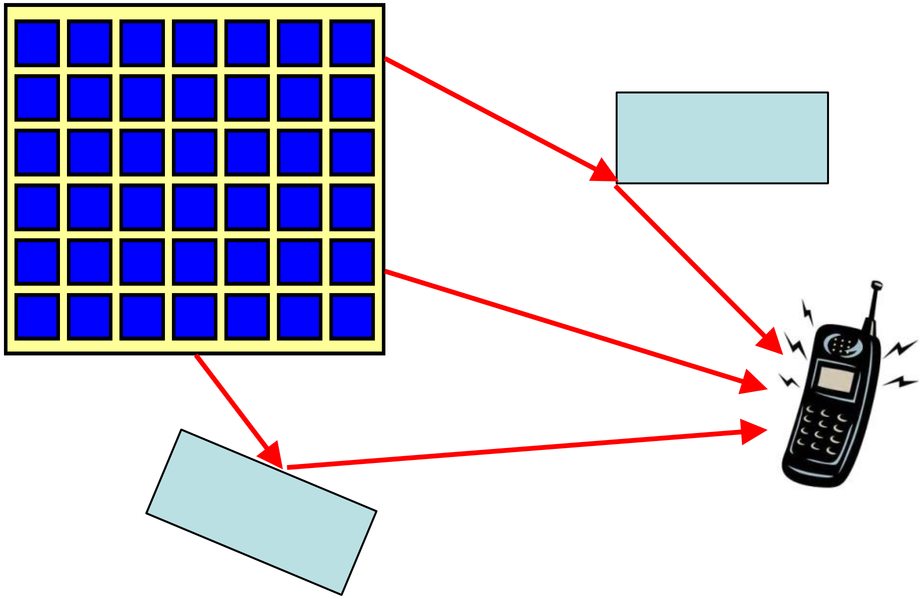

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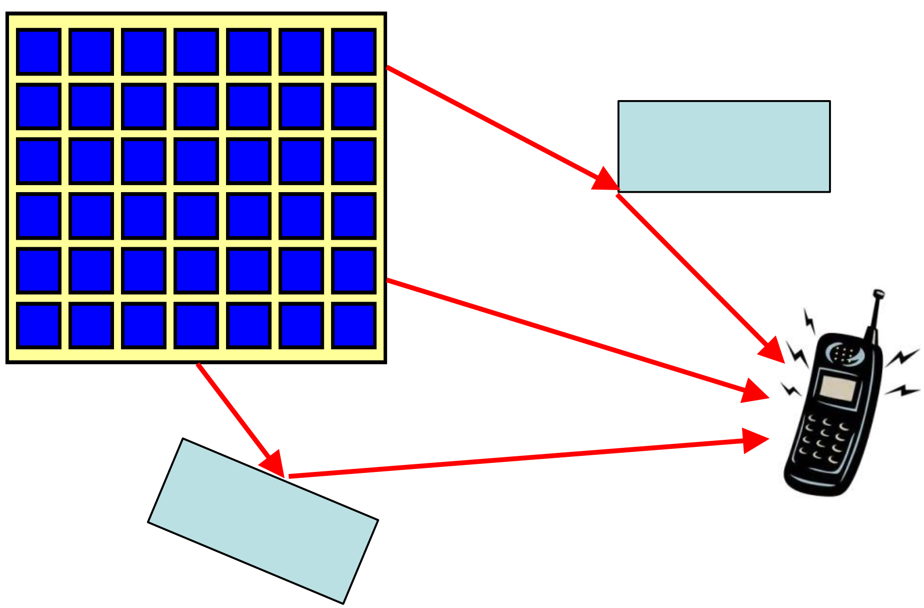

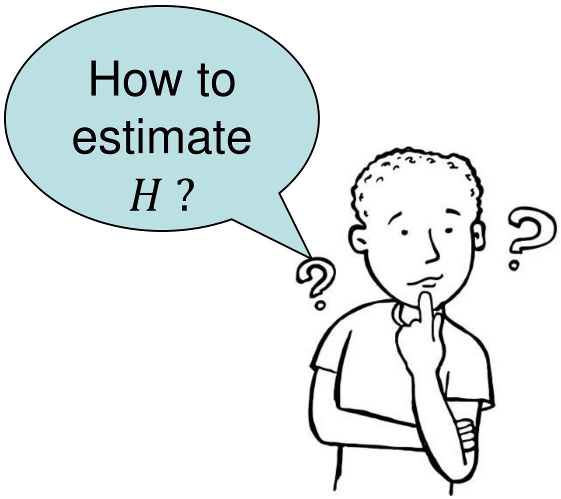

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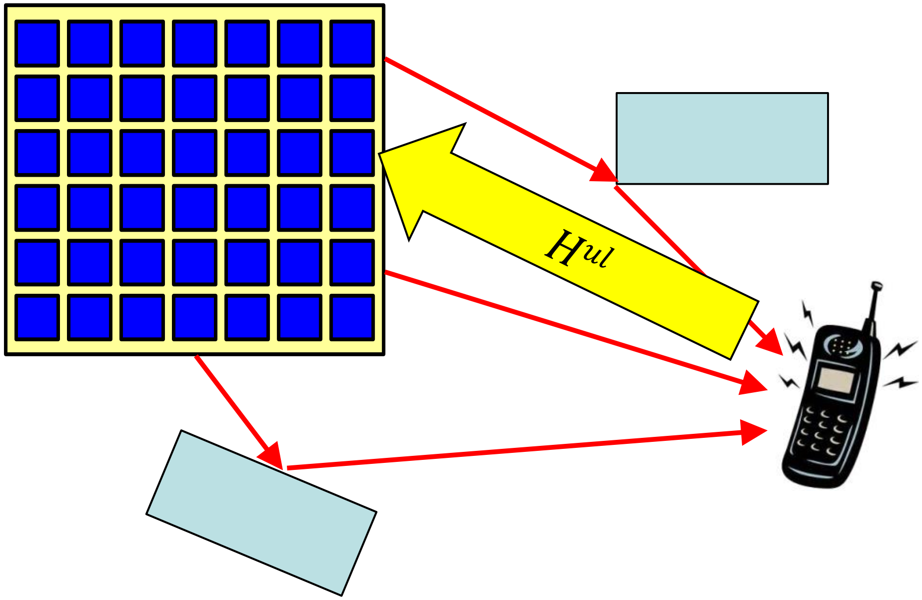

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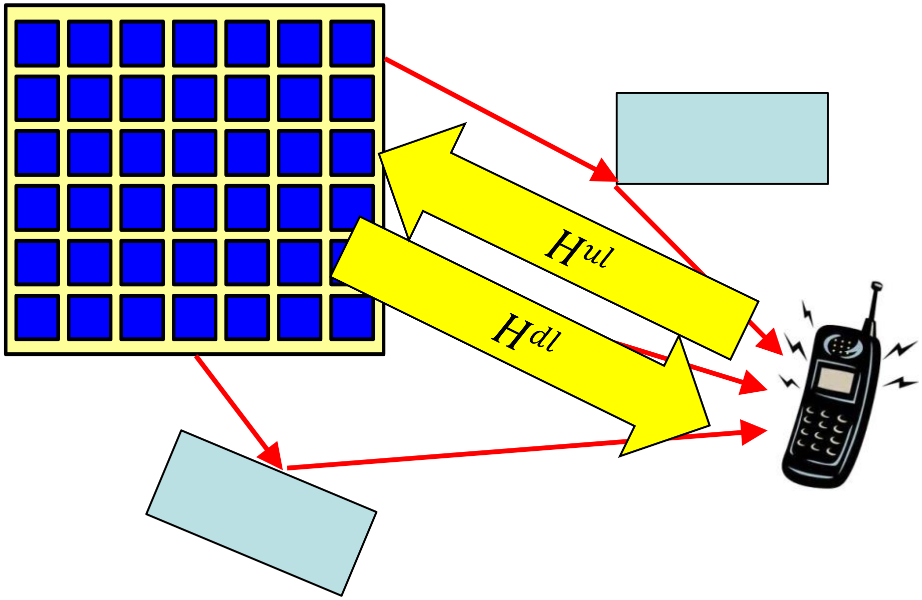

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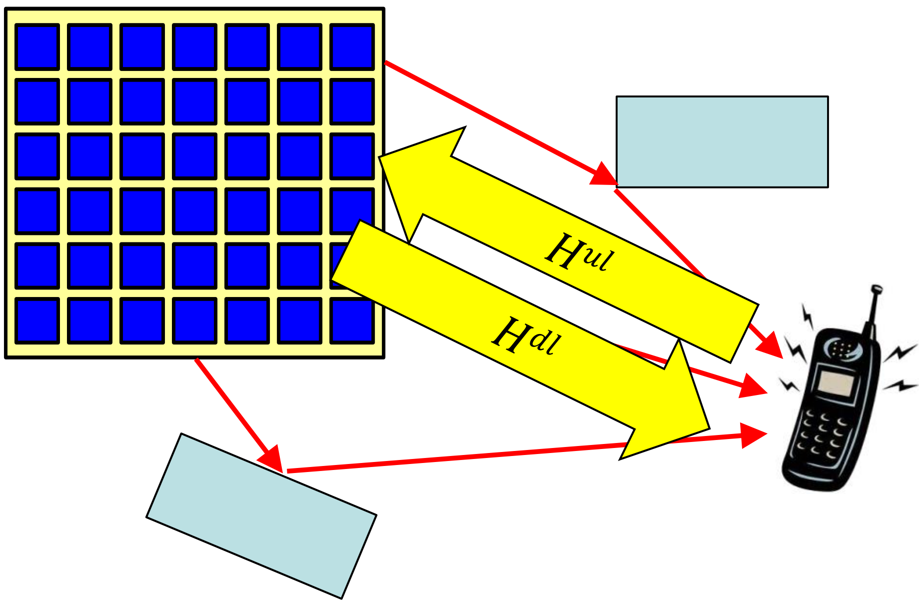

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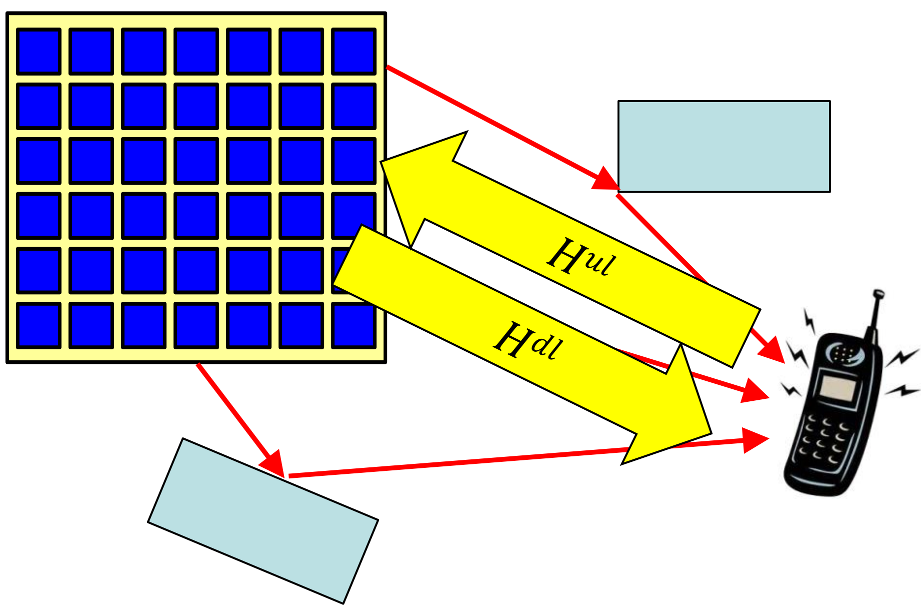

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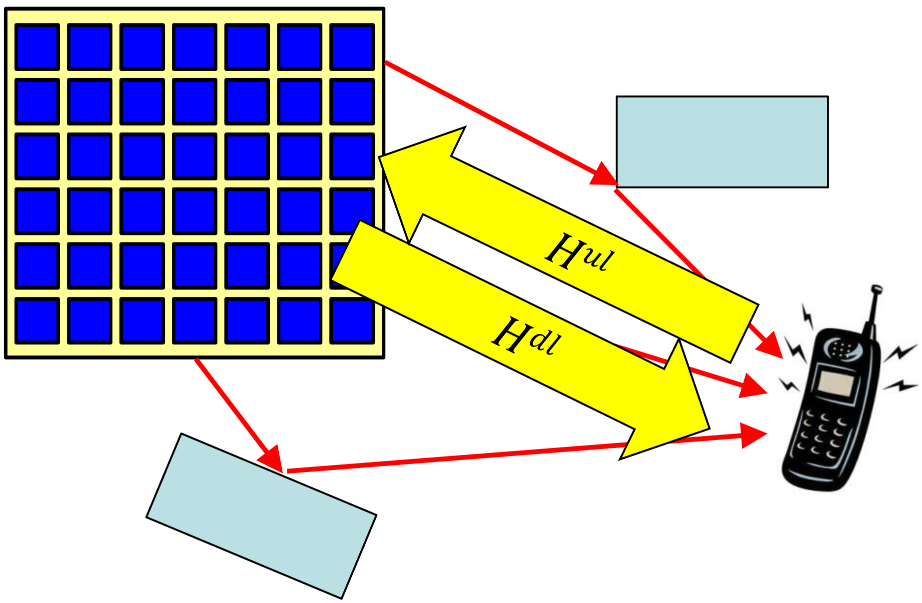

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Alternatively: Due to the limited number of paths,

$$H = f(\theta, \varphi)$$

DOA estimation algorithm

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  - Compared to ESPRIT, unitary ESPRIT processes real-value data from start to end.

Unitary ESPRIT algorithm

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- Array structure:

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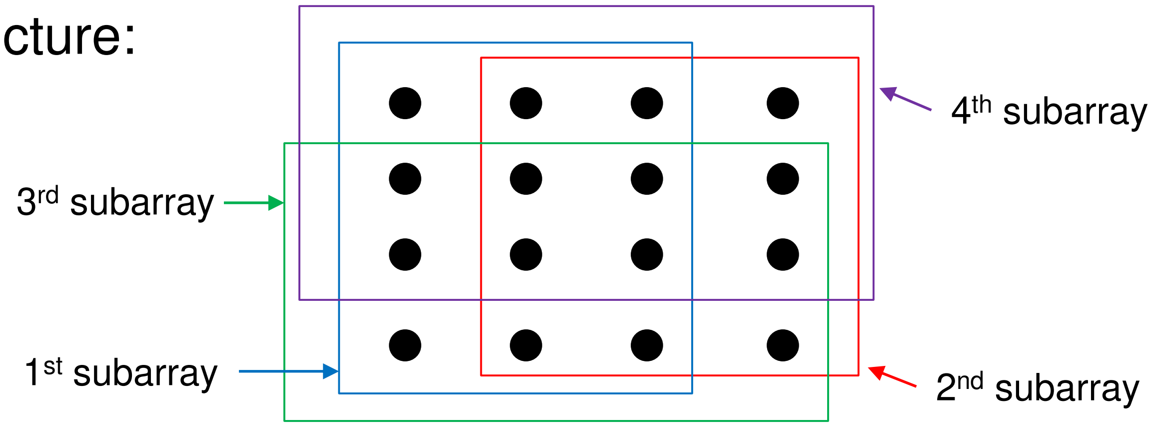

Unitary ESPRIT algorithm

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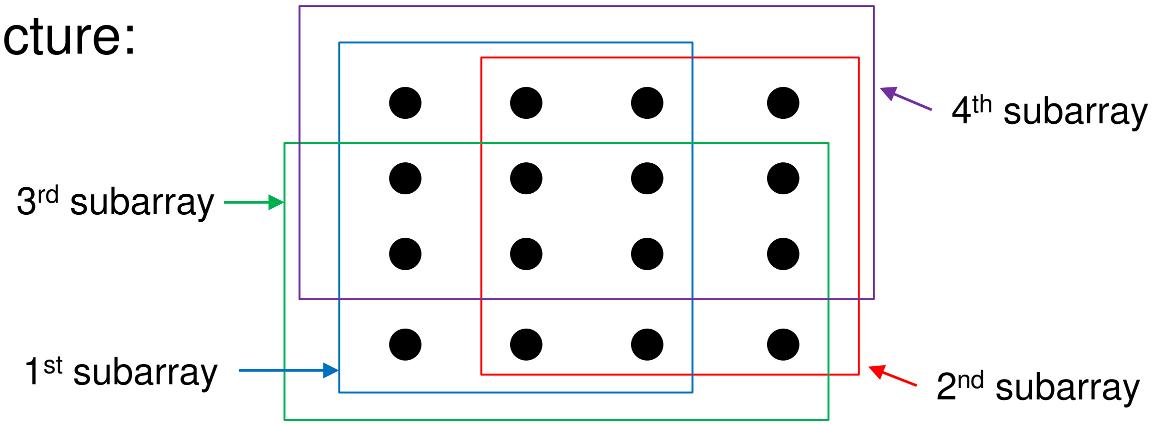

- This property leads to the rotational invariance of signal subspaces spanned by the data vectors associated with the spatially displaced subarrays [1].

[1] "Introduction to direction of arrival estimation" by Z.Chen , G.Gokeda, and Y.Yu

Unitary ESPRIT algorithm steps

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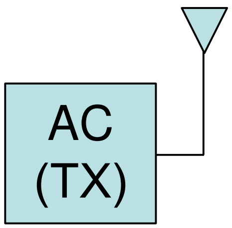
4. From the eigenvalues of the real-valued matrix obtained in step 3, extract the DOA information.

Channel model

Channel model



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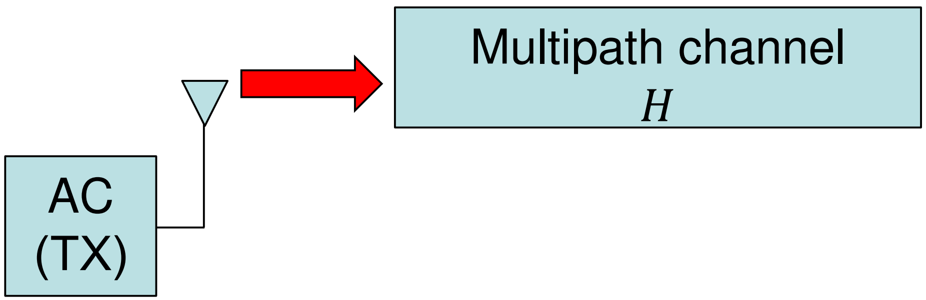

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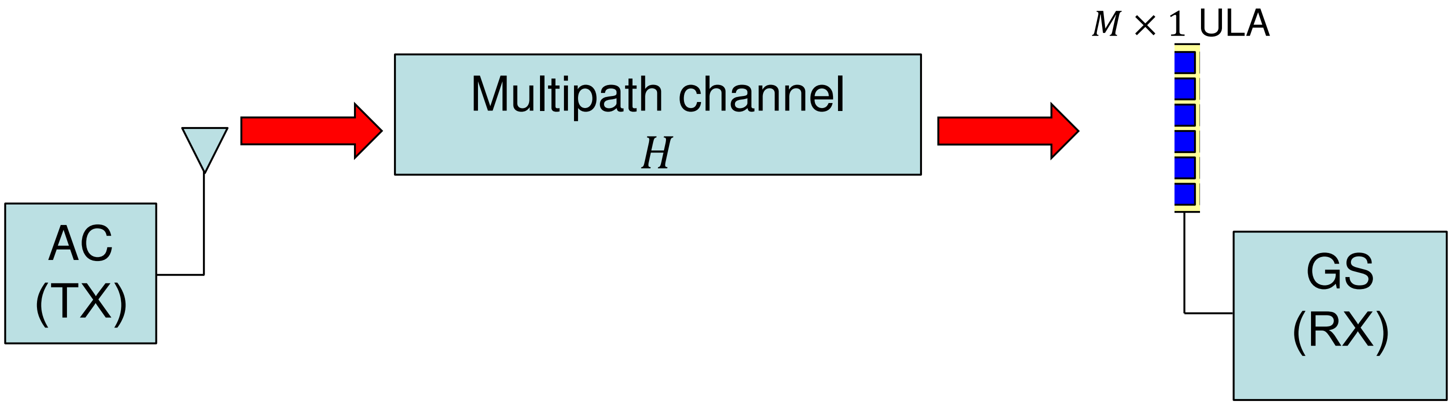

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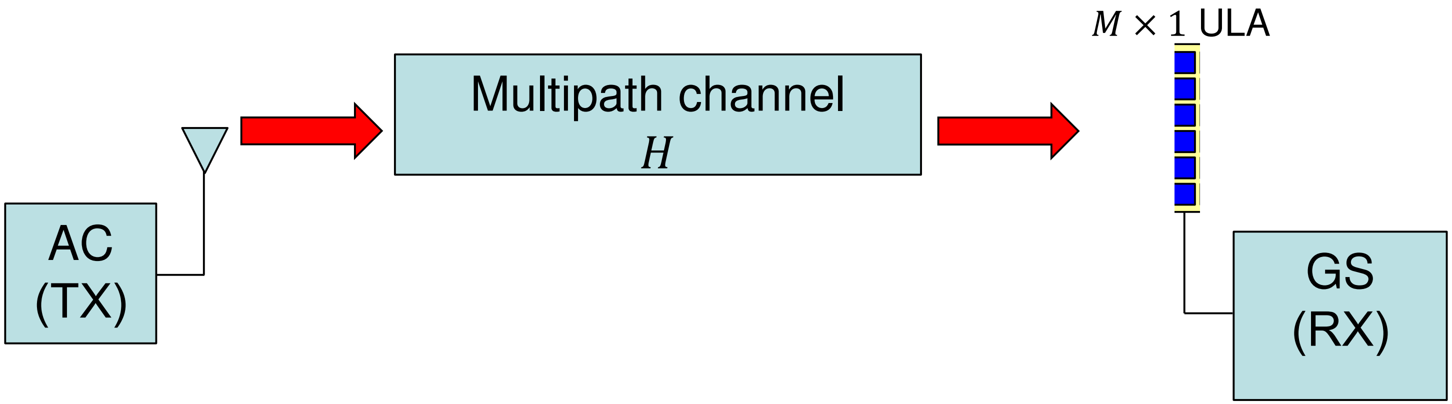

$$h(t) = \sum_{l=1}^{P} \alpha_l(t)\, a_l\, g_l(t - \tau_l)$$

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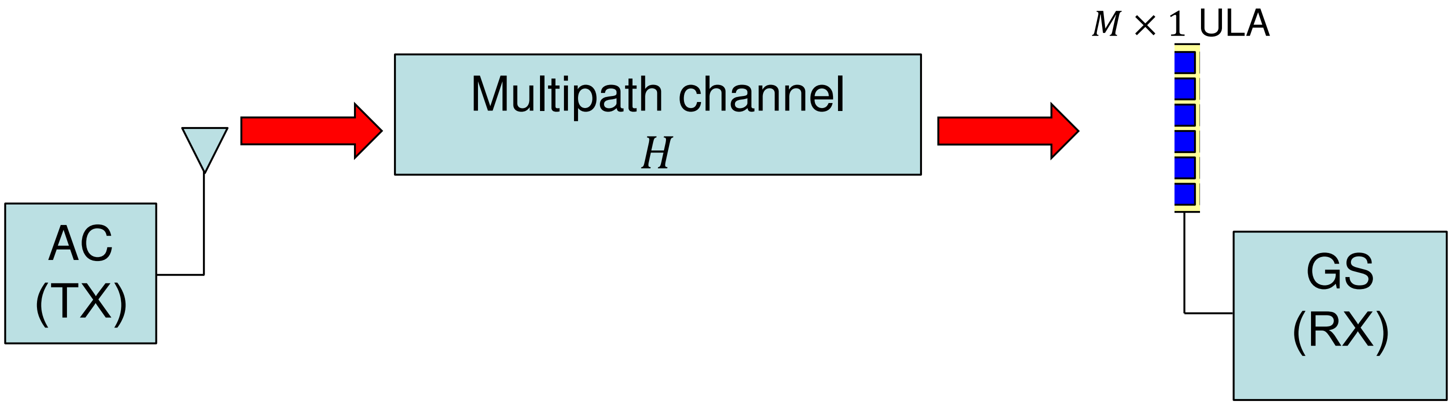

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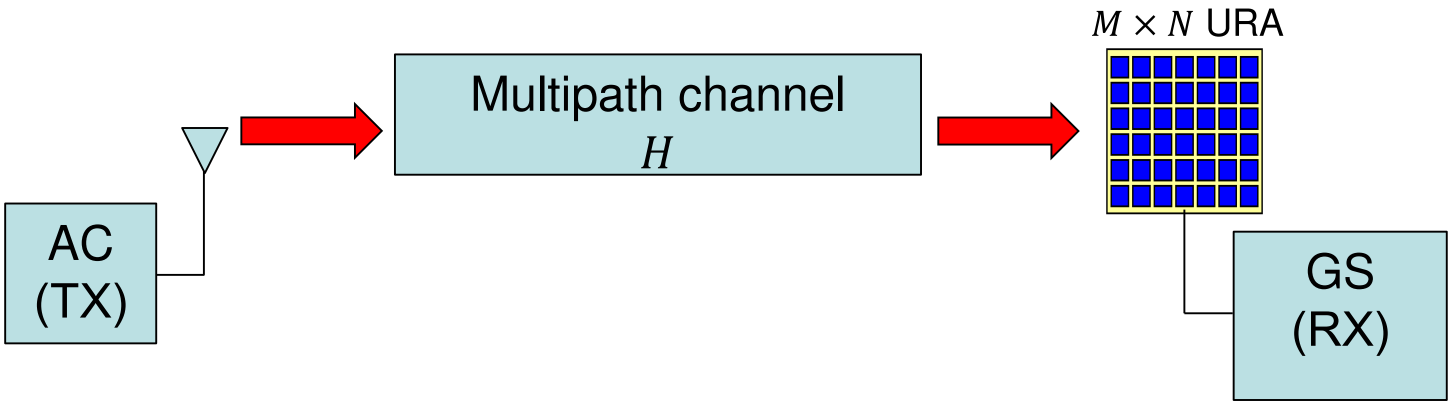

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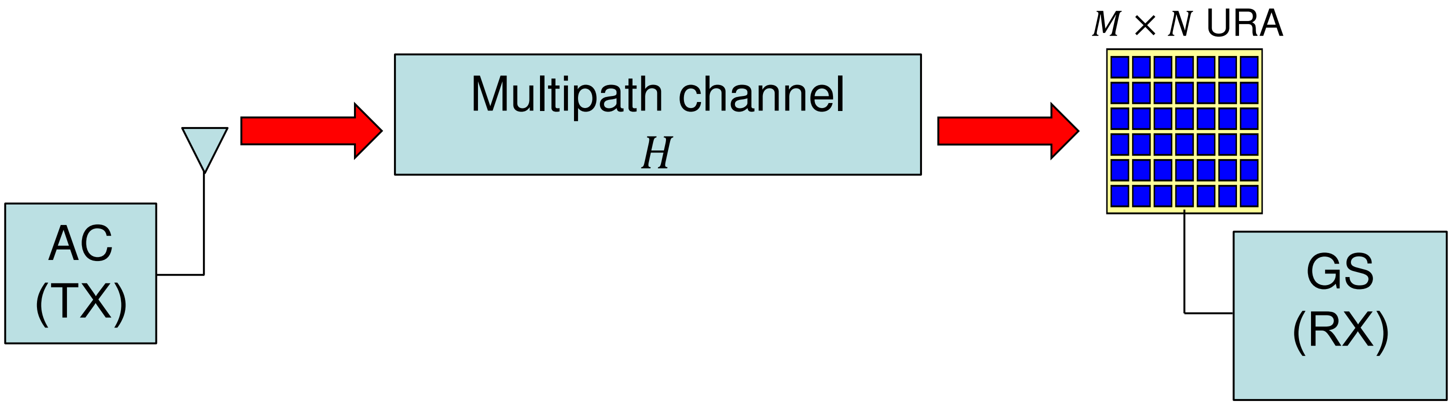

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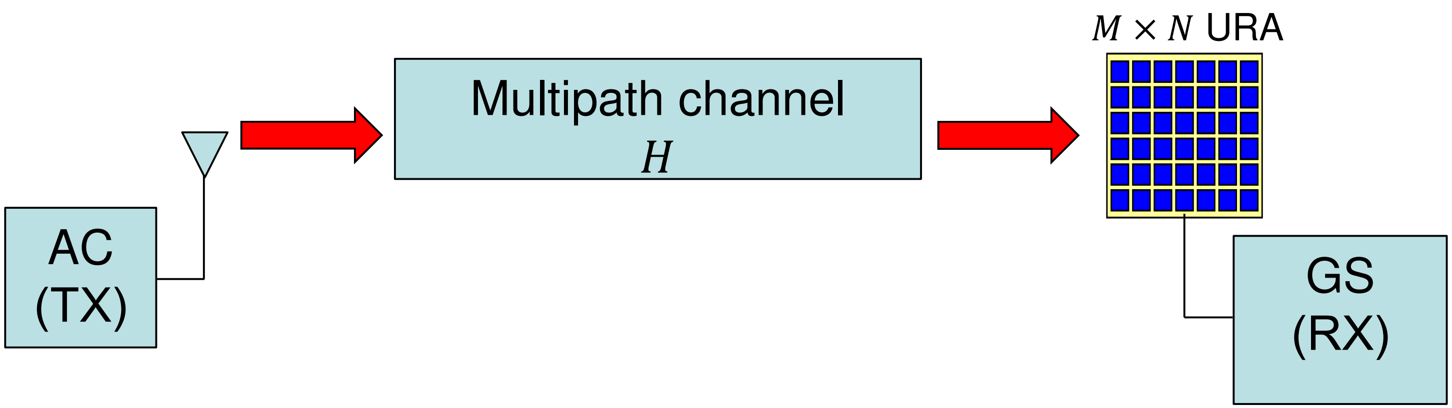

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$$\mu_l = \frac{2\pi}{\lambda}\cos\theta_l \qquad v_l = \frac{2\pi}{\lambda}\sin\theta_l\cos\phi_l$$

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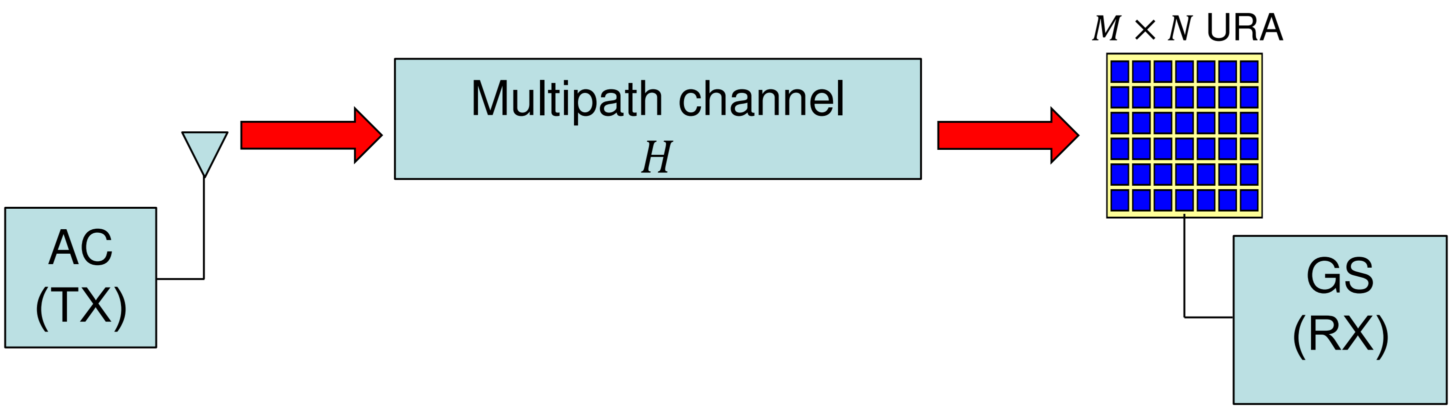

- By vector mapping: $a_l = vec\left(a(\mu_l)\, a(v_l)^T \right) = a(v_l) \otimes a(\mu_l)$

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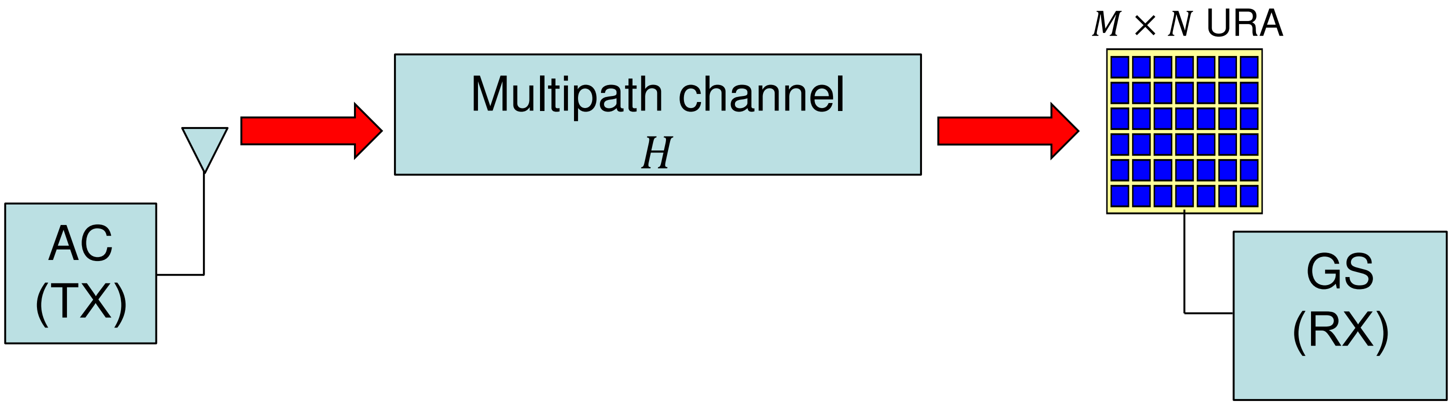

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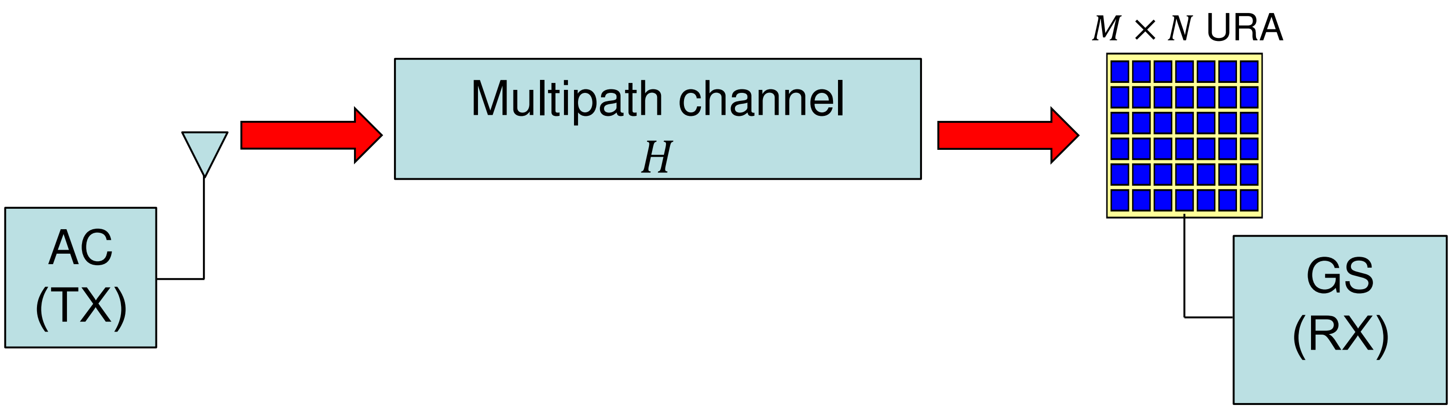

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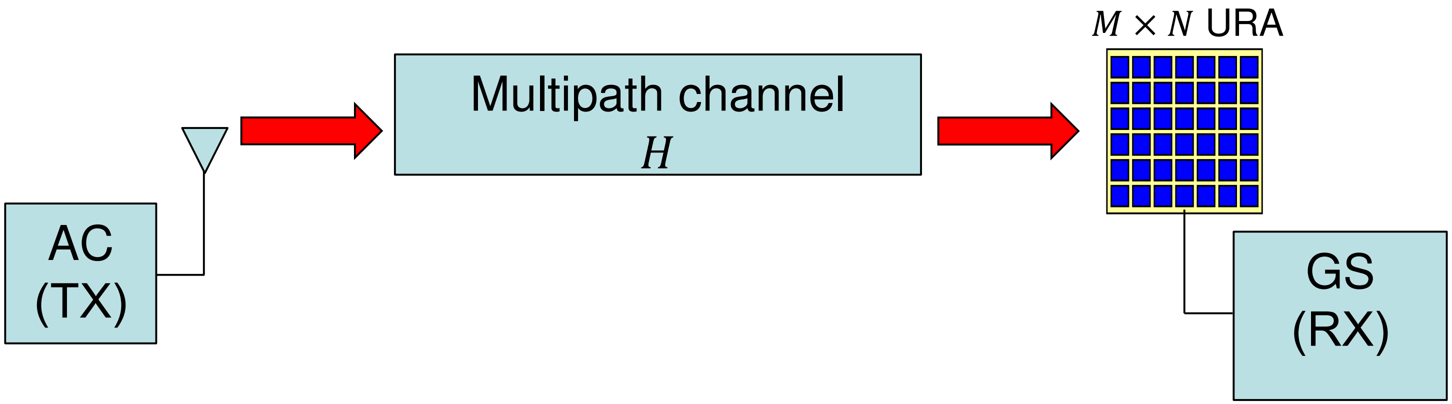

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A is the array response matrix ($MN \times P$).

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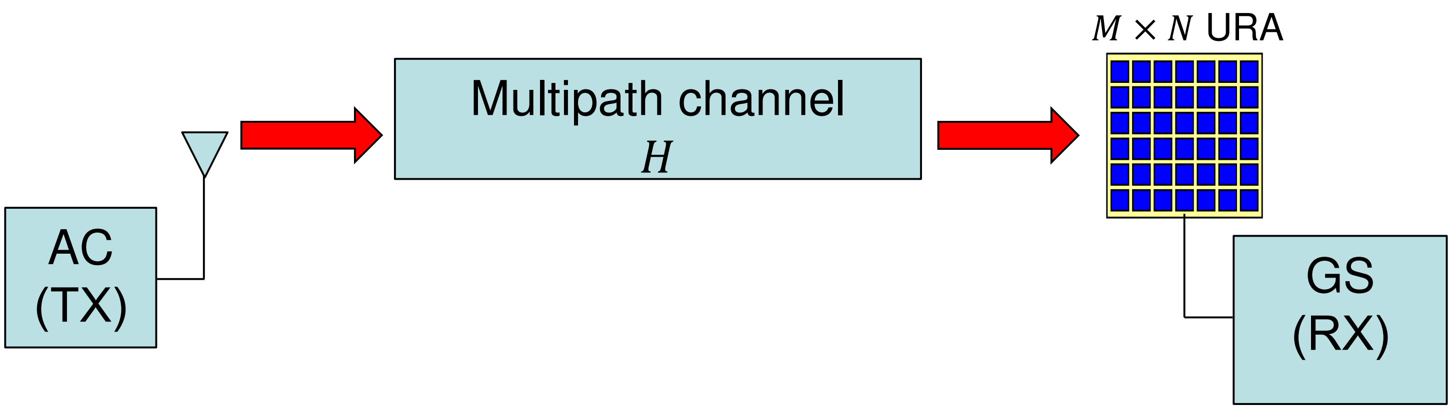

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D is the complex channel gain matrix (diagonal $P \times P$).

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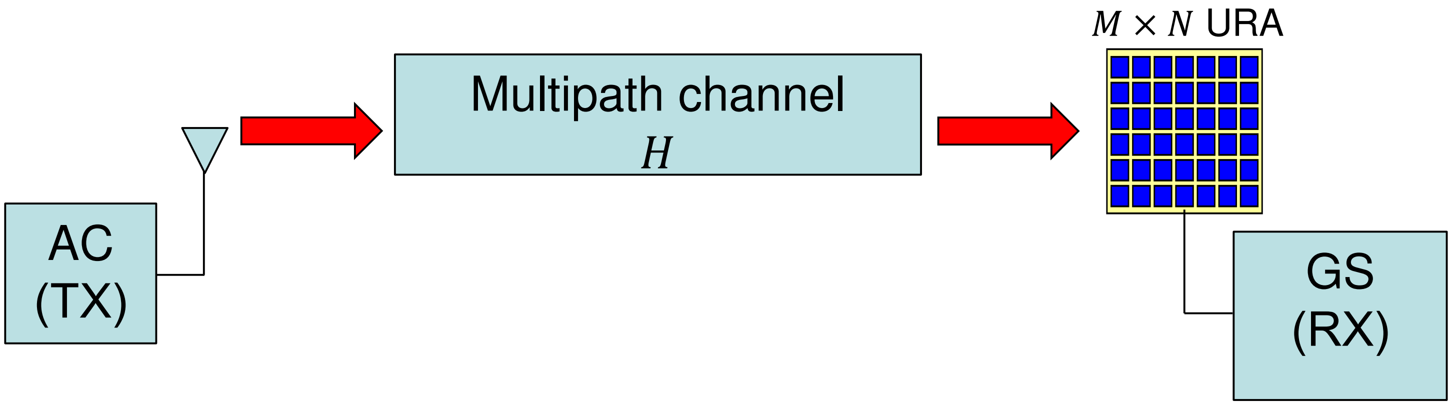

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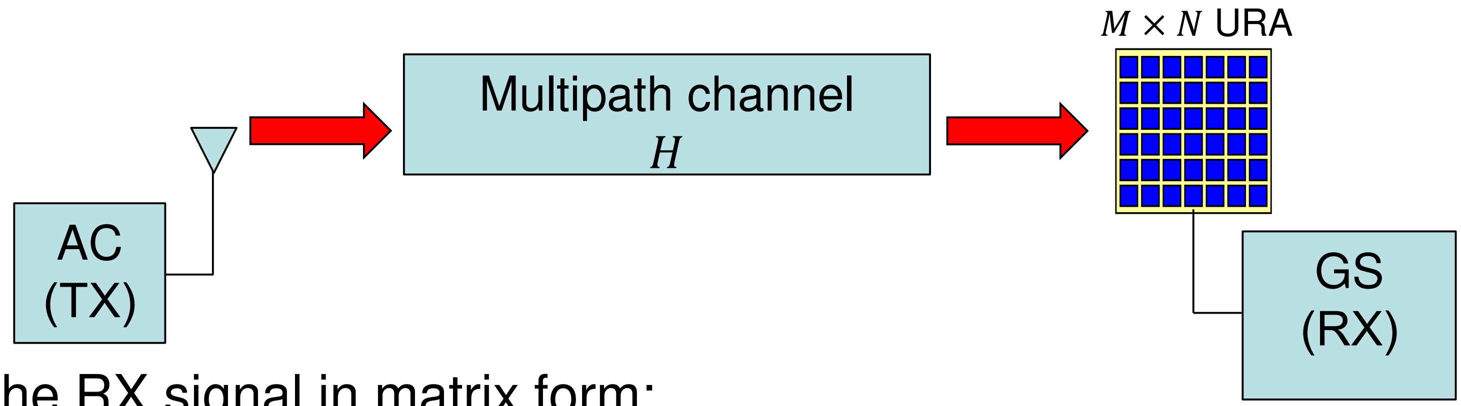

- The RX signal in matrix form:

$$Y = HS + W$$

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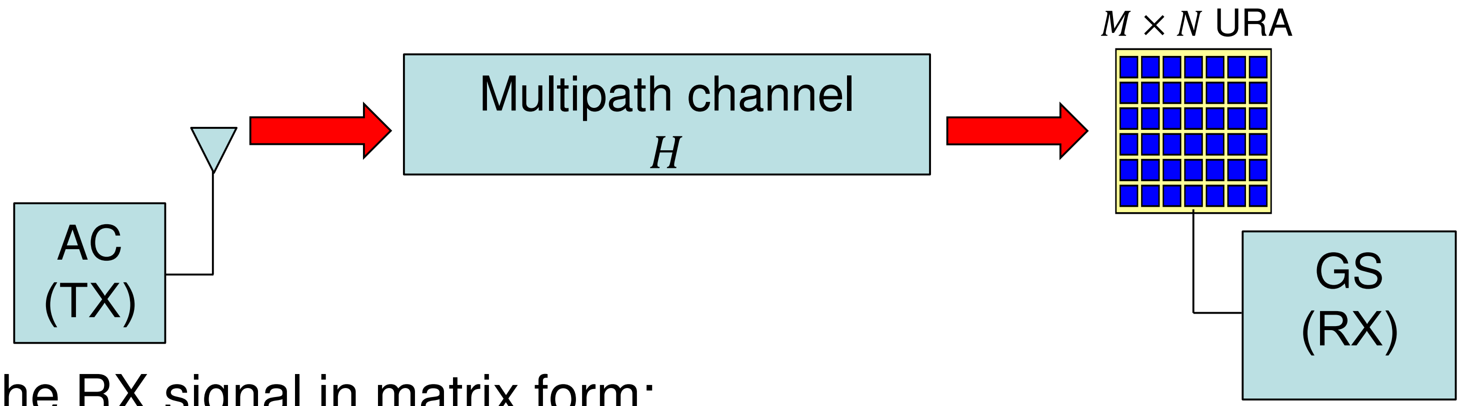

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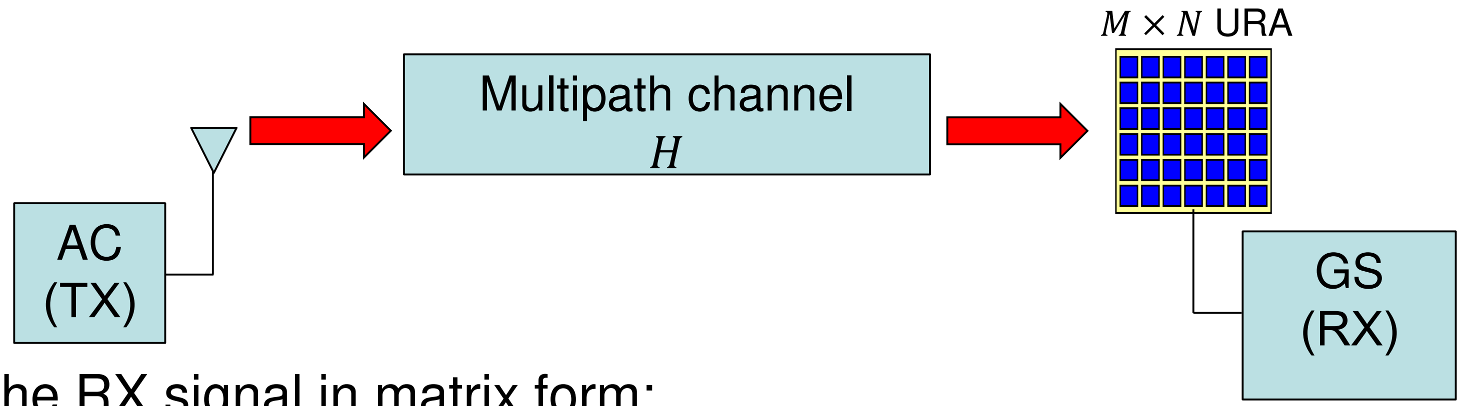

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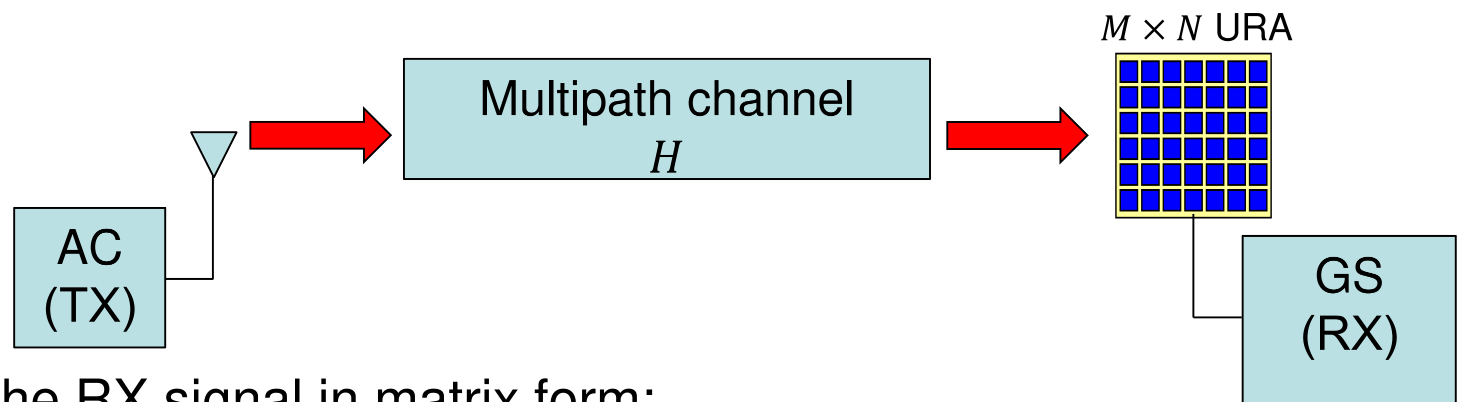

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F_{ψ} is the phase shift matrix ($P \times LB,\ B < V$)

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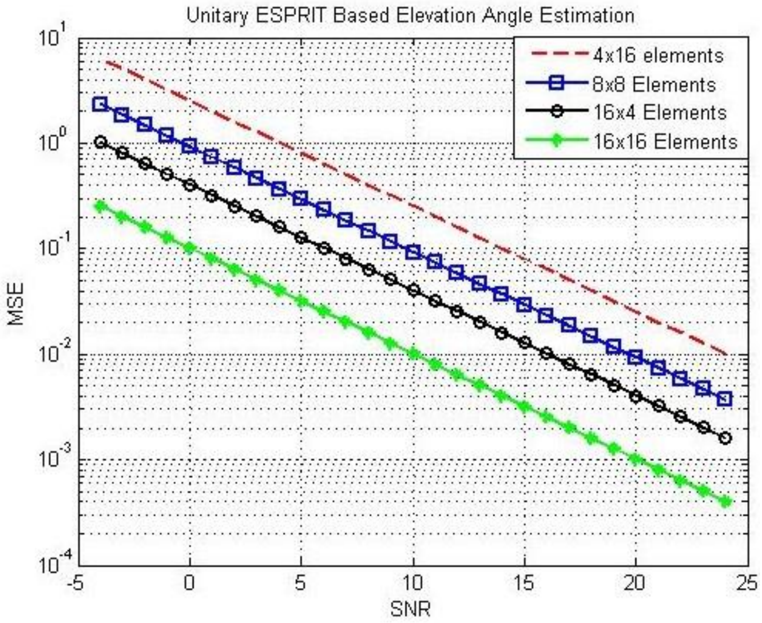

Elevation

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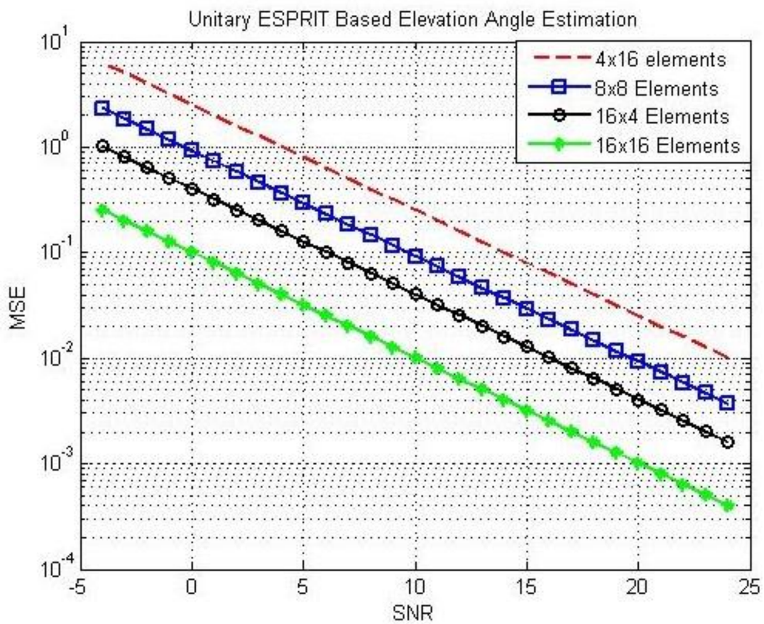

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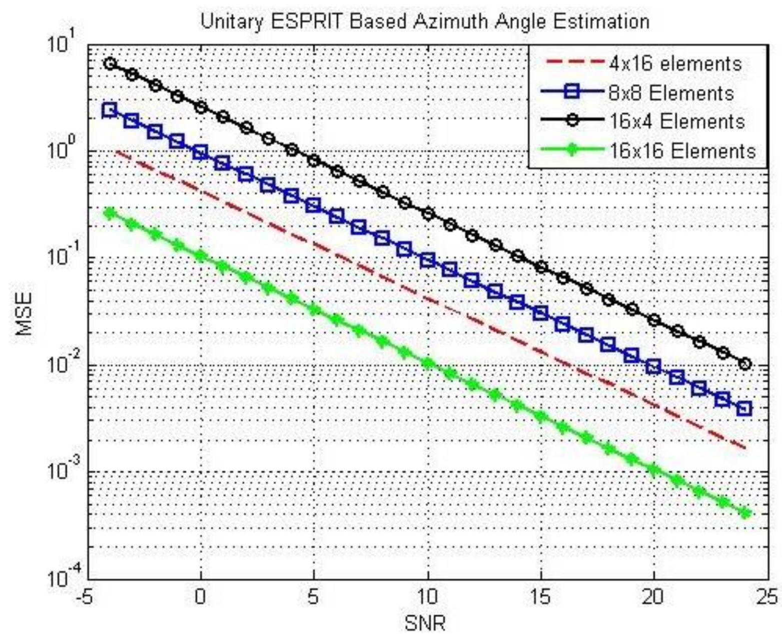

Azimuth

Achievable rate analysis

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- The achievable rate can written as:

$$R = \sum_{l=1}^{P} \log_2\left(1 + \frac{MN\,|\alpha_l|^2\,\left|a_l{}^T\,\mathrm{f}_l\right|^2\,p_l}{\sigma^2}\right)$$

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DOA estimation error: $\mathrm{f}_l = (a(v_l + \Delta v_l) \otimes a(\mu_l + \Delta\mu_l))^*$

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- Thus, the expected TX power for the l^{th} path is

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- If $\Delta v_l = 0$ and $\Delta\mu_l = 0$, the power allocation becomes the traditional water-filling solution.

Results

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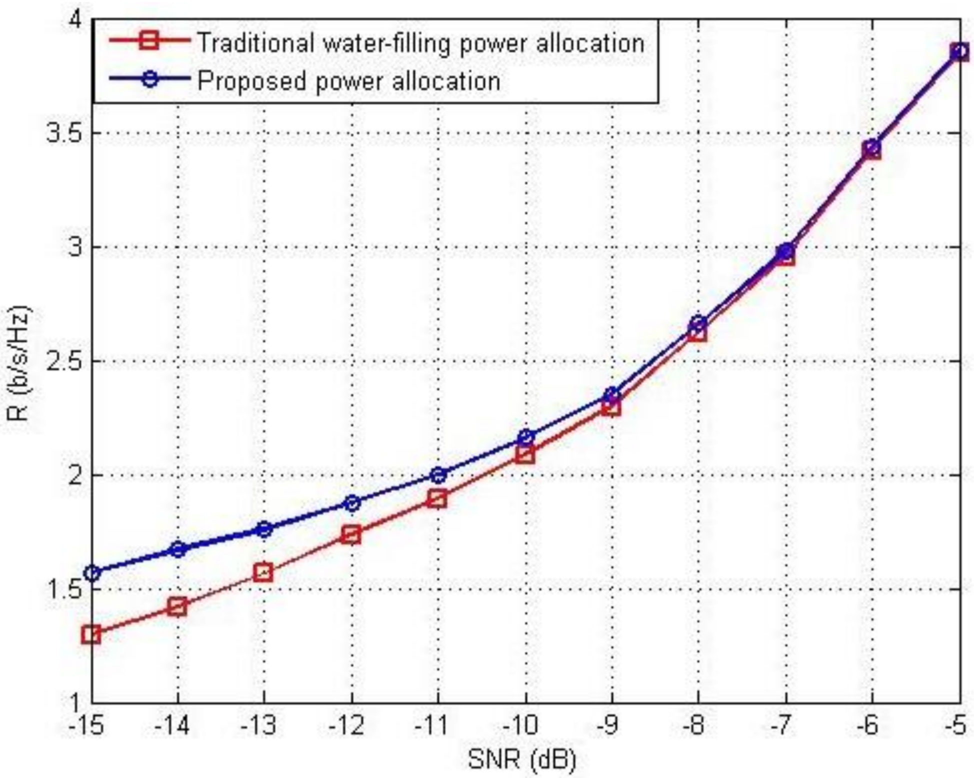
Traditional water-filling power allocation
Proposed power allocation
R (b/s/Hz)
SNR (dB)
4
3.5
3
2.5
2
1.5
1
-15
-14
-13
-12
-11
-10
-9
-8
-7
-6
-5

Future work

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- JADE for multi-cell MIMO systems.

Questions?