DOPING OF REPEAT–ACCUMULATE CODES FOR DIRTY PAPER CODING

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ABSTRACT

Costa's "writing on dirty paper"-channel model poses one of the remaining challenges to the coding community. Theory suggests that arbitrary interference on the channel (known at the transmitter) can be cancelled without a power penalty by appropriate precoding and shaping. We employ iterative quantization and decoding using systematically doped repeat-accumulate codes to materialize a significant portion of the promised "dirty paper"-capacity. Code design is done using the EXIT chart technique.

1. INTRODUCTION

It has recently been shown [1] that an information theoretic framework for the study of efficient known interference cancellation (precoding) techniques may be found in Costa's "Writing on dirty paper" [2]. The (generalized) dirty paper channel model is given by

$$Y = X + S + N \tag{1}$$

where S is arbitrary interference known at the transmitter (noncausally), N is a statistically independent Gaussian random variable with variance P_N, and P_X is the power of the transmitted encoder output.

If the interference S was known at the receiver, one could subtract it off the received signal leading back to an interference-free AWGN channel, and thus the interference would not pose a problem. One could similarly attempt to pre-subtract the interference at the transmitter, i.e., transmit $X' = X - S$. The received signal would then be $Y' = X' + S + N = X - S + S + N = X + N$, thus eliminating the interference. However, the problem with this "naive" approach stems from the power constraint: The average transmit power would be $E[X'^2] = E[X^2] + E[S^2]$. As the interference may be arbitrarily strong, this would entail a severe power penalty and hence a reduced transmission rate. Nonetheless, in [2] Costa showed that for Gaussian S the capacity is equal to $0.5\log(1 + P_X/P_N)$ and hence *the interference S does not incur any loss in capacity*. We treat the generalized model where S can be an arbitrary signal, deterministic or random, for which this result holds as well [1].

Willems suggested schemes for coding for the dirty paper channel (for causally known interference) in [3]. In [1] it was shown that the full capacity may be achieved using a scheme based on lattices and MMSE scaling. Related schemes were developed in the context of information embedding, e.g. [4]. In [5] a realization of the necessary lattice transmission scheme based on trellis shaping [6] and "syndrome dilution" was proposed. In [7] this approach was extended into a fully–fledged coding system by employing nonsystematic repeat–accumulate (RA) codes [8, 9] concatenated with a trellis shaping code.

In this paper we show how to improve the result of [7] by 0.6dB, by applying *systematic doping* [10] of the accumulator, and thus allowing to incorporate higher memory vector quantizer (VQ) shaping. The system presented offers a gain of around 2dB over an *optimal* one-dimensional interference canceling system.

2. LATTICE PRECODING

We review the lattice precoding approach proposed in [1]. Let Λ denote an n-dimensional lattice and let $\mathcal{V}$ denote its fundamental Voronoi region. Also let $\mathbf{U} \sim \text{Unif}(\mathcal{V})$, that is $\mathbf{U}$ is a random variable (dither) uniformly distributed over $\mathcal{V}$. The scheme is given by,

- *Transmitter:* The input alphabet is restricted to $\mathcal{V}$. For any $\mathbf{v} \in \mathcal{V}$, the encoder sends:

$$\mathbf{X} = [\mathbf{v} - \alpha\mathbf{S} - \mathbf{U}] \bmod \Lambda. \tag{2}$$

- *Receiver:* The receiver computes

$$\mathbf{Y}' = [\alpha\mathbf{Y} + \mathbf{U}] \mod \Lambda. \tag{3}$$

The resulting channel is a mod-Λ additive noise channel described by the following lemma:

Lemma 1 ([1]) *The channel from* $\mathbf{v}$ *to* $\mathbf{Y}'$ *defined by (1),(2) and (3) is equivalent in distribution to the mod-*Λ *channel*

$$\mathbf{Y}' = \mathbf{v} + \mathbf{N}' \mod \Lambda \tag{4}$$

with

$$\mathbf{N}' = (1-\alpha)\mathbf{U} + \alpha\mathbf{N} \mod \Lambda. \tag{5}$$

Taking a uniform input distribution and setting $\alpha = \frac{P_X}{P_X + P_N}$ yields an achievable rate of

$$\frac{1}{n} I(\mathbf{V};\mathbf{Y}') \geq \frac{1}{2}\log(1 + SNR) - \frac{1}{2}\log 2\pi e G(\Lambda) \tag{6}$$

per dimension, where $G(\Lambda) = \frac{1}{n}\frac{\int_{\mathcal{V}} \|\mathbf{x}\|^2 d\mathbf{x}}{|V|^{1+2/n}}$ is the normalized second moment of Λ. Thus, in principle, for a given lattice Λ, the gap to capacity of a precoding system may be made smaller than $0.5\log 2\pi e G(\Lambda)$. For optimal lattices for quantization we have $G(\Lambda) \rightarrow \frac{1}{2\pi e}$, and the gap goes to zero. Note that when Λ is one-dimensional, the lattice precoding scheme is based simply on scalar quantization (SQ) and is an extension of Tomlinson-Harashima precoding. Note also that while the gap to capacity of a scalar system is 1.53dB at high SNR, the lowest possible E_b/N_0–operating point is at 2.4dB. This means that the gap to capacity approaches 4dB at zero spectral efficiency (see SQ, Fig. 3).

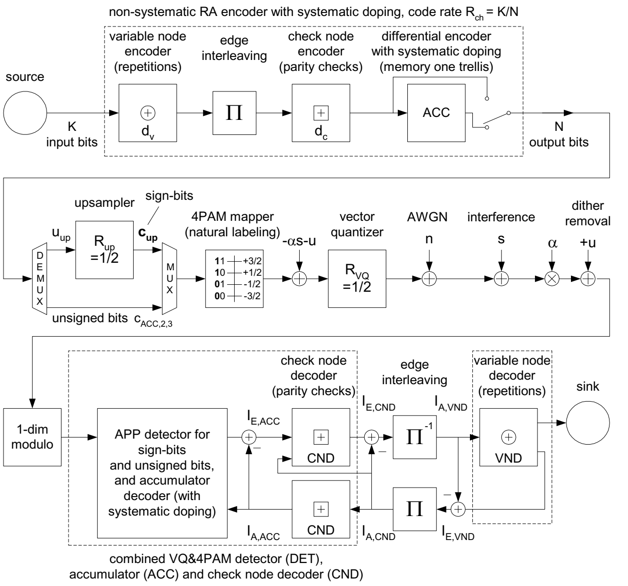

Figure 1: Dirty paper coding with nonsystematic repeat-accumulate codes (using inner systematic doping) and a vector quantizer (VQ); iterative quantization and decoding.

3. DESIGNED SYSTEM

3.1 System Overview

The transmitter is a concatenation of a nonsystematic repeat–accumulate (RA) code [9], performing the "coset dilution", and a trellis shaping code (i.e. the vector quantizer), as depicted in Fig. 1. The RA encoder is composed of an outer mixture of repetition codes of different rates (variable nodes), an edge interleaver, and an inner mixture of single parity check codes of different rates (check nodes), followed by a memory one differential encoder (accumulator, ACC). *Inner systematic doping* can be applied, that is, some of the coded bits of the accumulator output can be *substituted* by the corresponding systematic bits at the accumulator input. Code design is performed by appropriately choosing repetition and check node degree distributions. The encoded bits are grouped into triplets $(c_1, c_2, c_3)_{ACC}$ and demultiplexed into "upsampler" bits $u_{up} = c_{ACC,1}$ and unsigned bits $c_{ACC,2}, c_{ACC,3}$. The upsampler (replacing the inverse syndrome former in trellis shaping) has rate $R_{up} = 1/2$. The sign–bits $c_{up,1}, c_{up,2}$ generated by the upsampler, and the unsigned bits are mapped onto 4-PAM symbols using natural labeling. After adding the scaled interference and a uniformly distributed dither signal, the vector quantizer determines the minimum energy sequence by appropriately flipping the sign–bits (Viterbi decoding of a convolutional code of rate $1/2$ using a modulo metric). The quantization error vector is transmitted over the communication channel.

On the channel, white Gaussian noise is added, with double–sided noise power spectral density $P_N = N_0/2$ and mean zero. Interference is added. For 16-QAM (4-PAM per dim.) and $R_{VQ} = 1/2$, we have $E_s/N_0 = 2(1+0.5)R_{ch}E_b/N_0$. Thus, for simulation we set $P_N = E_s/(3R_{ch}2E_b/N_0)$, whereby E_s is the average energy per *complex* output symbol measured *after* the VQ at the transmitter, and R_{ch} is the rate of the RA code.

At the receiver, MMSE α–scaling is applied, and the dither signal u is removed; a one-dimensional modulo is performed prior to putting the signal into a soft in/soft out vector quantizer which performs an *a posteriori* probability (APP) detection of the sign–bits and the unsigned bits, respectively, using the BCJR algorithm on an appropriately defined trellis structure. The vector quantizer at the receiver, thus, can be viewed as an APP detector, computing *extrinsic* information which is forwarded to the RA decoder. The RA decoder is composed of an inner accumulator decoder (ACC), check node decoder (CND), and an outer variable node decoder (VND). Iterative quantization and decoding is performed by exchanging log–likelihood ratio values (L–values [11]) between inner VQ&ACC&CND– and outer VND–decoder.

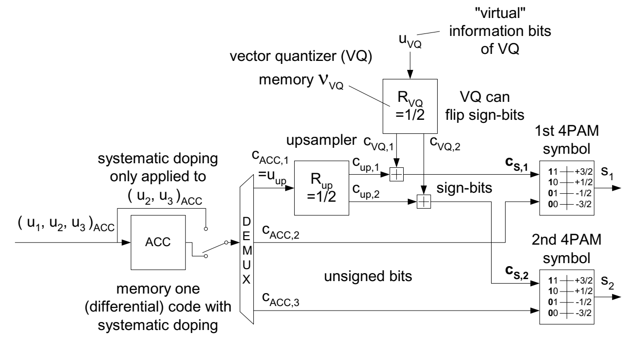

Figure 2: Joint accumulator (systematic doping), upsampler, vector quantizer and 4-PAM trellis processing.

Fig. 2 aids in understanding the structure of the joint trellis processing over accumulator trellis (memory $\nu_{ACC} = 1$), vector quantizer trellis (memory ν_{VQ}), upsampler, and modulo symbol metric based on two 4-PAM symbols per three hypothesized accumulator bits $(u_1, u_2, u_2)_{ACC}$. Note that *systematic doping* can be applied to the accumulator, i.e., some of the coded bits c_{ACC} are *substituted* by the corresponding systematic bits u_{ACC}. In this particular case, we only allow systematic doping of coded bits $c_{ACC,2}, c_{ACC,3}$.

By computing extrinsic information transfer curves [10] of the VQ detector (Monte–Carlo simulation using BEC *a priori* knowledge) for different α– and E_b/N_0–values, and numerically evaluating the corresponding area [12], we obtain the mutual information limits given in Fig. 3. To materialize these gains, we need to design an appropriate *iterative* quantization and decoding scheme

3.2 Triggering Convergence by Systematic Doping

Observe that the inner transfer curve of the joint VQ&ACC-processing block starts virtually at the origin for VQ memories greater 2 (see Fig. 4, left), thus preventing the iterative quantization and decoding scheme from starting to converge.

A simple yet effective means of solving this problem is to apply *systematic doping*. Feeding through some uncoded systematic (information) bits, i.e. bypassing the accumulator of the RA code, shifts up the inner transfer curve at the beginning, at the cost of losing some extrinsic output for higher *a priori* input (Fig. 4, right). Note that we only dope those bits that are mapped onto unsigned bits of the 4-PAM constellations. A doping ratio of systematic bits to coded bits of $i : c = 1 : 1$ (i.e. every other unsigned bit is a systematic bit

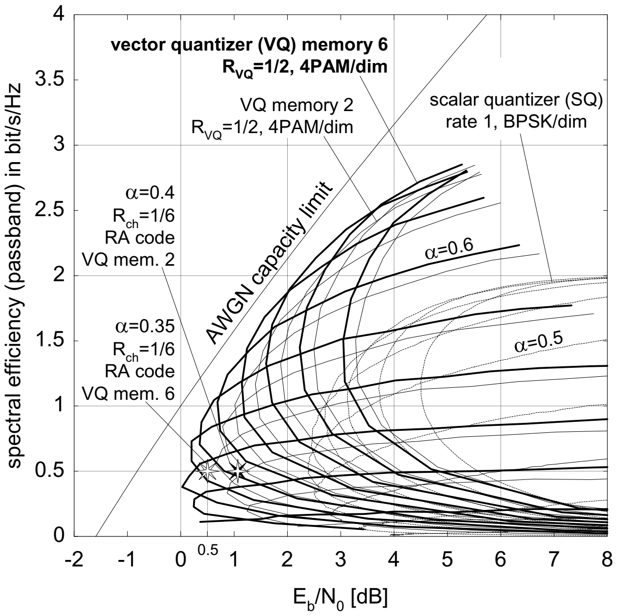

Figure 3: Mutual information limits of VQ and 4-PAM/dim. for different α–values (in steps of 0.1); $R_{VQ} = 1/2$, VQ of memory 2 and memory 6; RA code turbo cliff position at 1.1dB (memory 2 VQ) and 0.5dB (memory 6 VQ) at 0.5bit/s/Hz; scalar quantizer given as reference.

with respect to the accumulator) turned out to be sufficient to trigger convergence in the case of a memory 6 VQ.

It is interesting to note that we now use *two* forms of doping: 1.) A *biregular* CND (i.e. a fraction of the check nodes has degree $d_c = 1$) ensures that the inner ACC&CND-curve starts at a value $I_{E,ACC\&CND} > 0$, and thus allows to use a nonsystematic RA code. However, when combining the *ACC&CND* with a VQ of memory 6, the biregularity is not sufficient to enable convergence. 2.) In addition to that, we need to apply *systematic doping* to the ACC, and by this, in fact, making the RA code partially systematic again.

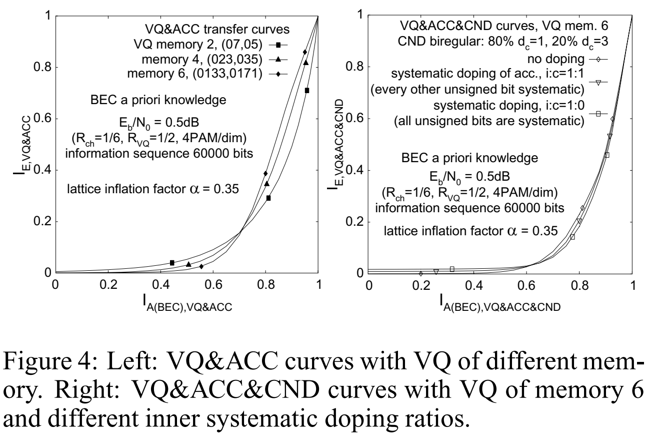

Figure 4: Left: VQ&ACC curves with VQ of different memory. Right: VQ&ACC&CND curves with VQ of memory 6 and different inner systematic doping ratios.

4. CODE DESIGN EXAMPLES

4.1 EXIT Curve of Outer VND Code Mixtures

The decoder output for a variable node of degree d_v is $L_{i,out} = \sum_{j\neq i} L_{j,in}$, where $L_{j,in}$ is the jth *a priori* L–value going into the variable node, and $L_{i,out}$ is the ith extrinsic L–value coming out of the variable node. The $L_{j,in}$ are modeled as the output L–value of an AWGN channel whose input was the jth interleaver bit transmitted using BPSK. The EXIT function of a degree–d_v variable node is then [9]

$$I_{E,VND}(I_{A,VND}, d_v) = J\left(\sqrt{(d_v - 1)} \cdot J^{-1}(I_{A,VND})\right) \quad (7)$$

with

$$J(\sigma) = 1 - \int_{-\infty}^{\infty} \frac{e^{-\left(\xi - \sigma^2/2\right)^2/2\sigma^2}}{\sqrt{2\pi}\sigma} \cdot \log_2\left[1 + e^{-\xi}\right] d\xi. \quad (8)$$

Some of these curves are plotted in Fig. 5 for different variable node degrees.

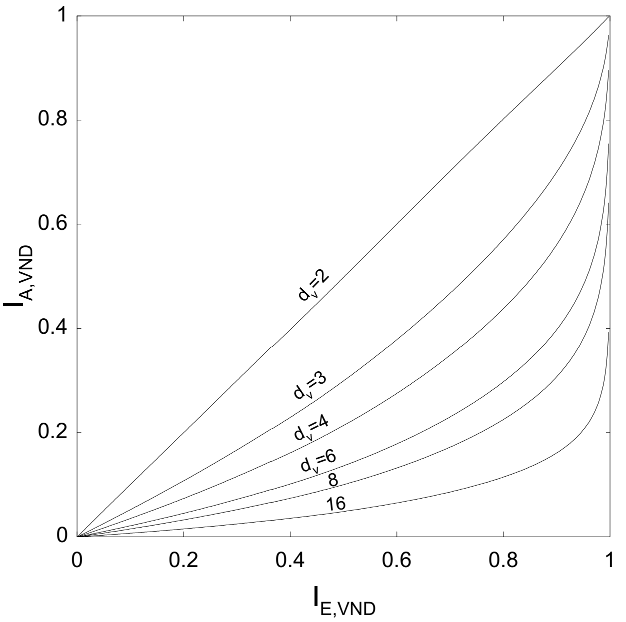

Figure 5: VND EXIT curves for nonsystematic RA codes.

Let D_v be the number of different variable node degrees, and denote these by $\widetilde{d}_{v,i}$, $i = 1, \ldots, D_v$. The average variable node degree is $\overline{d}_v = \sum_{i=1}^{D_v} a_{v,i} \cdot \widetilde{d}_{v,i}$, where $a_{v,i}$ is the fraction of variable nodes having degree $\widetilde{d}_{v,i}$. Let $b_{v,i}$ be the fraction of *edges* incident to variable nodes having degree $\widetilde{d}_{v,i}$. The EXIT curve of a mixture of codes is an average of the component EXIT curves [9, 13], and thus the VND curve writes as

$$I_{E,VND}\left(I_{A,VND}\right) = \sum_{i=1}^{D_v} b_{v,i} \cdot I_{E,VND}\left(I_{A,VND}, \widetilde{d}_{v,i}\right). \quad (9)$$

Only $D_v - 2$ of the $\widetilde{d}_{v,i}$ can be adjusted freely because we must enforce $\sum_i b_{v,i} = 1$ and $R_{ch} = \overline{d}_c / \overline{d}_v$, with $\overline{d}_c$ being the average check node degree.

4.2 Simulation Results

For a spectral efficiency of 0.5bit/s/Hz, we designed RA codes of rate $R_{ch} = 1/6$. To find appropriate VND degree distributions, the outer VND transfer curve is matched to the inner VQ&ACC&CND curve by means of curve fitting. We chose vector quantizers of rate $R_{VQ} = 1/2$ with memory 2 (see [7], baseline for comparison) and memory 6, with polynomials $(07_8, 05_8)$ and $(0133_8, 0171_8)$, respectively. A 4-PAM constellation was applied per dimension (natural labeling). The inner VQ&ACC&CND–curve was computed by Monte–Carlo simulation, assuming Gaussian *a priori* knowledge. The check node layer is biregular, with 80% of the check nodes being degree 1, and 20% being degree 3.

For the baseline system with VQ of memory 2, curve fitting at $E_b/N_0 = 1$dB yields a VND degree distribution of 64.36% variable nodes being degree 3, 31.24% degree 10, and 4.402% degree 76. We achieve convergence at 1.1dB, with $\alpha = 0.4$. No error floor was observed for 40 blocks simulated, which can be attributed to the fact that there are no degree 2 variable nodes, and the lowest variable node degree is 3. The iterations required varied from 60 to 90 iterations.

For the VQ of memory 6, curve fitting at $E_b/N_0 = 0.5$dB yields a VND degree distribution of 33.82% variable nodes being degree 2, 50% degree 3, 11.99% degree 10, and 4.187% degree 120. We achieve convergence at 0.5dB, with $\alpha = 0.35$ (only 1.3dB away from the AWGN capacity limit, see Fig. 3). After 10 blocks simulated, the error floor was $3 \cdot 10^{-6}$. The iterations required varied from 75 to 115 iterations. Note that the accumulator was doped, with $i : c = 1 : 1$, i.e., every other unsigned bit was systematic with respect to the accumulator.

Fig. 6 shows simulated decoding trajectories at 1.2dB and 0.6dB respectively. For the memory 2 VQ, the trajectory follows the individual transfer curves reasonably well. For the memory 6 VQ, there is a significant mismatch between predicted and actual behavior of the inner APP processing block. The inner curve was computed assuming a Gaussian *a priori* model; the extrinsic output turns out to be too optimistic for medium I_A–values. A closer look at the histograms shows that the distributions have a significant portion of reliability values clustered around zero (erasures), owing to the sign–bits which become available rather late (opposed to the unsigned bits), at high I_A–values. This effect is stronger the bigger the memory of the VQ. Thus, the Gaussian assumption is a poor model in this case, and a mixed Gaussian/erasure model would be more appropriate. However, by taking into account the overly optimistic behavior of the inner transfer curve for medium I_A–values in the curve fitting, good VND–distributions can still be found.

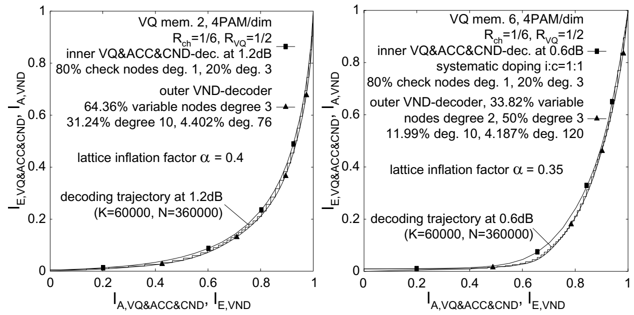

Figure 6: Rate $R_{ch} = 1/6$ nonsystematic RA codes designed by curve fitting, with inner VQ&4PAM, ACC, CND–curve and outer VND–curve; length $N = 3.6 \cdot 10^5$ bits. Left: $E_b/N_0 = 1.2$dB, VQ memory 2, no inner doping. Right: $E_b/N_0 = 0.6$dB, VQ memory 6, with inner doping.

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