TOPICAL REVIEW

Double-beta decay

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Abstract
We review recent developments in double-beta decay, focusing on what can
be learned about the three light neutrinos in future experiments. We examine
the effects of uncertainties in already measured neutrino parameters and in
calculated nuclear matrix elements on the interpretation of upcoming double-
beta decay measurements. We then review a number of proposed experiments.

1. Introduction

Double-beta ($\beta\beta$) decay is a second-order weak process in which two neutrons inside a
nucleus spontaneously transform into two protons. To conserve charge two electrons must be
emitted. If lepton number is also to be conserved two antineutrinos must be emitted as well.
This lepton-number-conserving process ($\beta\beta(2\nu)$ decay) has been observed in several nuclei.
Lepton number is not associated with a gauge symmetry, however, and so its conservation
is not sacrosanct. If lepton number is violated, e.g. through the propagation of Majorana
neutrinos, then a variant of the decay in which no neutrinos are emitted—$\beta\beta(0\nu)$ decay—may
also occur, though it has never been observed. In $\beta\beta(0\nu)$ decay a virtual neutrino is emitted by
one neutron and absorbed by the second. $\beta\beta(0\nu)$ decay can also occur through the exchange
of other particles, perhaps some predicted by supersymmetric models, between the nucleons.
Although we address this and other more exotic possibilities here, we are most interested in
$\beta\beta(0\nu)$ decay mediated by light Majorana neutrinos because its rate depends on the absolute
neutrino mass scale, a number on which we currently have only a generous upper limit.

In recent years, experimenters have discovered that neutrinos have nonzero masses and
mixings, and have pinned down many of the associated parameters. But although we now
know the differences between the squares of the masses, we do not know the mass of the
lightest neutrino, nor the pattern in which masses of the three active neutrinos are arranged.
$\beta\beta(0\nu)$ experiments have the potential to teach us about these matters, and in this review
we focus on the question of how much we can expect to learn from experiments in the next
decade.

Previous reviews have thoroughly addressed other aspects of $\beta\beta$ decay. Early papers by Primakoff and Rosen (1959), Haxton and Stephenson (1984) and Doi *et al* (1985), as well as more recent reviews by Tomoda (1991), Suhonen and Civitarese (1998), Vergados (2000) and Klapdor-Kleingrothaus (2000) presented the theoretical formalism in great detail. A comprehensive review by Faessler and Šimkovic (1998) in this journal devoted particular attention to $\beta\beta(0\nu)$ decay in supersymmetric models and to the calculation of the nuclear matrix elements governing all kinds of $\beta\beta$ decay in the quasiparticle random phase approximation (QRPA). We will treat these topics, particularly the first, more briefly. A recent review by Elliott and Vogel (2002) examined the impact of recent discoveries in neutrino physics on $\beta\beta(0\nu)$ decay, but since then several experiments (e.g. KamLAND and WMAP) have reported results and a number of papers interpreting the results have appeared, with the consequence that neutrino masses and mixings are better understood. We will focus most intently on the *additional* neutrino physics that can be extracted from $\beta\beta(0\nu)$ decay in light of these very recent results. The discussion will require us to examine the accuracy with which the $\beta\beta(0\nu)$ nuclear matrix elements can be calculated, in addition to surveying present and future experiments.

2. General theory of $\beta\beta$ decay

This subject has been covered extensively in the reviews listed above and we do not present it in great detail here. Since we focus on what can be learned about neutrinos from $\beta\beta(0\nu)$ decay, we must begin with a brief discussion of Majorana particles.

We define the left- and right-handed components of a Dirac 4-spinor by $\Psi_{L,R} = [(1 \mp \gamma_5)/2]\Psi$. In the standard model, only left-handed neutrinos interact. Because neutrinos are neutral, however, there is an additional way to construct left-handed neutrino fields. The charge conjugate field $\Psi^c \equiv i\gamma^2\Psi^*$ is also neutral; neutrino fields can be linear combinations of Ψ and Ψ^c and, since $(\Psi_R)^c$ is left-handed (and $(\Psi_L)^c$ is right-handed) we can define independent Majorana neutrino fields that are their own charge conjugates (antiparticles) via

$$\nu = \frac{\Psi_L + (\Psi_L)^c}{\sqrt{2}}, \qquad X = \frac{\Psi_R + (\Psi_R)^c}{\sqrt{2}}. \tag{1}$$

Mass terms in the Lagrangian can couple these two kinds of fields to themselves and each other; the most general mass term has the form

$$\mathcal{L}_M = -M_L \bar{\nu}\nu - M_R \bar{X}X - M_D(\bar{\nu}X + \bar{X}\nu). \tag{2}$$

The parameters M_L and M_R are 'Majorana' masses for the ν and X fields, and M_D is a 'Dirac' mass that couples the two. For N flavours of neutrinos these masses can be arranged in a matrix:

$$\mathcal{L}_M = -(\bar{\nu}\,\bar{X})\mathcal{M}\begin{pmatrix}\nu\\X\end{pmatrix} \qquad \mathcal{M} = \begin{pmatrix}\mathcal{M}_L & \mathcal{M}_D^T\\\mathcal{M}_D & \mathcal{M}_R\end{pmatrix}, \tag{3}$$

where the matrices $\mathcal{M}_L$, $\mathcal{M}_R$ and $\mathcal{M}_D$ are now $N \times N$. If $\mathcal{M}_L$ and $\mathcal{M}_R$ are zero, the Majorana ν pair with an X to form N Dirac neutrinos. At another extreme, in the 'see-saw' mechanism (Gell-Mann *et al* 1979, Yanagida 1979, Mohapatra and Senjanovic 1980)—here with one flavour, for simplicity—$M_L \ll M_D \ll M_R$ and the resulting eigenstates are Majorana neutrinos, the lightest of which has mass $m \sim M_D^2/M_R$.

In general the eigenstates of $\mathcal{M}$ represent Majorana neutrinos that are related to the fields ν and X by the 'mixing' matrix of eigenvectors:

$$\begin{pmatrix}\nu\\X\end{pmatrix} = \begin{pmatrix}U\\V\end{pmatrix}\begin{pmatrix}\phi\\\Phi\end{pmatrix}, \tag{4}$$

where U and V are $N \times 2N$ submatrices. We assume here that the states Φ are either absent or extremely heavy, in which case we can work with a nearly unitary $N \times N$ section of U that mixes the light neutrinos:

$$\nu_l \simeq \sum_m U_{lm}\phi_m. \tag{5}$$

The original states ν_l with definite flavour that enter the Lagrangian are linear combinations of the states ϕ_m with definite mass.

The matrix U nominally has N^2 parameters, $N(N-1)/2$ angles and $N(N+1)/2$ phases. N of the phases are unphysical (Kobzarev *et al* 1980), leaving $N(N-1)/2$ independent physical phases. For three active neutrinos, the mixing matrix can be written in the form

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}\,\mathrm{e}^{-\mathrm{i}\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}\,\mathrm{e}^{\mathrm{i}\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}\,\mathrm{e}^{\mathrm{i}\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}\,\mathrm{e}^{\mathrm{i}\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}\,\mathrm{e}^{\mathrm{i}\delta} & c_{23}c_{13} \end{pmatrix} \mathrm{diag}\{\mathrm{e}^{\frac{\mathrm{i}\alpha_1}{2}}, \mathrm{e}^{\frac{\mathrm{i}\alpha_2}{2}}, 1\}, \tag{6}$$

where s_{ij} and c_{ij} stand for the sine and cosine of the angles θ_{ij}, δ is a 'Dirac' phase analogous to the phase in the CKM matrix, and the other two phases α_1 and α_2 affect only Majorana particles. (They cannot be rotated away because Majorana neutrinos do not conserve lepton number.)

The neutrino masses and mixing matrix figure in the rate of neutrino-mediated $\beta\beta(0\nu)$ decay. If the weak quark–lepton effective low-energy Lagrangian has the usual V–A form, then the rate for that process is

$$\left[T_{1/2}^{0\nu}\right]^{-1} = \sum_{\mathrm{spins}} \int |Z_{0\nu}|^2 \delta(E_{e1} + E_{e2} - \Delta E) \frac{\mathrm{d}^3 p_1}{2\pi^3} \frac{\mathrm{d}^3 p_2}{2\pi^3}, \tag{7}$$

where $Z_{0\nu}$ is the amplitude for the process and ΔE is the Q-value of the decay.

The amplitude $Z_{0\nu}$ is evaluated in second-order perturbation theory and can be written as a lepton part contracted with a hadron part. The lepton part of the amplitude, containing outgoing electrons and exchanged virtual Majorana neutrinos of mass m_j, emitted and absorbed with amplitude U_{ej}, is

$$-\frac{\mathrm{i}}{4} \int \sum_j \frac{\mathrm{d}^4 q}{(2\pi)^4}\, \mathrm{e}^{-\mathrm{i}\cdot q(x-y)} \bar{e}(x)\gamma_\mu(1-\gamma_5) \frac{q^\rho\gamma_\rho + m_j}{q^2 - m_j^2}(1-\gamma_5)\gamma_\nu e^c(y) U_{ej}^2, \tag{8}$$

where q is the 4-momentum transfer. The term with q^ρ vanishes and the m_j in the denominator can be neglected for light neutrinos, so that the amplitude is proportional to

$$\langle m_{\beta\beta}\rangle = \left|\sum_j m_j U_{ej}^2\right| = |m_1|U_{e1}|^2 + m_2|U_{e2}|^2\,\mathrm{e}^{\mathrm{i}(\alpha_2-\alpha_1)} + |U_{e3}|^2\,\mathrm{e}^{\mathrm{i}(-\alpha_1-2\delta)}|. \tag{9}$$

The absolute value has been inserted for convenience, since the quantity inside it is squared in equation (7) and is complex if CP is violated.

The hadronic part of the amplitude must be evaluated between initial and final nuclear ground states, with the states in the intermediate nucleus summed over in the same way the virtual neutrino's momentum must be integrated over. For the amplitude to be appreciable, the wavelength of the virtual neutrino cannot be more than a few times larger than the nuclear radius R, i.e. only momenta $q \gtrsim 1/R \sim 100$ MeV will contribute. The excitation energies of the intermediate nuclear states generated by the hadronic part of the amplitude are all significantly smaller and so to good approximation the individual energies of those states in the 'energy denominator' can be neglected or replaced by an average value $\bar{E}$ (to which the

expression is not very sensitive) and the states can be summed over in closure. The result, in the allowed approximation for the weak hadronic current, is

$$\left[T_{1/2}^{0\nu}\right]^{-1} = G_{0\nu}(\Delta E, Z) \left| M_{0\nu}^{GT} - \frac{g_V^2}{g_A^2} M_{0\nu}^F \right|^2 \langle m_{\beta\beta} \rangle^2$$

$$\equiv G_{0\nu}(\Delta E, Z) |M_{0\nu}|^2 \langle m_{\beta\beta} \rangle^2. \tag{10}$$

Here $G_{0\nu}(\Delta E, Z)$ comes from the phase-space integral (which depends on the nuclear charge Z through the wavefunctions of the outgoing electrons), g_A and g_V are the weak axial-vector and vector coupling constants, and the M, which are nuclear matrix elements of Gamow–Teller-like and Fermi-like two-body operators, are defined as

$$M_{0\nu}^F = \langle f | \sum_{j,k} H(r_{jk}, \bar{E}) \tau_j^+ \tau_k^+ | i \rangle \tag{11}$$

$$M_{0\nu}^{GT} = \langle f | \sum_{j,k} H(r_{jk}, \bar{E}) \vec{\sigma}_j \cdot \vec{\sigma}_k \tau_j^+ \tau_k^+ | i \rangle. \tag{12}$$

The function H, which depends on the distance between nucleons and (quite weakly) on the average nuclear excitation energy in the intermediate nucleus, is sometimes called a 'neutrino potential' and has approximate form

$$H(r, \bar{E}) = \frac{2R}{\pi r} \int_0^\infty dq \frac{q \sin qr}{\omega(\omega + \bar{E} - [M_i + M_f]/2)}, \tag{13}$$

where M_i and M_f are the masses of the initial and final nuclei.

The approximate expression equation (10) is typically accurate to within about 30%, the largest corrections coming from 'induced' terms (weak magnetism, induced pseudoscalar) in the hadronic current (Šimkovic *et al* 1999). For $\beta\beta(2\nu)$ decay, the expression for the rate $\left[T_{1/2}^{2\nu}\right]^{-1}$ is similar to equation (10), the differences being in the phase-space factor $G_{2\nu}(\Delta E, Z) \neq G_{0\nu}(\Delta E, Z)$ and the matrix elements $M_{2\nu}^F$ and $M_{2\nu}^{GT}$, which do not contain the neutrino potential H but do contain energy denominators because, without the intermediate neutrino, closure is not a good approximation. For later reference, we give the relevant $\beta\beta(2\nu)$ matrix elements here:

$$M_{2\nu}^F = \sum_m \frac{\langle f | \sum_j \tau_j^+ | m \rangle \langle m | \sum_k \tau_k^+ | i \rangle}{E_m - [M_i + M_f]/2} \tag{14}$$

$$M_{2\nu}^{GT} = \sum_m \frac{\langle f | \sum_j \vec{\sigma}_j \tau_j^+ | m \rangle \langle m | \sum_k \vec{\sigma}_k \tau_k^+ | i \rangle}{E_m - [M_i + M_f]/2}. \tag{15}$$

Other mechanisms besides light-neutrino exchange can drive $\beta\beta(0\nu)$ decay, and we discuss them briefly later. No matter what the exchanged particles, however, the occurrence of $\beta\beta(0\nu)$ decay implies that neutrinos are Majorana particles with nonzero mass (Schechter and Valle 1982).

3. Phenomenology of neutrino properties and double-beta decay

The remarkably successful worldwide neutrino physics programme has revealed much about neutrinos over the past decade. We now know that they mix and we have initial values for the mixing matrix elements. We know the differences between the squares of the neutrino masses and the number of light active neutrino species. There is much we still do not know,

however. In this section we summarize what we have learned and its implications for $\beta\beta$ decay experiments that seek to learn more, focusing as mentioned above on the exchange of three light species of neutrinos. Other $\beta\beta$ possibilities are discussed later.

What aspects of still-unknown neutrino physics are most important to explore? Although the answer is to a certain degree a matter of opinion, it is clear that the absolute mass scale and whether the neutrino is a Majorana or Dirac particle are crucial issues. $\beta\beta$ decay is the only laboratory process that can test the absolute mass scale with a sensitivity near $\sqrt{\delta m_{atm}^2}$ (defined and discussed below). More importantly, whether the neutrino is Majorana or Dirac is a completely open question, and $\beta\beta$ decay is the only practical way to address it. Because future $\beta\beta$-decay experiments will be sensitive to a range of masses that includes $\sqrt{\delta m_{atm}^2}$ and therefore at least one neutrino, even null results will have a significant impact on our understanding. We believe that $\beta\beta(0\nu)$ decay should be part of any future experimental neutrino programme.

3.1. Neutrino-oscillation parameters

In this subsection, we define the various neutrino-oscillation parameters and discuss the ways their uncertainties affect our ability to extract $\langle m_{\beta\beta}\rangle$ from a $\beta\beta$ experiment. The oscillation experiments have provided data on the mixing-matrix elements and the differences in the squares of the mass eigenvalues $\left(\delta m_{ij}^2 \equiv \left(m_j^2 - m_i^2\right)\right)$. From the atmospheric-neutrino data, we have $\left|\delta m_{23}^2\right| \equiv \delta m_{atm}^2 = 2.0^{+1.0}_{-0.7} \times 10^{-3}$ eV^2 (90% CL) and $\theta_{atm} \equiv \theta_{23} \approx 45°$ (Saji 2004). The combined results of solar-neutrino experiments and reactor experiments (Ahmed *et al* 2004) give $\delta m_{12}^2 \equiv \delta m_{sol}^2 = 7.1^{+1.2}_{-0.6} \times 10^{-5}$ eV^2 (much less than δm_{atm}^2) and $\theta_{12} \equiv \theta_{sol} = 32.5^{+2.4}_{-2.3}$ degrees (68% CL). From reactor experiments, we have the limit $\theta_{13} < 9°$ (68% CL) (Hagiwara *et al* 2002). Note that other authors obtain modestly different results (see, e.g., Bahcall and Peña-Garay (2003), de Holanda and Smirnov (2003), Smy *et al* (2004)) for the solar-reactor parameters. The Super-Kamiokande collaboration has also presented an independent value for $\left|\delta m_{23}^2\right|$ of 2.4×10^{-3} eV^2 (Ishitsuka 2004). Finally the limits on θ_{13} depend on the specific value of δm_{23}^2. For these reasons and the fact that the precise values for these parameters are rapidly changing, the discussion to follow should be considered illustrative only.

The sign of δm_{sol}^2 is known; the lighter of the two eigenstates participating significantly in the solar oscillations, which we call ν_1, has the largest ν_e component. (The third eigenstate ν_3 contains very little ν_e.) We know that ν_1 is slightly lighter than ν_2, but we do not know whether ν_3 is heavier or lighter than this pair. If ν_3 is heavier, the arrangement of masses is called the 'normal hierarchy' (with two light neutrinos and a third significantly heavier one); if it is lighter the arrangement is called the 'inverted hierarchy'. When all three masses are significantly larger than $\sqrt{\delta m_{atm}^2}$ the hierarchy is referred to as 'quasidegenerate', no matter which eigenstate is the lightest. One of the largest questions left in the neutrino world is 'which of the hierarchies is realized in nature?'. The answer will tell us about the overall scale of the neutrino masses as well as the order in which they are arranged.

The central values of the oscillation parameters and equation (9) determine a range of $\langle m_{\beta\beta}\rangle$ values for a given value of m_1. Many authors have analysed the dependence (see, e.g., Vissani (1999), Bilenky *et al* (1999, 2004), Klapdor-Kleingrothaus *et al* (2001a, 2001b, 2001c), Matsuda *et al* (2001), Czakon *et al* (2002), Elliott and Vogel (2002), Feruglio *et al* (2002), Giunti (2003), Joaquim (2003), Pascoli and Petcov (2003, 2004a, 2004b), Sugiyama (2003), Murayama and Peña-Garay (2004)), and figure 1 shows the results. The bands indicate the range of possible values, which depend on the unknown phases in the mixing matrix. The borders indicate CP-conserving values of the phases, $e^{i(\alpha_1 - \alpha_2)} = \pm 1$.
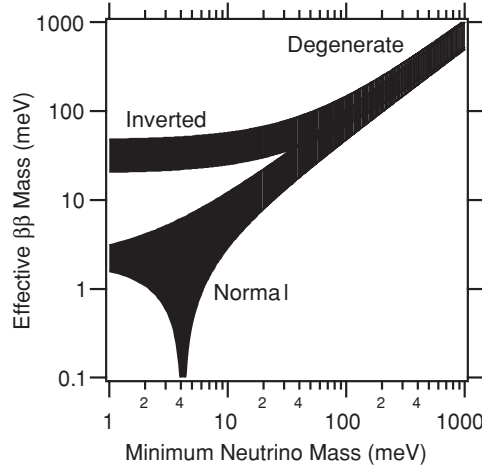
Figure 1. The effective Majorana mass $\langle m_{\beta\beta} \rangle$ as a function of the lightest neutrino mass.

The observation of $\beta\beta(0\nu)$ decay would have profound implications regardless of uncertainty in the deduced value of $\langle m_{\beta\beta} \rangle$. It would show that neutrinos are massive Majorana particles and that the total lepton number is not a conserved quantity. But it is interesting to ask whether one can use a measurement of $\langle m_{\beta\beta} \rangle$ to discern the correct hierarchy. At high values of the minimum neutrino mass, the mass spectrum is quasi-degenerate, and the bands in figure 1 are not resolved. For a minimum neutrino mass of about 50 meV, the degenerate band splits into two, representing the normal (m_1 lightest) and inverted (m_3 lightest) hierarchies. Figure 1 appears to imply that it would be straight-forward to identify the appropriate band at these low mass values. However, there are uncertainties in the oscillation parameters and the matrix elements that are not represented in the figure.

One can address the question of how difficult it would be to distinguish the hierarchies by comparing the maximum value for the normal hierarchy $\left(\langle m_{\beta\beta}\rangle_{\mathrm{max}}^{\mathrm{Nor}}\right)$ with the minimum value for the inverted hierarchy $\left(\langle m_{\beta\beta}\rangle_{\mathrm{min}}^{\mathrm{Inv}}\right)$ that can result from the parameter uncertainties. When the lightest neutrino mass is small, the expressions for these values are simple (Pascoli and Petcov 2003). When m_1 is near zero, $\langle m_{\beta\beta}\rangle_{\mathrm{max}}^{\mathrm{Nor}}$ occurs for constructive interference between the contributions from the m_2 and m_3 terms in equation (9), when CP is conserved:

$$\langle m_{\beta\beta}\rangle_{\mathrm{max}}^{\mathrm{Nor}} \approx \sqrt{\delta m_{\mathrm{sol}}^2}\sin^2\theta_{\mathrm{sol}}\cos^2\theta_{13} + \sqrt{\delta m_{\mathrm{atm}}^2}\sin^2\theta_{13}. \qquad (16)$$

From equation (16), it is clear that $\langle m_{\beta\beta}\rangle_{\mathrm{max}}^{\mathrm{Nor}}$ is maximal when θ_{13}, θ_{sol} and the δm^2 are as large as they can be. $\langle m_{\beta\beta}\rangle_{\mathrm{min}}^{\mathrm{Inv}}$ is minimal with the same conditions on θ_{13} and θ_{sol}, but the smallest allowed value for $\delta m_{\mathrm{atm}}^2$:

$$\langle m_{\beta\beta}\rangle_{\mathrm{min}}^{\mathrm{Inv}} = \sqrt{\delta m_{\mathrm{atm}}^2}\cos 2\theta_{\mathrm{sol}}\cos^2\theta_{13}. \qquad (17)$$

If we use the appropriate extremum values for the oscillation parameters in equations (16) and (17), we find $\langle m_{\beta\beta}\rangle_{\mathrm{max}}^{\mathrm{Nor}} \approx (9.1 \text{ meV})(0.327)(0.976) + (55 \text{ meV})(0.024) = 4 \text{ meV}$ and $\langle m_{\beta\beta}\rangle_{\mathrm{min}}^{\mathrm{Inv}} \approx (36 \text{ meV})(0.345)(0.976) = 12 \text{ meV}$. These numbers are sufficiently different, at least when using our low-confidence-level uncertainty ranges, that one could discriminate between the two solutions.

Table 1. A summary of the impact on $\langle m_{\beta\beta} \rangle$ in the normal and inverted hierarchies of the oscillation-parameter uncertainties. For the central values of the parameters, $\langle m_{\beta\beta} \rangle_{\max}^{\text{Nor}}$ and $\langle m_{\beta\beta} \rangle_{\min}^{\text{Inv}}$ are 2.4 meV and 19 meV, respectively. See Pascoli and Petcov (2003) for a similar analysis.

Oscillation parameter	Parameter range	Range in $\langle m_{\beta\beta} \rangle_{\max}^{\text{Nor}}$	Range in $\langle m_{\beta\beta} \rangle_{\min}^{\text{Inv}}$
$\sqrt{\delta m_{\text{sol}}^2}$	8.1–9.1 meV	2.3–2.6 meV	NA
$\sqrt{\delta m_{\text{atm}}^2}$	36–55 meV	3.2–3.7 meV (with $\theta_{13} = 9°$)	15.2–23.2 meV
θ_{sol}	30.1–34.9°	2.1–2.7 meV	15.5–22.4 meV
θ_{13}	0–9°	2.4–3.5 meV	18.6–19.0 meV

Since the precision of the oscillation parameters is likely to improve with future experiments, they would not appear to be a primary concern. Even so, it is interesting to see the effects of uncertainties in individual parameters. These effects can be determined by propagating the uncertainty in each parameter through to the uncertainty in $\langle m_{\beta\beta} \rangle$. The results are shown in table 1. It is clear that θ_{13} affects $\langle m_{\beta\beta} \rangle_{\max}^{\text{Nor}}$ a great deal. It is also clear that δm_{atm}^2 is critical for $\langle m_{\beta\beta} \rangle_{\min}^{\text{Inv}}$. Finally, θ_{sol} is important for both. In short, improved precision for θ_{13}, θ_{sol} and δm_{atm}^2 would help with the interpretation of a $\beta\beta(0\nu)$ experiment.

The parameter θ_{sol} has an effect in figure 1 that we have not yet mentioned. If it has the right value, cancellation can drive $\langle m_{\beta\beta} \rangle$ to very small values. But as long as solar mixing is substantially different from maximal, the cancellation is possible only over a narrow range of values for the lightest mass, and complete cancellation is not possible at all in the inverted hierarchy.

The uncertainty in $|M_{0\nu}|$ has been a source of concern for a long time. Typically, it has been assumed to contribute of a factor of 2–3 times $\langle m_{\beta\beta} \rangle$ to the uncertainty in $\langle m_{\beta\beta} \rangle$. This uncertainty clearly dwarfs any from the oscillation parameters and thus is the primary issue. We address it later.

3.2. The absolute mass scale

As already mentioned, we know that at least one neutrino has a mass greater than $\sqrt{\delta m_{\text{atm}}^2} \sim$ 45 meV; sensitivity to this value is therefore the goal of most future neutrino-mass experiments. In this subsection, we briefly compare the potential of other mass measurements.

There are many ways to measure the mass of the neutrino (see Bilenky *et al* (2003)) for a nice summary of the techniques). The best of these are $\beta\beta(0\nu)$ decay, β decay and cosmological observations. These three approaches are complementary in that they determine different combinations of the mass eigenvalues and mixing-matrix parameters (see equations (23)). A measurement of $\beta\beta(0\nu)$ decay determines a coherent sum of the Majorana neutrino masses because $\langle m_{\beta\beta} \rangle$ arises from exchange of a virtual neutrino. Beta decay measures an incoherent sum because a real neutrino is emitted. The cosmology experiments measure the density of neutrinos and thus a parameter proportional to the sum of the neutrino masses.

The present limit $\langle m_\beta \rangle \leqslant 2200$ meV (95% CL) comes from tritium beta decay (Lobashev *et al* 1999, Weinheimer *et al* 1999). This limit, when combined with the oscillation results, indicates that for at least one neutrino:

$$45 \text{ meV} \leqslant m_i \leqslant 2200 \text{ meV}. \tag{18}$$

The two β-decay experimental groups just referenced have joined forces to form the KATRIN collaboration (Osipowicz *et al* 2001). They propose to build a large spectrometer that exploits the strengths of both previous efforts. KATRIN hopes to reach a sensitivity to $\langle m_\beta \rangle$ near 200 meV (Bornschein 2003). Massive neutrinos would contribute to the cosmological matter density (Hannestad 2003a) an amount,

$$\Omega_\nu h^2 = \Sigma/92.5 \text{ eV}, \tag{19}$$

where Ω is the neutrino mass density relative to the critical density, $100h$ is the Hubble constant in km s^{-1} Mpc^{-1}, and $\Sigma \equiv m_1 + m_2 + m_3$ is the sum of the neutrino masses. The neutrinos are light, however, and cluster with cold dark components of the matter density only for scales larger than

$$k \sim 0.03\sqrt{(m_\nu/1 \text{ eV})\Omega_m} h \text{ Mpc}^{-1}. \tag{20}$$

For smaller values of k, perturbations are suppressed, and so measurements of the large scale structure (LSS) can provide constraints on the neutrino mass. Such constraints are rather weak, though, unless used in conjunction with precise determinations of the other various cosmological parameters, which also reflect the size of perturbations. Most recently the WMAP collaboration (Bennett *et al* 2003) has provided precise cosmological data that supplement LSS data from the 2dF galaxy survey (Elgarøy *et al* 2002), CBI (Person *et al* 2002), ACBAR (Kuo *et al* 2004) and the Lyman-α forest data (Croft *et al* 2002). These data have been used in various combinations to derive 95% CL limits on Σ of <0.75 eV (Barger *et al* 2003), <1.0 eV (Hannestad 2003b), <1.7 eV (Tegmark *et al* 2003), <0.69 eV (Spergel *et al* 2003), and <1.0 eV (Crotty *et al* 2004). There is at least one claim for a nonzero value: $\Sigma = 0.64$ eV (Allen *et al* 2003). An interesting paper by Elgarøy and Lahav (2003) points out the impact of the prior distributions on the resulting neutrino mass limit.

Future measurements by the Sloan Digital Sky survey (SDSS 2003) and the PLANCK satellite (PLANCK 2003) may obtain limits on Σ as low as 40 meV (Hu and Tegmark 1999). But the determination of Σ from cosmology is clearly complicated by the large number of correlated parameters that must be measured. Clean laboratory measurements of the neutrino mass will always be desirable. Ordinary β decay will be hard pressed to reach a sensitivity to $\sqrt{\delta m^2_{\text{atm}}}$, so $\beta\beta(0\nu)$ experiments are especially important.

3.3. The Majorana phases

Equation (9) shows the effective Majorana neutrino mass and its relation to the Majorana phases. When these relative phases are an integer multiple of π CP is conserved. In principle, the two relative phases ($\alpha \equiv (\alpha_2 - \alpha_1)$, $\beta \equiv (-\alpha_1 - 2\delta)$) have measurable consequences. In practice, determining them will be difficult. In this subsection, we discuss the physics of these phases. Many authors have examined the potential to combine measurements from $\beta\beta$ decay, tritium β decay, and cosmology to determine the Majorana phases. (See, e.g., Sugiyama (2003), Abada and Bhattacharyya (2003), Pascoli *et al* (2002a, 2002b).) We can illustrate the main ideas through a simplified set of hypothetical measurements. Figure 2 shows a two-neutrino-species example of such a set. We took the mixing matrix and δm^2 to be the best fit to the solar-neutrino data, with an arbitrary value for the Majorana phase α (of which there is only one) of 2.5 rad. We then made up values for Σ, $\langle m_{\beta\beta} \rangle$ and $\langle m_\beta \rangle$ assuming them to be the results of pretend measurements. Each curve in the m_2 versus m_1 graph is defined by one of these measurements. We chose the value of Σ (from cosmology) to be 600 meV, corresponding to a quasidegenerate hierarchy, and let $\langle m_\beta \rangle = 300$ meV and $\langle m_{\beta\beta} \rangle = 171$ meV. The m_2 versus m_1 curves from the 'measurements' of the oscillation parameters, Σ and β
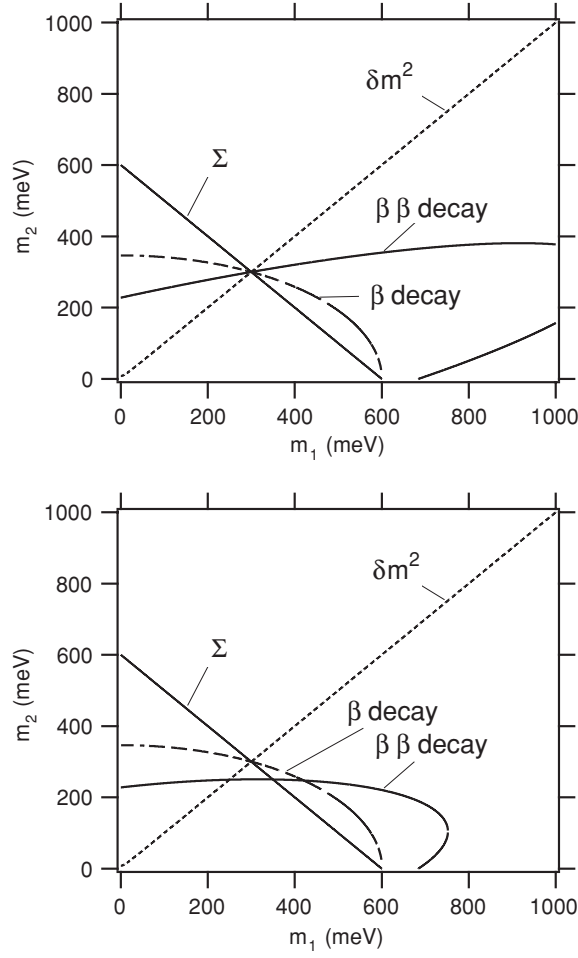
Figure 2. A consistency plot for the neutrino mass eigenvalues m_1 and m_2, for various hypothetical measurements. This set of curves indicates how measured values of Σ, $\langle m_{\beta\beta} \rangle$, $\delta m^2_{\mathrm{sol}}$ and $\langle m_\beta \rangle$ constrain the mass eigenvalues. The $\beta\beta(0\nu)$ curve has been drawn for an *incorrect* value of the phase in the bottom panel to indicate the sensitivity of this technique for extracting the CP-violating phase. See the text for further description of the parameters used to draw the curves.

decay are

$$
\begin{aligned}
m_2 &= \Sigma - m_1 \\
m_2 &= \sqrt{m_1^2 + \delta m_{12}^2} \\
m_2 &= \sqrt{(\langle m_\beta \rangle / U_{e2})^2 - (m_1 U_{e1} / U_{e2})^2}.
\end{aligned}
\tag{21}
$$

The $\beta\beta$ constraint is a little more complicated than that from β decay. The curve is also an ellipse but rotated with respect to the axes. The constraint is given by the solution to the quadratic equation resulting from the two-neutrino version of equation (9):
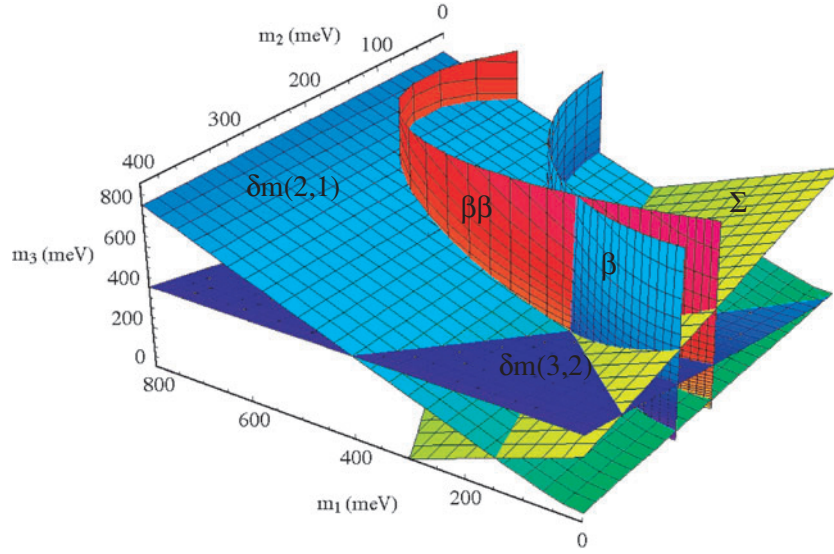
Figure 3. A consistency plot for the neutrino mass eigenvalues m_1, m_2 and m_3 for various hypothetical measurements. This set of curves indicates how measured values of Σ, $\langle m_{\beta\beta} \rangle$, oscillations and $\langle m_\beta \rangle$ constrain the mass eigenvalues. See the text for further description of the parameters used to draw the curves.

(This figure is in colour only in the electronic version)

$$
\begin{aligned}
a &= U_{e2}^4 \\
b &= 2m_1 U_{e1}^2 U_{e2}^2 \cos(\alpha) \\
c &= \left(m_1 U_{e1}^2\right)^2 - \langle m_{ee} \rangle \\
m_2 &= \left(-b \pm \sqrt{b^2 - 4ac}\right)/2a.
\end{aligned}
\tag{22}
$$

All of these equations express m_2 in terms of m_1 and measured parameters and all should intersect at one (m_1, m_2) point. However, because the point is overdetermined, the $\beta\beta$ ellipse will intersect only for a correct choice of α. This provides a way to determine α. In figure 2, we drew the $\beta\beta$ ellipse for $\alpha = 2.0$ rad and for the 'true' value of 2.5 rad to show how the intersection does indeed depend on a correct choice of the phase.

Although this two-species example is illuminating, it is overly simplistic. One needs to consider three species and two phases. In this case, the Σ constraint becomes a plane and the $\beta\beta$- and β-decay constraints become ellipsoids. Figure 3 shows a model three-species analysis with $\Sigma = 700$ meV, δm_{32}^2 positive, $\langle m_\beta \rangle = 232$ meV and $\langle m_{\beta\beta} \rangle = 159$ meV. (The phases were taken to be 2.0 and 2.5 rad, and U_{e3} was taken to be 0.03.) The surfaces, shown in figure 3, are defined by the equations for Σ, oscillations, $\langle m_\beta \rangle$ and $\langle m_{\beta\beta} \rangle$. Respectively, these are

$$
\begin{aligned}
\Sigma &= m_1 + m_2 + m_3 \\
\delta m_{21}^2 &= m_2^2 - m_1^2 \\
\delta m_{32}^2 &= m_3^2 - m_2^2 \\
\langle m_\beta \rangle^2 &= m_1^2 U_{e1}^2 + m_2^2 U_{e2}^2 + m_3^2 U_{e3}^2 \\
\langle m_{ee} \rangle^2 &= m_1^2 U_{e1}^4 + m_2^2 U_{e2}^4 + m_3^2 U_{e3}^4 + 2m_1 m_2 U_{e1}^2 U_{e2}^2 \cos(\alpha) \\
&\quad + 2m_1 m_3 U_{e1}^2 U_{e3}^2 \cos(\beta) + 2m_2 m_3 U_{e2}^2 U_{e3}^2 \cos(\alpha + \beta).
\end{aligned}
\tag{23}
$$

These surfaces intersect at a point but $\beta\beta$ decay is the only measurement of those used that is sensitive to the phases. Thus a second pair of phases will also produce a consistent result. (That additional ellipsoid is not shown here.) Two experiments that depend on the phases are required to unambiguously determine both. Furthermore, to keep the plots legible, this analysis ignores any uncertainty in the measured parameters. The uncertainties will be large, at least initially, and therefore conclusions about the phases will be weakened. Barger *et al* (2002), for example, have noted that a significant improvement in $|M_{0\nu}|$ is necessary before the interpretation can be successful. The articles listed above consider the uncertainties in their analyses.

When heavy right-handed Majorana neutrinos decay, they will violate lepton number. In the early universe, these decays would be out of equilibrium and could violate CP. The resulting net lepton number could be transferred to a net baryon number through standard weak interactions. Thus, this leptogenesis process (Fukugita and Yanagida 1986) could explain the baryon asymmetry in the universe. In principle, leptogenesis depends on the Majorana phases, so its understanding might provide the needed additional constraint. The baryon asymmetry can be expressed (Buchmüller *et al* 2002) in terms of the mass M_1 of the lightest of the heavy Majorana neutrinos Φ_1, the CP asymmetry ϵ in Φ_1 decays, an effective neutrino mass $\tilde{m}$, and the sum of the squares of the three light neutrino masses. Unfortunately, the relationship between ϵ and the low-energy phases relevant for $\beta\beta$ is model dependent. Many models have been studied in the literature, each with its own relationship and conclusions.

4. Calculating nuclear matrix elements

The observation of $\beta\beta(0\nu)$ decay would immediately tell us that neutrinos are Majorana particles and give us an estimate of their overall mass scale but without accurate calculations of the nuclear matrix elements that determine the decay rate, so it will be difficult to reach quantitative conclusions about masses and hierarchies. Theorists have tried hard to develop many-body techniques that will allow such calculations. They have tried to calibrate their calculations to related observables: $\beta\beta(2\nu)$ decay, ordinary β^+ and β^- decay, Gamow–Teller strength distributions, odd–even mass differences, single-particle spectra. They have tried to exploit approximate isospin and $SU(4)$ symmetries in the nuclear Hamiltonian, to extend well-known many-body methods in novel ways. In spite of all this effort, we know the matrix elements with limited accuracy. In this section, we review the state of the nuclear-structure calculations and discuss ways to improve them. While increased accuracy is not easy to achieve, we will argue that it is also not impossible.

Most recent attempts to calculate the nuclear matrix elements have been based on the neutron–proton quasiparticle random phase approximation or extensions to it. Of those that have not, the most prominent are based on the shell model. While the two methods have much in common—their starting point is a Slater determinant of independent particles—the kinds of correlations they include are complementary. The QRPA treats a large fraction of the nucleons as 'active' and allows these nucleons a large single-particle space to move in. But RPA correlations are of a specific and simple type best suited for collective motion. The shell model, by contrast, treats a small fraction of the nucleons in a limited single-particle space, but allows the nucleons there to correlate in arbitrary ways. That these very different approaches yield similar results indicates that both capture most of the important physics.

4.1. QRPA

The QRPA was developed by Hableib and Sorenson (1967) and first applied to double-beta decay by Huffman (1970). Both it and early shell model calculations had problems reproducing measured $\beta\beta(2\nu)$ rates until the realization by Vogel and Zirnbauer (1986) that in the QRPA the neutron–proton (np) particle–particle (i.e. pairing) interaction, which has little effect on the collective Gamow–Teller resonance, suppresses $\beta\beta(2\nu)$ rates considerably. Soon afterwards, Engel *et al* (1988) and Tomoda and Faessler (1987) demonstrated a similar though smaller effect on $\beta\beta(0\nu)$ decay. It was quickly realized, however, that the QRPA was not designed to handle realistic np pairing; the calculated half-lives were unnaturally sensitive to the strength of the pairing interaction. As a result, the rates of $\beta\beta$ decay, particularly $\beta\beta(2\nu)$ decay, were hard to predict precisely because a small change in a phenomenological parameter (the strength of np pairing) caused a large change in the lifetimes and eventually the breakdown (called a 'collapse') of the entire method when the parameter exceeds some critical value. Most recent work in the QRPA has aimed at modifying the undesirable aspects of the method so that its sensitivity to np pairing becomes more realistic.

One approach is 'second' QRPA (Raduta *et al* 1999) . The QRPA itself treats states in the intermediate nucleus as one-quasineutron+one quasiproton excitations of the initial and/or final ground states. The quasineutron–quasiproton pair is taken to obey boson statistics, inspiring the term 'quasiboson approximation' for the method. Second QRPA extends the structure of these states by adding another term in a boson expansion of the pair.

A larger number of papers have been based on the renormalized QRPA (RQRPA) (Hara 1964, Rowe 1968, Toivanen and Suhonen 1995). The quasiboson approximation is equivalent to replacing commutators by their expectation values in an independent-quasiparticle approximation to the initial and final ground states. The RQRPA uses the QRPA ground states to evaluate the commutators. Because the commutators in turn help fix the ground states, the two must be evaluated self-consistently, usually via iteration. A variant of this approach is the 'full RQRPA', in which the effects of isovector np pairing, artificially strengthened to account implicitly for isoscalar pairing, are included in the BCS calculation that defines the quasiparticles as well as in the subsequent QRPA calculation (Schwieger *et al* 1996, Šimkovic *et al* 1997). (Isovector np pairing was first introduced in this way in the unrenormalized QRPA (Cheoun *et al* 1993, 1995, Pantis *et al* 1996)). Another extension is the self-consistent RQRPA (SCQRPA). Here, the BCS calculation is modified and iterated together with the RQRPA calculation until the RQRPA ground state has the same number of quasiparticles as the BCS-like state on which it is based. All these methods reduce the dependence of the $\beta\beta(2\nu)$ matrix elements on the strength of the neutron–proton pairing interaction.

Unfortunately, it is not clear which of the methods is best. One might presume that the full RQRPA and the self-consistent RQRPA are better than the vanilla RQRPA, which in turn is better than the original unrenormalized QRPA. But the results of the RQRPA and full RQRPA appear to be quite sensitive to, e.g., the number of single-particle states in the model space (Šimkovic *et al* 1997). They also violate an important sum rule for single β strength, and studies in solvable models suggest that the reduced dependence (at least of the RQRPA) on neutron–proton pairing may be spurious (Engel *et al* 1997) resulting from an artificial reduction of isoscalar pairing correlations. And it is not clear that the full RQRPA's substitution of isovector pairing for isoscalar pairing is legitimate.

The self-consistent QRPA has been applied to $\beta\beta(0\nu)$ decay only once, and that application did not take into account the predictions of the same approach for $\beta\beta(2\nu)$ decay. The approximations that go into the second QRPA are of a different type,

and while one study indicates less dependence on model space size for that method (Stoica and Klapdor-Kleingrothaus 2001) there are not many other reasons to prefer one approach or the other.

Recently, the RQRPA was modified even further, so that the single β sum rule was restored. The resulting method, called the 'fully renormalized QRPA', has yet to be applied to $\beta\beta(0\nu)$ decay. Even more recently, Šimkovic et al (2003) raised the issue of nuclear deformation, which has usually been ignored in QRPA-like treatments of nearly spherical nuclei[3]. The authors argued that differences in deformation between the initial and final nuclei can have large effects on the $\beta\beta(2\nu)$ half-life. These ideas, too, have not yet been applied to $\beta\beta(0\nu)$ decay.

The profusion of RPA-based acronyms is both good and bad. The sheer number of methods applied gives us a kind of statistical sample of calculations (Bahcall et al 2004), which we will use below to get an idea of the theoretical uncertainty in the matrix elements. But the sample may be biased by the omission of non-RPA correlations in all but a few calculations. Other approaches that include correlations more comprehensively should be pursued.

4.2. Shell model

The obvious alternative to RPA, and the current method of choice for nuclear-structure calculations in heavy nuclei where applicable, is the shell model. It has ability to represent the nuclear wavefunction to arbitrary accuracy, provided a large enough model space is used. This caveat is a huge one, however. Current computers allow very large bases (millions of states), but in heavy nuclei this is still not nearly enough. Techniques for constructing 'effective' interactions and operators that give exact results in truncated model spaces exist but are hard to implement. Even in its crude form with relatively small model spaces and bare operators, however, the shell model offers advantages over the QRPA. Its complicated valence-shell correlations, which the QRPA omits (though it tries to compensate for them by renormalizing parameters) apparently affect the $\beta\beta$ matrix elements (Caurier et al 1996).

The first modern shell-model calculations of $\beta\beta$ decay date from Haxton and Stephenson (1984) and references therein. Only a few truly large-scale shell model calculations have been performed. The heavy deformed $\beta\beta$ nuclei, ^{238}U and ^{150}Nd, for example, require bases that are too large to expect real accuracy. Realistic work has thus been restricted to ^{48}Ca, ^{76}Ge and ^{136}Xe, though less comprehensive calculations have been carried out in several other nuclei (Suhonen et al 1997).

Large spaces challenge us not only through the problem of diagonalizing large matrices, but also by requiring us to construct a good effective interaction. The bare nucleon–nucleon interaction needs to be modified in truncated spaces (this is an issue in the QRPA as well, though a less serious one). Currently, effective interactions are built through a combination of perturbation theory, phenomenology and painstaking fitting. The last of these, in particular, becomes increasingly difficult when millions of matrix elements are required.

Related to the problem of the effective interaction is the renormalization of transition operators. Though the problem of the effective Gamow–Teller operator (Siiskonen et al 2001), which enters directly into $\beta\beta(2\nu)$ decay, has drawn some attention, very little work has been done on the renormalization of the two-body operators that govern $\beta\beta(0\nu)$ decay in the closure approximation. Shell-model calculations will not be truly reliable until they address this issue, which is connected with deficiencies in the wavefunction caused by neglect of single-particle levels far from the Fermi surface. Engel and Vogel (2004) suggest that

---

[3] Psuedo-$SU(3)$-based truncations have been used to treat it in well-deformed nuclei (e.g., in Hirsch et al (1996)).

significant improvement on the state of the art will be difficult but not impossible in the coming years.

4.3. Constraining matrix elements with other observables

The more observables a calculation can reproduce, the more trustworthy it becomes. And if the underlying model contains some free parameters, these observables can fix them. The renormalization of free parameters can make up for deficiencies in the model, reducing differences between, e.g., the QRPA and RQRPA once the parameters of both have been fit to relevant data. The more closely an observable resembles $\beta\beta(0\nu)$ decay, the more relevant it is.

Gamow–Teller distributions, both in the β^- and β^+ directions, enter indirectly into both kinds of $\beta\beta$ decay, and are measurable through (p, n) reactions. Aunola and Suhonen (1998) are particularly careful to reproduce those transitions as well as possible. Pion double charge exchange, in which a π^+ enters and a π^- leaves, involves the transformation of two neutrons into two protons, such as $\beta\beta$ decay, but the nuclear operators responsible are not the same in the two cases. Perhaps the most relevant quantity for calibrating calculations of $\beta\beta(0\nu)$ decay is $\beta\beta(2\nu)$ decay, which has now been measured in 10 different nuclei.

Two recent papers have tried to use $\beta\beta(2\nu)$ decay to fix the strength of np pairing in QRPA-based calculations. Stoica and Klapdor-Kleingrothaus (2001) used it only for the $J^\pi = 1^+$ channel relevant for $\beta\beta(2\nu)$ decay, leaving the np pairing strength unrenormalized in other channels. By contrast, Rodin *et al* (2003) renormalized the strength in all channels by the same amount. The results of the two procedures were dramatically different: Stoica and Klapdor-Keingrothaus (2001) found that the $\beta\beta(0\nu)$ matrix elements depended significantly on the theoretical approach (QRPA, RQRPA, FRQRPA, second QRPA) and, in some of the approaches, on the model space, while Rodin *et al* (2003) found almost no dependence on model-space size, on the form of the nucleon–nucleon interaction, or on whether the QRPA or RQRPA was used. The authors argued that fixing the np pairing strength to $\beta\beta(2\nu)$ rates essentially eliminates uncertainty associated with variations in QRPA calculations of $\beta\beta(0\nu)$ rates, though they left open the question of how close to reality the calculated rates were. Given all these considerations, can we meaningfully estimate the uncertainty in the $\beta\beta(0\nu)$ matrix elements? We address this question for a particular nucleus now.

4.4. How well can we estimate uncertainty? The case of ^{76}Ge

How accurate are existing calculations? One can answer provisionally by looking at the spread in the many predictions offered recently in the literature. We focus here on ^{76}Ge, which currently gives the strongest limit on $\langle m_{\beta\beta} \rangle$ (or perhaps even a value for it: see discussions), and figures prominently in several proposals for new experiments.

Table 2, taken in part from Civitarese and Suhonen (2003), shows predictions of most of the calculations in the literature for the *nuclear* contribution to the decay rate (Doi *et al* 1985), $C_{mm} \equiv \left[\langle m_{\beta\beta} \rangle^2 T_{1/2}^{0\nu} / m_e^2 \right]^{-1} = G_{0\nu} |M_{0\nu}|^2 m_e^2$, and for the effective neutrino mass $\langle m_{\beta\beta} \rangle$ that would be deduced by measuring a lifetime $T_{1/2}^{0\nu} = 4.0 \times 10^{27}$ years. Besides differing in method (by which they are grouped) the calculations use different model spaces, fit different observables, and adjust different parameters. The resulting C_{mm} vary by nearly two orders of magnitude, leading to an order of magnitude variation in the extracted neutrino mass. But some of the 'outliers', labelled with asterisks in the table, can be eliminated on the grounds that the corresponding calculations omit or misrepresent clearly important effects. The VAMPIR model used in the early calculation of Tomoda *et al* (1986) contains no np pairing correlations

Table 2. Predictions for ^{76}Ge. The quantity C_{mm}, defined in the text, is a measure of the nuclear contribution to the $\beta\beta(0\nu)$ decay rate. The values of $\langle m_{\beta\beta} \rangle$ are those that would be extracted by a calculation if the lifetime were 4.0×10^{27} years. The asterisks indicate approaches that omit important physics or have been superseded.

$C_{mm}(Y^{-1})$	$\langle m_{\beta\beta} \rangle$ (eV)	Method	Reference
1.12×10^{-13}	0.024	QRPA	Muto $et\ al$ (1989), Staudt $et\ al$ (1990)
6.97×10^{-14}	0.031	QRPA	Suhonen $et\ al$ (1992)
7.51×10^{-14}	0.029	Number-projected QRRA	Suhonen $et\ al$ (1992)
7.33×10^{-14}	0.030	QRPA	Pantis $et\ al$ (1996)
1.18×10^{-13}	0.024	QRRA	Tomoda (1991)
1.33×10^{-13}	0.022	QRPA	Aunola and Suhonen (1998)
8.27×10^{-14}	0.028	QRRA	Barbero $et\ al$ (1999)
1.85–12.5×10^{-14}	0.059–0.023	QRPA	Stoica and Klapdor-Kleingrogthaus (2001)
1.8–2.2×10^{-14}	0.060–0.054	QRRA	Bobyk $et\ al$ (2001)
8.36×10^{-14}	0.028	QRPA	Civitarese and Suhonen (2003)
1.42×10^{-14}	0.068	QRRA with np pairing	Pantis $et\ al$ (1999)
4.53×10^{-14}	0.038	QRPA with forbidden	Rodin $et\ al$ (2003)
8.29×10^{-14}	0.028	RQRPA	Faessler and Šimkovic (1998)
1.03×10^{-13}	0.025	RQRRA	Šimkovic $et\ al$ (1999)
6.19×10^{-14}	0.032	RQRRA with forbidden	Šimkovic $et\ al$ (1999)
5.5–6.3×10^{-14}	0.034–0.032	RQRRA	Bobyk $et\ al$ (2001)
2.21–8.83×10^{-14}	0.054–0.027	RQRPA	Stoica and Klapdor-Kleingrothaus (2001)
3.63×10^{-14}	0.042	RQRPA with forbidden	Rodin $et\ al$ (2003)
2.75×10^{-14}	0.049	Full RQRPA	Šimkovic $et\ al$ (1997)
3.36–8.54×10^{-14}	0.042–0.028	Full RQRPA	Stoica and Klapdor-Kleingrothaus (2001)
6.50–9.21×10^{-14}	0.032–0.027	Second QRPA	Stoica and Klapdor-Kleingrothaus (2001)
2.7–3.2×10^{-15}	0.155–143	Self-consistent QRPA*	Bobyk $et\ al$ (2001)
2.88×10^{-13}	0.015	VAMPIR*	Tomoda $et\ al$ (1986)
1.58×10^{-13}	0.020	Shell-model truncation*	Haxton and Stephenson (1984)
6.87–15.7×10^{-14}	0.031–0.020	Shell-model truncation*	Engel $et\ al$ (1989)
1.90×10^{-14}	0.059	Large-scale shell model	Caurier $et\ al$ (1996)

and so gives too large a decay rate. The shell-model truncation by Haxton and Stephenson (1984) was done in a way that minimized np pairing, and the upper limit in C_{mm} from Engel $et\ al$ (1989) was considered probably too large by the authors (though in their calculation they set $g_A = 1$, which here would move the upper limit into the middle of the range). At the other end of the spectrum, the very small self-consistent RQRPA decay rates of Bobyk $et\ al$ (2001) were obtained with a value of the np pairing strength that was not consistent with the measured $\beta\beta(2\nu)$ rate; when the strength is adjusted to reproduce $\beta\beta(2\nu)$ decay, the results for the $\beta\beta(0\nu)$ rate are close to those of the plain QRPA in the same reference. Without any further culling, the remaining C vary by about one order of magnitude, and the extracted $\langle m_{\beta\beta} \rangle$ vary by a factor of about three, from 0.022 to 0.068 eV for the lifetime we have chosen.

Even if some of the other calculations are objectionable, it is difficult to reduce the spread much below this factor of three without some real work. Aunola and Suhonen (1998), who among all the entries do the most extensive job of adjusting parameters to reproduce spectroscopic data and single-β transition strengths, obtain a low neutrino mass: 0.022 eV from our hypothetical lifetime. Caurier $et\ al$ (1996), who do the most complete shell-model calculation, obtain 0.059 eV, close to the maximum mass from calculations we have not removed. We cannot discount these two careful calculations without examining them more closely and so cannot further restrict the range. This is not to say that these calculations

cannot be questioned. It may not be appropriate, for example, to fit single-particle energies to β-decay rates as Aunola and Suhnen do. But for now, until someone investigates the situation further, the range of reasonable results should include these two calculations.

As mentioned above, Rodin *et al* (2003) argue that the variation in QRPA and RQRPA rates can be nearly eliminated by renormalizing a few parameters to reproduce pairing gaps and $\beta\beta(2\nu)$ decay rates. Indeed, the table shows that the QRPA and RQRPA numbers from that reference are close. But it has yet to be shown conclusively that $\beta\beta(2\nu)$ decay is more important than single-β decay or other observables, and Engel and Vogel (2004) demonstrate that in a two-level $SO(5)$-based model (Dussel *et al* 1970), at least, the procedure cannot fully eliminate model-space dependence when the QRPA is used.

4.5. Reducing the uncertainty

What can be done to improve the situation? In the near term, improvements can be made in both QRPA-based and shell-model calculations. First, existing calculations should be reexamined to check for consistency. One important issue is the proper value of the axial-vector coupling constant g_A, which is often set to 1 (versus its real value of 1.26) in calculations of β decay and $\beta\beta(2\nu)$ decay to account for the observed quenching of low-energy Gamow–Teller strength. What value should one use for $\beta\beta(0\nu)$ decay, which goes through intermediate states of all multipolarity, not just 1^+? Some authors use $g_A = 1$, some $g_A = 1.26$ and some $g_A = 1$ for the 1^+ multipole and 1.26 for the others. (Often authors do not reveal their prescriptions.) The second of these choices appears inconsistent with the treatment of $\beta\beta(2\nu)$ decay. Since the square of g_A enters the matrix element, this issue is not totally trivial. The striking results of Rodin *et al* suggest that an inconsistent treatment is responsible for some of the spread in table 2. More and better charge-exchange experiments would help teach us whether higher multipole strength is also quenched.

Next, the various versions of the QRPA should be tested against exact solutions in a solvable model that is as realistic as possible. The most realistic used so far are the $SO(5)$-based model first presented by Dussel *et al* (1970) and used to study the QRPA and RQRPA for Fermi $\beta\beta(2\nu)$ decay by Hirsch *et al* (1997), a two-level version of that model used by Engel and Vogel (2004) for the QRPA in Fermi $\beta\beta(2\nu)$ and $\beta\beta(0\nu)$ decay, and an $SO(8)$-based model (Evans *et al* 1981, Dussel *et al* 1986) used to test the QRPA and RQRPA for both Fermi and Gamow–Teller $\beta\beta(2\nu)$ decay by Engel *et al* (1997). It should be possible to extend the $SO(8)$ model to several sets of levels and develop techniques for evaluating $\beta\beta(0\nu)$ matrix elements in it. All these models, however, leave out spin–orbit splitting, which weakens the collectivity of np pairing. A generalization of pseudospin-based models like those of Ginocchio (1980), used in the fermion dynamical symmetry model (Wu *et al* 1994), might be useful provided techniques like those of Hecht (1993) to evaluate expectation values of operators outside the algebra can be extended. Such calculations should help us understand the virtues and deficiencies of QRPA extensions.

Along the same lines, we will need to understand the extent to which such methods can reproduce other observables, and their sensitivity to remaining uncertainties in their parameters. A very careful study of the first issue was made by Aunola and Suhonen (1998) and the second has been explored in several papers, most thoroughly by Stoica and Klapdor-Kleingrothaus (2001). These efforts must be extended. The work is painstaking, and perhaps not much fun as concocting still more variations of the QRPA, but it is crucial if we are to reduce theoretical uncertainty. Self-consistent Skyrme HFB+QRPA, applied to single-β decay by Engel *et al* (1999) and Gamow–Teller resonances by Bender *et al* (2002), may be helpful here; it offers a more general framework, in principal anyway, for addressing the

variability of calculated matrix elements. Solvable models can be useful here too, because they can sometimes supply synthetic data to which parameters can be adjusted (as in Engel and Vogel (2004)).

The best existing shell-model calculation produces smaller matrix elements than most QRPA calculations. Computer speed and memory is now at the point where the state of the shell-model art can be improved. The calculation of the $\beta\beta$ decay of ^{76}Ge by Caurier *et al* (1996) used the $f_{5/2}p_{3/2}p_{1/2}g_{9/2}$ model space, allowing up to eight particles (out of a maximum of 14) into the $g_{9/2}$ level. Nowadays, with the help of the factorization method (Papenbrock and Dean 2003, Papenbrock *et al* 2003), an accurate approximation to full shell-model calculations, we should be able to fully occupy the $g_{9/2}$ level, and perhaps include the $g_{7/2}$ and $f_{7/2}$ levels (though those complicate things by introducing spurious centre-of-mass motion). In addition, one can try through diagrammatic perturbation theory to construct effective $\beta\beta(0\nu)$ operators for the model space that are consistent with the effective interaction. Though perturbation theory has convergence problems, the procedure should at least give us an idea of the uncertainty in the final answers, perhaps also indicating whether result obtained from the 'bare' operators is too large or too small. Research into effective operators has been revived in recent years (Haxton and Song 2000) and we can hope to improve on diagrammatic perturbation theory. One minor source of uncertainty connected with renormalization (which also affects the QRPA) is short-range two-nucleon correlations, currently treated phenomenologically, following Miller and Spencer (1976).

Some *ab initio* calculations of nuclear properties are now possible (Carlson 1998). Although researchers have obtained accurate results only in nuclei with $A \leqslant 12$, we have reason to hope that 'nearly exact' variational or coupled-cluster methods (Kowalski *et al* 2003) will be applied to medium-heavy nuclei such as ^{76}Ge in the next 5 years. Accurate calculations in heavier nuclei are probably further off.

In short, much can be done and we would be well served by coordinated attacks on these problems. There are relatively few theorists working in $\beta\beta$ decay, and their efforts have been fragmented. More collaborations, postdoctoral and PhD projects, meetings, etc would make progress faster. *There is reason to be hopeful that the uncertainty will be reduced.* The shell-model matrix element may be too small because it does not include any particles outside the fp-shell. These particles, as shown by QRPA calculations, make the matrix element larger. We suspect that the results of a better shell-model calculation will be closer than the best current one to the QRPA results and that, as noted above, the spread in those results can be reduced. Finally, other nuclei may be more amenable to a good shell-model calculation than Ge. ^{136}Xe has 82 neutrons (a magic number) making it a particularly good candidate.

5. Other possible mechanisms for double-beta decay

Although the occurrence of $\beta\beta(0\nu)$ decay implies the existence of massive Majorana neutrinos (Schechter and Valle 1982), their exchange need not be the dominant contribution to the decay rate. Almost any physics that violates lepton number can cause $\beta\beta(0\nu)$ decay. A heavy Majorana neutrino can be exchanged, or supersymmetric particles, or a leptoquark. Right-handed weak currents, either leptonic or hadronic, can cause the absorption of an emitted virtual neutrino without the helicity flip that depends on the neutrino mass. These possibilities have been reviewed by Faessler and Šimkovic (1998) and Suhonen and Civitarese (1998). But now we know that there are light neutrinos and that next-generation $\beta\beta(0\nu)$ experiments may well allow us to learn something new about them. The other possibilities are more speculative and instead of analysing them in detail we confine ourselves to the question of whether it is possible, should $\beta\beta(0\nu)$ decay be observed, to determine which mechanism is responsible.

Light-neutrino exchange with right-handed currents is unique in this regard because the contraction of left- and right-handed currents gives rise to a term $q_\rho \gamma_\rho$ in the numerator of the neutrino propagator (see equation (8)) that cancels in the contraction of two left-handed or two right-handed currents. The extra q allows the electron to be emitted in a p-wave as well as an s-wave and introduces a contribution to the amplitude from nucleon recoil. In addition to increasing the phase space, these effects lead to a different single-electron energy distribution and opening-angle dependence than in $\beta\beta(0\nu)$ decay driven by the neutrino mass. An experiment sufficiently sensitive to the energies and paths of individual electrons could therefore determine whether right-handed currents were driving the decay.

The effects of right-handed currents can be shown to be negligible unless there exist some neutrinos with masses larger than a few MeV. Heavy Majorana neutrinos, however, are generated naturally in a variety of models via the see-saw mechanism. The heavy neutrinos complicate matters because they can mediate $\beta\beta(0\nu)$ decay themselves. It seems possible within, e.g., left–right symmetric models, for the exchange of these neutrinos via right-handed currents to compete with or even dominate the exchange of light neutrinos.

The exchange of heavy particles involves short-range propagators that give rise to decay rates of the same form as in the mass-driven mode: simple two-s-wave-electron phase space multiplied by the square of an amplitude[4]. The angular distributions and single-electron spectra will therefore be the same in all these processes. The only way to distinguish one from another is to take advantage of the different nuclear matrix elements that enter the amplitudes (leading to different total decay rates). Unknown parameters such as the effective neutrino mass or the trilinear R-parity-violating supersymmetric coupling (violation of R parity naturally accompanies Majorana neutrino-mass terms) also enter the rates, so several transitions would have to be measured. Šimkovic and Faessler (2002) argue that this is the best accomplished by measuring transitions to several final states in the same nucleus, but if the matrix elements can be calculated accurately enough one could also measure the rates to the ground states of several different nuclei.

The problems in determining the source of $\beta\beta(0\nu)$ decay are mitigated by constraints from other experiments on many extra-standard models. Some of these constraints will be much stronger once the Large Hadron Collider comes on line. If no signs of supersymmetry (for example) appear there, then supersymmetry probably does not exist. If supersymmetric particles are found, but neutralinos make it out of the detector without decaying, then R parity, which prevents the decay of supersymmetric partners into familiar particles, will not be strongly violated. The trilinear R-parity-violating coupling could then be ruled out as the source of double-beta decay. The presence of the LHC will make complementary experiments on $\beta\beta(0\nu)$ decay still more attractive.

Some theories posit the emission Goldstone bosons called 'Majorons', together with the emission of electrons, in $\beta\beta(0\nu)$ decay. If Majorons were emitted in a detector the total energy carried by the newly created electrons would vary from event to event. A long lifetime for this decay would make it difficult to detect above a $\beta\beta(2\nu)$-decay background. Double-beta decay has also been discussed as a test of special relativity and the equivalence principle (Klapdor-Kleingrothaus *et al* 1999). Finally, very recent attempts to unify the dark sector and neutrino physics (Kaplan *et al* 2004) posit a scalar field with variations on the scale of millimetres that couples to neutrinos. If such a field existed, the rate of $\beta\beta(0\nu)$ decay might depend on the density of matter in which the process occurred.

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[4] Two-nucleon correlations do not suppress the effects of heavy particles, which can be transmitted between nucleons by pions (Prezeau *et al* 2003, Faessler *et al* 1997).
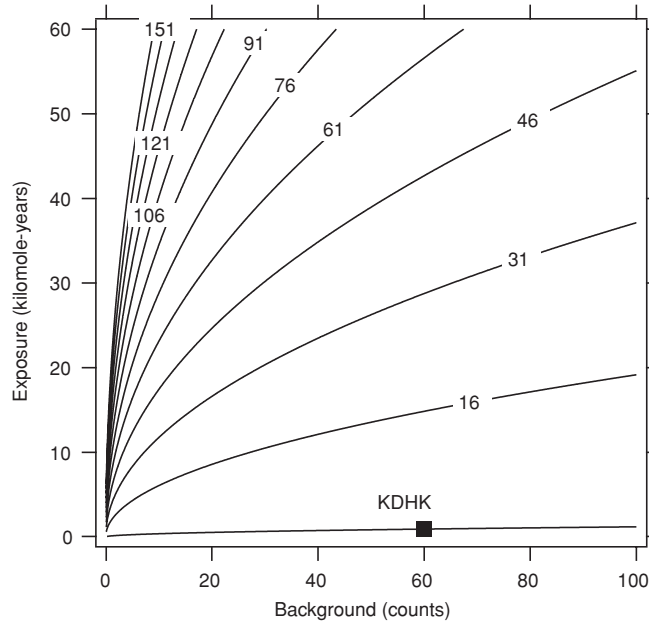
Figure 4. This contour plot shows the half-life, in units of 10^{25} years, for a peak of 5σ significance for a given exposure and background. The KDHK point is shown.

6. Experimental situation

If an experiment observes $\beta\beta(0\nu)$ it will have profound physics implications. Such an extraordinary claim will require extraordinary evidence. The recent claim (Klapdor-Kleingrothaus *et al* 2004a, 2004b) for an observation of $\beta\beta(0\nu)$ has been controversial (see discussion below). Also previous 'false peaks' in $\beta\beta$ spectra have appeared near a $\beta\beta(0\nu)$ endpoint energy (see discussion in Moe and Vogel (1994), p 273). One must ask the question: what evidence is required to convincingly demonstrate that $\beta\beta(0\nu)$ has been observed? Low-statistical-significance peaks ($\approx 2\sigma$) have faded with additional data, so one must require strong statistical significance (perhaps 5σ) (see figure 4). This will require a large signal-to-noise ratio that will most likely be accomplished by an ultra-low-background experiment whose source is its detector. Such experiments are usually calorimetric and provide little information beyond just the energy measurement.

How does an experiment demonstrate that an observed peak is actually due to $\beta\beta$ decay and not some unknown radioactivity? Additional information beyond just an energy measurement may be required. For example, although there is some uncertainty associated with the matrix elements, it is not so large that a comparison of measured rates in two different isotopes could not be used to demonstrate consistency with the Majorana-neutrino hypothesis. Alternatively, experiments that provide an additional handle on the signal, for example by measuring a variety of kinematical variables, demonstrating that two electrons are present in the final state, observing the γ rays associated with an excited state, or identifying the daughter nucleus, may lend further credibility to a claim. Experiments that provide this extra handle may require a significantly more complicated apparatus and therefore face additional challenges.

The exciting aspect of $\beta\beta$ research today is that many proposed experiments intend to reach a Majorana mass sensitivity of $\sqrt{\delta m_{\text{atm}}^2}$. Several different isotopes and experimental

Table 3. A summary of the recent $\beta\beta(0\nu)$ results. The $\langle m_{\beta\beta}\rangle$ limits are those deduced by the authors. All limits are at 90% confidence level unless otherwise indicated. The columns providing the exposure and background are based on arithmetic done by the authors of this paper, who take responsibility for any errors in interpreting data from the original sources.

Isotope	Exposure (kmol year)	Background (counts)	Half-life limit (year)	$\langle m_{\beta\beta}\rangle$ (meV)
^{48}Ca	5×10^{-5}	0	$>1.4 \times 10^{22}$	<7200–44700^a
^{76}Ge	0.467	21	$>1.9 \times 10^{25}$	$<350^b$
^{76}Ge	0.117	3.5	$>1.6 \times 10^{25}$	<330–1350^c
^{76}Ge	0.943	61	$=1.2 \times 10^{25}$	$=440^d$
^{82}Se	7×10^{-5}	0	$>2.7 \times 10^{22}(68\%)$	$<5000^e$
^{100}Mo	5×10^{-4}	4	$>5.5 \times 10^{22}$	$<2100^f$
^{116}Cd	1×10^{-3}	14	$>1.7 \times 10^{23}$	$<1700^g$
^{128}Te	Geochem.	NA	$>7.7 \times 10^{24}$	<1100–1500^h
^{130}Te	0.025	5	$>5.5 \times 10^{23}$	<370–1900^i
^{136}Xe	7×10^{-3}	16	$>4.4 \times 10^{23}$	<1800–5200^j
^{150}Nd	6×10^{-5}	0	$>1.2 \times 10^{21}$	$<3000^k$

[a] Ogawa *et al* (2004).
[b] Klapdor-Kleingrothaus *et al* (2001a, 2001b, 2001c).
[c] Aalseth *et al* (2002a).
[d] Klapdor-Kleingrothaus *et al* (2004a, 2004b).
[e] Elliott *et al* (1992).
[f] Ejiri *et al* (2001).
[g] Danevich *et al* (2003).
[h] Bernatowicz *et al* (1993).
[i] Arnaboldi *et al* (2004a).
[j] Luescher *et al* (1998).
[k] De Silva *et al* (1997).

techniques are being pursued actively and many of the programmes look viable. In this section we describe the current situation in experimental $\beta\beta$ decay.

6.1. Results to date

Table 3 lists the recent $\beta\beta(0\nu)$ results. The best limits to date come from the enriched Ge experiments. The two experiments had comparable results although the Heidelberg–Moscow result was marginally better. The $T_{1/2}^{0\nu}$ limits near 2×10^{25} years results in a $\langle m_{\beta\beta}\rangle$ limit near 300 meV, with an uncertainty of about a factor of 3 because of the uncertainty in $|M_{0\nu}|$. One recent paper (Zdesenko *et al* 2002) performed a joint analysis of the two experiments and found $T_{1/2}^{0\nu} > 2.5 \times 10^{25}$ years. Most of the results listed in table 3 are at least a few years old. The obvious exceptions to this are the Te and Cd results. CUORICINO continues to collect data.

6.1.1. A claim for the observation of $\beta\beta(0\nu)$. In early 2002, a claim for the observation of $\beta\beta(0\nu)$ was published (Klapdor-Kleingrothaus *et al* 2002a). The paper made a poor case for the claim and drew strong criticism (Aalseth *et al* 2002b, Feruglio *et al* 2002, Zdesenko *et al* 2002). The initial response to the criticism was emotional (Klapdor-Kleingrothaus 2002). In addition, one of the original co-authors wrote a separate reply (Harney 2001) that mostly defended the claim yet acknowledged some significant difficulty with the analysis. This author's name does not appear on later papers. More recently, however, supporting evidence for the claim has been presented and we recommend the reader study Klapdor-Kleingrothaus
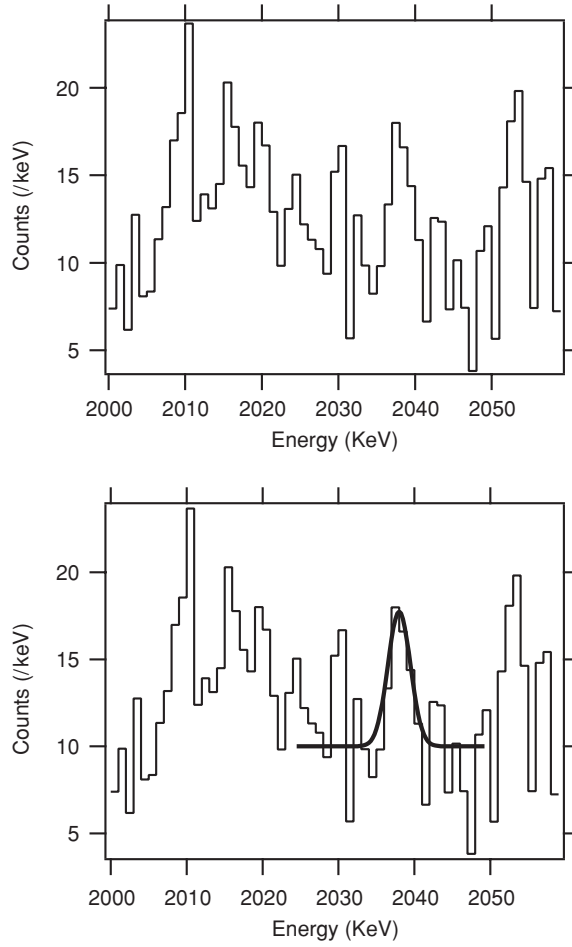
Figure 5. The spectrum from the Heidelberg–Moscow experiment upon which the claim for $\beta\beta(0\nu)$ is based. The data in the two panels are identical. The lower panel has a Gaussian curve to indicate the strength of the claimed $\beta\beta(0\nu)$ peak.

et al (2002b) for a good discussion of the initial evidence and Klapdor-Kleingrothaus *et al* (2004a, 2004b) for the most recent data analysis. Importantly, this later paper includes additional data and therefore an increase in the statistics of the claim. In this subsection we summarize the current situation. (We use the shorthand KDHK to refer to the collection of papers supporting the claim.)

Figure 5 shows the spectrum corresponding to 71.7 kg year of data from the Heidelberg–Moscow experiment between 2000 and 2060 keV (**Klapdor-Kleingrothaus *et al* 2004a, 2004b**). This spectrum is shown here to assist the casual reader in understanding the issues. However, the critical reader is encouraged to read the papers listed in the references as the authors analyse several variations of these data using different techniques. The fit about the expected $\beta\beta(0\nu)$ peak energy yields 28.75 ± 6.86 counts assigned to $\beta\beta(0\nu)$. The paper claims a significance of approximately 4σ for the peak, where the precise significance value depends on the details of the analysis. The corresponding best-fit lifetime, $T_{1/2}^{0\nu} = 1.19 \times 10^{25}$ years (**Klapdor-Kleingrothaus *et al* 2004a, 2004b**), leads to a $\langle m_{\beta\beta} \rangle$ of 440 meV with the matrix element calculation of Staudt *et al* (1990) chosen by the authors.

In the region between 2000 and 2100 keV, the KDHK analysis of 2002 found a total of seven peaks. Four of these were attributed to ^{214}Bi (2011, 2017, 2022, 2053 keV), one was attributed to $\beta\beta(0\nu)$ decay (2039 keV), and two were unidentified (2066 and 2075 keV). The KDHK analysis of 2004 does not discuss the spectrum above 2060 in detail. An additional possible feature may also be present near 2030 keV. A study (Klapdor-Kleingrothaus *et al* 2003b) comparing simulation to calibration with ^{214}Bi demonstrates that if the location of the Bi is known, the spectrum can be calculated. Furthermore, the relative strengths of the strong Bi lines at 609, 1764 and 2204 keV can be used to determine the location of the activity. Because the results of summing effects depend on the proximity of the activity, its location is critical for the simulation of the weak peaks near the $\beta\beta(0\nu)$ endpoint. The study also shows that the spectrum cannot be naively estimated, as was done in Aalseth *et al* (2002b). In fact, table VII in Klapdor-Kleingrothaus *et al* (2002b) finds, even with a careful simulation, that the expected strengths of the ^{214}Bi peaks in the 2000–2100 keV region are not predicted well by scaling to the strong peaks. That is, the measured intensities of the weak peaks are difficult to simulate without knowing the exact location of the activity. Furthermore, the deduced strengths of the weak lines are more intense than expected by scaling from the strong peaks, even though the activity location is chosen to best describe the relative intensities of the strong peaks.

Double-beta decay produces two electrons that have a short range in solid Ge. Therefore, the energy deposit is inherently localized. Background process, such as the γ rays from ^{214}Bi, tend to produce multiple energy deposits. The pulse waveform can be analysed to distinguish single-site events (SSE) from multiple-site events. Such an analysis by KDHK (Klapdor-Kleingrothaus *et al* 2003b, 2004a, 2004b) tends to indicate that the Bi lines and the unidentified lines behave as multiple-site events, whereas the $\beta\beta(0\nu)$ candidate events behave as SSE. Note, however, that the statistics are still poor for the experimental lines and this conclusion has a large uncertainty. Nonetheless, this feature of the data is very intriguing and clearly a strength of the KDHK analysis.

An analysis by Zdesenko *et al* (2003) points out the strong dependence of the result on the choice of the window width of the earlier 2002 analysis. The KDHK analysis argues that a small window is required because of the neighbouring background lines. Even so, their Monte Carlo analysis shows that the result becomes less stable for small windows (see figure 9 in Klapdor-Kleingrothaus *et al* 2002b). Zdesenko *et al* (2002) also remind us that the significance of a signal is overestimated when the regions used to estimate the background are comparable to the region used to determine the signal (Narsky 2000). The report of Klapdor-Kleingrothaus *et al* (2004a, 2004b) fits a wide region containing several peaks simultaneously after using a Bayesian procedure to identify the location of the peaks.

The claim for $\beta\beta(0\nu)$ decay was made by a fraction of the Heidelberg–Moscow collaboration. A separate group of the original collaboration presented their analysis of the data at *the IV International Conference on Non-Accelerator New Physics* (Bakalyarov *et al* 2003). They indicate that the data can be separated into two distinct sets with different experimental conditions. One set includes events that are described as 'underthreshold pulses' and one set that does not. Analysis of the two sets produce very different conclusions about the presence of the claimed peak. They conclude that the evidence is an experimental artefact and not a result of $\beta\beta(0\nu)$ decay. KDHK responds (Klapdor-Kleingrothaus *et al* 2004a, 2004b) that these corrupt data were not included in their analysis.

Traditionally, $\beta\beta$ experiments have ignored systematic uncertainties in their analysis. Only recently with the start-up of high-statistics $\beta\beta(2\nu)$ results has this situation begun to change. Historically, $\beta\beta(0\nu)$ results have always been quoted as upper limits based on low count rates. As a result, systematic uncertainties tended to be negligible in the final quoted

values. With a claim of a positive result, however, the stakes are dramatically raised. It is clear that it is difficult to produce a convincing result when the signal counts are comparable to expected statistical fluctuations in the background. The further presence of nearby unidentified peaks makes the case even harder to prove. Although KDHK does discuss some systematic uncertainties qualitatively and indicates they are small (in the position of the $\beta\beta(0\nu)$ peak, and the expected peak width, for example), there is no consideration of an uncertainty associated with the background model.

The next round of proposed $\beta\beta(0\nu)$ experiments are designed to reach $\sqrt{\delta m_{\mathrm{atm}}^2}$ and therefore will quickly confirm or repudiate this claim. This is fortunate since the feature near 2039 keV in the KDHK claim will likely require an experimental test. These experiments should provide a detailed listing of all identified systematic uncertainties and a quantified estimate of their size. Furthermore, because the stakes are very high and there will be many people who are biased, either for or against the KDHK claim, blind analyses should also become part of the experimental design.

6.2. Future possibilities for $\beta\beta(0\nu)$ experiments

The recent review by Elliott and Vogel (2002) describes the basics of experimental $\beta\beta(0\nu)$ decay in some detail. Therefore, we refer the reader to that article and only summarize the status of the various projects. Table 4 lists the proposals.

6.2.1. CANDLES.
The CANDLES collaboration has recently published the best limit on $\beta\beta(0\nu)$ decay of 1.4×10^{22} year in ^{48}Ca (Ogawa *et al* 2004). Using the ELEGANTS VI detector, this experiment consisted of 6.66 kg of $CaF_2(Eu)$ crystals surrounded by CsI crystals, a layer of Cd, a layer of Pb, a layer of Cu and a layer of LiH-loaded paraffin, all enclosed within an air-tight box. This box was then surrounded by boron-loaded water tanks and situated underground at the Oto Cosmo Observatory. This measurement successfully demonstrated the use of these crystals for $\beta\beta$ studies.

An improved version of this crystal technology, the CANDLES-III detector (Kishimoto *et al* 2004), is presently being constructed with 200 kg of CaF_2 crystals. These crystals have better light transmission than the $CaF_2(Eu)$ crystals. This design uses 60 10 cm^3 CaF_2 crystals, which are immersed in liquid scintillator. The collaboration has also proposed a 3.2 t experiment that hopes to reach 100 meV for $\langle m_{\beta\beta} \rangle$.

6.2.2. COBRA.
The COBRA experiment (Zuber 2001) uses CdZnTe or CdTe semiconductor crystals. These crystals have many of the advantages of Ge detectors but, in addition, operate at room temperature. Because the crystals contain Cd and Te, there are seven $\beta\beta$ and $\beta^+\beta^+$ isotopes contained. The final proposed configuration is for 64 000 1 cm^3 crystals for a total mass of 370 kg. The collaboration has already obtained 30 keV resolution at 2.6 MeV with these detectors and has published initial $\beta\beta$-decay studies (Kiel *et al* 2003). Background studies are the current focus of the efforts. Although it is tempting to focus on the naturally isotopic abundant ^{130}Te for $\beta\beta(0\nu)$ decay, the presence of the higher Q value ^{116}Cd creates a serious background from its $\beta\beta(2\nu)$ decay. Detectors enriched in ^{116}Cd are probably required to reach 45 meV.

6.2.3. CUORE.
The CUORICINO experiment uses 41 kg of TeO_2 crystals operated at 10 mK as bolometers. During the initial cool down, some of the cabling failed and hence not all crystals were active. As a result the initial run had contained about

Table 4. A summary of the $\beta\beta(0\nu)$ proposals. Background estimates were not available for all projects. The quantity of isotope includes the estimated efficiency for $\beta\beta(0\nu)$.

Collaboration	Isotope (kmol)	Anticipated background (counts year^{-1})	Detector description
CAMEO[a]	^{116}Cd (2)	Few	CdWO$_4$ crystals in liq. scint.
CANDLES[b]	^{48}Ca (0.04)		CaF$_2$ crystals in liq. scint.
COBRA[c]			CdTe semiconductors
CUORE[d]	^{130}Te (1.4)	$\approx$60	TeO$_2$ bolometers
DCBA[e]	^{82}Se (2)	$\approx$40	Nd foils and tracking chambers
EXO[f]	^{136}Xe (4.2)	<1	Xe TPC
GEM[g]	^{76}Ge (11)	$\approx$0.8	Ge detectors in LN
GENIUS[h]	^{76}Ge (8.8)	$\approx$0.6	Ge detectors in LN
GSO[i]	^{160}Gd (1.7)		Gd$_2$SiO$_5$ crystals in liq. scint.
Majorana[j]	^{76}Ge (3.5)	$\approx$1	Segmented Ge detectors
MOON[k]	^{100}Mo (2.5)	$\approx$8	Mo foils and plastic scint.
MPI bare Ge[l]	^{76}Ge (8.8)		Ge detectors in LN
Nano-crystals[m]	$\approx$100 kmol		Suspended nanoparticles
Super-NEMO[n]	^{82}Se (0.6)	$\approx$1	Foils with tracking
Xe[o]	^{136}Xe (6.3)	$\approx$118	Xe dissolved in liq. scint.
XMASS[p]	^{136}Xe (6.1)		Liquid Xe

[a] Bellini *et al* (2001).
[b] Kishimoto *et al* (2004).
[c] Zuber (2001).
[d] Arnaboldi *et al* (2004b).
[e] Ishihara *et al* (2000).
[f] Danilov *et al* (2000).
[g] Zdesenko *et al* (2001).
[h] Klapdor-Kleingrothaus *et al* (2001a, 2001b, 2001c).
[i] Danevich *et al* (2001), Wang *et al* (2000).
[j] Gaitskell *et al* (2003).
[k] Ejiri *et al* (2000).
[l] Abt *et al* (2004).
[m] McDonald (2004).
[n] Sarazin *et al* (2000).
[o] Caccianiga and Giammarchi (2001).
[p] Moriyama *et al* (2001).

10 kg of ^{130}Te (Arnaboldi *et al* 2004a). An initial exposure of 5.46 kg year, with an energy resolution of 9.2 keV FWHM resulted in a limit $T_{1/2}^{0\nu} > 7.2 \times 10^{23}$ year at 90% confidence level (Norman 2004). The background in the region of interest for this run was 0.22 ± 0.04 counts keV^{-1} kg^{-1} year^{-1}. The experiment has been suspended in order to fix the cabling. Afterwards, a 3 year run with the full mass will have a sensitivity of 10^{25} years.

The CUORICINO project is a prototype for the CUORE proposal. CUORE would contain 760 kg of TeO$_2$. With the anticipated improvement in background to better than 0.01 counts keV^{-1} kg^{-1} year^{-1}, the half-life sensitivity is $\approx 7 \times 10^{26}$ year or a few 10 of meV for $\langle m_{\beta\beta} \rangle$ (Arnaboldi *et al* 2004b).

6.2.4. DCBA. The drift chamber beta-ray analyser (DCBA) (Ishihara *et al* 2000) is a tracking chamber within a 1.6 kG magnetic field that can examine any $\beta\beta$ source that can be formed into a thin foil. On both sides of the foil are tracking regions filled with 1 atm He gas. The

future programme calls for 25 m^2 of source foils contained within each of 40 modules, for a total of approximately 600 kg of source material. The half-life sensitivity is estimated to be a few $\times 10^{26}$ year for ^{82}Se, ^{100}Mo or ^{150}Nd, assuming an enrichment of 90%.

6.2.5. EXO. The EXO project proposes to use 1–10 t of about 80% enriched liquid Xe as a time projection chamber (Danilov *et al* 2000). Development of a high-pressure gas TPC is being pursued in parallel. In addition to measuring the energy deposit of the electrons, the collaboration is developing a technique for extracting the daughter Ba ion from the Xe and detecting it offline. Observing the daughter in real time with the $\beta\beta$ decay is a powerful technique for reducing background. With a 1 t experiment, they anticipate sensitivity to a lifetime of 8×10^{26} year.

The collaboration has had some good progress on the research and development required to demonstrate that this technically challenging project is feasible. They have determined the energy resolution by using both ionization and scintillation measurements in liquid Xe. The resolution result $\sigma = 3\%$ stated in Conti *et al* (2003) was measured at 570 keV. Assuming a statistical dependence on energy this means about 1.5% resolution at the $\beta\beta(0\nu)$ energy of 2480 keV. They have also built an atom trapping system and have observed lone Ba ions in an optical trap. Furthermore, they have begun experiments to demonstrate that the ions are trapped and observable in an appreciable Xe gas background (Piepke 2004). Finally, using a ^{222}Ra source they are testing the Ba extraction technology. Ra and Ba have similar chemistry, but the radioactive decay of Ra makes it a convenient test material.

The EXO team is currently preparing a 200 kg enriched-Xe experiment to operate at the Waste Isolation Pilot Plant (WIPP). This prototype will not initially include Ba extraction.

6.2.6. MOON. The MOON project (Ejiri *et al* 2000) proposes to use 1 t of Mo enriched to 85% in ^{100}Mo. The beauty of ^{100}Mo is that it not only is a good $\beta\beta(0\nu)$ isotope, but also has a large charge–current cross section for low-energy solar neutrinos. Thus the MOON detector is being designed to perform both experiments. MOON measures individual β rays from $\beta\beta$ decay, which helps identify events arising from the light-neutrino-mass $\beta\beta$ mechanism and improve the background rejection.

The reference design for MOON is a collection of modules of interleaved plate and fibre scintillators sandwiching Mo foils. Each foil is about 20 mg cm^{-2}. Good position resolution is required to exploit the timing of the radioactive product produced in the solar–neutrino interaction. The position is determined by the fibre scintillators, whereas the scintillator plate provides the energy resolution ($\sigma \approx 2.2\%$ at 3 MeV). Other detector options are under consideration but a 1 kg prototype of the reference design is currently being built.

6.2.7. Majorana. The Majorana Collaboration proposes to field 500 kg of 86% enriched Ge detectors (Gaitskell *et al* 2003). By using segmented crystals and pulse-shape analysis, multiple-site events can be identified and removed from the data. Internal backgrounds from cosmogenic radioactivities will be greatly reduced by these cuts and external γ-ray backgrounds will also be preferentially eliminated. Remaining will be single-site events like that due to $\beta\beta$. The sensitivity is anticipated to be 4×10^{27} year.

Several research and development activities are currently proceeding. The collaboration is building a multiple-Ge detector array, referred to as MEGA, that will operate underground at the Waste Isolation Pilot Plant near Carlsbad, NM, USA. This experiment will investigate the cryogenic cooling of many detectors sharing a cryostat in addition to permitting studies of detector-to-detector coincidence techniques for background and signal identification.

A number of segmented crystals are also being studied to understand the impact of segmentation on background and signal. This SEGA programme consists of one 12-segment enriched detector and a number of commercially available segmented detectors. Presently, commercially available segmented detectors are fabricated from n-type crystals. Such crystals are much more prone to surface damage and thus more difficult to handle when packaging inside their low-background cryostats. Hence the collaboration is also experimenting with segmenting p-type detectors.

6.2.8. Bare Ge crystals. The GENIUS collaboration (Klapdor-Kleingrothaus *et al* 2001a, 2001b, 2001c) proposed to install 1 t of enriched bare Ge crystals in liquid nitrogen. By eliminating much of the support material surrounding the crystals in previous experiments, this design is intended to reduce backgrounds of external origin. Note how this differs from the background-reduction philosophy associated with pulse-shape analysis coupled with crystal segmentation. The primary advocates for this project indicate (Klapdor-Kleingrothaus *et al* 2004a, 2004b) that its motivation has been questioned by their own claim of evidence for $\beta\beta(0\nu)$ decay. Even so, the GENIUS test facility (**Klapdor-Kleingrothaus *et al* 2003a**) is being operated to demonstrate the effectiveness of operating crystals naked in liquid cryogen.

Another group at the Max Plank Institute in Heidelberg, however, is proposing to pursue a similar idea. They have recently submitted a letter of intent (**Abt *et al* 2004**) to the Gran Sasso Laboratory. They propose to collect the enriched Ge crystals from both the Heidelberg–Moscow and IGEX experiments and operate them in either liquid nitrogen or liquid argon. As a second phase of the proposal, they plan to purchase an additional 20 kg of enriched Ge detectors (most likely segmented) and operate with a total of 35 kg for about 3 years. Finally, they eventually plan to propose a large ton-scale experiment. It should be noted that this collaboration and the Majorana collaboration are cooperating on technical developments and if a future ton-scale experiment using ^{76}Ge proceeds these two groups will most likely merge and optimally combine the complementary technologies of bare-crystal operation and PSA segmentation.

6.3. Nanocrystals

Some elements may be suitable for loading liquid scintillator with metallic-oxide nanoparticles. Since Rayleigh scattering varies as the sixth power of the particle radius, it can be made relatively small for nanoparticles of radii below 5 nm. Particles of this size have been developed and commercial suppliers of ZrO_2, Nd_2O_3, etc are available. Absorption of the materials must also be taken into account, but some of the metal oxides such as ZrO_2 and TeO_2 are quite transparent in the optical region because of the substantial band gaps in these insulators. Some members of the SNO collaboration (**McDonald 2004**) have been studying a configuration equivalent to filling the SNO cavity with a 1% loaded liquid scintillator or approximately 10 t of isotope after the present heavy water experiment is completed. The group is currently researching the optical properties of potential nano-crystal solutions. In particular, one must demonstrate that sufficient energy resolution is achievable with liquid scintillator.

6.3.1. Super-NEMO. The recent progress of the NEMO-3 programme (**Sarazin *et al* 2000**) has culminated in excellent $\beta\beta(2\nu)$ results. In particular, the energy spectra from ^{100}Mo contain nearly 10^5 events and are nearly background free. These data permit, for the first time, a precise study of the spectra. In fact, there is hope that the data (**Sutton 2004**) will demonstrate whether the $\beta\beta(2\nu)$ transition is primarily through a single intermediate state or

through a number of states (Šimkovic *et al* 2001). The detector consists of several thin foils placed between Geiger-drift cells, surrounded by a scintillator calorimeter.

NEMO-3 began operation in February 2003 with several isotopes, ^{100}Mo being the most massive at 7 kg, and plans to operate for 5 years. The collaboration plans to increase the mass of ^{82}Se from 1 kg to 20 kg and begin an additional 5 year run. Presently, a Rn-removal trap is being installed to reduce the background, and operation should begin again by the summer of 2004. The anticipated sensitivities for $T_{1/2}^{0\nu}$ are 5×10^{24} year and 3×10^{25} year for Mo and Se, respectively. For the Se data, this corresponds to $\langle m_{\beta\beta} \rangle$ below 100–200 meV.

A much bigger project is currently being planned that would use 100 kg of source. The apparatus would have a large footprint however and the Frejus tunnel where NEMO-3 is housed would not be large enough to contain it. Currently the collaboration is studying the design of such a detector.

6.3.2. XMASS. The XMASS collaboration (**Suzuki 2004, Moriyama** *et al* **2001**) plans to build a 10 t natural Xe liquid scintillation detector. They expect an energy resolution of 3% at 1 MeV and hope to reach a value for $T_{1/2}^{0\nu} > 3.3 \times 10^{27}$ year. This detector would also be used for solar–neutrino studies and a search for dark matter.

6.3.3. Borexino CTF. In August of 2002, operations at the Borexino experiment resulted in the spill of scintillator. This led to the temporary closure of Hall C in the Gran Sasso Laboratory and a significant change in operations at the underground laboratory. As a result, efforts to convert the counting test facility (CTF) or Borexino itself into a $\beta\beta(0\nu)$ experiment (**Bellini** *et al* **2001, Caccianiga and Giammarchi 2001**) have been suspended (**Giammarchi 2004**).

6.4. The search for decays to excited states

Searches for $\beta\beta$ to excited states in the daughter atom have been performed in a number of isotopes but only observed in ^{100}Mo (the experimental situation is reviewed by Barabash (2000) and ^{150}Nd Barabash *et al* (2004)). These experiments typically search for the γ rays that characterize the excited states and therefore are not mode-specific searches. The interpretation therefore is that the measured rate (or limit) is for the $\beta\beta(2\nu)$ mode. These data may be very useful to QRPA nuclear theory because the behaviour of the nuclear matrix elements with respect to g_{pp} for the excited state decays is different than for transitions to the ground state (**Griffiths and Vogel 1992, Aunola and Suhonen 1996**). Thus, the excited state transitions probe different aspects of the theory and may provide insight into the physics of the matrix elements.

A further reason for interest in decays to the excited state, as mentioned earlier, is the potential ability to discover the process mediating the decay (**Šimkovic and Faessler 2002, Tomoda 2000**). However, the decay rate to an excited state is 10–100 times smaller than rate to the ground state (**Suhonen 2000a, 2000b**). Furthermore the structure of the excited state in the daughter nucleus is not as well understood as the ground state, and this increases the relative uncertainty in the nuclear matrix element.

6.5. The search for $\beta^+\beta^+$ modes of decay

The β^+ modes of decay have not received the attention of the β^- modes because of the greatly reduced phase space and corresponding long half-lives. However, their detection would provide additional matrix-element data. Furthermore, if the zero-neutrino mode were

detected, it might provide a handle on whether the decay is predominantly mediated by a light neutrino or by right-handed currents (Hirsch *et al* 1994).

Radiative neutrinoless double-electron capture is a possible alternative to traditional neutrinoless double-beta decay (Sujkowski and Wycech 2003, Wycech and Sujkowski (2004)). In this process, two electrons are captured from the atomic electron cloud and a radiated photon carries the full Q-value for the decay. A resonance condition can enhance the rate when the energy release is close to the 2P–1S energy difference. In this case, high-Z, low-energy-release isotopes are favoured (e.g. ^{112}Sn). Unfortunately the mass differences for the candidate isotopes are not known precisely enough to accurately predict the overlap between the two energies. If a favourable overlap does exist, however, the sensitivity to $\langle m_{\beta\beta}\rangle$ might rival that of $\beta\beta(0\nu)$ decay.

6.6. Towards a 100 kg experiment

The KDHK spectrum shows a feature very close to the $\beta\beta(0\nu)$ endpoint. This intriguing result will need to be confirmed or refuted experimentally. One can see the required operation parameters for a confirmation experiment from Klapdor-Kleingrothaus *et al* (2004a, 2004b). One needs about 75 kg year of exposure, and a background lower than about 0.5 counts kg^{-1} year^{-1}. Note that most of the proposals described above will all accomplish this very early on in their programme if they meet their design goals. If instead one designs an experiment only to test the claim (not to provide a precise measurement of the $T_{1/2}^{0\nu}$) then a 100 kg experiment could provide the answer after a modest run time.

If the KDHK result holds up, it will be a very exciting time for neutrino-mass research. A $\langle m_{\beta\beta}\rangle$ near 400 meV means that β-decay experiments and cosmology will be sensitive to the mass. As a result, one can certainly imagine a not-too-distant future in which we know the neutrino mass and its Majorana–Dirac character. Towards this goal, a precision measurement of $\langle m_{\beta\beta}\rangle$ will be required. To accomplish this, we will need more than one $\beta\beta$ experiment, each with a half-life measurement accurate to 10–20%. At this level the uncertainty will be dominated by the matrix element uncertainty even if future calculations can be trusted to 50%. With two experiments utilizing different isotopes, one might disentangle the uncertainty in $|M_{0\nu}|$.

6.7. Towards the 100 t experiment

The next generation of experiments hopes to be sensitive to $\sqrt{\delta m_{\mathrm{atm}}^2}$. If they fail to see $\beta\beta(0\nu)$ at that level, the target for the succeeding generation of efforts will be $\sqrt{\delta m_{\mathrm{sol}}^2}$. This scale is an order of magnitude lower and hence will require two orders of magnitude more isotopic mass, approximately 100 t of isotope.

A 100 t experiment will have to face the same technical challenges associated with radioactive backgrounds and energy resolution as today's proposals. (See, Elliott and Vogel (2002) for a discussion of these issues.) In addition, a background from solar neutrinos will also have to be considered. Solar neutrinos can result in a background via elastic scattering or charged current interactions. The rate ($R_{\beta\beta}$) of $\beta\beta(0\nu)$ events can be written as

$$R_{\beta\beta} = \frac{1}{M}\frac{\mathrm{d}N}{\mathrm{d}t} = \frac{\lambda N}{M} \approx \frac{420}{MW(g)}\left(\frac{10^{27}\,y}{T_{1/2}^{0\nu}}\right)\mathrm{year}^{-1}\,\mathrm{t}^{-1}, \tag{24}$$

where $MW(g)$ is the molecular weight of the $\beta\beta$ isotope and M is the mass of the target in tons. For pure ^{136}Xe, this would result in about 3 $\beta\beta(0\nu)$ events per year per ton for $T_{1/2}^{0\nu} = 10^{27}$ year.

Elastic scattering (ES) rates from ^{8}B solar neutrinos can be comparable to this $\beta\beta(0\nu)$ rate if the target material contains the isotope at a low fraction. For example, a 2% solution of Xe in liquid scintillator has been discussed as a possible $\beta\beta(0\nu)$ experiment. Of course the number of ES events within the $\beta\beta(0\nu)$ window depends on the resolution and therefore we need the cross section per unit energy $(\Delta\sigma/\Delta E)$(Bahcall 1989)

$$\frac{\Delta\sigma}{\Delta E} \approx 9 \times 10^{-48} \text{ cm}^2 \text{ keV}^{-1}. \tag{25}$$

The rate of ES events (R_8) is then given by

$$R_8 = \left(F_8 \frac{\Delta\sigma}{\Delta E} N_A \right) \left(\Delta E \frac{1}{MW_t(g)} N_e \right)$$

$$\approx (8 \times 10^{-4} \text{ keV}^{-1} \text{ year}^{-1} \text{ t}^{-1}) \left(\Delta E \frac{M}{MW_t(g)} N_e \right) \tag{26}$$

where F_8 is the ^{8}B neutrino flux (5×10^6 cm^{-2} s^{-1}), N_A is Avagadro's number, ΔE is the $\beta\beta(0\nu)$ energy window in keV, MW_t is the target molecular weight, and N_e is the number of electrons per molecule of the target. For a pure Xe target with an energy window of 50 keV, we find $R_8 \approx 0.02$ year^{-1} t^{-1}. This background is not a problem for any pure Xe detector that proposes a half-life sensitivity of 10^{28} year. It is significant for a detector with only 2% Xe at $T_{1/2}^{0\nu} = 10^{27}$ year.

Charge–current (CC) scattering of solar neutrinos, especially the large flux of solar pp neutrinos may also be a background for $\beta\beta(0\nu)$ decay. As pointed out by Raghavan (1997), some $\beta\beta$ isotopes make interesting targets for pp solar neutrino experiments because the reaction produces a radioactive isotope, the intermediate nucleus in the $\beta\beta$ process. The decay of this product nucleus provides a coincidence signature for the CC reaction. However for several reasons, this process must be considered as a background for $\beta\beta(0\nu)$ decay. First, the β-decay Q-value of the intermediate nucleus is larger than the $\beta\beta$ Q-value for most of the nuclei. Furthermore, the half-life of the intermediate nucleus is often too long for an effective coincidence identification. Finally, the end product β-decay daughter nucleus is the same as the $\beta\beta$ daughter.

The rate of CC events (R_{CC}) in an isotropic sample can be written as

$$R_{CC} = F_{CC} \langle \sigma_{CC} \rangle \frac{N_A}{MW(g)} f_E = 18 R_{SNU} \frac{f_E}{MW(g)} (\text{year}^{-1} \text{ t}^{-1}), \tag{27}$$

where F_{CC} is the solar neutrino flux, $\langle \sigma_{CC} \rangle$ is the spectrum-weighted CC cross section, f_E is the fraction of intermediate nucleus β decays that fall within the $\beta\beta(0\nu)$ energy window, and R_{SNU} is the rate of CC interactions in SNU (10^{-36} interactions s^{-1} per target atom). For example, the total $R_{CC} \approx 120$ year^{-1} t^{-1} (that is with $f_E = 1$) in the MOON proposal is much higher than $R_{\beta\beta} \approx 4$ year^{-1} t^{-1} for a half-life of 10^{27} year. In that case however, the intermediate nucleus has a convenient lifetime of 16 s, short enough that a delayed coincidence can identify the decay and separate it event by event from $\beta\beta(0\nu)$ candidate events.

R_{CC} in each $\beta\beta$ isotope must be considered separately. For example, ^{76}Ge, ^{116}Cd and ^{130}Te do not have a low-lying intermediate nucleus level reachable by the high-flux pp neutrinos, and in the case of Ge, ^{7}Be neutrinos. Alternatively, ^{82}Se, ^{176}Yb and ^{160}Gd have significant cross sections but the long-lived intermediate nucleus prevents easy anti-coincidence identification. In the important case of ^{136}Xe, little is known about the structure of the intermediate-nucleus (^{136}Cs) excited states. The decay of ^{136}Cs is to highly excited levels in ^{136}Ba that decay themselves via γ emission. This results in a relatively large f_E, but also a γ-ray cascade that might provide a signature to eliminate the potential background. ^{150}Nd is another isotope in which little is known about the intermediate-nucleus excited states.

7. Conclusion

If the reader retains anything from this review, it should be that the information recently acquired from oscillation experiments makes this an exciting time for $\beta\beta$ decay because the next generation of experiments will be sensitive to neutrino masses on the order of $\sqrt{\delta m^2_{\text{atm}}}$. If a nonzero rate is seen, we will know that neutrinos are Majorana particles. With some progress on the calculation of nuclear matrix elements—and we believe progress is possible—a nonzero rate in these experiments should allow the determination of the hierarchy realized by nature and the absolute mass scale. If no signal is seen, we should be able to say either that the hierarchy is normal or that neutrinos are Dirac particles.

Although none of the next-generation experiments is ready to operate, many of the new proposals are promising. We are hopeful that with concentrated effort from theorists and experimentalists, $\beta\beta$ decay will add to our growing understanding of neutrinos.

Acknowledgments

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