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Does the Efficient Market Hypothesis Hold?

Evidence from Six Transition Economies

ABSTRACT: *In this paper, a wavelet analysis of long-range dependence (LRD) based on the Hurst exponent is presented. An estimator is used to perform an analysis of LRD in the capital markets of six transition economies. The results suggest that we can divide the stock markets into two groups: markets with strong LRD (the Czech Republic, Hungary, Russia, and Slovenia), and markets with no or only a weak form of LRD (Poland and Slovakia). Additionally, if the Hurst exponent is estimated on a sliding time window, the results show some additional properties, which we believe are representative for the markets in transition economies.*

According to the efficient market hypothesis, an efficient capital market is one in which security prices adjust rapidly to the arrival of new information, and, therefore, the current prices of securities reflect all information about the security. Three sets of assumptions imply an efficient capital market: (a) an efficient market requires that

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a large number of competing profit-maximizing participants analyze and value securities, each independently of the others; (b) new information regarding securities comes to the market in a random fashion, and the timing of one announcement is generally independent of others; and (c) competing investors attempt to adjust security prices rapidly to reflect the effect of new information.

Although the price adjustment may be imperfect, it is unbiased. This means that sometimes the market will over- or underadjust, but an investor cannot predict which will occur at any given time. If we believe that the efficient market hypothesis is a valid proposition, then the current asset prices should reflect all generally available information. The efficient market hypothesis implies that since market prices reflect all available information, including information about the future, the only difference between the prices at time t and $t + 1$ are events that cannot possibly be predicted. Hence, in an efficient market, stock prices can be statistically tested for the random-walk hypothesis.

Fama (1965) concluded in an early survey that the stock market was efficient. Fama analyzed the distribution of a large data set and showed that empirical evidence seems to confirm the random-walk hypothesis—series of price changes have no memory. There are a number of additional studies where very little evidence for long memory has been found (references to these can be found in Hiemstra and Jones 1997). On the other hand, there are several studies where at least some evidence of long memory has been detected in stock market returns, see for example, studies by Barkoulas and Baum (1996), Barkoulas et al. (2000), Cheung and Lai (1995), Crato (1994), Henry (2002), Sadique and Silvapulle (2001), and Tolvi (2003). The results in these articles, however, are often conflicting, and are not robust to minor changes in the testing methods.

Based on the theoretical literature, one might expect to find long memory particularly in indices. One reason for this is the fact that long memory can be created by aggregating certain types of short-memory series (see, e.g., Granger 1980). Therefore, some studies also analyzed individual U.S. stocks (see, e.g., Barkoulas and Baum 1996; Hiemstra and Jones 1997) and found evidence of statistically significant long memory for some stocks listed on the New York Stock Exchange. On the other hand, Panas (2001) examines the daily returns of thirteen Greek stocks and finds statistically significant long memory in most of the series.

Because small markets do not always seem to behave as expected, and, more precisely, the efficient market hypothesis may not necessarily hold for the returns of stocks in small markets, it is more likely that long memory will be detected in them. This point was raised by Barkoulas et al. (2000), who examined weekly returns in the Greek stock market during the 1980s and found clear evidence of significant long memory (see also Sadique and Silvapulle 2001). Similarly, Wright (2001) examines some emerging market stock returns and finds that long memory is more often found in them than in developed markets.

The main goal of this paper is to analyze capital markets in six transition economies: the Czech Republic, Hungary, Poland, Russia, Slovakia, and Slovenia. We

try to apply some measures of long-memory process to aggregate financial time series in order to test whether there is evidence of random-walk behavior. The analysis will be based on the Hurst exponent.

The Hurst exponent is not so much calculated as estimated. A variety of techniques exist for the estimation, though accuracy can be a complicated issue. Therefore, we will compare standard procedures of estimating the exponent with a new approach based on discrete wavelet transform.

Testing software to estimate the Hurst exponent can be difficult. The best way to test algorithms toward estimating the Hurst exponent is to use a data set that has a known Hurst exponent value. In our research, we tested our software on artificial data sets with different statistical properties.

The evidence that financial time series are examples of long-memory processes is mixed. We will compare the results for selected transition economies with the results for some developed economies.

Analytical Framework

The Long-Range Dependence Phenomenon

The phenomenon of long-range dependence (LRD) has a long history and has remained a topic of active research in the study of economic and financial time series (Cutland et al. 1995; Gabaix et al. 2003; Lo 1991; Schwartz 1997). It is widespread in other areas in the physical and natural sciences (see, e.g., Beran 1994; Mandelbrot 1977), and has been extensively documented in hydrology, meteorology, and geophysics (see, e.g., Mandelbrot and Wallis 1969). More recently, LRD has also started to play an important role in the engineering sciences, especially in the analysis and performance modeling of traffic measurements from modern high-speed communications networks and semiconductor physics (for a recent bibliographical survey of this area, see Willinger et al. 1996). It is commonly accepted that the definition of LRD is the slow power-law decrease of the autocorrelation function of a wide-sense stationary process expressed as

$$\gamma_x(k) \sim c_\gamma k^{-(2-2H)}, \quad k \to +\infty \tag{1}$$

with $H \in (0.5,1)$, where $\gamma_x(k)$ is a covariance function, and c_γ is a positive constant. The Hurst parameter H measures LRD. An equivalent statement for the spectrum $\Gamma_x(\nu)$ of x is

$$\Gamma_x(\nu) \sim c_f |\nu|^{1-2H}, \quad \nu \to 0, \tag{2}$$

where $c_f = \pi^{-1} c_\gamma \Lambda(2H-1)\sin(\pi - \pi H)$, and Λ is the gamma function. Thus, LRD processes belong to the class of random processes that take the form $1/|\nu|^\alpha$ for a range of frequencies ν close to 0. The LRD phenomenon is also closely related to

the properties of scale invariance, self-similarity, and, hence, fractals, and is therefore often associated with statistically self-similar processes, such as the fractional Brownian motion (Mandelbrot and Van Ness 1968).

Standard Estimators

By definition, the LRD phenomenon is related to the power-law behavior of certain second-order statistics (variance, covariance, . . .) of the process with respect to the duration T of observation. Many estimators of H are therefore based on the idea of measuring the slope of a linear fit in a log-log plot. The so-called R/S (rescaled range statistics) estimators are famous examples of this approach (Beran 1994; Hampton 1996; Taqqu et al. 1996). In short, for a given signal $(x_i, i \geq 1)$ with partial sum $y(n) = \Sigma^n_{i=1}x_i$, $n \geq 1$, and $S^2(n) = n^{-1}\Sigma^n_{i=1}(x_i - n^{-1}y(n))^2$, $n \geq 1$, the rescaled adjusted range statistic or R/S statistic is defined by

$$\frac{R}{S}(n) = \frac{1}{S(n)}\left[\max_{0 \leq t \leq n}\left(y(t) - \frac{t}{n}y(n)\right) - \min_{0 \leq t \leq n}\left(y(t) - \frac{t}{n}y(n)\right)\right], \quad n \geq 1. \quad (3)$$

Hurst (1951) found that many naturally occurring empirical records appear to be well represented by the relation

$$E(R/S(n)) \sim c_1 n^H, \quad \text{as } n \to \infty, \quad (4)$$

with typical values of the Hurst parameter H in the interval $(0.5,1)$, and c_1 a finite positive constant that does not depend on n. On the other hand, if the observations x_i come from a short-range dependent model, then it is known that

$$E(R/S(n)) \sim c_2 n^{0.5}, \quad \text{as } n \to \infty, \quad (5)$$

where c_2 is independent of n and is finite and positive. The discrepancy between the last two relations is generally referred to as the "Hurst effect" or the "Hurst phenomenon."

Because the classical R/S analysis is sensitive to the statistical properties of a data set, a number of different versions of estimation procedures have been documented in the literature. The algorithm outlined in this paper uses two different procedures. In the first, nonoverlapping data regions, where the size of the data set is a power of two, are used; each subregion is a component power of two. The second uses overlapping regions and is not limited to data sizes that are a power of two.

Multiresolution Analysis and Discrete Wavelet Transform

A multiresolution analysis (MRA) consists in a collection of nested subspaces $\{V_j\}_{j \in Z}$, satisfying the following set of properties (Daubechies 1992):

1. $\cap_{j \in Z} V_j = \{0\}$, $\cup_{j \in Z} V_j$ is dense in $L^2(R)$,
2. $V_j \subset V_{j-1}$,
3. $x(t) \in V_j \Leftrightarrow x(2^j t) \in V_0$, and
4. there exists a function $\phi_0(t)$ in V_0, called the scaling function, such that the collection $\{\phi_0(t - k), k \in Z\}$ is an unconditional Riesz basis for V_0.

Similarly, the scaled and shifted functions $\{\phi_{j,k}(t) = 2^{-j/2}\phi_0(2^{-j}t - k), k \in Z\}$ constitute a Riesz basis for the space V_j. Performing a multiresolution analysis of the signal x means successively projecting it into each of the approximation subspaces V_j:

$$approx_j(t) = \left(Proj_{V_j} x\right)(t) = \sum_k a_x(j,k)\phi_{j,k}(t). \tag{6}$$

Because $V_j \subset V_{j-1}$, $approx_j$ is a coarser approximation of x than is $approx_{j-1}$. Therefore, the key idea of the MRA consists in examining the loss of information, that is, detail, when going from one approximation to the next, coarser one: $detail_j(t) = approx_{j-1}(t) - approx_j(t)$. The MRA analysis shows that the detail signals $detail_j$ can be directly obtained from projections of x onto a collection of subspaces, the W_j, called the wavelet subspaces. Moreover, MRA theory shows that there exists a function ψ_0, called the "mother wavelet," to be derived from ϕ_0, such that its templates $\{\psi_{j,k}(t) = 2^{-j/2}\psi_0(2^{-j}t - k), k \in Z\}$ constitute a Riesz basis for W_j:

$$detail_j(t) = \left(Proj_{W_j} x\right)(t) = \sum_k d_x(j,k)\psi_{j,k}(t). \tag{7}$$

Basically, the MRA consists in rewriting the information in x as a collection of details at different resolutions and a low-resolution approximation:

$$x(t) = approx_j(t) + \sum_{j=1}^{J} detail_j(t)$$
$$= \sum_k a_x(J,k)\phi_{J,k}(t) + \sum_{j=1}^{J}\sum_k d_x(j,k)\psi_{j,k}(t). \tag{8}$$

The $approx_j$, essentially being a coarser approximation of x, means that ϕ_0 needs to be a low-pass function. The $detail_j$, being an information differential, indicates rather that ψ_0 is a band-pass function, and therefore a small wave, a wavelet. More precisely, the MRA shows that the mother wavelet must satisfy $\int \psi_0(t)dt = 0$ and that its Fourier transform obeys $|\Psi_0(\nu)| \sim \nu^N$, $\nu \to 0$, where N is a positive integer, called the number of vanishing moments of the wavelet (Daubechies 1992).

Given a scaling function ϕ_0 and mother-wavelet ψ_0, the discrete (or nonredundant) wavelet transform (DWT) is a mapping from $L^2(R) \to l^2(Z)$ given by

$$x(t) \rightarrow \left\{ \left\{ a_x(J,k), k \in Z \right\}, \left\{ d_x(j,k), j = 1, ..., J, k \in Z \right\} \right\}. \tag{9}$$

These coefficients are defined through inner products of x with two sets of functions:

$$a_x(j,k) = \left\langle x, \overset{\circ}{\phi}_{j,k} \right\rangle$$

$$d_x(j,k) = \left\langle x, \overset{\circ}{\psi}_{j,k} \right\rangle, \tag{10}$$

where $\overset{\circ}{\psi}_{j,k}$ and $\overset{\circ}{\phi}_{j,k}$ are shifted and dilated templates of $\overset{\circ}{\psi}_0$ and $\overset{\circ}{\phi}_0$, called the dual mother wavelet and dual scaling function, and whose definitions depend on whether one chooses to use an orthogonal, semi-orthogonal, or bi-orthogonal DWT (Abry and Flandrin 1996; Aldroubi and Unser 1993). They can practically be computed by fast recursive filter-bank-based pyramidal algorithm, whose computational cost is extremely low (Daubechies 1992).

The Wavelet-Based H Estimator

The coefficient $|d_x(j,k)|^2$ measures the amount of energy in the analyzed signal about the time instant $2^j k$ and frequency $2^{-j}\nu_0$, where ν_0 is an arbitrary reference frequency selected by the choice of ψ_0. It has been suggested (Abry et al. 1993) that a useful spectral estimator can be designed by performing a time average of $|d_x(j,k)|^2$ at a given scale, that is

$$\hat{\Gamma}_x \left(2^{-j}\nu_0 \right) = \frac{1}{n_j} \sum_k \left| d_x(j,k) \right|^2, \tag{11}$$

where n_j is the available number of wavelet coefficients at octave j; essentially, $n_j = 2^{-j}n$, where n is the length of the data. $\hat{\Gamma}_x(\nu)$ is therefore a measure of the amount of energy that lies within a given bandwidth around the frequency ν and can be regarded as a statistical estimator for the spectrum $\Gamma_x(\nu)$ of x. In fact, one can show (Abry et al. 1995) that, when x is wide-sense stationary process, the expectation of $\hat{\Gamma}_x$ is

$$E\left(\hat{\Gamma}_x \left(2^{-j}\nu_0 \right) \right) = \int \Gamma_x(\nu) 2^j \left| \Psi_0 \left(2^j \nu \right) \right|^2 d\nu, \tag{12}$$

where Ψ_0 denotes the Fourier transform of the analyzing wavelet ψ_0. From this relation, one sees that $\hat{\Gamma}_x$ suffers from the standard convolutive bias, that is, the spectrum to be estimated is mixed within a frequency range corresponding to the frequency width of the analyzing window at scale j. The crucial point here is that

for LRD signals, this bias reduces naturally to a simple form, enabling an unbiased estimation of *H*. To see this, recall the spectral behavior (Equation (2)) and assume for the moment that this form holds for all frequencies. The bias Equation (12) can then be rewritten as

$$
\begin{aligned}
E\left(\hat{\Gamma}_x\left(2^{-j}v_0\right)\right) &= c_f \left|2^{-j}\right|^{(1-2H)} \int |v|^{(1-2H)} \left|\Psi_0(v)\right|^2 dv \\
&= \Gamma_x\left(2^{-j}v_0\right)\left|v_0\right|^{(2H-1)} \int |v|^{(1-2H)} \left|\Psi_0(v)\right|^2 dv.
\end{aligned}
\tag{13}
$$

From Equation (13), one sees that in the case of $1/|v|^\alpha$ processes, the standard convolutive bias turns into a multiplicative one. Moreover, this multiplicative constant is dependent on the analyzing scale *j*. It is therefore possible to design an estimator $\hat{H}$ of the parameter *H* from a simple linear regression of $\log_2(\hat{\Gamma}_x(2^{-j}v_0))$ on *j*, that is,

$$
\log_2\left(\hat{\Gamma}_x\left(2^{-j}v_0\right)\right) = \log_2\left(\frac{1}{n_j}\sum_k \left|d_x(j,k)\right|^2\right) = (2\hat{H}-1)j+\hat{c},
\tag{14}
$$

where $\hat{c}$ estimates $\log_2(c_f \int |v|^{(1-2H)}|\Psi_0(v)|^2 dv)$, provided that the integral

$$
\int |v|^{(1-2H)} \left|\Psi_0(v)\right|^2 dv
\tag{15}
$$

converges. Performing a weighted least-squares fit between the scales (octaves) j_1 and j_2 yields the following explicit formula for the estimator of *H*:

$$
\hat{H}(j_1,j_2) \equiv \frac{1}{2}\left[\frac{\sum_{j=j_1}^{j_2} S_j j\eta_j - \sum_{j=j_1}^{j_2} S_j\eta_j \sum_{j=j_1}^{j_2} S_j j}{\sum_{j=j_1}^{j_2} S_j \sum_{j=j_1}^{j_2} S_j j^2 - \left(\sum_{j=j_1}^{j_2} S_j j\right)^2} + 1\right],
\tag{16}
$$

where $\eta_j = \log_2((1/n_j)\sum_k |d_x(j,k)|^2)$, and the weight $S_j = (n\ln^2 2)/2^{j+1}$ is the inverse of the theoretical asymptotic variance of η_j (Abry et al. 1995).

Under Gaussian and quasi-decorrelation of the wavelet coefficient hypotheses, and in the asymptotic limit, a closed-form for the variance of the estimate of *H* can be obtained and given by

$$
\sigma_{\hat{H}}^2 = \operatorname{var}\hat{H}(j_1,j_2) = \frac{2}{n_{j_1}\ln^2 2}\frac{1-2^J}{1-2^{-(J+1)}(J+4)+2^{-2J}},
\tag{17}
$$

where $J = j_2 - j_1$ is the number of octaves involved in the linear fit, and $n_{j_1} = 2^{-j_1}n$ is the number of available coefficients at scale j_1. It can be shown that this variance is

the smallest possible as that equal to the Cramer–Rao bound for a given J. For further details on this estimator, see Abry et al. (1995).

From the above closed-form for the variance estimation, one can derive a confidence interval

$$\hat{H} - \sigma_{\hat{H}} z_\beta \le H \le \hat{H} + \sigma_{\hat{H}} z_\beta, \tag{18}$$

where z_β is the $1 - \beta$ quantile of the standard Gaussian distribution ($P(z \ge z_\beta) = \beta$). All the results presented below, both in numerical simulations and actual data analysis, were computed with $\beta = 0.025$ (95 percent confidence intervals), based on the above hypothesis.

Data

Artificially Generated Data

We used two types of data in our research: artificially generated data and real financial data. Artificially generated data are necessary for testing the source code of the developed software and for comparing the different approaches to the estimation of the Hurst exponent. Therefore, the artificial data must have a predefined (known) value of the Hurst exponent. The generation of data with a predefined value of the Hurst exponent can be accomplished by estimation of factional Gaussian noise (FGn) and fractional Brownian motion (FBm). FGn forms a stationary process $G_H(t)$ with the following properties (Beran 1994):

1. $G_H(t) = (1/\delta)(B_H(t + \delta) - B_H(t))$,
2. $G_H(t)$ is normally distributed with $N(0, \sigma\delta^{H-1})$, and
3. $E(G_H(t + \tau)G_H(t)) = \sigma^2 H(2H - 1)|\tau|^{2H-2}$ for $\tau >> \delta$.

FBm of the Hurst exponent $0 < H < 1$ is the zero-mean Gaussian process $B_H(t)$ with the following properties (Beran 1994):

1. $E(B_H(t)) = 0$,
2. $B_H(0) = 0$,
3. $B_H(t + \delta) - B_H(t)$ is normally distributed with $N(0, \sigma|\delta|^H)$,
4. $B_H(t)$ has independent increments, and
5. $E(B_H(t)B_H(s)) = \sigma^2/2(|t|^{2H} + |s|^{2H} - |t - s|^{2H}) \Rightarrow \mathrm{Var}(B_H(t)) = \sigma^2|t|^{2H}$.

$B_H(t)$ is nonstationary and does not admit a spectrum in the usual sense. However, it is possible to attach to it an average spectrum through the lens of wavelets

$$\Gamma_{B_H(t)} \cong |v|^{-(2H+1)}. \tag{19}$$

$B_H(t)$ is exactly self-similar, perfectly determined by H. There are several ways to create an approximation of FBm. One way, which we applied in our source

Figure 1. **Graphical Presentation of the Artificially Generated Data**
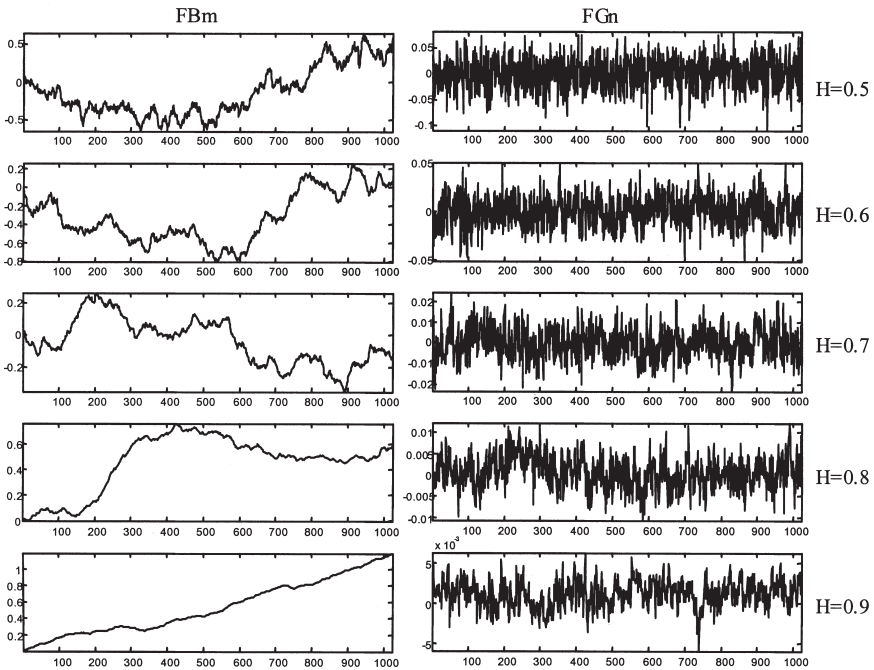
Notes: FBm—fractional Brownian motion; FGn—fractional Gaussian noise; H—Hurst exponent.

code, is by applying random midpoint displacement (RMD). Like all other procedures, the RMD procedure for generating FBm needs random numbers. A true random number is generated by a physical process, like counting the particles emitted by the decay of a radioactive element. Pseudo-random numbers are generated by software functions. They are referred to as pseudo-random because the sequence of numbers is deterministic. Given a particular function and a seed value, the same sequence of numbers will be generated by the function. For many applications, however, high-quality random numbers are needed, because this is the only way to avoid incorrect conclusions about the statistical properties of the observed estimator. The artificial data in this paper are based on random numbers generated by the GNU Scientific Library for Microsoft Visual C++.

We have generated data sets with a predefined Hurst exponent for $H = \{0.5, 0.6, 0.7, 0.8, 0.9\}$ and a different number of observations $T = \{2^9, 2^{10}, 2^{11}, 2^{12}, 2^{13}\}$. A graphical presentation of the data for $T = 2^{10}$ is shown in Figure 1.

Real Financial Data

Because the main objective of this paper is the analysis of selected capital markets in transition economies, we also need aggregate time series, which will represent the activity in the observed markets. As indicated in the introduction, we collected data for the Czech Republic, Hungary, Poland, Russia, Slovakia, and Slovenia. We decided to use those countries' major stock market indices (PX50, BUX, WIG20, RTS, SAX, SBI, respectively). In order to capture the real dynamics of the market, we use daily closing values.

Most financial models do not attempt to model closing values, but instead deal with returns on the instrument. This is heavily reflected in the literature on finance and economics (see, e.g., Campbell et al. 1997; Mantegna and Stanley 2000). The return is the profit or loss in buying a financial instrument, holding it for some period of time, and then selling it. The most common way to calculate a return is the log-return:

$$x_{t,t-\delta}^* = \log(x_t) - \log(x_{t-\delta}). \tag{20}$$

Another way to calculate the return is to use the percentage return:

$$x_{t,t-\delta}^* = \frac{x_t - x_{t-\delta}}{x_{t-\delta}}. \tag{21}$$

Both transformations yield similar results in our analysis. Because the log-return is most frequently used in the literature, the results in this paper will be presented for this type of data.

Some General Considerations About the
Statistical Properties of the Data

Procedures for the estimation of the Hurst exponent are very sensible to the statistical properties of the sample data. The testing of the stationarity hypothesis is particularly difficult in the presence of LRD, where many classical statistical approaches cease to hold. Even without LRD, however, there is the fundamental problem that there are an infinity of ways in which a process can be nonstationary. Normally, we must choose a particular model framework and test for stationarity only against the types of nonstationarity encompassed by it (an example is the augmented Dickey–Fuller test). To assist in the process, it is important to include a priori information concerning the known physics of the problem (Jagric 2003).

In contrast to standard procedures, the wavelet-based estimator seems to have a substantial robustness against an important class of nonstationarity, namely, the addition of deterministic trends (Jagric and Ovin 2004). This is a particularly important advantage in a LRD context, where it is very difficult in theory and in

practice to distinguish between real trends and long-term sample-path variations due to LRD.

Another fundamental feature of the wavelet-based analysis of the LRD phenomenon is that it can be used to meaningfully analyze the important class of nonstationary processes $X(t)$ with stationary increments, that is, $Y_\tau(t) = X(t + \tau) - X(t)$ is stationary. The properties of the estimations based on wavelet analysis for the study of processes with stationary increments are documented in detail in Flandrin (1989; 1992). The FBm, which we selected as part of the artificially generated data, is the canonical reference for such processes. If one would like to estimate the H parameter with classical procedures, it would be necessary to first compute the increments. With a wavelet estimator, one can work with the process itself.

Results

Estimating the Hurst exponent for a data set provides a measure of whether the data are a pure random walk or have underlying trends. Another way to state this is that a random process with an underlying trend has some degree of autocorrelation. When the autocorrelation has a very long decay, this kind of Gaussian process is sometimes referred to as a long-memory process.

Processes that we might naively assume are purely random sometimes turn out to exhibit Hurst-exponent statistics for long-memory processes. One example is seen in computer network traffic. We might expect that network traffic would be best simulated by having some number of random sources send random-sized packets into the network. Following this line of thinking, the distribution might be Poisson. As it turns out, the naive model for network traffic seems to be wrong. Network traffic is best modeled by a process that displays a nonrandom Hurst exponent (Paxson and Floyd 1994).

As outlined in the introduction, we first tested the developed software on the artificially generated data, which we divided into two major groups. The first group includes data that have the properties of an FBm process (see Table 1). As the results suggest, standard estimation procedures fail. All estimates of the Hurst exponent (H1, H2, and H3) are far from true values of the exponent. This holds for both short and long samples.

In contrast to the results obtained by standard procedures, it seems that the results for the wavelet estimation of the exponent are quite robust. Due to the properties of the FBm, the number of vanishing moments (estimations based on Daubechies 1 [or Harr], Daubechies 2, and Daubechies 5 wavelets) does not influence the quality of the results. The results are only improved if the number of observations in the sample is changed. The developed software also correctly indicated the presence of self-similarity.

The results for the second group of artificially generated data are presented in the Table 2. The data incorporate the FGn process, which is stationary. Therefore, the standard procedures can be normally applied. If we carefully examine the re-

Table 1

Estimation of the Hurst Exponent for Artificial Data—Group 1

Estimations

Artificial data sets			Wavelet—D1			Wavelet—D2			Wavelet—D5			RSA		RSA October
Process	H	Size	H-LRD	H-SS	Confidence intervals	H-LRD	H-SS	Confidence intervals	H-LRD	H-SS	Confidence intervals	H1	H2	H3
FBm	0.5	9		0.46	0.37–0.54		0.46	0.37–0.54		0.44	0.38–0.55	0.69	0.66	1.01
		10		0.45	0.39–0.50		0.49	0.43–0.54		0.43	0.37–0.50	0.67	0.69	1.00
		11		0.44	0.40–0.48		0.45	0.41–0.48		0.39	0.35–0.43	0.62	0.51	0.98
		12		0.46	0.43–0.48		0.44	0.42–0.47		0.45	0.42–0.48	0.59	0.45	1.00
		13		0.47	0.45–0.48		0.45	0.43–0.47		0.43	0.41–0.45	0.61	0.56	0.99
	0.6	9		0.48	0.40–0.56		0.41	0.32–0.50		0.46	0.36–0.57	0.50	0.56	0.97
		10		0.59	0.54–0.64		0.58	0.53–0.64		0.51	0.44–0.57	0.57	0.53	1.00
		11		0.57	0.53–0.60		0.60	0.56–0.63		0.57	0.52–0.61	0.53	0.56	1.00
		12		0.58	0.55–0.60		0.58	0.56–0.61		0.57	0.54–0.59	0.58	0.58	0.98
		13		0.57	0.55–0.59		0.57	0.56–0.59		0.55	0.53–0.56	0.52	0.44	1.00
	0.7	9		0.71	0.63–0.79		0.77	0.68–0.85		0.72	0.62–0.83	0.43	0.35	0.97
		10		0.64	0.58–0.69		0.63	0.57–0.68		0.62	0.55–0.68	0.50	0.51	1.00
		11		0.69	0.66–0.73		0.71	0.67–0.75		0.69	0.65–0.73	0.43	0.42	1.00
		12		0.65	0.63–0.68		0.67	0.64–0.70		0.66	0.63–0.69	0.48	0.45	0.99
		13		0.69	0.67–0.71		0.66	0.64–0.68		0.65	0.64–0.67	0.42	0.42	1.00

FBm H	size	H-LRD	95% CI	H-SS	95% CI	H1	95% CI	H2	H3	
0.8	9	0.76	0.68–0.84	0.74	0.65–0.83	0.78	0.68–0.89	0.31	0.27	0.99
	10	0.80	0.74–0.85	0.74	0.68–0.80	0.71	0.65–0.78	0.29	0.16	0.99
	11	0.82	0.79–0.86	0.77	0.73–0.81	0.75	0.70–0.79	0.26	0.15	1.01
	12	0.78	0.76–0.81	0.77	0.75–0.80	0.75	0.72–0.77	0.35	0.35	1.00
	13	0.78	0.77–0.80	0.77	0.75–0.79	0.75	0.73–0.76	0.31	0.20	0.97
0.9	9	0.87	0.79–0.95	0.98	0.89–1.07	0.98	0.88–1.09	0.38	0.49	0.99
	10	0.89	0.84–0.94	0.88	0.83–0.94	0.87	0.81–0.94	0.24	0.18	1.00
	11	0.83	0.80–0.87	0.81	0.77–0.85	0.86	0.82–0.90	0.35	0.25	1.01
	12	0.86	0.84–0.88	0.87	0.84–0.89	0.88	0.86–0.91	0.25	0.21	0.99
	13	0.86	0.84–0.88	0.88	0.87–0.90	0.88	0.86–0.90	0.27	0.33	0.98

Notes: FBm—fractional Brownian motion process; H—Hurst exponent; size—number of observations ($2^n = \{9, 10, 11, 12, 13\}$); confidence intervals—95 percent; H-LRD—wavelet (D1, D2, D5) estimation of the Hurst exponent for LRD process; H-SS—wavelet (D1, D2, D5) estimation of the Hurst exponent for LRD and self-similar process; H1—RSA estimation of the Hurst exponent based on regression on all data; H2—RSA estimation of the Hurst exponent based on regression on segmented means; H3—RSA estimation of the Hurst exponent based on octave structure of the data.

Table 2

Estimation of the Hurst Exponent for Artificial Data—Group 2

Artificial data sets			Estimations										
			Wavelet—D1			Wavelet—D2			Wavelet—D5			RSA		RSA October
Process	H	Size	H-LRD	H-SS	Confidence intervals	H-LRD	H-SS	Confidence intervals	H-LRD	H-SS	Confidence intervals	H1	H2	H3
FGn	0.5	9	0.62		0.53–0.70	0.56		0.47–0.65	0.58		0.48–0.69	0.60	0.58	0.48
		10	0.58		0.53–0.63	0.61		0.56–0.67	0.58		0.51–0.64	0.66	0.58	0.52
		11	0.58		0.54–0.61	0.56		0.52–0.60	0.55		0.51–0.59	0.63	0.55	0.51
		12	0.57		0.54–0.59	0.56		0.53–0.58	0.54		0.52–0.57	0.64	0.59	0.52
		13	0.58		0.56–0.59	0.55		0.53–0.57	0.53		0.51–0.55	0.63	0.52	0.52
	0.6	9	0.61		0.53–0.69	0.59		0.50–0.68	0.56		0.45–0.67	0.68	0.64	0.62
		10	0.70		0.65–0.76	0.69		0.63–0.75	0.65		0.59–0.72	0.64	0.61	0.59
		11	0.69		0.65–0.73	0.68		0.64–0.72	0.66		0.62–0.71	0.70	0.62	0.60
		12	0.69		0.66–0.71	0.69		0.66–0.71	0.66		0.63–0.68	0.65	0.55	0.60
		13	0.68		0.66–0.70	0.66		0.65–0.68	0.64		0.63–0.66	0.68	0.59	0.60
	0.7	9	0.85		0.77–0.93	0.83		0.74–0.92	0.88		0.77–0.98	0.67	0.54	0.71
		10	0.76		0.71–0.82	0.77		0.72–0.83	0.73		0.66–0.79	0.70	0.65	0.72
		11	0.81		0.78–0.85	0.80		0.76–0.84	0.80		0.76–0.84	0.74	0.68	0.67
		12	0.77		0.75–0.80	0.77		0.75–0.80	0.76		0.73–0.79	0.72	0.67	0.70
		13	0.76		0.75–0.78	0.75		0.74–0.77	0.75		0.73–0.77	0.76	0.67	0.70

0.8	9	0.89	0.81–0.97	0.90	0.81–0.99	0.80	0.70–0.91	0.66	0.58	0.85
	10	0.84	0.79–0.90	0.87	0.81–0.93	0.87	0.81–0.94	0.80	0.79	0.75
	11	0.86	0.82–0.90	0.86	0.82–0.90	0.87	0.82–0.91	0.81	0.83	0.76
	12	0.88	0.85–0.90	0.86	0.83–0.88	0.87	0.84–0.90	0.76	0.69	0.82
	13	0.86	0.84–0.88	0.85	0.84–0.87	0.85	0.83–0.87	0.82	0.81	0.81
0.9	9	1.06	0.98–1.14	1.03	0.95–1.12	1.03	0.93–1.14	0.74	0.68	0.81
	10	0.97	0.91–1.00	1.01	0.95–1.07	1.01	0.95–1.08	0.79	0.75	0.80
	11	0.91	0.88–0.95	0.94	0.90–0.98	0.94	0.90–0.98	0.80	0.79	0.83
	12	0.95	0.93–0.98	0.96	0.94–0.99	0.97	0.94–0.99	0.80	0.81	0.81
	13	0.97	0.95–0.98	0.97	0.95–0.99	0.97	0.95–0.98	0.82	0.78	0.80

Notes: FGn—fractional Gaussian noise process; H—Hurst exponent; size—number of observations ($2^n = \{9, 10, 11, 12, 13\}$); confidence intervals—95 percent; H-LRD—wavelet (D1, D2, D5) estimation of the Hurst exponent for LRD process; H-SS—wavelet (D1, D2, D5) estimation of the Hurst exponent for LRD and self-similar process; H1—RSA estimation of the Hurst exponent based on regression on all data; H2—RSA estimation of the Hurst exponent based on regression on segmented means; H3—RSA estimation of the Hurst exponent based on octave structure of the data.

sults for RSA (rescaled range statistic adjusted) and RSA based on octave data structure, we find an interesting property. The results suggest that both procedures cannot give correct estimates if the Hurst exponent exceeds a value of 0.8. This seems to be the case for all data sets—the number of observations does not change the results. If we consider only the data sets with $H < 0.8$, the best results are achieved with RSA, which is based on octave structure.

As expected, the results for the wavelet-based estimations are stable. Based on the results in Table 1 and Table 2, we can conclude that the wavelet estimation of the Hurst exponent gives better results. The results are robust for different sample sizes and some types of nonstationarity. We also experimented with time series, which include (or are contaminated by) different types of deterministic trends. The results suggest that if we use a sufficient number of vanishing moments, we can obtain extremely stable results.

Let us now turn to the analysis of real financial data. As explained above, all indices are transformed into the log-return format. Basic descriptive statistics for the financial data in the log-return format are presented in Table 3. Let us first look at the skewness. The skewness of a symmetric distribution, such as the normal distribution, is zero. However, none of the series seems to be symmetric. There are two series (PX50 and SBI) with positive skewness, which means that the distributions have a long right tail. All other series (BUX, WIG20, RTS, SAX) have negative skewness, which implies that the distributions have a long left tail.

The next statistic that is important for our analysis is the kurtosis. The kurtosis of the normal distribution is three. In our case, all series have a kurtosis that exceeds a value of three. This means that the distributions are peaked (leptokurtic) relative to the normal distribution.

Because both descriptive statistics (skewness and kurtosis) indicate deviations from normal values, we can expect that the observed distributions are not normally distributed. Therefore, we calculated the Jarque–Bera statistic, a test statistic for testing whether the series is normally distributed. The reported probability is the probability that a Jarque–Bera statistic exceeds (in absolute value) the observed value under the null hypothesis—a small probability value leads to the rejection of the null hypothesis of a normal distribution. In our case, the results for all series indicate a rejection of the null hypothesis. These results are also confirmed by the quantile–quantile (QQ) plots (Figure 2). If we plot the quantiles of the chosen series against the quantiles of the normal distribution, we can detect strong deviation, especially at the tails. In all cases, the plots indicate an S-shape curve, which is a typical sign of a nonnormal distribution in financial time series. Such results are similar to the results of other studies (see, e.g., Henry 2002), and indicate that the stock markets under study exhibit some common features.

Finally, we also performed a simple unit root test in order to test the stationarity of the selected series. We used the augmented Dickey–Fuller (ADF) test. The results for the ADF test statistic are compared with MacKinnon critical values for the rejection of hypothesis of a unit root at the 1 percent significance level. In our

Table 3

Basic Descriptive Statistics for Financial Data

Country Index	Czech Republic PX50	Hungary BUX	Poland WIG20	Russia RTS	Slovakia SAX	Slovenia SBI
Mean	0.000145	0.000315	0.000081	0.000331	0.000020	0.000293
Median	0.000069	0.000197	−0.000081	0.000621	0.000000	0.000213
Maximum	0.066840	0.059132	0.064430	0.067563	0.041578	0.082225
Minimum	−0.032860	−0.078317	−0.102519	−0.091647	−0.049874	−0.050435
Standard deviation	0.006189	0.007227	0.009741	0.013245	0.006085	0.005967
Skewness	1.342059	−0.865260	−0.436134	−0.344032	−0.408682	0.416943
Kurtosis	17.177770	17.696320	11.924810	7.971277	9.488938	25.34591
Jarque–Bera	22,270.17	30,775.32	8,476.89	2,342.39	3,928.12	60,087.56
Probability	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
ADF test statistic	−20.030770	−25.360830	−21.477880	−20.291380	−21.190350	−21.688550
1 percent critical value*	3.967100	3.966300	3.967100	−3.967600	3.967600	−3.966700
Sum	0.372433	1.062091	0.205729	0.738939	0.044992	0.844974
Sum squared deviation	0.098289	0.176112	0.239990	0.391359	0.081570	0.102665
Start date	September 7, 1993	January 2, 1991	April 18, 1994	September 1, 1995	July 3, 1995	January 7, 1993
End date	July 3, 2004	June 30, 2004	August 3, 2004	August 5, 2004	July 30, 2004	July 16, 2004
Observations	2,567	3,373	2,530	2,232	2,204	2,884

* MacKinnon critical values for rejection of hypothesis of a unit root.

Figure 2. **The Quantile–Quantile (QQ) Plot for Financial Data**
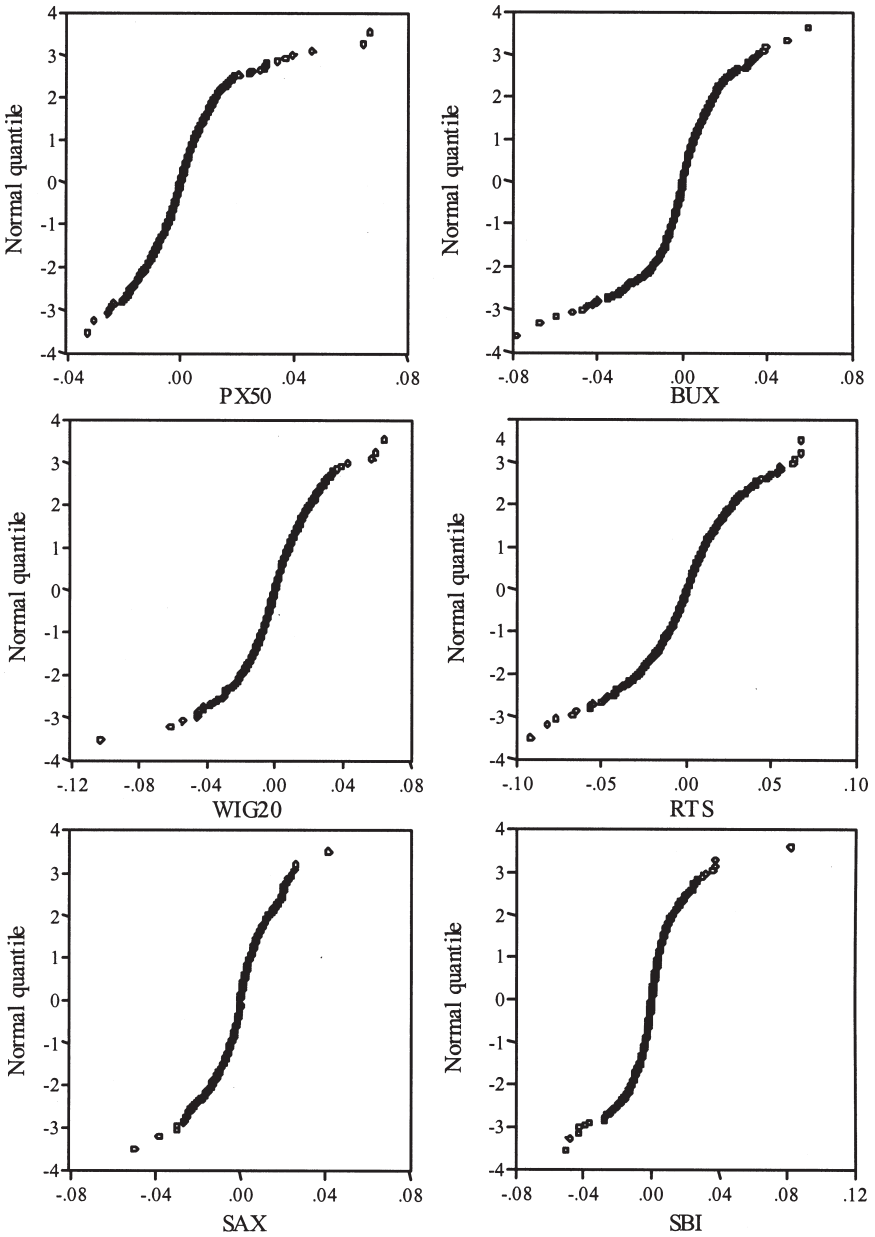
case, all series seem to be stationary, at least in the form that is incorporated in the ADF test.

After the examination of basic statistical properties of the selected time series, we can now perform the estimations of the Hurst exponent. The exponent was estimated for every series for ten different numbers of vanishing moments. This enabled us to test if the estimations are stable and can be treated as reliable. Additionally, we can also make some conclusions about the statistical properties based on the relation between the number of vanishing moments and the Hurst exponent.

The results for the estimated Hurst exponent are presented in Figure 3. The weighted average estimation of the Hurst exponent gives a surprising picture of the stock markets. The results indicate that we can make two groups of markets. In the first group are the markets where strong long-range dependence can be detected. This is the case for the Czech Republic ($\overline{H}_{PX50} = 0.645 \pm 0.035$), Hungary ($\overline{H}_{BUX} = 0.626 \pm 0.030$), Russia ($\overline{H}_{RTS} = 0.648 \pm 0.039$), and Slovenia ($\overline{H}_{SBI} = 0.656 \pm 0.033$). The second group consist of markets where no or only an extremely weak form of LRD could be detected. This is the case for Poland ($\overline{H}_{WIG20} = 0.569 \pm 0.036$) and Slovakia ($\overline{H}_{SAX} = 0.525 \pm 0.039$).

Based on the weighted average estimates of the exponent, the strongest evidence for LRD was detected in Slovenia. Because the Slovenian stock market is small in comparison to other markets in the group, this could be the reason for its inefficiency. This is also supported by the results for Poland, which has the largest stock market in the group and exhibits only an extremely weak form of LRD. However, the results for other countries do not support this hypothesis. We tried to find an explanation for these results in different measures of market size (e.g., market capitalization, number of domestic firms listed, number of foreign firms listed); however, none of the statistical measures could successfully explain the results. We think that the explanation of the different behaviors of the stock markets is more complex and involves the historical development, institutional regulations, and properties of the respective national economic systems.

As many studies suggest (Jagric 2003), the properties of the transition economies are changing. Therefore, it is reasonable to assume that the Hurst exponent is not constant throughout the observed period. In order to test this hypothesis, we estimated the exponent on a sliding time window of size $T = 2^9$ for each stock market. The results are presented in Figure 4.

The graphs in Figure 4 reveal some interesting features. As expected, the exponent is not constant throughout the observed period. The only exception to this is in Slovakia, where the exponent seems not to have any long-run direction of change. This can be explained by the fact that the average value of the exponent for the whole observed time period is already near 0.5 (random walk). In contrast, all other markets have clearly identifiable patterns in the changes of the estimated Hurst exponent.

The results for the Czech Republic indicate a clear tendency of lowering the exponent. At the beginning the exponent was around 0.9, but its value lowered to

Figure 3. **Estimated Hurst Exponent for the Selected Stock Market**
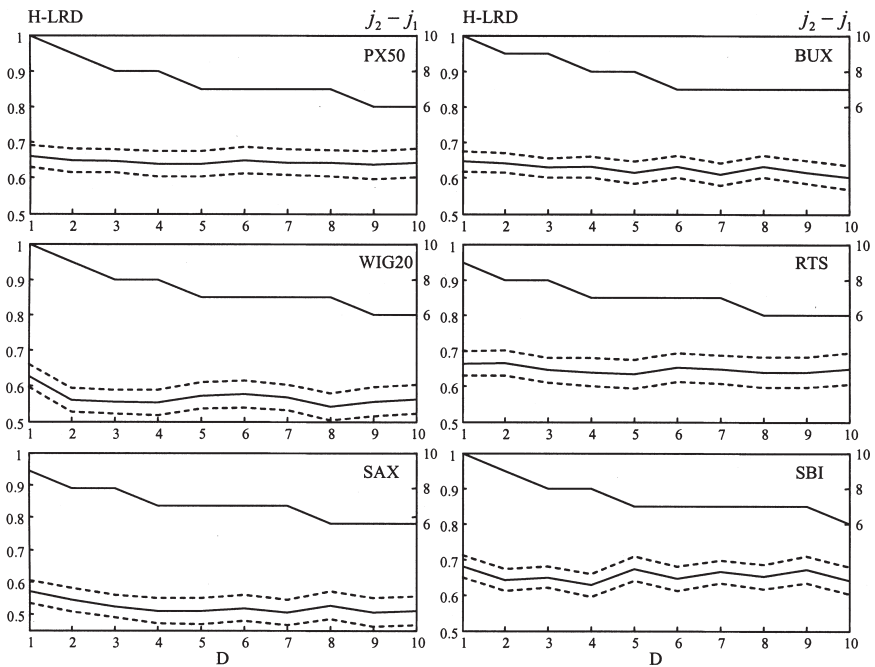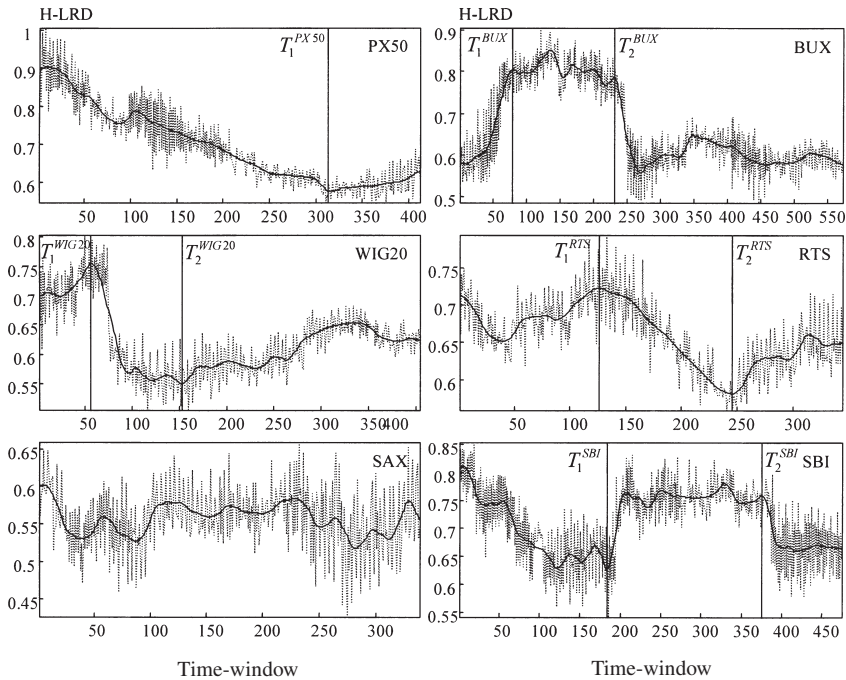

Notes: H-LRD—wavelet estimation of the Hurst exponent for a different number of vanishing moments with 95 percent confidence interval; D—number of vanishing moments (type of the selected wavelet), $j_2 - j_1$—number of octaves included in the estimation.

the point T_1^{PX50} (August 21, 2000–September 13, 2002), where the exponent reached the minimum. After the point T_1^{PX50}, the exponent rebounded and slowly increased for the rest of the observed period. It is also important to note that the lowering of the exponent was interrupted in the time window from February 7, 1996 to June 25, 1998. During this time, the exponent slightly increased, and volatility increased considerably. If we look at the history of the Czech stock market, we see that a domestic currency crisis occurred in the middle of this window (May 7, 1997–August 25, 1997). This might be considered evidence for the influence of currency crisis on the behavior of stock market participants.

The exponent for Hungary is mostly constant at around 0.6. The only exception is the time span between the points T_1^{BUX} (July 8, 1992–August 24, 1994) and T_2^{BUX} (April 8, 1995–August 18, 1997), when the exponent increased drastically. The end of this period (point T_2^{BUX}) was marked by an important event—foreigners were granted permission to trade on the derivatives market of the stock exchange in January 1997.

Figure 4. **Estimated Hurst Exponent for the Selected Stock Markets—Sliding Time-Windows**

Notes: H-LRD—wavelet estimation of the Hurst exponent (dotted line—estimations; solid line—filtered values), window size = 2^9, time step = five trading days.

As in the case of the Czech Republic, Russia, and Slovenia, the exponent for Poland was extremely high at the beginning of the observed period. It was even increasing up to the point T_1^{WIG20} (July 21, 1995–August 8, 1997). After the point T_1^{WIG20}, the exponent sharply dropped and rebounded at the point T_2^{WIG20} (June 25, 1997–July 15, 1999). From this point, the exponent slowly increased for the rest of the observed period.

The estimated Hurst exponent for Russia was initially around 0.7. Around the point T_1^{RTS} (March 12, 1998–March 27, 2000), the volatility of the exponent increased considerably. Examining the history of the Russian stock market, we identify one major event during this period—the Russian currency crisis of 1998. As in the case of the Czech Republic, the crisis had an impact on the behavior of the market participants, since after the point T_1^{RTS} the exponent dropped. The rebound occurred at the point T_2^{RTS} (August 2, 2000–August 19, 2002). From this point on, the exponent increased again, though it seems that the exponent stopped increasing as it reached the value of approximately 0.65.

With the beginning of normal trading on the Ljubljana stock market (January 7, 1993), the SBI index indicates the presence of an extremely strong long-memory process. The estimated value of the Hurst exponent is over 0.8. However, the nature of the index changed, in conjunction with the value of the exponent lowering to the point T_1^{SBI}, which represents a window from August 27, 1996 to September 9, 1998. In the middle of this window, there were some important changes on the Ljubljana exchange. First, there was the imposition of a tax on capital gains deriving from trading in securities for individuals (January 1, 1997). Second, by decree of the Bank of Slovenia, custody accounts were made obligatory for all foreign portfolio investments. Furthermore, these custody accounts and foreign currency reserves to the amount of funds on the account should appear on the balance sheet of the Slovene bank maintaining such an account (February 10, 1997). Third, there was a relaxation of limitations for foreign portfolio investors, imposed by the Slovene central bank in February 1997. Since then, the Bank allows foreign investors to buy Slovene securities without balancing a foreign exchange position for the amount invested. Foreign investors have to undertake the obligation not to sell the securities for a period of seven years (June 30, 1997).

After the point T_1^{SBI}, the estimated value of the Hurst exponent increased again and reached the maximum at point T_2^{SBI}, which represents a window from June 26, 2000 to July 18, 2002. As in the case of the point T_1^{SBI}, there were some important changes that took place in the middle of the time window. First, the stock exchange market segments A and B are merged into a single official market (January 1, 2001). Second, the Bank of Slovenia lifted all restrictions on foreign portfolio investments in the Slovenian capital market (July 1, 2001).

The results for Slovenia and Hungary are similar and can therefore be treated as strong evidence of the influence of foreign investors on the domestic capital market. It seems to us that the market becomes more efficient if there are no restrictions on foreign investors.

As mentioned above, most of the stock markets exhibit large values of the exponent at the beginning of the observed time period. We think that the lack of adequate infrastructure, both physical and legal, as well as the poor disclosure of accurate information are possible sources of inefficiency. Most of these institutional failures in transition economies have been removed in recent years, which may be the reason for the lower values of the exponent at the end of the observed period. However, their management control systems are still not as effective as the control systems observed in more developed capital markets. Given the short life of these equity markets, market imperfections in these transition economies can be a source of acute inefficiency, because they interfere with the rapid processing of information.

One important characteristic to note of some Eastern European equity markets, which can be of direct relevance to the theory of efficiency, is the existence of price restrictions on price movements. Such price limits certainly hinder the infor-

mation transmission mechanism, as price can only go down once a ceiling is reached. Gordon and Rittenberg (1995) noted that these regulations may promote inefficiency by encouraging investors to base their investment decisions simply on the presence of regulations alone.

Conclusions

This paper presented a wavelet analysis of LRD. We found that an estimator of the Hurst exponent can be implemented efficiently, allowing for the direct analysis of very large data sets. The estimator is highly robust against the presence of deterministic trends and facilitates their detection and identification. Computational and numerical comparisons are made against traditional estimators based on R/S analysis. The estimator is used to perform a thorough analysis of the LRD in the capital markets of six transition economies: the Czech Republic, Hungary, Poland, Russia, Slovakia, and Slovenia.

Peters (1992) suggests that a Hurst exponent value of $0.5 < H < 1.0$ shows that the efficient market hypothesis is incorrect—returns are not randomly distributed and there is some underlying predictability. The results in this study are mixed. If we estimate the exponent for the whole observed period, we can divide stock markets into two groups. The first group represents the stock markets where quiet strong LRD is detected (the Czech Republic, Hungary, Russia, and Slovenia). These are similar to results observed in the case of the S&P 500 (Bollerslev and Mikkelsen 1996; Peters 1992) and seven Asia-Pacific stock markets (Pandey et al. 1995). The second group included stock markets where no or only a weak form of LRD was detected (Poland and Slovakia).

However, if the Hurst exponent is estimated on a sliding time window, the results show some additional features about the observed stock markets. Most of the stock markets exhibit excessively high values of the exponent at the beginning of the observed period. In most cases, important events on the markets triggered the process of the lowering of the exponent. These include not only events like currency crises, which also had an influence on foreign markets, but also changes in institutional regulations.

Based on the results of this study and similar findings by Hampton (1996), we think that the estimated Hurst exponent may be sensitive to the frequencies of the data employed: daily, weekly, monthly, and so forth. Additionally, the format of the time series has an important impact on the results. The estimated Hurst exponent is always larger for log-return series than for original time series, where traditional methods for the estimation of the exponent fail.

Long memory may have important implications for the construction of trading systems. Hampton illustrated the use of the Hurst exponent for trading. In his particular example, he uses the ten-day average of the estimated Hurst exponent as a technical index. If this index is higher than 0.66, then it is time to buy. If this

index is lower than 0.43, then it is time to sell. Therefore, a future direction for research in this regard is the matter of how to make use of the property of long memory to design efficient trading strategies.

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