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Dominant Point Detection for Planar Data

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Abstract. Dominant point detection was considered to extract a number of important points from a set of data points collected from some scientific phenomena or analytical studies. The extracted data points shall be able to reflect the original shape of the data. Here, a method to detect the dominant points is proposed using an exclusive formula which involved eigenvalues of the covariance matrix. Concept of region of support which played a vital role in dominant point detection is discussed. A few samples where the data points are regularly spaced and irregularly spaced are used to test the efficiency of the method. The graphical results are presented to show shape preservation of the dominant points to the shape of the data.

INTRODUCTION

Dominant point detection (or corner detection) methods have been applied in image processing and shape preserving curve fitting. Basically, the approaches can be categorized as gray-level and boundary-based [1]. Gray-level approaches directly work on gray-level images by matching corner templates or by computing gradients at edge points whereas boundary-based approaches detect corners on the boundaries of objects [2]. In this paper, boundary-based approach is our focus as the information of a shape is prominently represented by its boundary corner. Subri *et al*. [3] has claimed that corner detection is able to serve the purpose in simplifying the analysis of images by drastically reducing the amount of data. The aim of corner detection is to detect some potential feature points which sufficiently describe the shape of an image [4]. Therefore, instead of using large number of points, a good corner detection method is able to preserve the shape of the image with small amount of detected dominant points.

Suppose we consider a curve which consists of a set of data points collected from the boundary of an image. Two linear segments connecting any three points form a corner. Very often, significant corners are identified as the points of the boundary with local maximum curvature. The curvature measures the rate at which the curve bends away from the tangent line at a point [5]. This measure of significance is similar to as measuring the variation of tangent lines. The common steps for dominant point detection techniques are first, determining significant regions of measure for the data points which are usually identified as region of support, then estimate the sharpness of the angles formed in the region of support, finally locate the points which have local sharpest angle as the significant corners.

Many existing corner detection methods are reported by using curvature-based approach. Sun [1] and Wu [6] used K-cosine value as curvature measure to detect dominant points. Teh and Chin [7] presented a parallel algorithm for detecting dominant points on a digital closed curve. They first determine region of support, followed by computation of relative curvature-based measure by a process of non-maximum suppression. Tsai *et al*. [2] had proposed a measure for corner detection based on the eigenvalues of the covariance matrices of boundary points over a region of support. According to [2], curvature-based methods may detect many spurious corners for an object with circular arcs of varying radii. Zhu *et al*. [8] also presented an auto-corner detection based on eigenvalues of the covariance matrices of boundary points over multi-region of support. Curvature product graph is used to detect

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corners, thus the method is free from the impact of human and threshold setting. Tsai [9] and Subri *et al*. [3] proposed a neural network approach to detect dominant points. In [9], curvature is measured by using neural network to identify the included angles at boundary points. In [3], a neural network classifier is presented to detect the corners of chain code series. The results show that there are strength and drawbacks of using neural network in corner detection of chain code series. The strength is that it makes corner detector more sensitive in detecting a corner whereas the drawbacks of the method is that it can be classified as tedious, and trial and error process. Wang *et al*. [4] used bending value to assess the degree of possibility of a point being a corner.

In this paper, the data points extracted from boundary of images are considered. After region of support of every data points has been determined, dominant points are detected by using a formula which involved both eigenvalues of covariance matrices. The paper is organized as follow, we first discuss the method in determining region of support. Then, we discuss about the eigenvalues of covariance matrix. Some reviews on detecting dominant points using eigenvalues of covariance matrix are given. After that, we introduce the new measure of curvature via an exclusive formula involving eigenvalues. The proposed measure is able to extract significant points which preserve the shape of the boundary data. The results of our proposed method are illustrated by using six images. The paper ends with a conclusion.

REGION OF SUPPORT (ROS)

Region of support is an important concept to be applied while detecting dominant points. Before we start to extract dominant points using any measure of significance, it is advisable to determine the regions of interest of every boundary data point of an image first. The connected sequences of points on either side of a point of interest are identified as the "arms" of the point. Set of points that come before the point of interest is regarded as "left arm" whereas set of points which come after the point of interest is regarded as "right arm" irrespective of the direction of traversal of the curve [10]. A point may have symmetric ROS in which same number of points is considered on both left and right arms, otherwise it may have asymmetric ROS. Guru and Dinesh [10] claimed that it is more reasonable and natural to have asymmetric ROS. Teh and Chin [7] emphasized that besides accuracy of the measure of significance, precise determination of ROS is also a main factor that affects the performance of dominant point detection. Wu [11] used the adaptive bending value to determine the ROS for each point on the curve. Teh and Chin [7] determined the ROS for each point based on its local geometric properties. Wu [6] modified the Teh-Chin's method such that the ROS for each point depends on the ROS for the previous point. That is, the determination of the ROS for each point can be done dynamically by using the information of the previously found ROS.

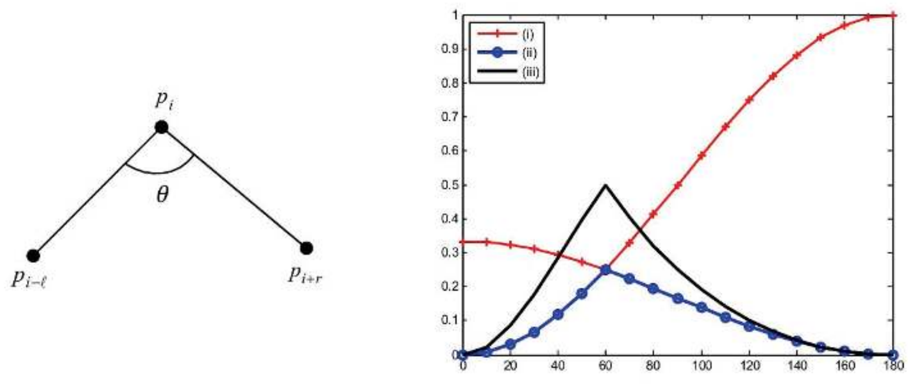
Some researchers proposed corner detection method based on fixed size of symmetric ROS. For instance, the same size of symmetric ROS for every data point was fixed up in [2] before the corner was detected. Teh and Chin [7] had pointed out this approach caused the difficulty in such a way that there is seldom any basis for selecting suitable values for the parameters to successfully determine true corner points as features describing a curve varies enormously in size and extent.

In this paper, an asymmetric ROS from [12] is adopted. The method proposed in [12] involved simple calculation. Furthermore, it is scale and rotation invariant. The main concept is trying to obtain a left or right arm with the longest line segment but minimum error. The error is referred to the sum of squared perpendicular distance from each data point in an arm to the line segment that joins the point of interest and the end point of the arm. Then, a function F = *length-error* is used to determine the size of ROS. The method started its F calculation within the point of interest and the second neighboring point. The support region is outwards to its corresponding left or right side until termination condition is achieved. That is, the value of previous F is greater than the current value of F. We have applied the method on few boundary data. One of the tests was finding the size of ROS for a diamond shape. Due to the original method being less accurate in detecting the ROS for the points which are next to a sharp corner, here, a minor modification has been made. Our calculation on F started with the very next neighboring point, i.e. the first point next to the point of interest which gives zero error.

EIGENVALUES OF COVARIANCE MATRIX

Once the region of support is determined, we can start to detect dominant points based on the corresponding region of support. Let's consider the sequence of n data points describing the boundary W of an object,

$$W = \{p_i(x_i, y_i),\ i = 1, 2, 3, ..., n\}\ .$$



(a) (b)

FIGURE 1. (a) Angle formed by $p_{i-\ell}$, p_i and p_{i+r}. (b) Eigenvalues against angle (in degree).(i) λ_L (ii) λ_S (iii) λ_R

Suppose $L_i(p_i)$ and $R_i(p_i)$ be the left arm and right arm of ROS of point p_i respectively.

$$L_i(p_i) = \{p_j \mid j = i-1, i-2, ..., i-\ell+1, i-\ell\}, R_i(p_i) = \{p_j \mid j = i+1, i+2, ..., i+r-1, i+r\},$$

where ℓ and r denote the size of the left and right arms respectively. Let $S(p_i)$ be defined as

$$S(p_i) = L(p_i) \cup R(p_i) = \{p_j \mid j = i-\ell, i-\ell+1, ..., i-1, i, i+1, ..., i+r-1, i+r\}.$$

The covariance matrix C of a segment $S(p_i)$ is given by

$$C = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$$

where $c_{11} = \frac{1}{\ell+r}\sum_{j=i-\ell}^{i+r}(x_j - c_x)^2$; $c_{12} = c_{21} = \frac{1}{\ell+r}\sum_{j=i-\ell}^{i+r}[(x_j - c_x)(y_j - c_y)]$; $c_{22} = \frac{1}{\ell+r}\sum_{j=i-\ell}^{i+r}(y_j - c_y)^2$; $c_x = \frac{1}{\ell+r+1}\sum_{j=i-\ell}^{i+r}x_j$; $c_y = \frac{1}{\ell+r+1}\sum_{j=i-\ell}^{i+r}y_j$. The covariance matrix is symmetric. There are two eigenvalues λ_L and λ_S for the matrix C,

$$\lambda_L = \frac{1}{2}\left[c_{11} + c_{22} + \sqrt{(c_{11} - c_{22})^2 + 4c_{12}^2}\right], \quad \lambda_S = \frac{1}{2}\left[c_{11} + c_{22} - \sqrt{(c_{11} - c_{22})^2 + 4c_{12}^2}\right], \tag{1}$$

where $\lambda_L \geq \lambda_S$.

To extract the shape information about a boundary curve, the eigenvalues of the matrix C can be used. If $S(p_i)$ is a straight line, the λ_S will be zero, regardless of the length and orientation of the line segment. If $S(p_i)$ is a full circle, then λ_S and λ_L are equal. Besides that, if the shape of $S(p_i)$ is an ellipse, then $\sqrt{\lambda_L}$ and $\sqrt{\lambda_S}$ are the semimajor and semiminor axial lengths of the ellipse [2].

Eigenvalue λ_S has been utilized as a measure for corner detection in [2]. The authors claimed that the data points with sharper angles have larger λ_S than smoother ones. The λ_S of a corner point is usually a local maximum among the λ_S of the points on the boundary segment [13]. According to Yahya [14], λ_S is not only affected by the sharpness of curves but also by the size of support. It is not suitable in detecting sharp corner for equidistant data points produced from circular arcs of various radii and curved corners of different sharpness and sizes. Thus, the ratio

$$\lambda_R = \frac{\lambda_S}{\lambda_L + \lambda_S}$$

was proposed to replace eigenvalue λ_S as the measure for the sharpness. Yahya [14] had compared the λ_S and λ_R in measuring the curvature of circular arc with various radii and sizes of support, and claimed that this ratio λ_R reflects the effect of sharpness and range of support very well. Though the λ_R has better performance than λ_S in measuring curvature, a drawback can be identified for both λ_R and λ_S. Consider a set $S(p_i)$ of three points, i.e. $\ell = r = 1$ in Figure 1(a), three points $p_{i-\ell}$, p_i and p_{i+r} form an angle. Then, the graphs of the eigenvalues against angle are shown in Figure 1(b) with unit length of arms.

The values of λ_S and λ_R are increasing for $\theta \in (0^\circ, 60^\circ)$, but decreasing for $\theta \in (60^\circ, 360^\circ)$ while λ_L acting contrary. This indicates the values of λ_S and λ_R do not exactly reflect the sharpness of corner. From this observation, we conclude that either λ_S or λ_R is suitable to measure the curvature only in certain circumstance.

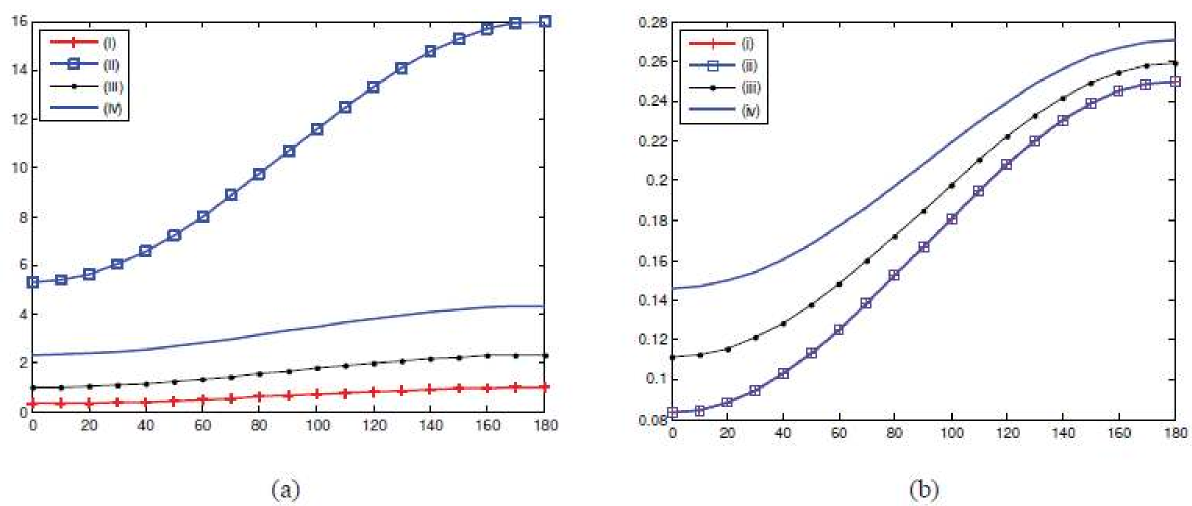


FIGURE 2. (a) The graphs of λ_{sum} with different lengths of arms. (b) The graphs λ_C with different lengths of arms.

(i) $\left|\overline{p_{i-\ell}p_i}\right| = 1; \left|\overline{p_i p_{i+r}}\right| = 1$ (ii) $\left|\overline{p_{i-\ell}p_i}\right| = 4; \left|\overline{p_i p_{i+r}}\right| = 4$ (iii) $\left|\overline{p_{i-\ell}p_i}\right| = 1; \left|\overline{p_i p_{i+r}}\right| = 2$ (iv) $\left|\overline{p_{i-\ell}p_i}\right| = 1; \left|\overline{p_i p_{i+r}}\right| = 3$

DOMINANT POINT DETECTION

Based on the example in Figure 1(a), λ_S and λ_R are not ideal measures to estimate curvature of data point p_i. Therefore, the sum of the eigenvalues

$$\lambda_{sum} = \lambda_L + \lambda_S = c_{11} + c_{22}$$

is proposed as a new measure of curvature. The graph of λ_{sum} is shown in Figure 2(a) with different length of arms. Clearly, the value of λ_{sum} is gradually increasing when the angle increases from 0^o to 180^o. Hence, λ_{sum} is an appropriate measure for detecting sharp corner. If the value of λ_{sum} is smaller than a predetermined threshold, the point p_i will be selected as a dominant point.

The range of λ_{sum} varies with the lengths $\left|\overline{p_{i-\ell}\,p_i}\right|$ and $\left|\overline{p_i\,p_{i+r}}\right|$. This makes the λ_{sum} inefficient in detecting corners for asymmetric ROS and irregular data points. In order to minimize the range of measure, an improved measure is introduced by

$$\lambda_C = \frac{\lambda_L + \lambda_S}{\left(\left|\overline{p_{i-\ell}\,p_i}\right| + \left|\overline{p_i\,p_{i+r}}\right|\right)^2} \,. \tag{2}$$

Note that the angles θ of 0^o and 180^o produce zero for λ_S and λ_R. Hence, these two angles cannot be differentiated by λ_S and λ_R. However, λ_C has the smallest value for angle 0^o and increases gradually for the larger angles up to the largest value for angle 180^o, see Figure 2(b). Therefore, λ_C can effectively detect any different angle from 0^o and 180^o. In other words, the sharper the angle θ, the smaller the value of λ_C. Hence, λ_C is a reasonable measure to detect sharp corners.

Besides that, λ_C is also suitable for detecting dominant points in both regular and irregular spaced data since it is not affected much by the lengths of arms. For the cases $\left|\overline{p_{i-\ell}\,p_i}\right| = \left|\overline{p_i\,p_{i+r}}\right|$, the graphs of λ_C are identical. As shown in Figure 2(b) the values λ_C are very close to each other, even though they are calculated by using different lengths of arms of ROS. Hence, it is easier to adjust the threshold for λ_C compared to λ_{sum} which can exclude those false corner of large angle and lengths.

The algorithm may detect consecutive data points as dominant points. We believe that these dominant points carry the similar information about the shape of the boundary image since they are near to each other. Therefore, we replace them by selecting a point which has smallest value of λ_C. The algorithm for the proposed dominant points detection is as follow:

Step 1: Determine region of support.

Step 2: Form the covariance matrix C for every segment $S(p_i)$ and determine eigenvalues λ_L and λ_S as in (1).

Step 3: Find the value of λ_C as defined in (2).

Step 4: The point p_i is chosen as dominant point if its λ_C exceeds a predetermined threshold.

Step 5: The consecutive dominant points are replaced by a point among them which gives smallest value of λ_C.

TABLE 1. Data of six images.

Data	Number of Data Points	Number of Dominant Points
Hypocycloid	100	10
Epicycloid	100	10
Shape T	252	8
Maple	280	21
Shark	209	15
Leaf	246	12

THE RESULTS

The proposed method is applied on several images. Here, six selected images (Table 1) are displayed in order to show the effectiveness of the method (see Figure 3). The “o” in the figure indicates the selected dominant points. Table 1 shows the total number of data points and the number of dominant points detected for every data set used. The dominant points are joined with the line segments to show the preservation of the shape of the data. Figure 3(a) and Figure 3(b) show two examples of irregularly spaced data whereas the rest (Figure 3(c)-3(f)) show regularly spaced data.

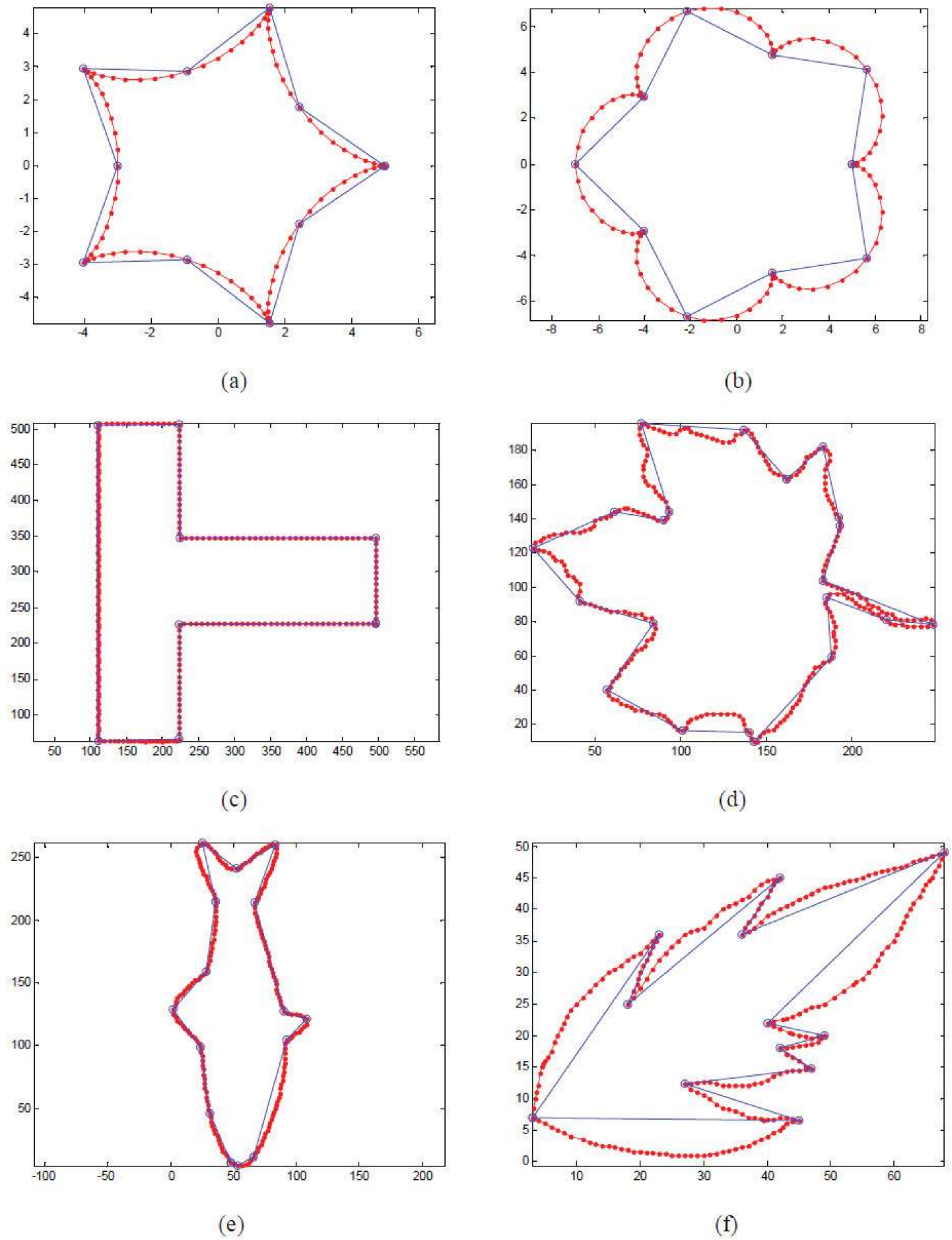


FIGURE 3. Examples of dominant points detection. (a) hypocycloid (b) epicycloid (c) shape T (d) maple (e) shark (f) leaf

CONCLUSION

An exclusive formula was introduced which involves the eigenvalues of covariance matrix to detect dominant points from the boundary data. The results show that the proposed method is not only able to detect the sharp corner but preserves the original shape of the boundary data at the same time. In future study, the method is hoped to be enhanced so that it will be more precise in application for the asymmetric ROS. Furthermore, the work can be used to establish dominant points detection for 3D data.

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