Doorway Passing of an Intelligent Wheelchair by Dynamically Generating Bézier Curve Trajectory

Sen Wang, Ling Chen, Huosheng Hu and and Klaus McDonald-Maier

***Abstract*— Door passing is the basic capability of an intelligent wheelchair. This paper presents a novel strategy to address the door passing issue by dynamically generating the Bézier curve based trajectory. It consists of door finding, optimization based trajectory generation and tracking control, which are executed repeatedly to increase the ability of passing the door and improve the performance. Whenever the door is detected, the optimization method produces a new smooth reference trajectory in real time for the wheelchair to follow. The proposed approach is tested in reality to verify its feasibility and efficiency, and the experimental results show its good performance in terms of the accuracy of finding the door and passing the doorway.**

I. INTRODUCTION

Door passing is considered as the fundamental capability of intelligent wheelchairs that are operated in an indoor environment. The wheelchair users are normally elderly and disabled, who may suffer some types of disabilities such as Parkinson's disease. It is difficult for them to operate wheelchair using a traditional joystick in an obstacle-free environment, not to mention to pass through the constrained doorway. Therefore, there is great demand that the intelligent wheelchair should autonomously travel through confined and narrow doorways without user intervention or carer supervision.

In general, this problem can be addressed by two typical methods - obstacle avoidance and trajectory planning. Among all the methodologies solving obstacle avoidance, potential field based method gains the most popularity [1], [2]. It has the disadvantage of falling into a local minimum as well as being undulating in some singularity point which is unacceptable for the wheelchair, as human being sitting on the wheelchair will feel uncomfortable in this circumstance. Meanwhile, some existing wheelchairs provide the door passage mode, such as the TAO one [3]. However, to the best of our knowledge, they all avoid the door frame at a low moving speed and the curvature of the traveling trajectory is not smooth or continuous [3].

Recently, a trajectory planning based method has been used to drive the wheelchair to follow a desired path. Due to the small ratio of the door width to the wheelchair width (only about 300 mm access space for the wheelchair [4]), it is difficult for the wheelchair to pass through the door at every heading direction. A frontier point method integrated with simultaneous localization and mapping was used for door passing of wheelchair in [5]. However, it needed to continuously generate the mean frontier points, which makes the curves non-smooth.

The authors are with School of Computer Science and Electronic Engineering, University of Essex, Colchester CO4 3SQ, United Kingdom swangi, lcheno, hhu, kdm at essex.ac.uk

In order to produce a smooth trajectory, trajectory planning based on circular arc [6], spline [7] and Bézier curves [8], [9] can be adopted. Jolly $et.\ al$ [8] proposed an efficient, Bézier curve based approach for the trajectory planning of a mobile robot, considering the boundary conditions, velocity limitation, etc. Choi $et.\ al$ in [9] presented two trajectory planning algorithms based on Bézier curve for autonomous vehicles with constraints from waypoints and corridor width. However, it regarded the vehicle as a particle, which was infeasible for the door passing of the wheelchair. In [10], we proposed a Bézier curve based method, where the trajectory was produced only once during the whole period of door passing, which suffered from the accumulated errors of odometry and further caused inaccuracy of door passing.

This paper intends to propose an efficient strategy for an intelligent wheelchair to pass through the narrow doorway in real time and with the greatest possible success. Compared to our previous work [10], this new strategy, which only uses the onboard sensors without the need of the global coordinates, enables the wheelchair to pass the doorway more accurately and efficiently by dynamically generating Bézier curve trajectory.

The rest of this paper is organized as follows. Section II gives a brief problem description. The door passing strategy is elaborated in Section III, where door finding strategy, trajectory planning based on Bézier curve as well as the wheelchair control are presented. Section IV presents the experimental results which verify the feasibility of the proposed door passing strategy in practice. Finally, a brief conclusion and future work are presented in Section V.

II. PROBLEM DESCRIPTION AND MAIN IDEAS

A. Problem Description

We consider the door passing problem of a wheelchair as a trajectory planning problem. As long as the wheelchair is able to follow the designed path which passes through the doorway without colliding with the door wall as well as other obstacles, then the door passing problem will be well solved. Fig.1 shows the schematic description of the door passing process of a wheelchair, where the red line represents the desired trajectory. The wheelchair in the corridor intends to traverse the two doors. There are two frames which are global frame$\{G\}$ and local frame $\{W\}$. P_s represents the wheelchair's current position which will be updated as (0, 0) in the local frame $\{W\}$ every time when the door is found, and P_d is the middle position of the door. H_s and H_d are the

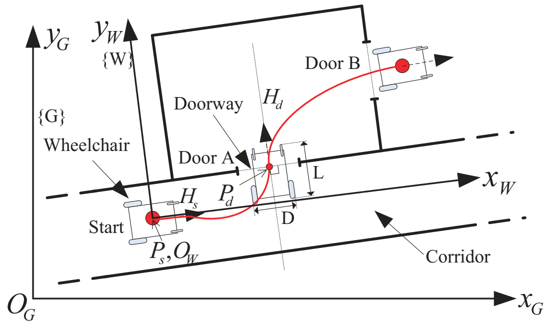

Fig. 1. The schematic description of door passing process of a wheelchair.

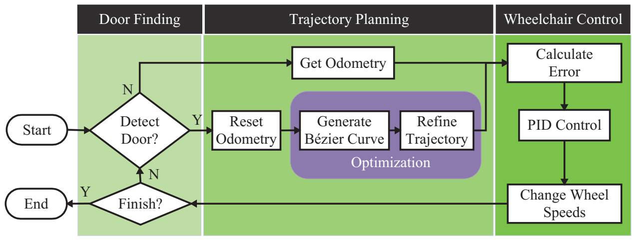

Fig. 2. Algorithm scheme.

headings corresponding to the aforementioned two positions respectively. Unless otherwise stated, positions and headings throughout the paper are all based on the local frame {W}.

In order to make the wheelchair pass the doorway successfully, the door passing strategy should comply with the following criteria:

- The actual trajectory of the wheelchair has to be graceful and smooth without oscillating or ambiguity.
- Due to the mechanical constraints of the wheelchair, the curvature of the trajectory needs to be limited to certain ranges so that the turning rate will not be too high.
- The heading of the wheelchair is perpendicular to the door plane when the wheelchair arrives at the P_d point.

Because this work aims to conquer the drawbacks of our previous work [10], we propose a new doorway passing strategy by dynamically generating Bézier curve trajectory, the detail of which are: first, finding the position of the door continuously; once the door is found, generating a new Bézier curve based trajectory by using the refined door position; then, controlling the wheelchair to follow this new trajectory.

B. *Bézier curve*

In this study, the cubic Bézier curve which is defined by four control points $P_0(A_0, B_0), P_1(A_1, B_1), P_2(A_2, B_2)$ and $P_3(A_3, B_3)$ is adopted for trajectory planning because of its efficient capacity of producing smoothing curves. For a cubic Bézier curve, the polygon formed by connecting the Bézier points with lines, starting with P_0 and finishing with P_3, is called the Bézier polygon (or control polygon). The convex hull of the Bézier polygon contains the Bézier curve. The reasons for why Bézier curves can be ideally used for trajectory planning in robotics rest with the following properties of Bézier curve:

- The curve begins at P_0 and ends at P_3. This is the so called *endpoint interpolation property*.
- The start (end) of the curve is tangent to the first (last) section of the Bézier polygon.

These two properties are exactly the requirements of trajectory planning given two points (start and goal points). The cubic Bézier curve can be expressed as $B(t) = (x(t), y(t)), t \in [0, 1]$ where

$$x(t) = A_0(1-t)^3+3A_1(1-t)^2t+3A_2(1-t)t^2+A_3t^3 \quad (1a)$$

$$y(t) = B_0(1-t)^3+3B_1(1-t)^2t+3B_2(1-t)t^2+B_3t^3 \quad (1b)$$

III. Door Passing Strategy

We propose a practically efficient algorithm to realize smooth door passing by dynamically generating Bézier curve trajectory. Fig.2 shows the flowchart of the proposed algorithm, in which *Door Finding*, *Trajectory Planning based on Bézier curve* and *Wheelchair Control* are the three major operations. The elaboration of each operation is presented in this section.

A. *Door Finding*

In this subsection, a strategy of door finding is presented to obtain the coordinates P_d of the middle position of the door as well as the slope K_w of the wall. P_d and K_w will then be used as the input parameters for generating Bézier curve in III-B. *Segmentation* and *Line Fitting* are the main procedures applied to achieve the goal of calculating P_d and K_w.

1) Segmentation: Laser Range Finder is used to detect the door. The purpose of segmentation is to divide one scan of laser data points into several different subsets so that points of each subset belong to a straight line. For example, in Fig.3, a full scan of laser data points are divided into 6 subsets.

Consider a full scan of 180° as an ordered sequence of N measurements points (P), where each scanned point can be defined in polar coordinates (r_n, α_n), that is:

$$P = \left\{ P_n | P_n = \begin{pmatrix} r_n \\ \alpha_n \end{pmatrix}, n \in [1, N] \right\} \quad (2)$$

Then the Cartesian (x_n, y_n) can be obtained as:

$$P = \{P_n | P_n = (x_n, y_n) = (r_n cos\alpha_n, r_n sin\alpha_n)\} \quad (3)$$

First, an Adaptive Breakpoint Detector (ABD) [11] is adopted to find continuous points groups by detecting breakpoints where the discontinuity has occurred. As can be seen in Fig.3, subset 1 and 2 are in the same continuous point group, subset 3 alone is in another group and subset 4, 5 and 6 belong to another group which is different from the other two groups.

The general principle of ABD is: if $D(r_i, r_{i+1}) > D_{thd}$, then segments are separated, otherwise segments are not

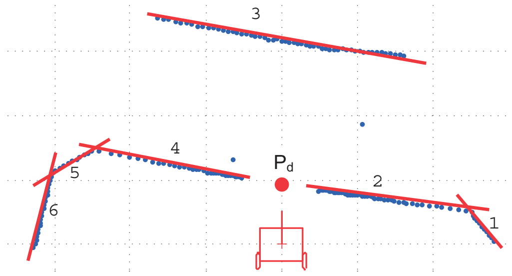

Fig. 3. A full scan of laser data points and its expected segmentations

segmented, where D_{thd} is the threshold condition and $D(r_i, r_{i+1})$ is the Euclidean distance between two consecutive scanned points

$$D(r_i, r_{i+1}) = \sqrt{r_i^2 + r_{i+1}^2 - 2r_i r_{i+1} cos\Delta\alpha} \tag{4}$$

and the threshold condition is expressed as:

$$D_{thd} = r_i \frac{sin\Delta\alpha}{sin(\lambda - \Delta\alpha)} + 3\delta_r \tag{5}$$

where λ is an auxiliary parameter and δ_r is a residual variance to encompass the stochastic behaviour of the sequence scanned points P_n and the related noise associated to r_n. For this project, we take λ as $10°$, δ_r as 10 mm provided by the laser, and $\Delta\alpha$ as $1°$.

Then, an algorithm named Iterative End-Point Fit (IEPF) [12] is applied to separate each continuous group into different subsets so that points of each subset belong to the same straight line. The process of IEPF starts by connecting the first and last data points of a cluster via a straight line. Then for all data points between them, a perpendicular distance $d_\perp$ to the line is calculated. If the maximum perpendicular distance $d_\perp$ to the line is greater than a threshold d_{th}, the corresponding point which has the maximum perpendicular distance is determined as the break point. The same process starts by connecting the first data point and the new generated break point and connecting the new generated break point and the last point. This is done recursively until the last point or the maximum perpendicular distance between extreme points is less than the threshold d_{th}. All the points determined by this method are defined as the breaking points of the group, including two end points of the group. Thus, all the points between two neighboring breaking points are considered to be on the same line and in the same subset.

For every two subsets, we calculate the distance D_{rf} between the rear end point (P_r) of the first subset and the front end point (P_f)of the second subset. If D_{rf} is equal to the width of the door, then the door is found and $P_d = (P_r + P_f)/2$. As can be seen in Fig.3, the door is found between subset 2 and 4.

2) Line Fitting: Now that the door position is found, next step is to calculate the slope K_w of the wall which the found door rests on. The calculation of K_w can be realized by fitting the straight line composed by the laser data points of the wall. The purpose of line fitting is to determine the line parameters. One form of parameters representing a straight line is

$$y = kx + q \tag{6}$$

where q and k are the y-intercept and the slope of a line respectively. Suppose there are a set of points $R = \{(x_i, y_i), i = 1 : k\}$ that are needed to be fitted to a specific line, then parameters k and q of this line can be calculated by the Least Square based line fitting algorithm which is represented as:

$$S_x = \sum_{i=1}^{k} x_i; \quad S_y = \sum_{i=1}^{k} y_i; \tag{7a}$$

$$S_{xx} = \sum_{i=1}^{k} x_i^2; \quad S_{yy} = \sum_{i=1}^{k} y_i^2; \quad S_{xy} = \sum_{i=1}^{k} x_i y_i; \tag{7b}$$

$$k = \frac{-S_x S_{yy} + S_y S_{xy}}{S_y S_{xx} - S_x S_{xy}}; \tag{7c}$$

$$q = \frac{-S_{xy}^2 - S_{xx} S_{yy}}{S_y S_{xx} - S_x S_{xy}} \tag{7d}$$

where (7c) provides the right solution of the slope of the fitted straight line.

For the scenario shown in Fig.3, laser points in subset 2 and 4 are found as points of the wall. Substituting Cartesian coordinates of all the laser points in subset 2 and 4 to (7) yields the slope K_w of the wall.

B. Trajectory Planning based on Bézier Curve

Because the odometry suffers from the accumulated error and the ratio of the door width to the wheelchair width is very small, it is unsuitable to perform the door passing only by using the odometry to estimate the current position. Therefore, the laser range finder is introduced to detect the door and then refine the reference trajectory continuously.

Since the computational complexity of the trajectory planning method proposed in [10] is high, which means it cannot be used in this real-time updating method, a new method which can produce the Bézier curve based trajectory in real time is presented. Its key idea is that the control points which totally define the shape of the Bézier curve are determined using an optimization method. The merit of this method lies in generating the smoothest Bézier curve in real time, satisfying the various constraints produced by the wheelchair, the environment and the real-time requirement.

The trajectory planning strategy is continuously performed with the relative positions between the wheelchair and the door. In Fig.1, the control points of $\overset{\frown}{P_s P_d}$ are denoted as P_i, $i \in 0, 1, 2, 3$. Then, the trajectory planning is required to generate the cubic Bézier curves which connect the points P_s and P_d with the orientations H_s and H_d. In other words, when the door is detected, the wheelchair uses the position P_s, the calculated middle position of the door P_d and the slope of the wall K_w to perform the optimization based control point estimation. Since P_s and P_d correspond to the known control points P_0 and P_3 respectively, the parameter estimation only needs to calculate P_1 and P_2, which completely determine the shape of the curve once given P_0 and P_3. Note that $H_d = -1/K_w$ as H_d is desired to be perpendicular to the wall.

In order to meet the aforementioned requirements, two types of constraints are devised for the optimization according to the environment and the reality:

1) Orientations: Since the trajectory planning begins with the heading H_s, the slope of tangent line of the designed Bézier curve at position P_0 should equal H_s. Similarly, the tangent line at P_3 should be the normal of the door at the middle in order to overcome the issue caused by the small ratio of door width to the wheelchair's. Therefore, the possible ranges of P_1 and P_2 can be further reduced to one dimension. In other words, the P_1 and P_2 must respectively locate on the tangent lines at positions P_0 and P_3 due to the second property of Bézier curve in Section II-B. Moreover, P_1 has to be searched along the direction of H_s, while P_2 only can lie on the opposite direction of H_d. Hence, the constraints are

$$\|P_0 - P_c\| = \|P_1 - P_0\| + \|P_1 - P_c\| \tag{8a}$$

$$\|P_3 - P_c\| = \|P_2 - P_3\| + \|P_2 - P_c\| \tag{8b}$$

where P_c is the intersection point of lines P_0P_1 and P_2P_3, and $\|\cdot\|$ denotes the 2-norm. These two restrictions ensure that the Bézier curve determined by P_1 and P_2 can join the start and end points P_0 and P_3 with the desired orientations H_s and H_d.

2) Complete convexity: The curvature limitation described in II-A requires that there is no sharp bend or sudden change of curvature in the curve. Thus, the Bézier curve is better to be completely convex. It has been known that a Bézier curve is completely convex if its control polygon is convex [13]. Therefore, P_1 and P_2 have to be constrained to guarantee the complete convexity of the curve. The cross product method is introduced in this proposed strategy to fulfill this convex requirement. The cross products ω_1 and ω_2 of vectors $\overrightarrow{P_0P_1}$, $\overrightarrow{P_1P_2}$ and $\overrightarrow{P_2P_3}$ are

$$\omega_1 = \overrightarrow{P_0P_1} \times \overrightarrow{P_1P_2} \quad \omega_2 = \overrightarrow{P_1P_2} \times \overrightarrow{P_2P_3} \tag{9}$$

Therefore, the constraint of the complete convexity is

$$sign(\omega_1) \cdot sign(\omega_2) > 0 \tag{10}$$

where $sign(\omega_i)$, $i = 1, 2$, is the rotation direction of the ω_i.

The constrained optimization problem is to find the control points P_1 and P_2 which make the curve smooth. Then, the curvature and the rate of its change should be as small as possible. According to (1), the curvature of a Bézier curve with respect to t is

$$\kappa(t) = \frac{1}{\rho(t)} = \frac{\dot{x}(t)\ddot{y}(t) - \dot{y}(t)\ddot{x}(t)}{(\dot{x}^2(t) + \dot{y}^2(t))^{3/2}} \tag{11}$$

where $\rho(t)$ is the radius of curve. Therefore, P_1 and P_2 can be computed by the following constrained optimization problem, which is subjected to (8) and (10):

$$\min_{P_1,P_2} \quad \int_0^1 [(\kappa(t))^2 + (\dot{\kappa}(t))^2]dt \tag{12a}$$

$$s.t. \qquad (8) \quad (10) \tag{12b}$$

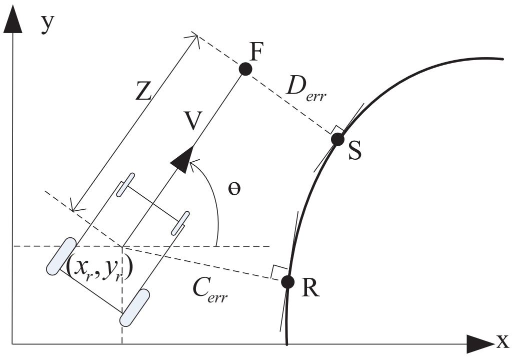

Fig. 4. The position error of the wheelchair [9].

where $\dot{\kappa}(t)$ is the first derivative of curvature. Thus, the calculated P_1 and P_2 can meet the various requirements and provide an optimal trajectory planning for the door passing.

When the wheelchair reaches the doorway, it is expected to be at the center of the door with heading being perpendicular to the door. However, because of the disturbances and the control errors, the final position and orientation of last step cannot coincide with the desired ones even though the errors may be small under an efficient control strategy. In order to satisfy the third requirement in II-A, the true heading H_d^a needs to be adjusted to the perpendicular direction of the door before crossing the doorway.

C. Wheelchair Control

In order to drive the wheelchair to track the desired trajectory, a feedback control principle using PID controller is adopted. The position error between the actual position of the wheelchair and the reference trajectory is used as the input of the PID controller. As can be seen in Fig.4, a point F ahead along the heading of the wheelchair with a distance of Z is used to define the position error, and F is projected onto the reference trajectory at point S such that $\overline{FS}$ is perpendicular to the tangent at S. The position error is then represented by the distance D_{err} between point F and S. The cross track error C_{err} is defined by the shortest distance between the desired trajectory and the position of the center of the gravity of the wheelchair (x_r, y_r). Note that when door is found, (x_r, y_r) is updated as $(0, 0)$ in the frame {W}; if the door is not found, (x_r, y_r) can be obtained from the odometry information which has been accumulated since the last time when door is found.

In this paper, because the longitudinal velocity V cannot decrease the errors between the wheelchair and the reference trajectory, the angular rate ω is selected as the control output of the PID controller, and V is considered as constant. The discretized PID controller can be expressed as

$$\omega = k_p D_{err}^k + k_i T_s \sum_{i=1}^{k} D_{err}^i + \frac{k_d}{T_s}(D_{err}^k - D_{err}^{k-1}) \tag{13}$$

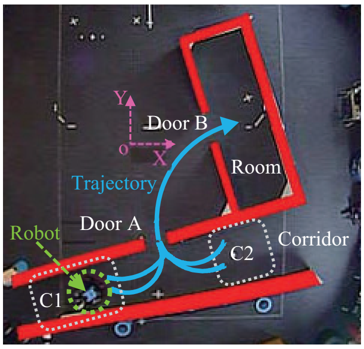

Fig. 5. Experiment scenario.

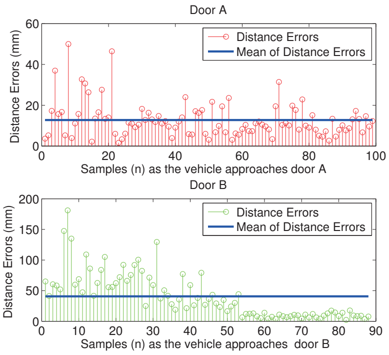

Fig. 6. Distance errors between detected door position and the true door position.

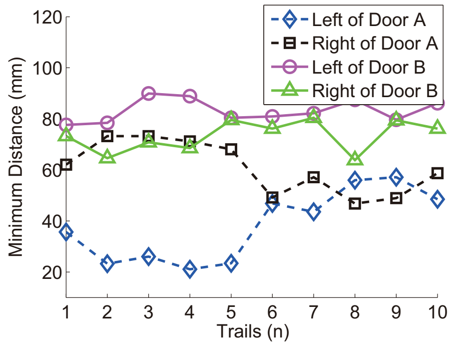

Fig. 7. Minimum distances of the robot to the edges of Door A and B.

where k_p, k_i and k_d are proportional, integral and derivative gains respectively, D_{err}^k is the current position error, and D_{err}^{k-1} is the position error of last time instance. The reason for why D_{err} rather than C_{err} is used as the input of the PID controller lies in the fact that the required k_p for compensating the same quantity of C_{err} is much larger than that of D_{err}, and the larger k_p is more likely to cause system oscillation.

IV. Experimental Results

In order to verify our door passing method, the proposed algorithm was tested in reality. For simplicity, the Pioneer robot which is also controlled by changing the speeds of the two wheels is used instead of the wheelchair.

The experiment scenario is illustrated in Fig.5, where the (red) bold solid lines representing the walls enclose the corridor and the room. There are two doors Door *A* and Door *B*, and two starting regions *C1* and *C2* which are enclosed by the (white) dotted rectangles. With the different start poses, the robot respectively departs from *C1* and *C2* for 5 times to pass the Door *A* and Door *B*, and let $T_i, i = 1, \ldots, 10$ denote these 10 trials ($T_i, i = 1, \ldots, 5$ are from the $C1$ and $T_i, i = 6, \ldots, 10$ are from the $C2$). The (blue) solid curves are the trajectories of the robot when passing the doors. The robot, Door *A* and Door *B* are $420mm$, $500mm$ and $520mm$ in width respectively. The ground truth information including global coordinates (x_g, y_g) and heading of the robot is obtained by the VICON system to evaluate the performance of the proposed strategy. In this experiment, the k_p, k_d and k_i of the PID controller are selected to be 0.04, 0.001 and 0.001 respectively.

In order to verify the accuracy of door finding strategy, the vehicle is driven from start point to door *A* and door *B*, during which the proposed door finding algorithm is executed to detect the door position, meanwhile the detected door position is recorded. Then the distance errors between the detected door position and the true door position are calculated as shown in Fig.6. The mean values of distance errors of door *A* and *B* are $12.8mm$ and $40.7mm$ respectively, which are accurate enough to be acceptable when compared to the door width ($500mm$ for door *A* and $520mm$ for door *B*). It can be seen that although the distance errors are relatively high at the beginning of the trajectory, they decrease to low as the vehicle approach the doors. Therefore, the robot can track the updated trajectories, reaching the middle of the door at last.

Fig.7 shows the minimum distances of the robot to each side of the edges of the two doors when passing the doorway. The smaller the minimum distances are, the more likely for the robot to bump into the door. However, as we can see in Fig.7, for all the ten trials of the door passing scenarios, the minimum distances are large enough for the robot passing through the doors safely without bumping into them.

When the robot is passing the doorway, the orientation and the offset between the robot and the middle of the door are the two key parameters which determine whether the door passing can be successfully conducted. The desired offset is zero and the orientation should be perpendicular to the door plane. However, due to the noises in reality, there exist errors. Fig.8 presents the offset of the 10 trails, where the figures in the upper row and the lower row are about the Door *A* and Door *B* respectively, and the panels on the

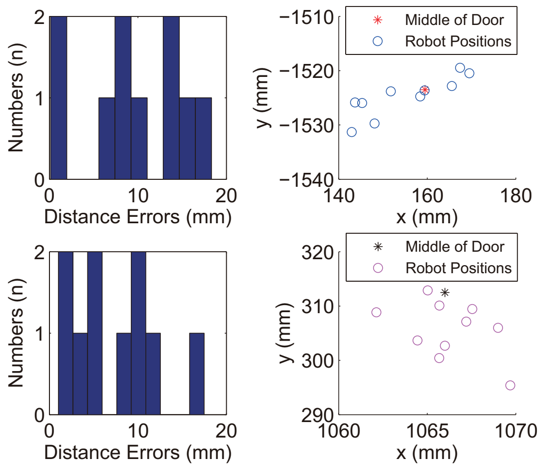

Fig. 8. Offset between the robot and the middle of the door when it is passing the doors.

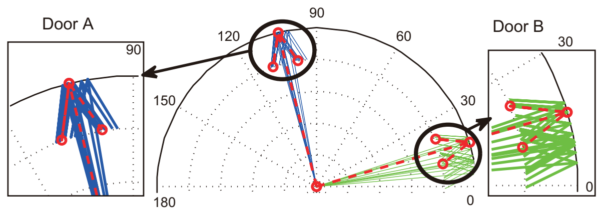

Fig. 9. Headings of Robot When Passing the Doorway

left are the histograms of the offsets and the panels on the right are the distribution of the desired and true positions. It can be seen that most of the offsets are under $10mm$, which satisfies the requirements. Even the $20mm$ offset is acceptable because the robot still has enough free space to pass the door successfully. Actually, the real results must be better since the VICON system may not sample the exact position where the center of the robot matches with the middle position of the door when the robot is passing the doorway. Therefore, the accuracy of the position control of the proposed door passing strategy is high, although suffering from the various noises in practice. In Fig.9, the real orientations are compared with the two expected ones, which are depicted as the (red) dashed arrows. As can be seen, the real headings are close to the desired ones although the Door B's are not very accurate. However, they can also meet the requirements since the orientation has less influences than the offset.

Fig.10 presents the robot's global ground truth trajectories of 10 trials of door passing experiment. It is obvious that no matter which start position and heading the robot is at, the robot is able to find and pass the two doors both smoothly and accurately without bumping into the door edges, which verifies the high accuracy and effectiveness of our proposed door passing strategy.

V. Conclusions

In this paper, a practically effective algorithm is proposed to solve the door passing problem for an intelligent wheelchair by dynamically generating Bézier curve trajectory. It aims to conquer the drawbacks of our previous work [10] and improve both accuracy and computational efficiency of the door passing strategy through continuously operating the process of *Finding door→ Generating Bézier curve trajectory→ Following the generated trajectory*. The experiments including 10 trials of door passing are conducted to verify the high accuracy and effectiveness of our proposed door passing strategy. Our future work will focus on implementing the proposed approach on a real wheelchair.

Acknowledgments

The authors gratefully acknowledge the support of the UK EPSRC Global Engagements grant EP/K004638/1 and the EU Interreg IV A 2 Mers Seas Zeen Cross-border Cooperation Programme SYSIASS project: Autonomous and Intelligent Healthcare System (http://www.sysiass.eu/). Sen Wang and Ling Chen have been financially supported by scholarships from China Scholarship Council and University of Essex. Our thanks also go to Robin Dowling for his technical support during the research.

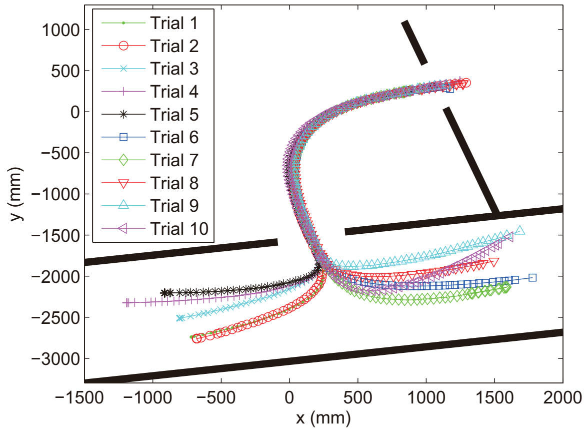

Fig. 10. Trajectories of 10 trials of door passing experiment

References

[1] J. Barraquand, B. Langlois, and J. Latombe, "Numerical potential field techniques for robot path planning," *IEEE Trans Syst Man Cybern*, vol. 22, no. 2, pp. 224–241, 1992.

[2] F. Arambula Cosío and P. Castañeda, "Autonomous robot navigation using adaptive potential fields," *Math Comput Model*, vol. 40, no. 9-10, pp. 1141–1156, 2004.

[3] R. Simpson, "Smart wheelchairs: A literature review." *J Rehabil Res Dev*, vol. 42, no. 4, p. 423, 2005.

[4] BSI, *Design of buildings and their approaches to meet the needs of disabled people-Code of practice*, British Standards Institution. Std. BS 3800, 2009.

[5] F. Cheein, C. De La Cruz, T. Bastos, and R. Carelli, "Slam-based cross-a-door solution approach for a robotic wheelchair," *Int J Adv Robot Syst*, vol. 7, no. 2, pp. 155–164, 2010.

[6] A. Scheuer and T. Fraichard, "Continuous-curvature path planning for car-like vehicles," in *Proceedings of the 1997 IEEE/RSJ International Conference on Intelligent Robots and Systems*, vol. 2. IEEE, 1997, pp. 997–1003.

[7] J. Connors and G. Elkaim, "Analysis of a spline based, obstacle avoiding path planning algorithm," in *IEEE 65th Vehicular Technology Conference*. IEEE, 2007, pp. 2565–2569.

[8] K. Jolly, R. Sreerama Kumar, and R. Vijayakumar, "A bezier curve based path planning in a multi-agent robot soccer system without violating the acceleration limits," *Robot Auton Syst*, vol. 57, no. 1, pp. 23–33, 2009.

[9] J. Choi, R. Curry, and G. Elkaim, "Path planning based on bézier curve for autonomous ground vehicles," in *Advances in Electrical and Electronics Engineering-IAENG Special Edition of the World Congress on Engineering and Computer Science*. IEEE, 2008, pp. 158–166.

[10] L. Chen, S. Wang, H. Hu, and K. McDonald-Maier, "Bézier curve based trajectory planning for an intelligent wheelchair to pass a doorway," in *9th United Kingdom Automatic Control Council International Conference on Control*, 2012, pp. 339–344.

[11] G. Borges and M. Aldon, "Line extraction in 2d range images for mobile robotics," *Journal of Intelligent and Robotic Systems*, vol. 40, no. 3, pp. 267–297, 2004.

[12] R. Duda and P. Hart, "Pattern classification and scene analysis," *A Wiley-Interscience Publication, New York: Wiley, 1973*, vol. 1, 1973.

[13] P. Laurent, P. Sablonnière, and L. Schumaker, *Curve and surface design: Saint-Malo 99*, ser. Innovations in applied mathematics. Vanderbilt University Press, 2000.